# **REGIONALIZED FLOOD FREQUENCY ANALYSIS FOR BASINS IN PUERTO RICO**

by

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## ABSTRACT

The main goal of this research is to provide a flood regionalization study for Puerto Rico based on recent advances in watershed classification and parameter estimation techniques. The regional flood frequency was achieved using the "index-flood" procedure in combination with the regional *L*-moment algorithm. The main assumption of the "index flood" procedure is that sites form a homogeneous region, and for this matter the heterogeneity measure based on *L*statistics were used to determine the homogeineity of a region. The hybrid cluster analysis was used as a regionalization procedure to group inland streams into four clusters using physical and climatological characteristics of the basins. Relative root mean square error values lower than 1% demonstrated the high accuracy in the procedure to estimate the quantile curves for the regions. The procedure recommended to estimate quantiles for ungaged basins couples the relationship between the derived regional growth curves and the estimated mean peak streamflow at site.

### RESUMEN

El objetivo principal de esta investigación es proveer un análisis de regionalización de frecuencia de inundaciones para Puerto Rico basado en los avances recientes en clasificación de cuencas y nuevas técnicas de estimados de parámetros. El análisis regional fue ejecutado utilizando el procedimiento "índice de flujo" en combinación con el algoritmo regional de momentos *L*. La hipótesis principal del procedimiento "índice de flujo" es que las cuencas forman una región homogénea. El algoritmo de conglomerado híbrido es utilizado para el procedimiento de regionalización, agrupando las cuencas en cuatro regiones según las características físicas y climatológicas. Valores de la raíz cuadrada del error cuadrático medio menores a 1% fueron obtenidos, demostrando la alta precisión en el proceso para estimar los cuantiles regionales. El proceso recomendado para estimar los cuantiles para cuencas sin aforar relaciona las curvas de crecimiento derivadas mediante el análisis regional y el promedio del caudal máximo anual estimado para el lugar.

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# LIST OF SYMBOLS

А	Area
AMC	Antecedent Moisture Condition
CDA	Contributing Drainage Area
CL	Channel Length
CN	Curve Number
$D_i$	Discordancy Measure
DR	Depth-to-rock
GEV	General Extreme Value Distribution
GLO	Generalized Logistic Distribution
GNO	Generalized Normal Distribution
GPA	Generalized Pareto Distribution
GS	Ground Slope
Н	Heterogeneity measure
$l_1$	Sample L-location
$l_2$	Sample L-scale
$\lambda_1$	L-location or mean of distribution
$\lambda_2$	L-scale
L-CV	Coefficient of L-variation
MAR	Mean Annual Rainfall
NRCS	Natural Resources Conservation Service
PE3	Pearson Type 3
RMSE	Root Mean Square Error
RI-T	Average T-year 24 hour Rainfall Intensity
SCS	Soil Conservation Service
SP	Average Soil Permeability
t	Sample coefficient of L-variation
<i>t</i> <sub>3</sub>	Sample L-skewness
$t_4$	Sample L-kurtosis

τ	Coefficient of L-variation
τ3	L-skewness
τ4	L-kurtosis
TSVQ	Tree-Structure Vector Quantization
USGS	United States Geological Survey
VC	Vegetative Cover
$Z_i^{DIST}$	Goodness-of-fit measure

### 1. INTRODUCTION

#### **1.1** Justification

An important public issue regarding natural disasters has been the adequate prediction of catastrophic flooding in populated areas. The process of climate change has increased awareness of the potential for more frequent major flooding. Indeed, the occurrence of unprecedented flooding all across the world prompted in the United States a workshop on Hydrologic Frequency Analysis (Olsen, Kiang, & Waskom, 2010). Here, federal agencies discussed the next step for developing best practices for hydrologic frequency analysis for changing conditions. Although, there was agreement that changes occur in hydrology, there was no consensus on which methods could be used to model either past or potential future hydrological changes.

The island of Puerto Rico is constantly affected by floods and formulating adequate flood management policies is a constant challenge. An integral part of such efforts is the ability to estimate and predict the magnitude and frequency of major flood events. Appropriate technologies for assessing the magnitude and frequency of expected floods are urgently needed in the policy making process. In Puerto Rico, a large percentage of streams are not monitored and there is no reliable way to assess the flooding potential of such systems without recurring to the use of regionalized methods. For flood frequency analyses, the use of traditional rainfall/runoff procedures such as the Natural Resources Conservation Service (NRCS) method (NRCS, 1986) are not reliable since such methods are based on foreign parameters and fixed hydrological responses, in the form of the universal synthetic unit hydrograph. Additional uncertainty in this procedure is introduced by using the T-year rainfall to generate an assumed T-year runoff peak, based on the assumption that both return periods are exactly similar.

A regionalized flood frequency analysis is based on actual streamgage data and results in a more efficient use of the available historical discharge time series for a site. In this type of analysis, the available gage data at monitored streams serves as the basis for estimating the corresponding parameters related to ungaged sites. The United States Geological Survey (USGS) has published three reports addressing the estimation of magnitude and frequency of floods in Puerto Rico based on streamgage records. The procedure developed by the USGS is based on a regional regression analysis approach. The three reports - López & Fields (1970), López, Colón-Diepa, & Cobb (1979), and Ramos-Gines (1999) – all follow the same procedure, each updating the earlier one. The fundamental methodology of these studies consists of estimating the parameters of the Log-Pearson extreme value distribution, with subsequent regionalization of the parameter through regressions with catchment and climatic attributes. The regression-based approach of these studies yielded relatively high standard errors and require the determination of several parameters (such as, contributing drainage area, depth-to-rock, mean annual rainfall, average permeability of soils, vegetative cover, main channel slope, main channel length, average of the ground

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slope and the average i-year 24-hour rainfall intensity), some of which are difficult to estimate in a practical context.

## 1.2 Objectives

The objective is to perform a new flood regionalization study for Puerto Rico based on recent advances in watershed classification and parameter estimation techniques of flood frequency distributions.

### 2. LITERATURE REVIEW

#### **2.1** Flood Frequency Analyses in Puerto Rico

The USGS has published three reports on the estimation of the magnitude and frequency of river floods in Puerto Rico. Their procedure is based on regionalizing flood estimates using regressions with basin physical and climatologic attributes. The reports by López & Fields (1970) and López, Colón-Diepa, & Cobb (1979) presented regressions for estimating peak flows for the 5-, 10-, 25-, 50-, and 100-year return periods. In the latest report, Ramos-Ginés (1999) presented updated regressions for up to a 500-year return period. The lowest standard errors achieved with the USGS procedure are in the range of 22.8 to 29.5 percent.

In the USGS reports, the flood frequency curves for gaged streams were constructed following the procedures in Bulletin 17B from the Interagency Advisory Committee on Water Data (1982), which prescribes the Log-Pearson Type III distribution for estimating flood-frequency parameters. Flood frequency relations for ungaged sites are constructed by regressing the peak flows with catchment attributes such as drainage area and mean annual rainfall. Ramos-Gines (1999) added the depth-to-rock as a significant parameter. Depth-to-rock is not easily estimated at ungaged locations, and alternate expressions are provided that do not include this parameter, albeit with higher standard estimation errors.

Using a different approach to regionalization, Segarra (1998) developed generalized flood frequency relations for ungaged basins by grouping catchments The use of the Log-Pearson distribution was into homogeneous regions. abandoned in favor of the Generalized Extreme Value (GEV) distribution for determining parameters for gaged sites. The study grouped basins into four homogenous regions, as opposed to the single homogenous region employed by López, Colón-Diepa, & Cobb (1979) and Ramos-Ginés (1999). However. Ramos-Ginés argued that the standard errors related to the regression-based approach were lower than those resulting from the approach of Segarra (1998). In part, this result was explained by the fact that the study gleaned the physical catchment data from large-scale printed topographic quadrangles, and this procedure may have introduced scaling errors in the measures. The use of geographical information systems-based tools should result in much more accurate physical parameter estimates that could improve the estimation errors arising from this particular aspect.

#### **2.2** Other Flood Frequency Analyses

Four additional previous studies on flood frequency analysis were evaluated. The studies treat the use of stochastic rainfall-runoff model, kinematic wave peak discharge, and regional frequency analysis using *L*-moments.

The first study was performed on six basins in Sicily by Aronica & Candela (2007). They proposed deriving frequency distributions of peak flows using a

semi-distributed stochastic rainfall-runoff model. The authors argue that the benefit of this approach are that the rainfall-runoff model is not complex, has a limited number of parameters, and does not require a major calibration effort resulting in a robust tool for those catchments which are partially or poorly gauged. The authors selected a Monte Carlo simulation approach to determine a derived flood distribution for its practical applicability and flexibility. For ungaged catchments, or catchments where the recording period is not sufficiently long, the authors used previous works where regionalization had been employed by means of cluster analysis. Two modules were defined for the Monte Carlo simulation, (1) a stochastic rainfall generator module, and (2) a catchment response module. For the rainfall module, the Two Components Extreme Value distribution was used and simple relations between the rainfall duration were used to estimate the parameters. The Rational Method was used to define the basin response module; therefore, the effective depth of rainfall was needed to determine the peak discharge. For a simpler procedure the Soil Conservation Service - Curve Number (SCS-CN) method was used to transform the storm depth into effective rainfall (h<sub>e</sub>).

The results of the study by Aronica and Candela (2007) demonstrated that for areas greater than 200 km<sup>2</sup> derived distributions are not entirely reliable. The method provides a good estimation for smaller basins and does not require extensive recorded data. The method is less reliable for areas larger than 200 km<sup>2</sup>. In the second study, Cadavid, Obeysekera, & Shen, (1991) derived floodfrequency distributions applicable to small watersheds in which overland flow is considered an important runoff component, such as urban catchments. The cited work is based on the use of the kinematic wave to determine peak discharge and time to peak, as a function of precipitation and watershed characteristics. Even though the model included variables representing catchment geometry and dynamics, the results were not comparable with recorded peak discharges and times to peak data.

Saf (2009) performed a flood frequency analysis for a region of Turkey based on *L*-moments probabilistic method for parameter estimation. The methodology used in the study follows the steps proposed by Hosking & Wallis (1997), and encompassed 36 stations with more than 10 years of data. To define homogenous regions, the author used the *L*-moment diagram and the  $Z_i^{DIST}$ statistics criteria. In the study, a Monte Carlo simulation was used to address the accuracy of the estimates selected. These simulated results were compared to the observed estimates, and the accuracies of the procedure to estimate quantiles were determined. The greater relative root-mean-square error (RMSE) was 0.192 percent for a recurrence of 100 years, which can be considered a relatively low level. Federal agencies in the United States commonly use Bulletin 17B or a derivative to determine flood quantiles. Lim & Voeller (2009) have compared this method to the *L*-moments approach. The authors performed a frequency analysis using the *L*-moments flood index method on the North basin of the Red River, which flows within the Canadian province of Manitoba, and through three states of the United States: North Dakota, Minnesota, and South Dakota. The authors conclude that the *L*-Moments based index flood method (LMFI) offers several advantages over the older Bulletin 17B approach but suggest further studies to adapt the method for the consideration of the estimation of index floods, particularly for ungaged basins.

The regional flood frequency analysis using *L*-moments is now considered a standard technique in hydrologic studies, as evidenced by its use in the preparation of Atlas 14 (Bonnin, et al., 2006), the new rainfall frequency atlas for Puerto Rico. Therefore, this analysis could provide better flood quantile estimations in Puerto Rico when compared to the results obtained from the Bulletin 17B on the USGS report. The conclusions of Lim & Voeller (2009) and Saf (2009) demonstrate that the procedure can provide low quantile estimation errors.

## 3. *L*-MOMENTS

The most recent approach developed for the estimation of flood distribution parameters uses *L*-moments. The "*L*" on the *L*-moments stands for the construction of moments from linear functions of order statistics. It has been applied at many locations, including the United States (Lim & Voeller, 2009), Turkey (Saf, 2009), India (Kumar & Chatterjee, 2005), and China (Yang, Hao, & Sun, 2009), and has become a standard technique for flood quantile parameter estimation.

### **3.1** Method of *L*-moments

*L*-moments were introduced by Hosking & Wallis (1990), and have several theoretical advantages over other moment estimation procedures. Compared with other moment estimation procedures, *L*-moments are more robust with respect to the outliers, are less subject to bias in estimation, and can approximate their asymptotic normal distribution more closely for finite examples than other methods (Yang, Hao, & Sun, 2009). For a finite sample  $X_{i:n}$  denote the *i*th smallest observation, and let  $X_{1:n} \leq X_{2:n} \leq ... \leq X_{n:n}$  be an ascending ordered sample. To determine the distributional parameters of a population of size *n*, the L-moments are defined by,

$$\lambda_r = r^{-1} \sum_{j=0}^{r-1} (-1)^j {\binom{r-1}{j}} E(X_{r-j:r}), \qquad r = 1, 2, 3, \dots$$
(3.1)

where,

$$E(X_{r:n}) = \frac{n!}{(r-1)! - (n-r)!} \int_0^1 x(u) u^{r-1} (1-u)^{n-r} \, du.$$
(3.2)

Equation 3.1 defines the measurements of the shape parameter of the distribution:  $\lambda_1$  represent the location parameter of the distribution,  $\lambda_2$  is the measure of the scale parameter of the distribution,  $\lambda_3$  is the measure of skewness parameter of the distribution, and  $\lambda_4$  is the measure of the kurtosis of the distribution. Each of the parameters are described by the following expressions:

$$\lambda_1 = E(X_{1:1}) \tag{3.3}$$

$$\lambda_2 = \frac{1}{2}E(X_{2:2} - X_{1:2}) \tag{3.4}$$

$$\lambda_3 = \frac{1}{3}E(X_{3:3} - 2X_{2:3} + X_{1:3}) \tag{3.5}$$

$$\lambda_4 = \frac{1}{4}E(X_{4:4} - 3X_{3:4} + 3X_{2:4} - X_{1:4}).$$
(3.6)

Of this group of *L*-moments,  $\lambda_1$  (mean) and  $\lambda_2$  (*L*-scale) are mostly used to summarize the probability distribution. *L*-moment ratios are also useful in the definition of probability distribution. *L*-moment ratios are dimensionless *L*moments defined by:

$$\tau_r = \lambda_r / \lambda_2, \quad r = 3, 4, 5 \dots$$
 (3.7)

When  $\lambda_1$  is used as the divisor of  $\lambda_2$  the coefficient of L-variation ( $\tau$ ) is obtained. In the case were the measure of skewness is divided by the measure of scale, the L-skewness ( $\tau_3$ ) is obtained. Similarly, when the measure of kurtosis is divided by the measure of scale, the *L*-kurtosis ( $\tau_4$ ) is obtained. Sample *L*-moments are defined in general as:

$$l_{\rm r} = \frac{1}{\rm r} {\binom{n}{r}}^{-1} \sum_{i=1}^{n} \left[ \sum_{j=0}^{r-1} (-1)^j {\binom{r-1}{j}} {\binom{i-1}{r-1-j}} {\binom{n-i}{j}} x_{i:n} \right].$$
(3.8)

The estimates of the *L*-moment ratios obtained from the observed data are defined then by:

$$t_r = \frac{l_r}{l_2}, \quad r = 3,4,5....$$
 (3.9)

The coefficient of L-variation for the sample (t) is obtained then by:

$$t = \frac{l_2}{l_1}.$$
 (3.10)

The values of  $l_r$ ,  $t_r$  and t are the natural estimators of  $\lambda_r$ ,  $\tau_r$ , and  $\tau$ , respectively.

## 4. REGIONAL FLOOD FREQUENCY ANALYSIS BASED ON L-MOMENTS

The main idea of the Regional Flood Frequency Analysis (RFFA) is to use the data from a grouping of statistically similar regions to estimate site quantiles. The aim of regional frequency analysis is not to fit a distribution to a particular data set, but to obtain quantile estimates of the distribution from which future data values will arise (Hosking & Wallis, 1997).

A screening of the data used on a RFFA is recommended to eliminate sites with erroneous data values, trends and outliers that bias in the *L*- moments of a sample. The discordancy measure ( $D_i$ ), in terms of the *L*-moments is suggested by Hosking and Wallis (1997). A discordant site is a point in a three dimensional space defined by the *L*-CV, *L*-skewness and *L*-kurtosis that lies outside a concentric ellipse determined by the covariance matrix of the sites *L*-moments ratios. For site *i*, the measure indicates the discordancy between the site *L*moment ratio and the regional average *L*-moment ratio. To define the discordancy measure let *N* represent the number of sites on a region,  $\mathbf{u}_i = \left[t^{(i)} \quad t_3^{(i)} \quad t_4^{(i)}\right]$ , and T be the transpose of a matrix (Hosking & Wallis, 1997):

$$D_i = \frac{1}{3} N (\mathbf{u}_i - \overline{\mathbf{u}})^T \mathbf{A}^{-1} (\mathbf{u}_i - \overline{\mathbf{u}})$$
(4.1)

$$\boldsymbol{A} = \sum_{i=1}^{N} (\mathbf{u}_{i} - \overline{\mathbf{u}}) (\mathbf{u}_{i} - \overline{\mathbf{u}})^{T}$$
(4.2)

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$$\overline{\mathbf{u}} = N^{-1} \sum_{i=1}^{N} \mathbf{u}_i \,. \tag{4.3}$$

Table 1 presents the corresponding critical value for regions with a certain number of sites. Regions with *N* sites where their  $D_i$  value is greater than  $D_{crit}$  can be eliminated from the regionalization procedure for a regional flood frequency analysis.

Table 1. Critical values for discordancy measures.(From Hosking & Wallis, 1997)

Number of sites in Region	Critical value (D <sub>crit</sub> )	Number of sites in Region	Critical value (D <sub>crit</sub> )
5	1.333	11	2.632
6	1.648	12	2.757
7	1.917	13	2.869
8	2.140	14	2.971
9	2.329	≥15	3
10	2.491		

A site with  $D_i$  values greater than 3 is always assumed as a discordant site. Only Regions with a number of sites greater than 5 are presented on Table 1 because for regions with fewer sites the  $D_i$  statistic is not functional. Hosking and Wallis (1997) showed that  $D_i \leq (N-1)/3$ ; therefore, for  $N \leq 3$  the matrix A given by Equation 4.2 is singular and  $D_i$  cannot be calculated. For regions with  $N \geq 7$  the rate of change on  $D_{crit}$  when adding a site is lower than for  $5 \leq N < 7$ .

#### 4.1 Heterogeneity Measure

Several modern hydrologic regionalization procedures can be used to cluster inland streams into homogenous groupings. Once regions are formed, a homogeneity test is performed in order to compute heterogeneity measures and assure that homogeneous regions are obtained. One of the advantages of using *L*moment based methods for testing homogeneity of a particular region is that they avoid assumptions about the form of the underlying probability distribution of the observed data (Rao & Srinivas, 2008). The homogeneity test evaluates the dispersion of the sample *L*-moment ratios between sites. The measures of dispersion are computed as follows:

a. weighted standard deviation of L-CVs (V):

$$V = \left\{ \sum_{i=1}^{N} n_i (t^{(i)} - t^R)^2 \middle/ \sum_{i=1}^{N} n_i \right\}^{1/2}$$
(4.4)

b. weighted average distance of L-CV/ L-skewness distances ( $V_2$ ):

$$V_{2} = \sum_{i=1}^{N} n_{i} \left\{ \left( t^{(i)} - t^{R} \right)^{2} + \left( t^{(i)}_{3} - t^{R}_{3} \right)^{2} \right\}^{1/2} / \sum_{i=1}^{N} n_{i}$$
(4.5)

c. weighted average distance of L-skewness/L-kurtosis (V<sub>3</sub>)  

$$V_{3} = \sum_{i=1}^{N} n_{i} \left\{ \left( t_{3}^{(i)} - t_{3}^{R} \right)^{2} + \left( t_{4}^{(i)} - t_{4}^{R} \right)^{2} \right\}^{1/2} / \sum_{i=1}^{N} n_{i}$$
(4.6)

where *N* is the number of sites in a region,  $n_i$  is the record length of site *i*, and  $t_4^{(i)}$ ,  $t_3^{(i)}$  and  $t_4^{(i)}$  are the *L*-moment ratios of site *i*. The values of  $t^R$ ,  $t_3^R$ , and  $t_4^R$  are a regional weighted average of the site's *L*-moment ratios.

Three heterogeneity measures  $H_1$ ,  $H_2$ , and  $H_3$  are obtained by substituting each dispersion measure into Equation 4.7 in which  $\mu_{V_i}$  and  $\sigma_{V_i}$  are the mean and the standard deviation of the  $N_{sim}$  values of the measure of dispersion  $V_i$ .

$$H_i = \frac{\left(V_i - \mu_{V_i}\right)}{\sigma_{V_i}} \tag{4.7}$$

The values of the heterogeneity measure are then used to determine whether a region is homogeneous or heterogeneous. The following values of  $H_i$ where suggested by Hosking and Wallis (1997) regarding each range:  $H_i < 1$  as "acceptably homogenous",  $1 \le H_i < 2$  as "possibly heterogeneous", and  $H_i \ge 2$  as "definitely heterogeneous". The accuracy of the quantiles estimate is improved by a lower *H*.

#### 4.2 Goodness-of-fit Measure

An appropriate regional frequency distribution must be selected once the homogenous regions are defined. The aim is to find a distribution that will yield accurate quantile estimates for each site (Hosking & Wallis, 1997). The goodness-of-fit test is necessary to test whether the flow data follows a GEV or a GPA distribution, or any other distribution used in hydrology. It is intended to make an overall comparison of the observed and hypothetical frequencies and determine if it is the best fit (Kottegoda & Rosso, 1997). Hosking & Wallis (1997) used the *L*-kurtosis of the fitted distribution ( $\tau_4^{Dist}$ ) and *L*-kurtosis of the region ( $t_4^R$ ) to define the goodness-of-fit measure for each distribution:

$$Z^{DIST} = (\tau_4^{DIST} - t_4^R + B_4) / \sigma_4 \tag{4.8}$$

where  $B_4$  is bias of  $\tau_4$ ,

$$B_4 = N_{sim}^{-1} \sum_{m=1}^{Nsim} (t_4^{[m]} - t_4^R), \qquad (4.9)$$

And,  $\sigma_4$  is the standard deviation of L-kurtosis values from the simulation,

$$\sigma_4 = \left[\frac{\sum_{m=1}^{Nsim} \left(t_4^{[m]} - t_4^R\right)^2 - N_{sim} B_4^2}{N_{sim} - 1}\right]^{\frac{1}{2}}.$$
(4.10)

On the equations,  $N_{sim}$  refers to the number of simulated regional data sets generated using a kappa distribution in a similar manner to the heterogeneity statistics. For more details on the use of kappa distribution on the simulation see section 5.2.3 of Hosking and Wallis (1997). Values of  $Z^{DIST}$  lower than 1.64 are considered acceptable, at a confidence level of 90%.

### 4.3 Index Flood

The basic assumption of the index flood method is that the flood distribution of a homogeneous region is the same except for a site specific scale or an index factor. Therefore, a regionalized frequency analysis with application of the index flood procedure can be used to define the quantiles of particular sites in a region. At an homogeneous region with N sites, the quantile function at site i is defined as follows:

$$Q_i(F) = \mu_i q(F), \quad i = 1, \cdots, N$$
 (4.11)

where q(F) is the dimensionless T-year flow value estimated from the region (regional growth curve), and  $\mu_i$  is the index factor. Hosking & Wallis (1997) recommended the mean of the observed annual peak streamflow data at site *i* as the index flood. They also recommended a method in which the parameters are estimated separately at each site, where *L*-moment parameters are denoted by  $\hat{\theta}_k^{(i)}$ . The weighted regional average of the site estimates is defined as follows:

$$\hat{\theta}_{k}^{R} = \sum_{i=1}^{N} n_{i} \,\hat{\theta}_{k}^{(i)} \bigg/ \sum_{i=1}^{N} n_{i}$$
(4.12)

Substituting these estimates into q(F) results in the estimated regional growth curve  $\hat{q}(F) = q(F; \hat{\theta}_1^R, ..., \hat{\theta}_p^R)$  where *p* refers to the undetermined parameters. The site-*i* quantile estimates are obtained from the estimates of  $\mu_i$  and q(F):

$$\hat{Q}_i(F) = \hat{\mu}_i \hat{q}(F) \tag{4.13}$$

For ungaged basins, an index flood  $(\hat{\mu}_i)$  relates these basins to clusters where data is available. The parameters obtained from the *L*-moments are then used to estimate quantiles q(F) for each recurrence interval. Depending on the distribution fitted to each region, the parameters estimated with the *L*-moments are used to define the regional growth curve equation. Substitution the regional parameters into the distribution growth curve yields q(F). For the set of distributions considered in this study the quantile functions are defined as follows:

• Generalized Extreme Value (GEV):

$$x(F) = \xi - \frac{\alpha}{k} (1 - (-\log F)^k)$$
(4.14)

Where the location parameter  $(\xi)$ , scale parameter  $(\alpha)$ , and shape parameter (k) functions are defined as:

$$k \approx 7.8590c + 2.9554c^2 \tag{4.15}$$

$$c = \frac{2}{3+\tau_3} - \frac{\log 2}{\log 3}$$
(4.16)

$$\alpha = \frac{\lambda_2 k}{(1-2^{-k})\Gamma(1+k)} \tag{4.17}$$

$$\xi = \lambda_1 - \alpha \{1 - \Gamma(1+k)\}/k$$
(4.18)

• Pearson Type 3 (Pe3):

The distribution's parameters are the first three (ordinary) moment ratios: the mean ( $\mu$ ) as location parameter, the standard deviation ( $\sigma$ ) as scale parameter, and the skewness ( $\gamma$ ) as shape parameter. If  $\gamma \neq 0$ , it follows that  $\alpha = 4/\gamma^2$ ,  $\beta = \frac{1}{2}\sigma|\gamma|$ , and  $\xi = \mu - 2\sigma/\gamma$ . The probability density function for this distribution is as follows:

$$f(x) = \frac{|x - \xi|^{\alpha - 1} exp(-|x - \xi|/\beta)}{\beta^{\alpha} \Gamma(\alpha)}.$$
(4.19)

To obtain the values of the parameters the value of  $\alpha$  is first determined, depending on the values possessed by the skewness ( $\tau_3$ ). If  $0 < |\tau_3| < 1/3$ , and  $z = 3\pi\tau_3^2$ , then

$$\alpha \approx \frac{1 + 0.2906z}{z + 0.1882z^2 + 0.0442z^3}.$$
(4.20)

On the contrary, if  $1/3 \le |\tau_3| < 1$ ,  $z = 1 - |\tau_3|$  then

$$\alpha \approx \frac{0.36067z - 0.59567z^2 + 0.25361z^3}{1 - 2.78861z + 2.56096z^2 - 0.77045z^3}$$
(4.21)

Consequently, the parameters for the Pearson Type 3 distribution are defined as follows,

$$\gamma = 2\alpha^{-1/2} sign(\tau_3) \tag{4.22}$$

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$$\sigma = \lambda_2 \pi^{1/2} \alpha^{1/2} \Gamma(\alpha) / \Gamma\left(\alpha + \frac{1}{2}\right)$$
(4.23)

$$\mu = \lambda_1 \tag{4.24}$$

where  $\lambda_1$ ,  $\lambda_2$ , and  $\tau_3$  refers to the mean, L-scale, and L-skewness as defined in Section 3.1.

• Wakeby:

$$x(F) = \xi + \frac{\alpha}{\beta} \left( 1 - (1 - F)^{\beta} \right) - \frac{\gamma}{\delta} \left( 1 - (1 - F)^{\delta} \right)$$
(4.25)

The Wakeby distribution is considered a more robust distribution because it has five parameters:  $\xi$ ,  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$ , allowing for a wider range of distribution shapes. Hosking and Wallis (1997) defined the expression to calculate the parameters values in terms of the L-moments. In the case where  $\xi$  is unknow:

$$(N_2C_3 - N_3C_2)z^2 + (N_1C_3 - N_3C_1)z + (N_1C_2 - N_2C_1) = 0$$
(4.26)

where,

$$N_{1} = 3\lambda_{2} - 25\lambda_{3} + 32\lambda_{4}, \quad C_{1} = 37\lambda_{2} - 85\lambda_{3} + 203\lambda_{4} - 125\lambda_{5},$$

$$N_{2} = -3\lambda_{2} + 5\lambda_{3} + 8\lambda_{4}, \quad C_{2} = -7\lambda_{2} + 25\lambda_{3} + 7\lambda_{4} - 25\lambda_{5}, \quad (4.27)$$

$$N_{3} = 3\lambda_{2} + 5\lambda_{3} + 82, \quad C_{3} = 7\lambda_{2} + 5\lambda_{3} - 7\lambda_{4} - 5\lambda_{5}.$$

The parameters  $\beta$  and  $-\delta$  are the roots of equation 4.26, with  $\beta$  being the largest root value. With this information the rest of the parameters are obtained from:

$$\alpha = \frac{(1+\beta)(2+\beta)(3+\beta)\{(1+\delta)\lambda_2 - (3-\delta)\lambda_3\}}{\{4(\beta+\delta)\}},$$
(4.28)

$$\gamma = -\frac{(1-\delta)(2-\delta)(3-\delta)\{(1-\beta)\lambda_2 - (3+\beta)\lambda_3\}}{4(\beta+\delta)}, \qquad (4.29)$$

$$\xi = \lambda_1 - \frac{\alpha}{1+\beta} - \frac{\gamma}{1-\beta}.$$
(4.30)

The GEV and Pe3 distributions were considered for this study due to their extensive use in hydrology. The GEV has demonstrated accepted standard errors when fitted to a specific site in Puerto Rico as presented by Sánchez (1995). Similarly, the log-Pearson Type 3 distribution was used on the last report by the USGS as part of the procedure stated in Bulletin 17B by U.S. Interagency Advisory Committee on Water Data (1982).

In this section it has been shown the general steps to compute a flood frequency curve for a defined region. To relate the flood quantiles of a region to a site within the region, the index flood procedure is used. Before all these procedures are applied, sites must be grouped into regions. For this reason, next hydrological regionalization procedures taken under consideration for the analysis are presented.

### 5. HYDROLOGICAL REGIONALIZATION

Estimating the flood frequency relation for an ungaged site requires performing a regional flood frequency analysis. The procedure is based on the similarity of gaged basins to ungaged ones. "The process of identifying similar watersheds for pooling peak flow information is known as regionalization" (Rao & Srinivas, 2008). Several methods have been developed for this purpose, such as methods of residual and cluster analysis.

The method of residuals, a procedure used frequently by the United States Geological Survey (USGS), relies on the formation of homogeneous regions based on the positive and negative signs of residuals pertaining to a regional regression model that relates flood quantiles at each gauged site to watershed characteristics. Under this method, the delineation of homogenous regions on occasions are arranged to coincide with geographic and/or hydrologic boundaries. Bulletin 17B by U.S. Interagency Advisory Committee on Water Data (1982) describes the procedure for performing the regional analysis. It involves the use of residuals from a regression equation relating the t-year flood level Q<sub>t</sub>, at each gauged site to the physical and climatic characteristics of the watershed. Consequently, the t-year flood levels are estimated from a log-Pearson Type III probability distribution as fitted to the log-transformed maximum annual flood series at each gauged site using a regionalized coefficient of skewness. A more appropriate technique for regionalization to be considered in this study is cluster analysis. Clustering is a process by which a set of feature vectors is divided into clusters or groups such that the feature vectors within a cluster are as similar as possible. Examples of feature vectors are the following: drainage area, average basin slope, main stream slope, stream length, runoff coefficient, geographical location attributes, mean annual rainfall, and precipitation intensities.

Cluster analysis is considered the most practical method to group sites in homogenous regions (Hosking & Wallis, 1997). Clustering algorithms are classified as hard clustering and fuzzy clustering. Hard clustering is structured in that a feature vector has a membership to a cluster of one. On the contrary in fuzzy clustering a feature vector can belong to more than one cluster. Hard clustering can be subdivided as hierarchical and partitional clustering, as seen on Figure 1. Under the hierarchical classification exists the following categories: single linkage, complete linkage, average linkage, and Ward's linkage. These clustering methods differ on how the nearest neighbor to a chosen cluster is defined. For single linkage the distance between clusters is the smallest distance between a pair of feature vectors, for the complete linkage it is the greatest distance between a pair of feature vectors, and for the average linkage it is the average distance between all pairs of feature vectors. In Ward's cluster method the objective function minimizes the sum of squares of deviations of the feature vectors from the centroid of their respective clusters. From these hierarchical

algorithms, the average-link clustering, and Wards method tend to form clusters of roughly the same size, producing good results according to Hosking & Wallis (1997).





The partitional clustering is categorized in three algorithms: K-means, Kmedoids and K-modes. In the K-means, the mean of feature vectors within the cluster is the representative centroid, while for the K-medoids it is the median of feature vectors. The K-modes is the result of a modified K-means algorithm. The mode of feature vectors is the representative centroid.

### 5.1 Hybrid Clustering

Rao & Srinivas (2008) describe other types of cluster analysis: hybrid cluster analysis, fuzzy cluster analysis and artificial neural networks. The hybrid clustering analysis mentioned by Rao & Srinivas (2008) is a combination between the hierarchical and partitional clustering. Hierarchical clustering has a disadvantage in that the feature vector cannot move from one cluster to another minimizing the objective function, an advantage that exists in the partitional clustering algorithms where investigated: single linkage, complete linkage, and Ward's linkage combined with the partitional cluster, K-means. The main objective of the K-means analysis is to minimize an objective function F, defined by:

$$F = \sum_{k=1}^{K} \sum_{j=1}^{n} \sum_{i=1}^{N_k} d^2 (x_{ij}^k - x_{\cdot j}^k)$$
(5.1)

where *d* denotes distance between clusters, *K* denotes the number of clusters,  $N_k$  represents the number of feature vectors in cluster *k*;  $x^{k}_{ij}$  denotes the rescaled value of attribute *j* in the feature vector *i* assigned to cluster *k*; and  $x^{k} \cdot_{j}$  is the mean value of attribute *j* for cluster *k*. It was found that the hybrid cluster between Ward's linkage and K-means provides the lowest *F* values. Even though the procedure eventually did not provide fully homogeneous areas, the results were considered acceptable. For a regional flood frequency analysis, all regions must be homogenous to attain the best result.
Chipman & Tibshirani (2006) propose another use of the hybrid clustering algorithm in a study to cluster gene expression data of breast cancer tumors. The algorithm is a merge between the agglomerative and divisive clustering methods, in addition to taking into consideration the mutual cluster. The mutual clusters are formed by individuals that are closer to each other than to any other individual (Chipman & Tibshirani, 2006). The main idea is that the mutual points should not be separated. By definition the distance (d) between any point on cluster S must be less than any other point not located in S:

$$\forall x \in S, y \notin S, d(x, y) > diameter(S) \equiv max_{w \in S, z \in S} d(w, z).$$
(5.2)

The agglomerative or bottom-up method forms clusters by joining individuals with the nearest distance, while the divisive or top-down method forms clusters by successively separating the individuals from a unified group. The algorithm used for the top-down method is the K-means with K = 2 being applied in a recursive way, also known as tree-structure vector quantization (TSVQ). The algorithm proposed by Chipman & Tibsshirani (2006) first defines the mutual clusters, then performs a top-down clustering via K-means, and lastly applies the K-means algorithm within each mutual cluster. The distance metrics used to define the cluster in K-means is the square Euclidean distance because it is suitable for clusters with spherical shapes.

#### 5.2 Fuzzy Clustering

Another analysis proposed by Rao & Srinivas (2008) is fuzzy clustering, which has the flexibility of allowing basins to be grouped into more than one cluster simultaneously. This method is dependent in iterative optimization of a fuzzy objective function. The objective function is defined as

Minimize 
$$J(U, V: X) = \sum_{i=1}^{c} \sum_{k=1}^{N} u_{ik}^{\mu} d^{2}(x_{ik}, v_{i})$$
 (5.3)

where,

$$d^{2}(x_{ik}, v_{i}) = (x_{ik} - v_{i})^{T} A_{i}(x_{ik} - v_{i})$$
(5.4)

and, it is subjected to the following constraints,

$$\sum_{i=1}^{c} u_{ik} = 1 \qquad \forall k \in \{1, 2, \dots, N\}$$
(5.5)

$$0 < \sum_{k=1}^{N} u_{ik} < N \qquad \forall i \in \{1, 2, \dots, c\}.$$
(5.6)

On the equations, *d* represent the distance,  $x_{ik}$  denotes the rescaled value of  $y_{ik}$ , which in turn represents the value of attribute *i* in a *k*-th feature vector,  $v_i$  characterizes the centroid of one of the *i* cluster,  $\mu$  is a fuzzifier, and  $u_{ik}$  denotes the membership of a feature vector in a *i*-th fuzzy cluster.

To identify optimal allocation of feature vectors on c clusters, the author describes the following validity measures: partition coefficient, partition entropy, fuzziness performance index, normalized classification entropy, extended Xie-Beni, Fukuyama-Sugeno, and Kwon's index. During the process of application of the algorithm the more effective indices must be selected for identification of optimal regions. Rao & Srinivas (2008) found that the fuzzy clustering algorithm generated less effort in adjusting the clusters to find homogenous regions.

Bhaskar & O'Connor (1989) compared the methods mentioned earlier, residuals and cluster analysis, in the state of Kentucky. The method of residuals was performed using statistics of hydrologic variables, discriminant analysis, and a regression analysis relating the t-year log-Pearson Type-III flood quantile to watershed physical characteristics. On the cluster analysis, regions were delineated using the coefficient of variation of the log-transformed maximum annual flood series (LCV), and the specific mean annual flood (QSP). Comparing the results, it was concluded that cluster regions have lower variability within each of the regions with respect to the mean and median of hydrologic variables. The discriminant analysis indicated that the regions formed by the method of residual display different watershed characteristics than those formed by the method of residual discrimination values.

From the methods discussed above, the hybrid cluster analysis by Chipman & Tibshirani (2006) will be used as the procedure to group sites into regions in Puerto Rico. This method is simple and the definition of the mutual clusters is convinient when ungaged basins need to be assigned to a region. Although Rao & Srinivas (2008) identified the fuzzy clustering method as an easier method for adjustment, a site can be grouped in more than one cluster simultaneously. The

characteristic of this method can create difficulties if used in a flood frequency analysis since it is expected that a site is only contained by one region.

# 6. AREA OF STUDY

The study area encompasses a total of 166 USGS stations with peak streamflow data. A screening of the data was performed, eliminating sites affected by hydraulic structures, short records, and broken records. McCuen & Galloway (2010) made a study that provides a graphical approach to assess the expected accuracy in 2-, 10-, and 100-year magnitudes resulting from a log-Pearson Type III analysis. With a 90 % confidence the relative error for stations with n = 10 was approximately 1.6, while for n = 25 it was 1.0. This suggest that in order to minimize the introduction of large errors in the analysis due to short records, only sites with records greater than 25 years should perhaps be selected. In addition, all stations located downstream of a flow regulation structure and channels were not included in the study. Another consideration for the selection of the stations was the number of years without data. Only those stations missing one year where included in the analysis, leaving a total of 30 stations. These are listed in Table 2, and located in Figure 2 on a map of Puerto Rico. Most of the stations conglomerated are in the eastern part of Puerto Rico, while the others are more scattered across the southern and western regions.

id	Site Number	Station Name
1	50028000	RIO TANAMA NR UTUADO
2	50028400	RIO TANAMA AT CHARCO HONDO
3	50031200	RIO GRANDE DE MANATI NR MOROVIS
4	50034000	RIO BAUTA NR OROCOVIS
5	50035000	RIO GRANDE DE MANATI AT CIALES
6	50038100	RIO GRANDE DE MANATI AT HWY 2 NR MANATI
7	50038320	RIO CIBUCO BLW COROZAL
8	50039500	RIO CIBUCO AT VEGA BAJA
9	50043000	RIO DE LA PLATA AT PROYECTO LA PLATA
10	50047850	RIO DE BAYAMON NR BAYAMON
11	50050900	RIO GRANDE DE LOIZA AT QUEBRADA ARENAS
12	50051310	RIO CAYAGUAS AT CERRO GORDO
13	50055000	RIO GRANDE DE LOIZA AT CAGUAS
14	50056400	RIO VALENCIANO NR JUNCOS
15	50057000	RIO GURABO AT GURABO
16	50061800	RIO CANOVANAS NR CAMPO RICO
17	50063800	RIO ESPIRITU SANTO NR RIO GRANDE
18	50064200	RIO GRANDE NR EL VERDE
19	50065500	RIO MAMEYES NR SABANA
20	50067000	RIO SABANA AT SABANA
21	50071000	RIO FAJARDO NR FAJARDO
22	50075000	RIO ICACOS NR NAGUABO
23	50090500	RIO MAUNABO AT LIZAS
24	50092000	RIO GRANDE DE PATILLAS NR PATILLAS
25	50112500	RIO INABON AT REAL ABAJO
26	50115000	RIO PORTUGUES NR PONCE
27	50124200	RIO GUAYANILLA NEAR GUAYANILLA
28	50138000	RIO GUANAJIBO NR HORMIGUEROS
29	50144000	RIO GRANDE DE ANASCO NR SAN SEBASTIAN
30	50147800	RIO CULEBRINAS AT HWY 404 NR MOCA

Table 2. USGS stations evaluated in this study



Figure 2. Location of USGS stations selected for the flood frequency analysis.

### 7. RESULTS AND DISCUSSIONS

#### 7.1 Sample *L*-moments and *L*-moment ratios

The *L*-moments and *L*-moment ratios  $(l_1, t, t_3, t_4 \text{ and } t_5)$  of each USGS station are presented in Table 13 in Appendix A.1, where its *L*-statistics are displayed. These were calculated for the 30 stations in Table 2 using the R source code (Hosking, 2008). The *L*-moment ratio diagram is presented on Figure 3. In the diagram it is observed that the *L*-moment ratios are spread through the values of 0.03 to 0.6 for the *L*-skewness, and from 0.1 to 0.47 for the *L*-kurtosis.



Figure 3. L-moment ratio diagram for the 30 USGS sites selected.

Using the *L*-moment statistics results, the discordancy measure and the heterogeneity measures were computed. The discordancy measure  $(D_i)$  was calculated using Equation 4.1 to determine which stations have potential errors in the data at the particular site. Considering Puerto Rico as a single region, only one station presented a value greater than 3.0, the critical value for regions with

more than 15 stations. Station number 50075000, located on Rio Icacos near Naguabo presented a value of 3.47, and for this reason it was excluded from the study. The exclusion of discordant sites from the analysis, the use of records with potential data errors can be avoided. The heterogeneity measures  $H_1$ ,  $H_2$ , and  $H_3$  were computed from Equation 4.7, yielded values of 9.64, 8.62, and 5.73, respectively. These values demonstrate that the area of study is heterogeneous and must be divided into sub-regions.

### 7.2 Cluster Analysis

The cluster analysis was performed using only site characteristic data which includes physical and climatological basin characteristics. Hosking & Wallis (1997) strongly recommend the use of site characteristics because at-site statistics are used to determine the homogeneity of the group, and the assessment can be affected when at-site statistics are used on both procedures. Most of the site characteristics for this analysis were obtained from the USGS report by Ramos-Ginés (1999). In this USGS report the following characteristics were described: location, contributing drainage area (CDA), depth-to-rock (DR), mean annual rainfall (MAR), average permeability of soils (SP), vegetative cover (VC), main channel slope (CS), main channel length (CL), average of the ground slope (GS), and the average T-year 24-hour rainfall intensity (RI-T). MAR values were obtained from a mean annual rainfall raster from climate data collected at weather stations from 1990 to 2000. The correlation coefficient between the mean annual flow and the sites characteristics was calculated to determine which characteristics must be selected for further analysis. Results are shown in the following table.

Site Characteristics	Correlation	Site Characteristics	Correlation
MAF vs CDA	0.920	MAF vs RI-10	-0.347
MAF vs CS	-0.495	MAF vs RI-25	-0.372
MAF vs MAR	0.0129	MAF vs RI-50	-0.321
MAF vs SP	-0.220	MAF vs RI-2*CDA	0.911
MAF vs VC	-0.095	MAF vs RI-5*CDA	0.912
MAF vs CL	0.881	MAF vs RI-10*CDA	0.911
MAF vs GS	0.014	MAF vs RI-25*CDA	0.904
MAF vs RI-2	-0.395	MAF vs RI-50*CDA	0.903
MAF vs RI-5	-0.415	MAF vs MAR*CDA	0.917

 Table 3. Correlation between mean annual flow (MAF) and site characteristics.

Table 3 shows that the mean annual flow is highly correlated to the contributing drainage area (correlation = 0.920), and also to the channel length (correlation = 0.881). Since the contributing drainage area has a greater effect on the floods, it was selected as one of the independent attributes for the cluster analysis. Another attribute considered in the analysis was the rainfall. The mean annual rainfall (MAR) demonstrates a high correlation value when the volume of rainfall within the area of the basin was evaluated. In Table 3, channel slope presents a low correlation value (correlation = -0.495). Even though the channel slope correlation did not yield an acceptable value, it was nevertheless included in the cluster analysis having then two physical characteristics for the analysis are presented in Table 4.

### Table 4. Selected sites characteristics used in regionalization.

id	Site Number	Latitude	Longitude	CDA	MAR	CS
				( <b>mi</b> <sup>2</sup> )	(in)	(ft/mi)
1	50028000	18.30056	66.78278	18	72.32	139.7
2	50028400	18.41444	66.71444	22.2	63.11	102.4
3	50031200	18.29583	66.41306	55.2	68.50	86.25
4	50034000	18.23611	66.455	16.7	74.61	89.62
5	50035000	18.32389	66.46	134	68.19	76.76
6	50038100	18.43111	66.52694	165	59.76	54.57
7	50038320	18.35361	66.33528	15.2	62.99	198
8	50039500	18.44806	66.37472	81.6	59.33	73.6
9	50043000	18.16028	66.22889	63.2	59.37	69.37
10	50047850	18.33556	66.13694	41.7	59.33	59.79
11	50050900	18.11944	65.98944	5.99	72.76	334.4
12	50051310	18.1575	65.95806	10.2	77.56	55.37
13	50055000	18.2425	66.00944	89.6	63.23	105
14	50056400	18.21611	65.92611	16.4	69.17	126
15	50057000	18.25833	65.96806	60.1	66.34	171.3
16	50061800	18.31889	65.88917	10.2	66.97	332.4
17	50063800	18.36028	65.81361	8.64	64.80	364.5
18	50064200	18.345	65.84167	7.34	65.59	449.6
19	50065500	18.32944	65.75111	6.8	62.60	492.2
20	50067000	18.33111	65.73111	3.91	60.28	406.1
21	50071000	18.29889	65.695	14.8	52.95	281.2
22	50090500	18.02722	65.94	5.29	66.10	225.9
23	50092000	18.03444	66.03278	18.4	60.94	227
24	50112500	18.08611	66.56278	9.68	60.51	476.2
25	50115000	18.07917	66.63361	8.8	58.15	280
26	50124200	18.04444	66.79806	18.9	50.83	238.6
27	50138000	18.14333	67.14917	120	62.36	75.35
28	50144000	18.28472	67.05139	134	74.84	81.67
29	50147800	18.36167	67.0925	71.3	65.91	53.41

(CDA and CS characteristics adapted from Ramos-Ginés, 1999)

It has been recommended that the variables selected for the cluster analysis be transformed in order to avoid scale differences (Rao & Srinivas, 2006;

Chipman & Tibshirani, 2006). The procedure to assign the transformations can be arbitrary, and characteristics that are deemed more important can be assigned a greater value. The transformations applied to the group of characteristics in this study were chosen following an application presented by Hosking & Wallis (1997) for the formation of regions for Appalachian streamflow data. A nonlinear transformation was achieved by applying a logarithmic transformation to the drainage area; in this manner, sites with outlier values can themselves form a cluster. The variables were standardized by dividing the individual characteristics' values by the standard deviation of the transformed characteristic. Two different approaches (a and b) were followed in terms of the importance of the characteristics of sites assigned to the clusters. Approach a assumes that all characteristics have the same importance. On the contrary, approach b took under consideration that the contributing drainage area is highly correlated with the mean annual flow; therefore, the area was multiplied by 4.0 to assign an importance value equal to the other characteristics all together. The R program algorithm used for both approaches to cluster analysis is presented on Appendices B.1 for approach a, and B.2 for approach b. Let  $X_{gh}$ , be the nomenclature to identify the clusters evaluated where X denotes the cluster, g denotes approach and h denotes the number of clusters that the data was divided.

A dendrogram is a visual representation of relationship between sites. The height on the dendrogram represents the difference in distance between sites. The dendrograms illustrated in Figure 4 shows that the differences in approaches had

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an impact in the manner that the sites were assigned to each cluster. The Euclidean distance between those sites seems to change within both approaches. Other sites, such as sites 16 and 21, demonstrated to be mutual in both methods, demonstrating that the characteristics between these sites are very similar. If the island were divided into only two regions, approach *a* (characteristics equally important) would have a similar number of sites in both regions (14 sites for Cluster  $A_{a2}$  and 15 sites for Cluster  $B_{a2}$ ). Under approach *b*, assuming that area is the most important characteristic, Cluster  $A_{b2}$  would be larger, comprising 18 sites, while Cluster  $B_{b2}$  would include of 11 sites.



Comparing Clusters A and B between both approaches, sites 1, 2, 4, and 7 would belong to Cluster  $B_{a2}$  under approach *a*, and to Cluster  $A_{b2}$  under approach *b*. Further differences are found when the sites are grouped into four clusters. Due to the limitation in the number of sites available, it is not recommended to use more than four clusters. The next step is to evaluate whether the clusters are homogeneous by applying the heterogeneity measures to each cluster, and then, decide which is the best configuration for grouping the sites.

#### **7.3** Discordancy and heterogeneity measures

Now that the gauging stations have been grouped into regions, the discordancy and heterogeneity measures must be computed to determine if the regions can be classified as homogeneous. As discussed earlier, the aim is to define the most probable homogeneous regions because this represents the similarity of the *L*-moments between the sites. The test for heterogeneity was carried out for both approaches and the results are displayed in Table 5. In approaches *a* and *b* it is not suitable to split the island in two regions because the resulting clusters are "definitely heterogeneous" ( $H_i > 2$ ). When four regions are considered, the heterogeneity values are lowered considerably in Cluster  $A_{a4}$  ("acceptably homogeneous"), Cluster  $B_{a4}$  ("possibly heterogeneous") and Cluster  $D_{b4}$  ("possibly heterogeneous"). Because of the higher values of the heterogeneity measure in the remaining clusters, those regions are classified as "definitely heterogeneous". Considering the lower values of  $H_i$  under approach *a*, this approach is selected for further modification until the values of  $H_i$  are lowered.

Number of Regions	Cluster	$H_1$	$H_2$	$H_3$				
Chara	Characteristics Equally Important							
2 Deciona	Cluster A <sub>a2</sub>	3.48	5.41	4.27				
2 Regions	Cluster B <sub>a2</sub>	7.82	6.2	4.41				
	Cluster A <sub>a4</sub>	-0.3	1.27	1.17				
4 Deciena	Cluster B <sub>a4</sub>	1.3	1.36	0.92				
4 Regions	Cluster C <sub>a4</sub>	4.54	3.8	2.45				
	Cluster D <sub>a4</sub>	5.31	4.74	3.12				
	Area Important							
2 Deciona	Cluster A <sub>b2</sub>	4.32	5.78	4.31				
2 Regions	Cluster B <sub>b2</sub>	7.19	5.06	4.28				
	Cluster A <sub>b4</sub>	3.38	4.29	3				
4 Deciena	Cluster B <sub>b4</sub>	2.88	4.1	3.5				
4 Regions	Cluster C <sub>b4</sub>	7.08	5.07	3.47				
	Cluster D <sub>b4</sub>	1.82	2.34	2.12				

Table 5. Heterogeneity measures for approaches a (characteristics equally<br/>important) and b (area important).

Hosking & Wallis (1997) recommended that regions resulting from clustering analysis be adjusted to obtain lower *H* values for the RFFA. In this study lower *H* values were obtained transferring one or more sites from one region to another, and eliminating one or more sites from the data set. Cluster  $C_{a4}$ was divided and various sites were transferred to Cluster  $A_{a4}$ . In addition, one mutual cluster from Cluster  $A_{a4}$  was transferred to Cluster  $B_{a4}$ . Despite the fact that Cluster  $D_{a4}$  is "definitely heterogeneous", when sites were transferred to another cluster, the  $H_i$  values increased. These sites are located on western Puerto Rico and the number of sites that meets the requirement for sites selection on this study is considerably low, with only 3 sites. The USGS station 50038320 was eliminated from the cluster analysis since its incorporation into any cluster increased the heterogeneity measure. The final structuring of clusters (Cluster 1, Cluster 2, Cluster 3, and Cluster 4), and their *L*-moment ratio diagram is presented in Appendix C.1. The diagrams in this appendix shows the sites *L*-skewness and *L*-kurtosis, and the curves for each distribution. The diagrams provide a image representation of which distribution better fits the sites in each specific region. In addition,

Figure 5 illustrates the location of the individual sites and their corresponding cluster. Finally, as show on Table 6, Cluster 1 is considered "acceptably homogeneous", Cluster 2 and Cluster 3 as "possibly heterogeneous", and Cluster 4 as "definitely heterogeneous".

Cluster	Heterogeneity Measures				
Cluster	$H_1$	$H_2$	$H_3$		
Cluster 1	0.66	0.68	-0.11		
Cluster 2	1.29	1.25	0.82		
Cluster 3	1.31	1.54	1.6		
Cluster 4	5.1	4.75	3.15		

 Table 6. Heterogeneity measures for final selection of clusters.

For Cluster 1 the heterogeneity measure obtained from the measured based on *L*-skewness and *L*-kurtosis ( $H_3$ ) has a negative value. Hosking and Wallis (1997) explain that negative *H* values indicates that there is less dispersion between the at-site sample *L*-CV values when compared to what is expected from an homogeneous region. They recommend that for values lower than -2, the data must be examined for possible cross-correlation between sites frequency distributions. Consequently, the value of -0.11 is not a major concern. Hereafter, the groups of sites will be referred to as regions instead of clusters.



Figure 5. Assignment of sites to clusters

Once the regions were defined, the goodness-of-fit measure was calculated for each. Values of  $Z^{Dist}$  were computed using the L-kurtosis of the sample region and each fitted distribution. As shown in Table 7, Regions 1 and 4 possess acceptable Z values for the GEV distribution, while the Pearson Type 3 fits best Regions 2 and 3.

Dogion	<b>Distributions</b> Z values					
Region	GEV	GNO	Pe3	GPA		
Region 1	-0.54	-1.4	-2.86	-1.65		
Region 2	2.32	1.81	0.84	-1.56		
Region 3	3.01	1.96	0.14	0.44		
Region 4	-0.9	-1.36	-2.16	-1.63		

Table 7. Goodness-of-fit-measures

It was initially desired to use the same distribution for all regions under study, but this could not be achieved since not all regions under same distribution comply with the critical value  $Z_{crit} = 1.64$ . The GEV has provided a good fit to flood data at many locations but not on a regional level in all cases. Even though the GEV distribution fits Cluster 4, it is nevertheless a heterogeneous region. The Wakeby distribution is mentioned by Hosking & Wallis (1997) to be a more robust frequency distribution, and may well provide a better estimate for a heterogeneous area. Because of this reason the Wakeby distribution was chosen to fit Region 4.

#### 7.4 Regional Quantiles

The index flood was used as the basis for the estimation of the regional growth curves. The dimensionless T-year flood value, q(F), was estimated for each region. The values are presented in Table 8.

Region		Fitted				
	5 year	10 year	25 year	50 year	100 year	Distribution
1	1.30	1.91	3.01	4.16	5.68	GEV
2	1.45	1.85	2.33	2.69	3.03	Pe3
3	1.56	2.22	3.12	3.80	4.48	Pe3
4	1.33	1.95	3.03	4.10	5.43	Wakeby

 Table 8. Estimated quantiles for clustered regions in Puerto Rico for various return periods.

Regions 1 and 4 have similar regional quantile values from 5- to 100- years return period, and also a higher upper tail when compared to the other regions. In a GEV distribution the upper tail behavior is related to the shape parameter k. For values of k < 0 the tail weight is heavier for a GEV distribution than for a Pe3 distribution. Hosking and Wallis (1997) define the tail weight as "the rate at which quantile increases as return period is extrapolated beyond the range of the data". For a Wakeby distribution the tail is heavier when  $\delta > 0$ . In Region 1, k = -0.41, and in Region 2,  $\delta = 0.33$ , thus it is expected that these regions have heavier tails than Regions 2 and 3.

#### 7.5 Quantiles Estimation Errors

In order to assess the overall deviation of the estimated quantiles from the true quantiles, the regional average root mean square error (RMSE) was

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computed. Hosking & Wallis (1997) assert that the relative RMSE is the criterion to which they give most weight in judging whether one estimation procedure is superior to another. The relative RMSE of the site-*i* quantile is defined by the following:

$$R_{i}(F) = \left[ M^{-1} \sum_{m=1}^{M} \left\{ \frac{\hat{Q}_{i}^{[m]}(F) - Q_{i}(F)}{Q_{i}(F)} \right\}^{2} \right]^{\frac{1}{2}} \times 100\%$$
(7.1)

where *M* is the total of repetitions,  $\hat{Q}_i^{[m]}(F)$  is the site-*i* quantile estimate for nonexceedance probability *F*, and  $Q_i(F)$  is the site-*i* quantile true value implied by the specified distribution. For an evaluation of the performance of an estimation over a region, the regional average relative RMSE of the estimated quantile is calculated as follows:

$$R^{R}(F) = N^{-1} \sum_{i=1}^{N} R_{i}(F)$$
(7.2)

The available source code used to determine the relative RMSE is based on a Monte Carlo simulation using the same characteristics from the region under study. On the simulation, the *L*-moment ratios at the individual simulated sites are in accordance with the heterogeneity measure for the region (Hosking & Wallis, 1997). The observed regional estimates and the simulated values were compared, determining the accuracy of the procedure in estimating quantiles.

Table 9 shows the percent values obtained for the regional average relative RMSE. This values are considerably low for all regions, but it was expected that

Region 4 would yield a higher RMSE value (RMSE=0.79) because of its heterogeneous classification.

Dogion	Quantile RMSE %					
Region	5 years	10 years	25 years	50 years	100 years	
Region 1	0.048	0.026	0.120	0.301	0.601	
Region 2	0.010	0.029	0.061	0.087	0.114	
Region 3	0.008	0.037	0.095	0.145	0.198	
Region 4	0.084	0.086	0.179	0.398	0.790	

 Table 9. Regional average relative root mean square error values for various return periods.

These RMSE values are not comparable with the RMSE results from the USGS latest report (Ramos-Ginés, 1999). The approach used on the USGS report is based on the differences between the estimated T-year quantile at site *i* based on at-site streamflow record ( $Q_{rec,i}$ ), and the regional regression estimate at site *i* ( $Q_{rec,i}^*$ ). The USGS used formulas to determine the RMSE were the following:

$$RMSE_{log} = \left[\frac{\sum \left(logQ_{rec,i} - logQ_{rec,i}^*\right)^2}{N_p}\right]^{\frac{1}{2}}$$
(7.3)

and in percent,

$$RMSE_{\%} = 100 \left( e^{(5.302)(RMSE_{log})} - 1 \right)^{\frac{1}{2}}$$
(7.4)

Thus, the meaning of the RMSE computed by the USGS is different from that used on this study. Ramos-Ginés (1999) claims that the regression equation used to estimate the 100-year flood quantile yielded a RMSE of 43 percent, the smallest value obtained when compared to previous studies. Therefore, the 43 percent refers to the RMSE between two estimated quantiles and not to the accuracy of the individual estimate, unlike the results obtained through the Monte Carlo simulation.

The empirical quantile of the distribution of estimates is obtained by calculating the average on a region of the ratio between estimated and true quantile values, and accumulating a histogram of the values resulting from the ratio. Then, the 90% error bounds for the true quantile lie within the interval,

$$\frac{\hat{Q}(F)}{U_{0.5}(F)} \le Q(F) \le \frac{\hat{Q}(F)}{L_{0.5}(F)}.$$
(7.5)

In Figure 6, the continuous line represents the regional quantile values while the dotted lines indicates the lower and upper error bounds of the regional quantile values for a 90% confidence. The regional growth curves illustrate how the error bounds increase for return periods approximately greater than 20 years. Also, for Region 1 and 4 the upper bounds in the 100-year return period are wider. For Region 1 the GEV parameter k is less than 0, thus when the distribution is fitted to the data, the data will not lie close to the upper bound, as opposed to when k > 0 (Hosking and Wallis, 1997). The wider upper bound in these regions is also related to the heavier tails that both regions have.



Figure 6. Regional growth curves and error bounds for 90% confidence

#### 7.6 Comparison of regional and at-site estimation

At-site estimates were performed to compare and evaluate the differences with the regional estimates. It was assumed that the GEV distribution was the best fit for each site. The resulting percent errors were calculated and presented in Table 24 on Appendix C.3, page 117. These values are used to compare the magnitude of the difference between the two quantile values. The formula for the percent error, where the at-site quantile ( $q_i(F)$ ) is considered as the real value and the regional site quantile ( $\hat{q}_i(F)$ ) as the estimated value, is given by:

% error = 
$$\frac{\hat{q}_i(F) - q_i(F)}{q_i(F)}$$
. (7.6)

For USGS station 50028000 from Region 1, located in Tanamá River, the percent error for 100-year estimate is equal to 95%. Figure 7 is shows the plot of the site quantile estimated from the regional growth curve (black line) and the atsite quantile (red line) obtained from the GEV distribution. For return periods less than 50-years the at-site quantile fits the data accurately, but for the 100-year return period the regional estimate quantile curve is closer to the data. Even though the percent error is high for this station, the regional estimate is a better estimate for the higher return period at this station, as seen on Figure 7. Other consideration when analyzing extreme data, like annual peak streamflow, is the accuracy of the flow value measured at the river. For most extreme events, the magnitude of the flows is computed via approximate means since these flows are usually beyond the range of the streamflow gage calibration curves for the river.

Contrary to station 50028000 in Region 1, other stations that comprise the region have lower percent errors at 100-year return periods. For example, the USGS station 50092000 in Rio Grande de Patillas, with a percent error of only - 4%. Regions 2 and 3 also are composed by USGS stations with percent errors less than 20% between at-site and regional quantiles. In general, for the 5-, 10-, and 25-year return periods, the standard error values are considerably lower for most stations.



Figure 7. USGS station 50028000 at-site and estimated quantiles.

The percent errors were considerably high for all the three sites that comprise Region 4. The higher value corresponds to USGS station 50147800 at Culebrinas River with an error of 218%, which indicates an overestimated regional quantile. USGS station 50138000 at Guanajibo River also possesses higher percent errors at the 100-year return period with a value of 44%. These results suggest that for Region 4 basins in western Puerto Rico, another parameter estimation may be called for. In addition, a possible reason for the high percent error values in some of the stations may be related to the assumption that the GEV was the appropriate distribution for all the sites.

# 8. APPLICATION TO UNGAGED BASINS

Following the same procedure as for the index flood method, the quantiles for an ungaged basin can be estimated. Only two requisites would be required to obtain the ungaged site quantile: the regional flood frequency relationship, and the sample mean of the data at site *i*, considered in this study as the scale value or index flood. Next are listed the parameters for the respective distribution of the regions, calculated using the L-moments and L-moments ratios of the specific region.

- Region 1: GEV Parameters
  - $\xi = 0.53016$
  - $\alpha = 0.37084$
  - *k* = -0.41599
- Region 2: Pe3 Parameters
  - μ = 1
  - $\sigma = 0.632164$
  - *γ* = 1.306465
- Region 3: Pe3 Parameters
  - $\mu = 1$
  - $\sigma = 0.950069$
  - $\gamma = 2.104329$
- Region 4: Wakeby Parameters
  - $\xi = -0.105$
  - α = 6.176
  - $\beta = 14.454$
  - $\gamma = 0.473$
  - $\delta = 0.330$

With the use of these parameters the dimensionless regional quantile function is defined for each distribution. For Region 1, characterized by a GEV distribution, with F being the nonexceedance probability, the function desired is as follows:

$$q(F) = 0.530 - \frac{0.371}{-0.416} (1 - (-\log F)^{-0.416})$$
(8.1)

Similarly, the q(F) function for Region 4 with a nonexceedance probability *F*, characterized by a Wakeby distribution is:

$$q(F) = -0.105 + \frac{6.176}{14.454} (1 - (1 - F)^{14.454}) - \frac{0.473}{0.330} (1 - (1 - F)^{0.330}).$$
(8.2)

An exception is presented at Region 2 and Region 3 where the best fitted distribution, as determined by the goodness-of-fit measure, is the Pearson Type 3. As stated earlier, the probability density function of this distribution does not possess an analytical solution. Therefore, tabulated values of the q(F) must be used. Table 18 to Table 21, from Appendix C.2, provide these values with an *F* sequence from 0.2 to 0.99 with an increase of 0.01. For the estimation of the scale factor  $\mu_i$ , the linear regression model developed by Sanchez (1995) was used. The regression takes into consideration the contributing drainage area (CDA) in mi<sup>2</sup>, the 5-year return period 24 hr rainfall ( $X_{25}$ ) in inches. The last two variables values are obtained from NOAA Atlas 14 (Bonnin, et al., 2006). With a correlation coefficient of 93 percent, the formula derived was:

$$\log(Q_{mean}) = 2.02 + 0.76 * \log(CDA) + 0.318x_5 - 0.135x_{25}$$
(8.3)

#### 8.1 Accuracy of the index flood method for ungaged basin analysis

To test for accuracy of the regional flood frequency analysis on an ungaged basin, one station that was used on the analysis will be considered as ungaged. The station was extracted from its corresponding region and the procedure to determine the regional quantile was repeated without it. The scale factor  $\mu_i$  in Equation 4.13 was substituted by  $Q_{mean}$  value. The USGS station 50092000 at Rio Grande de Patillas belonging to Region 1 was selected for the test. The contributing drainage area for the station is 18 mi<sup>2</sup>, and the 5-year and the 25-year return period 24 hr rainfall are 6.84 and 11.4 inches, respectively. A mean annual flow value of 4075 cfs is obtained by substituting the characteristic values in the regression function. This value is not exactly comparable with the mean annual peak streamflow data obtained from the record data of the USGS station.

Table 10. Comparison of at-site and regional quantile estimates for site50092000 for various return periods.

Drocoduro	Quantile						
rrocedure	5-year	10-year	25-year	50-year	100-year		
At- site	8269.7 cfs	12029.03 cfs	18629.61 cfs	25342.15 cfs	34068.24 cfs		
Regional Growth Curve	1.298138	1.908236	3.012798	4.169084	5.710218		
Q <sub>mean</sub> equation	4075 cfs						
Site estimate	5289.91 cfs	7776.06 cfs	12277.15 cfs	16989.02 cfs	23269.14 cfs		
Percent Error	-36%	-35%	-34%	-33%	-32%		
Q <sub>mean</sub>	6217.364 cfs						
Site Estimate	8071.00 cfs	11864.20 cfs	18731.66 cfs	25920.71 cfs	35502.50 cfs		
Percent Error	-2%	-1%	1%	2%	4%		

Table 10 shows the at-site quantile, the regional growth curve of Region 1 with station 50092000 excluded from the analysis, the site quantile estimate from the regional analysis, and the percent error. High values of the percent error results from the use of the  $Q_{mean}$  estimated from the regression equation of Equation 8.3. The percent error values decrease significantly when the mean annual peak streamflow data from the record data was used as the index flood on Equation 4.13. The larger percent error of 4% is obtained at the 100-year return period. The graph in Figure 8 depicts graphically the similarity between the atsite estimate (red line) and the quantile estimated from the regional growth curve and the mean annual peak streamflow of the record data.



Figure 8. Comparison of USGS station 5009200 at-site and estimated quantiles with index flood equal to record data mean annual peak.

Among those stations that were excluded from this study, USGS station 50073200 was chosen randomly in order to evaluate the accuracy of the proposed methodology on stations with short record. The USGS station 50073200 at Rio Daguao, located at eastern Puerto Rico, has a 15 year annual peak streamflow record. This station was excluded from the analysis due to its short period of record. A GEV distribution was fitted to the data for the at-site quantiles. The contributing drainage area for the station is 2.26 mi<sup>2</sup>, and the 5-year return period 24 hr rainfall and the 25-year return period 24 hr rainfall are 8.5 and 11.5 inches, respectively. A mean annual flow value of 2,751.84 cfs is obtained from regression equation.

To determine into which region Station 50073200 can be grouped into, the location of the site was evaluated, and in this case was assigned to Region 2. This information was validated implementing the hybrid cluster analysis. It was found that Station 50073200 forms a mutual cluster with Stations 50051310 and 50056400, therefore it is grouped into Region 2. Using the mean annual flow value of 2,751.84 cfs, and the q(F) values from Table 8 for Region 3, the estimated site quantiles for site 50073200 were computed and are presented in Table 11. The percent error between the at-site and regional estimated quantile is considerably high for all the return periods. This can be related to the bias from a short record and the estimated mean annual streamflow. The percent errors for this station are comparable with Site 50065500 from Region 2, included on this analysis in Table 24.

Drogoduro	Quantile (cfs)					
Frocedure	5-year	10-year	25-year	50-year	100-year	
At- site	4929.57	6972.38	10005.41	12634.81	15615.22	
Site estimate	3915.67	4988.85	6429.10	7561.90	8743.63	
Percent Error	-21%	-28%	-36%	-40%	-44%	

Table 11. Comparison of at-site and regional quantile estimates for site50073200 for various return periods.

Figure 9 depicts the underestimated quantile (black line). If the  $Q_{mean}$  value obtained from the regression equation were nearer average of the annual peak streamflow value of the station, a better estimate would be produced.



Figure 9. Comparison of USGS station 50073200 at-site and estimated quantiles.

The results obtained from the comparison of quantile estimates in USGS station 50092000 emphasize the importance of the estimate of the index flood in a regional frequency analysis. In addition, it was discussed that for the estimation

of quantiles with short records the procedure provides relatively good results despite the error involved in the index flood estimation with the regression equation and the short record.

#### 8.2 Application at an ungaged site

The ungaged river Unibón, located in the Cibuco river watershed at northern Puerto Rico, was selected to apply the regional flood frequency analysis. The river is located inside Region 3 and using the clustering analysis, it was determined that Region 3 is the best option to allocate this river. A mean peak flow of 3,481.91 cfs for this river was estimated with Equation 8.3. With the use of q(F) for Region 3, the quantile estimate for 100-year at Unibón river is 15,583 cfs.

Table 12. Regional quantile estimated at Unibón River for various returnperiods.

<b>S:</b> to			Quantile		
Site	5-year	10-year	25-year	50-year	100-year
Regional	1.561548	2.228793	3.11895	3.79612	4.47551
Unibón	5437.17 cfs	7760.46 cfs	10859.90 cfs	13217.75 cfs	15583.32 cfs

It is evident that the procedure to determine the quantile estimate for an ungaged basin is very simple. Once the site characteristics are found, in this case CDA, MAR, CS,  $X_5$ , and  $X_{25}$ , the quantiles for nonexceedance values from 0.2 to 0.99 are easily obtained from the information provided in Table 18 to Table 21, from pages 111 to 114. In cases where only the 5-, 10-, 25-, 50-, and 100-year return periods are needed, Table 8 is the best option if only the return periods mentioned are needed.

# 9. CONCLUSIONS

The objectives of this study were to perform a new flood regionalization study for Puerto Rico based on recent advances in watershed classification and parameter estimation techniques of flood frequency distributions. They were accomplished with the use of the hybrid clustering algorithm and the L-moments. Consequently, a regional flood frequency analysis based on the method of Lmoments was carried out using annual peak streamflow data. The following results and conclusions were obtained from this analysis:

- According to the heterogeneity measures applied in the study, Puerto Rico is definitely heterogeneous when considered as a single region.
- The grouping of gage sites in Puerto Rico that provided the more acceptable heterogeneity measures was a four-region grouping, from which only one region is considered as "acceptably homogeneous", two as "possibly heterogeneous", and one as "definitely heterogeneous".
- The general extreme value distribution provided the best fit only to Region 1, while for Regions 2 and 3 the best fit turned out to be the Pearson Type 3 distribution. For Region 4, a Wakeby distribution was applied following an observation from the related literature that this five-parameter distribution could perhaps provide a more robust fitting.
- Monte Carlo simulations demonstrated that, based on the root mean square error (RMSE), that the regionalized flood frequency procedure applied to the available peak streamflow data is accurate. The larger RMSE for a 100-year flood was obtained in Region 4 with a 0.79%. This value is very

low, but when the percent error between at-site and estimated quantile was calculated, errors higher than 40% were obtained between the few sites that comprise the region. Region 4 was a region in which a larger percent of the sites yielded higher errors. This can be explained by the relatively high heterogeneity values obtained for this region.

- The determination of quantiles at ungaged sites is straightforward with the index flood method. The accuracy of the procedure is dependent on the quality of the relation available for the mean annual peak streamflow estimates.
- Presently, the available annual peak flow records are of relatively short durations, mostly not exceeding 50 years. Most of the available stream gage data have one or two very large peaks notably different from the rest of the series. This situation introduces considerable uncertainty in the regionalization process that can only be remedied over time as more annual peaks are incorporated in the data base.

## 10. **Recommendations**

The accuracy of the flood frequency study is conditioned on the quality of several fundamental parameters, particularly the procedure for ungaged basins which relies on the availability of a relation of estimating the mean annual peak streamflow. Further effort is required to update and further qualify the relation of the mean annual flood with catchment morphological parameters. This could improve the predictive feature of flood frequency quantiles for ungaged basins.

The particular nature of the annual peak flow series available on the island most surely induces a high degree of uncertainty in flood estimation for the higher return periods. Island peak flow data is characterized by nearly uniform values interspersed with a few inordinate extreme flows. Because of the relatively short periods of record, the presence of a small number of seemingly outlying peaks, in some cases exceeding a tenfold increase in magnitude over the nominal values, will certainly bias the analysis. Only the long-term sampling of annual data will serve to reduce the attendant estimation uncertainty when predicting the high-end flows such as the 100-year flood or above.

Although it is impossible to completely eliminate estimation uncertainty, improvements in the theoretical analysis of extreme value events should provide more accurate procedures. However, these must be accompanied by more accurate measurements of extreme floods. At present, river overbank peak flows are computed by indirect means different from the estimation procedures for instream flows. Thus, peak flow estimates for major floods are subject to
considerable irreducible uncertainty. Improvements in peak flood estimation should also accompany developments in the statistical field.

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## **APPENDICES**

## A. *L*-statistics

A.1 *L*-moments values for sites considered in the study

Site number	n	Mean	L-CV	L-skewness	L-kurtosis	t <sub>5</sub>	Discordancy
		$l_1$	Т	t <sub>3</sub>	t <sub>4</sub>		Di
50028000	51	5569.02	0.29617	0.20777	0.25144	0.21541	1.42
50028400	31	4486.48	0.37890	0.33365	0.18638	0.08602	0.21
50031200	44	13010.82	0.47029	0.37404	0.17192	0.03374	0.24
50034000	35	5380.91	0.54813	0.52798	0.32130	0.17050	0.48
50035000	59	26879.66	0.49182	0.42747	0.23867	0.11786	0.15
50038100	49	37825.10	0.47579	0.28507	0.08060	0.02400	1.09
50038320	35	8318.00	0.32641	0.09325	0.09913	0.10460	1.27
50039500	52	9118.27	0.49765	0.40671	0.13028	0.02354	0.97
50043000	34	18647.35	0.52648	0.44025	0.19170	0.04021	0.6
50047850	29	9964.90	0.49289	0.47211	0.37245	0.33854	0.87
50050900	34	18647.35	0.52648	0.44025	0.19170	0.04021	0.6
50051310	33	6562.06	0.36682	0.13380	0.03269	-0.02624	0.97
50055000	51	25410.59	0.35568	0.19348	0.18716	0.12659	0.82
50056400	40	11524.00	0.37903	0.25601	0.14736	0.06262	0.11
50057000	50	22066.00	0.46875	0.27484	0.04561	0.02049	1.41
50061800	42	6398.10	0.37232	0.21450	0.09969	-0.00227	0.29
50063800	43	8112.09	0.33322	0.18157	0.05008	-0.00541	0.61
50064200	34	7121.18	0.38878	0.36902	0.19075	0.00354	0.43
50065500	34	10115.88	0.27903	0.03393	0.09977	0.05075	1.92
50067000	31	4392.26	0.28203	0.24851	0.12489	0.02820	1.24
50071000	49	8766.94	0.30263	0.27176	0.19660	0.02170	0.74
50075000	29	1228.28	0.28921	0.35666	0.14367	-0.01587	3.47
50090500	33	3719.06	0.48895	0.50481	0.35860	0.20820	0.52
50092000	44	6217.36	0.45113	0.44262	0.31344	0.20285	0.28
50112500	47	2119.51	0.48920	0.57703	0.37459	0.27159	0.97
50115000	34	3291.97	0.50299	0.57291	0.44179	0.30883	1.31
50124200	30	4747.27	0.53965	0.51630	0.22447	0.03212	0.84
50138000	36	12319.72	0.69533	0.69272	0.47665	0.31642	2.98
50144000	48	24100.42	0.49549	0.59750	0.42004	0.28244	1.25
50147800	43	24749.53	0.18356	0.01996	0.09839	-0.00002	1.95

# A.2 R Code for L-moment estimation and GEV fitting to considered stations Station 50028000

```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(1370, 1480, 1670, 1680, 1730, 1850, 1960, 2570, 2910, 3080, 3120,
3400,
3490, 3490, 3650, 4060, 4120, 4210, 4300, 4470, 4490, 4700, 4910, 5300,
5360, 5410, 5420, 5440, 5540, 5700, 5860, 6040, 6060, 6400, 6420, 6480, 6670, 6670, 6770, 6860, 7240, 7340, 7510, 7530, 7840, 7940, 8590, 8950,
10270, 12200, 23500)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
```

```
xi alpha k
4134.56854991 2248.80294880 -0.05803271
name n l_1 t t_3 t_4 t_5
1 1 51 5569.02 0.2961749 0.2077667 0.2514385 0.2154058
```



```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(1020, 1060, 1360, 1610, 1880, 1900, 2080, 2160, 2320, 2340, 2600,
2920,
3010, 3150, 3420, 3530, 3540, 3800, 3970, 4120, 4430, 5140, 5450, 6440,
6730, 7380, 7811, 8010, 9200, 11700, 15000)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
```

```
xi alpha k
4134.56854991 2248.80294880 -0.05803271
name n l_1 t t_3 t_4 t_5
1 151 5569.02 0.2961749 0.2077667 0.2514385 0.2154058
```



```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(606, 1280, 1920, 2870, 2920, 3630, 3650, 3670, 3910, 4540, 4670,
5320, 5960, 6040, 6270, 6330, 6680, 6680, 7120, 7280, 7290, 7300, 7610,
7650, 9480, 10600, 11700, 12400, 12600, 12800, 12900, 13800, 15300,
19000, 22500, 24100, 24200, 28000, 28500, 30000, 32700, 35000, 47700,
48000)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
         xi
                    alpha
                                     k
```

```
6945.0287427 6167.8088742 -0.2948494
name n l_1 t t_3 t_4 t_5
1 1 44 13010.82 0.4702914 0.3740437 0.1719246 0.0337389
```



```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(182, 924, 996, 1060, 1180, 1560, 1570, 1580, 1650, 1700, 1850, 1960,
2040, 2070, 2300, 2430, 2540, 2560, 2790, 3310, 3710, 3760, 4230, 4650,
4820, 5920, 6410, 7420, 7480, 9580, 10800, 11400, 17800, 25900, 28200)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
        xi
                  alpha
                                  k
2282.196023 2046.747849
                          -0.491876
              1 1
                          t
                                  t 3
                                                        t 5
 name n
                                              t 4
```

```
1 1 35 5380.914 0.5481346 0.5279825 0.3213005 0.1705034
```



```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(1060, 2760, 2920, 5220, 5620, 6120, 6430, 6470, 6880, 7250,
7490, 7870, 7950, 8360, 9800, 9900, 10200, 10300, 10800, 11300,
11600, 12100, 13000, 13700, 13700, 14400, 14600, 15300, 16600, 17700,
17700, 17800, 18900, 19900, 20200, 22400, 24800, 25200, 26300, 26500, 27300, 27500, 28000, 29100, 32400, 36200, 36400, 38100, 42100, 46000, 48200, 56500, 72100, 74300, 75400, 77300, 78900, 125000, 128000)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
                          alpha
                                               k
             xi
```

```
13431.0323536 11844.6651534 -0.3654108
name n l_1 t t_3 t_4 t_5
1 1 59 26879.66 0.4918223 0.427472 0.2386716 0.1178635
```



```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(1260, 1700, 5000, 5470, 6000, 6610, 6960, 7100, 8550, 11000,
11100, 11300, 11300, 11600, 12100, 13400, 13600, 15500, 15800, 17000,
19800, 21440, 27200, 30190, 30200, 30800, 31030, 31500, 34500, 34800,
37600, 37900, 43000, 43300, 44200, 53000, 58700, 62000, 68110, 70800,
76000, 77600, 81400, 81760, 90000, 97250, 104000, 107000, 136000)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
```

```
xi alpha k
21011.9337738 21565.2004962 -0.1715951
name n l_1 t t_3 t_4 t_5
1 1 49 37825.1 0.4757868 0.2850661 0.080603 0.02400219
```



```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(220, 1300, 2800, 2850, 2950, 3260, 3720, 3910, 4340, 4700, 4890,
5400, 5550, 5600, 6770, 6930, 7650, 7940, 8860, 9790, 10000, 10300, 10400, 10800, 11000, 11400, 11700, 11700, 11900, 12200, 12400, 12500,
13600, 20400, 21400)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
           xi
                      alpha
                                        k
```

```
6290.4113289 4335.8473559 0.1230376
name n l_1 t t_3 t_4 t_5
1 135 8318 0.3264056 0.09325012 0.09913084 0.104595
```



```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(1040, 1570, 2050, 2140, 2160, 2210, 2220, 2240, 2400, 2500, 2500,
2500, 2500, 2600, 2640, 2650, 2730, 2790, 2810, 3450, 3670, 3900, 4500,
4600, 4640, 5820, 6000, 6050, 6740, 6800, 7020, 7650, 7710, 7950, 9100,
9700, 10100, 10100, 12100, 12300, 12900, 15000, 16600, 20000, 20200, 22200, 25000, 26400, 28000, 29400, 30300, 34000)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
           xi
                      alpha
                                          k
4544.6214125 4261.8981829
                              -0.3382752
 name n
               1 1
                              t
                                   t_3
                                                   t 4
                                                               t 5
1
     1 52 9118.269 0.4976546 0.4067054 0.1302836 0.02353612
         35000
                                                                   0
                   Observations
               0
                   GEV Fit
         30000
                                                       o
                                                    0
         25000
     Mean Annual Peak streamflow, cfs
                                                  0
                                                0
                                               0
         20000
                                             00
          15000
                                            0
         10000
         5000
                                              Return period
                   0 0000000000000000000
                                           5
                                                 10
                                                      20
                                                              50
                                                                   100
                                                              1
          0
                                               2
               -2
                       -1
                               0
                                       1
                                                       3
                                                               4
                                                                       5
                               Reduced variate, -\log(-\log(F))
```

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```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(1150, 2680, 2700, 4170, 4420, 4440, 5100, 5370, 5400, 5630, 6200,
6690, 6840, 8490, 8960, 9580, 9760, 11100, 11430, 12400, 13200, 13600, 14100, 18100, 26500, 27000, 27600, 28900, 30800, 48000, 54500, 59600,
66000, 73600)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
           xi
                      alpha
                                         k
```

```
8607.8616707 8536.9758774 -0.3819274

name n l l t t 3 t 4 t 5

1 1 34 18647.35 0.5264753 0.4402481 0.1916981 0.04021034
```



```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(677, 685, 2720, 2810, 3250, 3390, 3640, 3650, 4070, 4120, 4190, 4470, 5240, 5500, 6290, 6980, 9330, 9580, 11400, 11500, 11600, 11900, 12100,
12800, 13490, 13600, 17000, 28000, 65000)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
           xi
                      alpha
                                         k
```

```
4883.8709707 3951.8037006 -0.4225494
name n l l t t 3 t 4 t 5
1 1 29 9964.897 0.4928871 0.4721144 0.3724457 0.3385429
```



74

```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(1770, 2320, 2660, 2820, 2960, 3470, 3540, 3620, 3640, 3690, 3780,
3850, 4030, 4290, 4330, 4430, 4760, 5000, 5230, 5650, 6510, 7000, 7170,
7260, 7320, 8470, 8580, 8950, 9830, 11700, 18200, 44000, 45000)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
```

```
xi alpha k
4037.5730816 2036.0649793 -0.5904211
name n l_1 t t_3 t_4 t_5
1 1 33 8055.455 0.4687065 0.6107171 0.4699207 0.3364215
```



```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(528, 1150, 1400, 1450, 1690, 2380, 2690, 3060, 3380, 3750, 4030,
4190, 4330, 4530, 4740, 5490, 5920, 6020, 6100, 7090, 7300, 8150, 8180,
8860, 9610, 9830, 11200, 12100, 12600, 12700, 13200, 14200, 14700)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
          xi
                     alpha
                                        k
4.650539e+03 3.650788e+03 5.702728e-02
                                    t 3
                                                             t 5
 name n
              1 1
                            t
                                                 t 4
    1 33 6562.061 0.3668188 0.1337961 0.03268929 -0.0262432
1
        15000
                                                             0
                  Observations
              0
                                                     0
                 GEV Fit
              _
                                              0
                                            0
                                           0
    Mean Annual Peak streamflow, cfs
                                         0
        10000
        5000
                                            Return period
                      0
                     0
                                         5
                                                                100
                                2
                                              10
                                                    20
                                                           50
                  0
                                                                 1
        0
             -2
                             0
                                     1
                                             2
                                                     3
                                                             4
                                                                    5
                     -1
```

Reduced variate,  $-\log(-\log(F))$ 

```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(1720, 2700, 2830, 4470, 6900, 7600, 7780, 8740, 11700, 11800, 11800,
12800, 12900, 13200, 13500, 13700, 15600, 17200, 18400, 18700, 20000, 20100,20500, 22800, 22800, 24300, 25100, 26700, 26800, 27700, 27800, 28000, 28000, 28400, 28900, 29100, 29800, 31100, 33100, 33800, 34700, 36400, 37100, 40200, 40800, 43300, 48300, 49000, 62800, 71500, 83000)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
```

```
xi alpha k

1.767257e+04 1.259459e+04 -3.632759e-02

name n l_1 t t_3 t_4 t_5

1 151 25410.59 0.355681 0.1934845 0.1871577 0.1265918
```



```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x \! = \! c \, (1480, \ 1970, \ 2260, \ 3180, \ 3500, \ 3610, \ 3820, \ 5060, \ 5360, \ 5710, \ 6290,
6610, 7000, 7150, 7150, 7220, 7340, 7460, 8780, 9380, 9460, 9930, 9940,
12100, 12100, 12700, 13000, 13400, 14400, 14500, 15100, 17400, 18000,
20500, 21200, 23000, 23300, 24900, 25700, 40000)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
           xi
                      alpha
                                         k
                               -0.1296562
7542.0006493 5506.3315684
  name n 11
                                   t 3
                          t
                                               t 4
                                                           t 5
     1 40 11524 0.3790262 0.2560089 0.1473628 0.06261552
1
        40000
                  Observations
               0
                  GEV Fit
        30000
     Mean Annual Peak streamflow, cfs
                                                         0
                                                     ю
        20000
        10000
                          _568<sup>26568000</sup>
                                              Return period
                       ഷ്
                                                10
                                                              50
                                                                   100
                                 2
                                           5
                                                      20
                                                                    0
              -2
                      -1
                              0
                                      1
                                               2
                                                       3
                                                               4
                                                                       5
```

Reduced variate,  $-\log(-\log(F))$ 

```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(900, 1620, 3220, 4320, 4450, 4910, 5010, 5170, 5190, 5350, 6110, 6140, 6290, 6440, 7130, 7180, 7340, 7840, 8700, 8790, 9700, 10100, 14000,
14100, 14300, 15100, 16200, 17200, 19300, 23300, 25300, 27200, 28600,
29400, 29400, 32300, 32600, 34800, 36500, 38200, 41500, 41600, 42100,
43500, 48600, 50600, 54900, 62100, 64100, 74600)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
```

```
xi alpha k
12482.6869459 12621.4248390 -0.1569379
name n l_1 t t_3 t_4 t_5
1 150 22066 0.468747 0.274842 0.04560659 0.02048972
```



```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(258, 673, 1210, 1760, 2070, 2300, 2510, 2910, 3159, 3320, 3440, 3460,
3590, 3630, 3680, 3770, 4010, 4040, 4050, 5070, 5090, 5140, 5260, 6080,
6220, 6320, 6910, 6960, 8200, 8870, 9260, 9400, 10100, 10200, 10400,
10600, 10600, 12500, 13400, 15000, 16000, 17300)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
```

```
xi alpha k
4311.6277949 3213.9029720 -0.0681788
name n l_1 t t_3 t_4 t_5
1 142 6398.095 0.3723224 0.2144962 0.09968504 -0.00226767
```



1 1

name n

t.

```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x \! = \! c \left( 1400, \ 1700, \ 2580, \ 2700, \ 2710, \ 3290, \ 3400, \ 3710, \ 4100, \ 4300, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \ 4320, \
4700, 4710, 4830, 4830, 4940, 5140, 5150, 5400, 5960, 6800, 7400, 7460,
7880, 7960, 8310, 8440, 8870, 9230, 9520, 11450, 11900, 12790, 13110,
13200, 13260, 13500, 13520, 15080, 15600, 15700, 16770, 21200)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
                               xi
                                                              alpha
                                                                                                                  k
5829.31998967 3834.26011873
                                                                                      -0.01804257
                                                                                                   t 3
                                                                                                                                   t 4
```

t. 5



```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x = c (1070, \ 1510, \ 2320, \ 2870, \ 2980, \ 3050, \ 3100, \ 3420, \ 3950, \ 4000, \ 4100,
4140, 4240, 4450, 4540, 4540, 4730, 5590, 5710, 5780, 5790, 5880, 6180,
6360, 7290, 8230, 12200, 12600, 13700, 14000, 14800, 17400, 19600, 22000)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
         xi
                  alpha
                                  k
```

```
4384.2647902 2820.1880739 -0.2880861

name n l_1 t t_3 t_4 t_5

1 1 34 7121.176 0.3887779 0.3690195 0.1907469 0.003542907
```



```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(1570, 2110, 2730, 3280, 5190, 5420, 5560, 6020, 6130, 6130, 6320,
7140, 8030, 9700, 9830, 9920, 10300, 10600, 10900, 11200, 11300, 11400, 12160, 12600, 12800, 13000, 13800, 13900, 15300, 15600, 15700, 18000,
19800, 20500)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
```

```
xi alpha k
8225.059556 4819.307922 0.223768
name n l_1 t t_3 t_4 t_5
1 1 34 10115.88 0.2790321 0.03392621 0.09976632 0.05075375
```



```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(1730, 1760, 1810, 2120, 2260, 2320, 2570, 2760, 2830, 2960, 2970, 3030, 3310, 3470, 3940, 4120, 4290, 4350, 4480, 4500, 4520, 4580, 4660,
4840, 5330, 7340, 8050, 8170, 8480, 9010, 9600)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
          xi
                     alpha
                                       k
3270.5869614 1581.4666506
                              -0.1186952
                                  t 3
               1 1
                                            t 4
                                                        t 5
 name n
                           t
1
     1 31 4392.258 0.282026 0.248514 0.124889 0.02819502
                0
                    Observations
                                                                   0
                   GEV Fit
                _
                                                      0
         8000
                                                  0
                                                0
                                               0
     Mean Annual Peak streamflow, cfs
         6000
                                             0
                                   0000000000
         4000
         2000
                     0
                       0
                                                 Return period
                                    2
                                             5
                                                    10
                                                          20
                                                                  50
                                                                       100
                                                                         1
         0
                                                  2
                                                          3
                                                                            5
               -2
                        -1
                                0
                                         1
                                                                   4
```

Reduced variate,  $-\log(-\log(F))$ 

```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x \! = \! c \left( 1430, \ 2300, \ 2600, \ 2940, \ 3770, \ 4320, \ 4670, \ 4710, \ 5220, \ 5320, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5530, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \ 5500, \
5590, 5800, 6000, 6030, 6080, 6110, 6110, 6270, 6360, 6410, 6690, 6710, 7310, 7340, 7600, 7700, 7850, 7900, 7950, 8480, 8570, 9150, 9200, 10500,
10600, 10700, 10800, 11200, 11800, 14500, 14500, 14560, 15100, 15300,
19400, 19600, 21700, 23300)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
```

```
xi alpha k
6315.1051988 3254.9817827 -0.1524962
name n l_1 t t_3 t_4 t_5
1 1 49 8766.939 0.302625 0.2717584 0.1966049 0.02169503
```



Reduced variate,  $-\log(-\log(F))$ 

```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(482, 567, 641, 650, 752, 760, 771, 779, 783, 791, 811, 859, 859, 872, 893, 1000, 1110, 1120, 1220, 1260, 1260, 1500, 1790, 1850, 1900, 2180,
2480, 2820, 2860)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
         xi
                  alpha
                                  k
```

```
879.5941976 371.1879713 -0.2713537
name n 1 1 t t 3 t 4 t 5
1 1 29 1228.276 0.2892115 0.3566612 0.1436683 -0.01587123
```



5000

0

-2

```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(297, 549, 883, 1010, 1080, 1160, 1410, 1530, 1550,
                                                                1690,
                                                                        1760,
2010, 2030, 2060, 2070, 2080, 2170, 2300, 2520,
                                                                2550,
                                                                         2680,
2680, 3030, 3140, 4310, 5060, 6280, 6560, 6780, 7400, 8950, 9950,
23200)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
         xi
                   alpha
                                   k
1819.4651256 1344.3461292
                          -0.4634029
                         t
             1 1
                                 t 3
                                          t. 4
                                                    t. 5
 name n
1
    1 33 3719.061 0.4889487 0.5048077 0.3585974 0.2082034
                                                            0
                 Observations
              0
                 GEV Fit
        20000
    Mean Annual Peak streamflow, cfs
        15000
        10000
```

Return period 20000000000000000000 5 10 20 50 -1 0 1 2 3

0

Reduced variate,  $-\log(-\log(F))$ 

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5

Т

4

```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
          987, 1400, 1600, 1750, 1800, 1890, 2050, 2400, 2410, 2460,
x=c(667,
2630, 2640, 2850, 3230, 3270, 3320, 3700, 3820, 3960, 4400, 4500, 4600,
4730, 4950, 5380, 5550, 5600, 5630, 5720, 6230, 6360, 6530, 6830, 8130,
8290, 8370, 8440, 9190, 14700, 17400, 19900, 22400, 30900)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
```

```
xi alpha k
3346.3653130 2425.3694359 -0.3849781
name n l_1 t t_3 t_4 t_5
1 1 44 6217.364 0.4511339 0.4426193 0.313444 0.202851
```



88

```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(544, 570, 625, 643, 647, 691, 705, 737, 746, 746, 772, 786,
794, 860, 881, 895, 945, 979, 981, 1080, 1100, 1110, 1110, 1120,
1120, 1200, 1230, 1270, 1310, 1460, 1510, 1830, 1850, 1880, 2300,
2800, 2800, 3410, 3470, 3480, 3780, 3960, 4190, 4330, 5650, 5720, 19000)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
          xi
                     alpha
                                        k
1020.3724859
              622.9538200
                              -0.5508642
               1 1
                             t
                                     t. 3
                                                 t. 4
                                                            t. 5
  name n
1
     1 47 2119.511 0.4892023 0.5770297 0.3745948 0.2715939
                                                                        0
                    Observations
                0
                    GEV Fit
         15000
     Mean Annual Peak streamflow, cfs
         10000
                                                               0
         5000
                                              000<sup>0</sup>,
                                                     0
                                                  Return period
                                            \infty
                                   and the second second
                                              5
                                                           20
                                                                   50
                                                                        100
                                                     10
                       _____
                                                                          1
         0
               -2
                        -1
                                 0
                                          1
                                                   2
                                                            3
                                                                     4
                                                                             5
```

Reduced variate,  $-\log(-\log(F))$ 

```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x{=}c\,(500,\ 574,\ 800,\ 923,\ 980,\ 1020,\ 1060,\ 1200,\ 1470,\ 1510,\ 1530,\ 1530,
1550, 1700, 1780, 1810, 1880, 1920, 1940, 2330, 2380, 2800, 2850, 3010,
3050, 3050, 3070, 3800, 4120, 4260, 6790, 10640, 13100, 21000)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
          xi
                    alpha
                                     k
1537.7340516 1007.5147569
                            -0.5459791
                                   t 3
              1 1
                           t
                                              t 4
                                                        t 5
 name n
1
    1 34 3291.971 0.5029942 0.5729141 0.4417894 0.3088251
                                                                 о
         20000
               0
                   Observations
                   GEV Fit
        15000
    Mean Annual Peak streamflow, cfs
                                                        о
         10000
                                                    0
                                                 ο
        5000
                                               0
                                              Return period
                                     00000
                       000000000000000000000000
                                           5
                                                 10
                                                       20
                                                               50
                                                                    100
         0
              -2
                      -1
                               0
                                       1
                                               2
                                                        3
                                                                4
                                                                        5
```

Reduced variate,  $-\log(-\log(F))$ 

```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(753, 978, 1120, 1190, 1210, 1320, 1400, 1450, 1500, 1640, 1690, 1780,
1800, 1810, 2010, 2170, 2697, 2800, 3440, 3720, 4260, 4300, 4890, 5860,
8130, 11900, 14500, 14700, 18700, 18700)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
        xi
                 alpha
                                 k
2063.162484 1836.104988
                         -0.477566
              1 1
                                    3
                                                      t 5
                         t
                                            t 4
 name n
                                  ÷
1
    1 30 4747.267 0.5396494 0.5162951 0.2244669 0.03211737
                  Observations
                                                     0
                                                             0
              0
              _
                  GEV Fit
        15000
                                                 0
                                              0
    Mean Annual Peak streamflow, cfs
                                            0
        10000
                                          0
        5000
                                             Return period
                   5
                                               10
                                                     20
                                                            50
                                                                 100
                                                                  1
        O
                                              2
                                                     3
                                                                     5
             -2
                     -1
                                      1
                                                             4
                              0
```

Reduced variate,  $-\log(-\log(F))$ 

```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(5880, 3020, 7480, 4940, 2920, 4710, 5640, 5000, 2200, 3740, 3040, 4340, 5040, 1970, 2820, 2740, 2710, 5390, 2170, 2590, 4150, 1190, 5020,
3060)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
```

```
xi alpha k
3167.7821129 1382.8525947 0.1150973
name n l_1 t t_3 t_4 t_5
1 1 24 3823.333 0.2278155 0.09805627 0.06111027 0.006931629
```



92

name n

1

```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(1230, 1340, 1340, 1350, 1500, 1700, 1860, 2300, 2320, 2360,
2370, 2650, 2760, 2770, 2780, 2920, 3030, 3310, 3410, 3850,
3960, 4720, 5230, 5310, 7400, 7510, 8930, 11300, 11400, 15000,
23600, 30400, 40000, 40600, 53000, 128000)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
         хi
                  alpha
                                  k
3182.7699802 3414.0809301
                         -0.6836616
                        t
                                                  t 5
             1 1
                                         t. 4
```

t 3

1 36 12319.72 0.6953266 0.6927246 0.4766538 0.3164175



```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(4670, 5950, 6980, 7410, 7450, 7830, 8000, 8020, 8090, 8800, 8960,
9130, 9300, 9320, 9710, 10700, 11600, 12000, 12400, 12400, 12500, 12600,
12700, 13100, 13800, 14200, 14400, 15000, 15600, 15900, 16000, 17800,
19300, 20300, 21300, 21400, 25400, 28600, 29700, 32300, 32700, 38700, 50800, 51200, 53600, 77200, 140000, 162000)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
          хi
                     alpha
                                        k
11409.3167332
              6724.9077589
                               -0.5749957
              1 1
                                  t 3
 name n
                                             t 4
                                                       t 5
                          t
1
    1 48 24100.42 0.4954893 0.5975035 0.4200445 0.2824378
                  Observations
               0
        150000
                  GEV Fit
                                                          0
    Mean Annual Peak streamflow, cfs
        000001
                                                      0
        50000
                                                0
                                                 0
                                              Return period
                      5
                                                      20
                                                              50
                                                                   100
                                                10
         0
              -2
                                               2
                                                       3
                                                               4
                                                                       5
                      -1
                              0
                                       1
```

Reduced variate,  $-\log(-\log(F))$ 

```
# Call the packages with the functions needed for the analysis.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
# Vector of Peak annual streamflow data for the station.
x=c(4850, 12000, 15000, 15100, 15700, 15800, 16600, 17100, 17130, 18600,
18600, 18700, 19600, 20100, 20400, 20500, 21600, 22500, 23300, 24100,
24500, 24600, 24700, 25800, 26700, 26800, 26800, 28400, 28400, 28950,
29000, 30000, 31300, 31400, 31800, 32800, 33000, 33100, 33800, 34700,
36900, 41200, 42300)
# Computes L-moments
lmom <- samlmu.s(x, nmom=5, sort.data=TRUE)</pre>
# Computes GEV Parameters
pelgev(lmom)
regsamlmu(x, nmom = 5, sort.data = TRUE, lcv = TRUE)
# Extreme-value plot
evplot(x)
# Fit a GEV distribution
# Adds the quantile function of a distribution to an extreme-value plot.
evdistq(quagev, pelgev(lmom))
        xi
                 alpha
                                 k
```

```
2.179453e+04 7.868015e+03 2.482463e-01

name n 1 1 t t_3 t_4 t_5

1 43 24749.53 0.1835649 0.01996185 0.0983888 -2.191976e-05
```



### **B.** Cluster Analysis

#### **B.1** Approach *a*: equal importance characteristics **R** input

```
# Data contains site characteristics
data(DrainageAttributes)
# Take a sequence of vector, matrix or data frames arguments and combine
by columns using cbind
transformed <- cbind(a1 = log10(DrainageAttributes$CDA),</pre>
a2 = DrainageAttributes$lat,
a3 = DrainageAttributes$long,
a4 = DrainageAttributes$CS,
a5 = DrainageAttributes$MAR)
#Divide the column by the standard deviation of each value
transformed <- apply(transformed, 2, function(x) x/sd(x))</pre>
# Clustering by Hybrid's method
(hyb1<-hybridHclust(transformed, themc=NULL, trace=FALSE))
par(mfrow=c(1,1))
plot(transformed, pch = as.character(1:nrow(transformed)), asp = 1)
plot(hyb1)
```

```
transformed
```

	a1	a2	a3	a4	a5
[1,]	2.578375	148.9371	154.2597	0.9883656	2.873245
[2,]	2.765458	149.8640	154.1018	0.7244713	2.848865
[3,]	3.578006	148.8987	153.4057	0.6102114	2.432457
[4,]	2.511504	148.4126	153.5025	0.6340539	2.620995
[5,]	4.369150	149.1270	153.5141	0.5430705	2.699335
[6,]	4.554792	149.9996	153.6687	0.3860781	2.673655
[7,]	2.427549	149.3689	153.2260	1.4008332	2.610593
[8,]	3.926681	150.1375	153.3171	0.5207137	2.538753
[9,]	3.698738	147.7955	152.9803	0.4907869	2.196135
[10,]	3.327818	149.2219	152.7679	0.4230092	2.220190
[11,]	1.596863	147.4632	152.4272	2.3658516	3.243493
[12,]	2.071701	147.7729	152.3547	0.3917380	3.250644
[13,]	4.010111	148.4646	152.4734	0.7428661	2.687633
[14,]	2.495333	148.2499	152.2809	0.8914393	2.942483
[15,]	3.653872	148.5935	152.3778	1.2119329	2.626846
[16,]	2.071701	149.0863	152.1955	2.3517017	3.241868
[17,]	1.923633	149.4231	152.0210	2.5788065	6.001015
[18,]	1.778171	149.2988	152.0858	3.1808818	5.615163
[19,]	1.710003	149.1722	151.8767	3.4822732	5.846284
[20,]	1.216352	149.1858	151.8305	2.8731230	3.421303
[21,]	2.403760	148.9235	151.7470	1.9894661	3.875093
[22,]	1.486004	146.7126	152.3130	1.5982233	2.621320
[23,]	2.597981	146.7714	152.5273	1.6060057	2.846264
[24,]	2.025024	147.1919	153.7515	3.3690745	3.149874
[25,]	1.940002	147.1354	153.9151	1.9809762	3.074785
[26,]	2.621899	146.8528	154.2950	1.6880747	2.537128
[27,]	4.270713	147.6576	155.1060	0.5330948	2.302431
[28,]	4.369150	148.8082	154.8801	0.5778083	2.908027
[29,]	3.806313	149.4345	154.9751	0.3778712	3.039353
## **B.2** Approach *b*: area important R input

```
library(lmom)
library(lmomRFA)
library(hybridHclust)
# Data contains site characteristics
data(DrainageAttributes)
# Take a sequence of vector, matrix or data frames arguments and combine
by columns using cbind
transformed <- cbind(a1 = log10(DrainageAttributes$CDA),</pre>
a2 = DrainageAttributes$lat,
a3 = DrainageAttributes$long,
a4 = DrainageAttributes$CS,
a5 = DrainageAttributes$MAR)
#Divide the column by the standard deviation of each value
transformed <- apply(transformed, 2, function(x) x/sd(x))</pre>
\#CDA importance is equal to the other variables together
transformed[,1] <- transformed[,1] *4</pre>
# Clustering by Hybrid's method
(hyb1<-hybridHclust(transformed, themc=NULL, trace=FALSE))</pre>
par(mfrow=c(1,1))
plot(transformed, pch = as.character(1:nrow(transformed)), asp = 1)
plot(hyb1)
```

transformed

LL ans	LOTIMEA				
	a1	a2	a3	a4	a5
[1,]	10.313500	148.9371	154.2597	0.9883656	2.873245
[2,]	11.061830	149.8640	154.1018	0.7244713	2.848865
[3,]	14.312023	148.8987	153.4057	0.6102114	2.432457
[4,]	10.046015	148.4126	153.5025	0.6340539	2.620995
[5,]	17.476600	149.1270	153.5141	0.5430705	2.699335
[6,]	18.219168	149.9996	153.6687	0.3860781	2.673655
[7,]	9.710198	149.3689	153.2260	1.4008332	2.610593
[8,]	15.706722	150.1375	153.3171	0.5207137	2.538753
[9,]	14.794952	147.7955	152.9803	0.4907869	2.196135
[10,]	13.311272	149.2219	152.7679	0.4230092	2.220190
[11,]	6.387451	147.4632	152.4272	2.3658516	3.243493
[12,]	8.286805	147.7729	152.3547	0.3917380	3.250644
[13,]	16.040445	148.4646	152.4734	0.7428661	2.687633
[14,]	9.981333	148.2499	152.2809	0.8914393	2.942483
[15,]	14.615490	148.5935	152.3778	1.2119329	2.626846
[16,]	8.286805	149.0863	152.1955	2.3517017	3.241868
[17,]	7.694532	149.4231	152.0210	2.5788065	6.001015
[18,]	7.112684	149.2988	152.0858	3.1808818	5.615163
[19,]	6.840013	149.1722	151.8767	3.4822732	5.846284
[20,]	4.865410	149.1858	151.8305	2.8731230	3.421303
[21,]	9.615039	148.9235	151.7470	1.9894661	3.875093
[22,]	5.944016	146.7126	152.3130	1.5982233	2.621320
[23,]	10.391926	146.7714	152.5273	1.6060057	2.846264
[24,]	8.100094	147.1919	153.7515	3.3690745	3.149874
[25,]	7.760006	147.1354	153.9151	1.9809762	3.074785
[26,]	10.487595	146.8528	154.2950	1.6880747	2.537128
[27,]	17.082853	147.6576	155.1060	0.5330948	2.302431
[28,]	17.476600	148.8082	154.8801	0.5778083	2.908027
[29,]	15.225251	149.4345	154.9751	0.3778712	3.039353

#### **B.3** Evaluation of clusters heterogeneity and discordancy measure

## **Example R Input**

```
# Call the packages with the functions needed for the analysis of
#regional frequency. First will be obtained the regional L-moments, then
#is identified the homogeneity of the cluster and which distribution is
#the best fit. Finally, the regional quantiles and site quantiles are
#obtained.
# R functions for use with the method of L-moments.
library(lmom)
# R functions for regional frequency analysis using L-moments.
library(lmomRFA)
#Computes a regional weighted average of L-moments.
regavlmom(regdata cluster) # Weight proportional to record length
regavlmom(regdata_cluster, weight=1) # Equal weights
# Fit a generalized extreme value distribution to the cluster
rfit <- regfit(regdata cluster, "gev")</pre>
rfit # Print details of the fitted distribution
rfit$index # Display the Index flood values for each site
# Plot the regional growth curve
evplot(rfit)
# Discordancy Measure, Heterogeneity measure and Goodness-of-fit measure
regtst(regdata cluster3 1, nsim=1000)
# Compute cluster quantiles for T=5,10,25,50 and 100
regquant(c(0.8000, 0.9000, 0.9600, 0.9800, 0.99), rfit)
# Compute quantiles for T=5,10,25,50 and 100
sitequant(c(0.8, 0.9, 0.96, 0.98, 0.99), rfit)
```

## **R** Ouputs

#### Approach *a*: Two Regions

#### Cluster A<sub>a2</sub>

```
l 2 Column5
    1 1
                           Column6
                                     Column7
1.0000000 0.4008452 0.3502243 0.2223710 0.1066801
     1 1 1 2 Column5 Column6 Column7
1.0000000 0.4031775 0.3523911 0.2231883 0.1066351
Regional frequency distribution: gev
Parameters:
      xi
               alpha
0.6080621 0.4243489 -0.2625835
50050900 50051310 50056400 50061800 50063800 50064200 50065500 50067000
8055.455 6562.061 11524.000 6398.095 8112.093 7121.176 10115.880 4392.258
50071000 50090500 50092000 50112500 50115000 50124200
8766.939 3719.061 6217.364 2119.511 3291.971 4747.267
Discordancy measures (critical value 2.97)
1.28 1.11 0.15 0.36 0.63 0.47 2.56 1.86 1.02 0.52 0.23 0.55 1.06 2.20
Heterogeneity measures (based on 1000 simulations)
3.48 5.41 4.27
Goodness-of-fit measures (based on 1000 simulations)
 glo gev gno pe3 gpa
1.41 0.56 -0.50 -2.33 -2.04
    0.8 0.9 0.96
                            0.98
                                     0.99
1.388118 1.910002 2.734940 3.494233 4.400175
```

	0.8	0.9	0.96	0.98	0.99
50050900	11181.924	15385.937	22031.189	28147.635	35445.410
50051310	9108.917	12533.551	17946.845	22929.369	28874.215
50056400	15996.675	22010.865	31517.452	40267.538	50707.614
50061800	8881.313	12220.375	17498.408	22356.433	28152.736
50063800	11260.545	15494.115	22186.090	28345.541	35694.627
50064200	9885.035	13601.462	19475.991	24883.046	31334.419
50065500	14042.038	19321.353	27666.328	35347.239	44511.640
50067000	6096.974	8389.222	12012.563	15347.572	19326.703
50071000	12169.548	16744.873	23977.055	30633.725	38576.064
50090500	5162.497	7103.415	10171.410	12995.265	16364.518
50092000	8630.437	11875.179	17004.119	21724.917	27357.488
50112500	2942.132	4048.271	5796.736	7406.065	9326.219
50115000	4569.645	6287.672	9003.344	11502.913	14485.248
50124200	6589.768	9067.290	12983.492	16588.056	20888.805

# Cluster B<sub>a2</sub>

1 1 1 2 Column5 Column6 Column7 1.0000000 0.4433404 0.3496511 0.2107934 0.1201420 1 1 1 2 Column5 Column6 Column7 1.0000000 0.4468896 0.3564544 0.2181153 0.1266889 Regional frequency distribution: gev Parameters: xi alpha k 0.5666625 0.4698778 -0.2618006 50028000 50028400 50031200 50034000 50035000 50038100 50038320 50039500 5569.020 4486.484 13010.820 5380.914 26879.660 37825.100 8318.000 9118.269 50043000 50047850 50055000 50057000 50138000 50144000 50147800 18647.350 9964.897 25410.590 22066.000 12319.720 24100.420 24749.530 Discordancy measures (critical value 3.00) 1.04 0.92 0.20 0.32 0.08 0.85 1.35 0.94 0.37 0.64 0.71 1.08 2.77 2.03 1.70 Heterogeneity measures (based on 1000 simulations) 7.82 6.20 4.41 Goodness-of-fit measures (based on 1000 simulations) glo gev gno pe3 gpa 2.15 1.19 -0.01 -2.08 -1.76 0.8 0.9 0.96 0.98 0.99 1.429872 2.006896 2.918383 3.756800 4.756601 0.8 0.9 0.96 0.98 0.99 50028000 7962.985 11176.444 16252.53 20921.69 26489.60 50028400 6415.097 9003.907 13093.28 16854.82 21340.41 50031200 18603.805 26111.363 37970.55 48879.05 61887.27 50034000 7694.017 10798.935 15703.57 20215.02 25594.86 50035000 38434.468 53944.683 78445.13 100981.51 127855.81 50038100 54085.044 75911.043 110388.12 142101.34 179918.89 50038320 11893.674 16693.361 24275.11 31249.06 39565.40 50039500 13037.956 18299.418 26610.60 34255.51 43371.96 50043000 26663.320 37423.293 54420.10 70054.37 88698.00 50047850 14248.525 19998.512 29081.38 37436.13 47399.04 74157.83 95462.51 120868.03 50055000 36333.886 50996.412 50057000 31551.551 44284.168 64397.03 82897.55 104959.15 50138000 17615.620 24724.397 35953.66 46282.72 58599.99 50144000 34460.511 48367.037 70334.25 90540.46 114636.07 50147800 35388.655 49669.734 72228.60 92979.04 117723.63

#### Approach *a*: Four Regions

#### Cluster A<sub>a4</sub>

Heterogeneity measures (based on 1000 simulations)  $-0.30\ 1.27\ 1.17$ 

Goodness-of-fit measures (based on 1000 simulations) glo gev gno pe3 gpa 0.28 -0.01 -0.89 -2.38 -1.23 0.8 0.9 0.96 0.98 0 99 1.323591 1.939232 3.032021 4.154852 5.627516 0.9 0.98 0.8 0.96 0.99 50050900 10662.124 15621.399 24424.312 33469.225 45332.20 50051310 8685.482 12725.361 19896.309 27264.393 36928.10 50056400 15253.058 22347.714 34941.014 47880.516 64851.49 50090500 4922.514 7212.124 11276.272 15452.149 20929.07 50092000 8229.244 12056.914 18851.180 25832.228 34988.31 501125002805.3654110.2246426.4038806.25511927.58501150004357.2226383.8979981.32613677.65318525.62 50124200 6283.438 9206.054 14393.815 19724.193 26715.32

#### Cluster B<sub>a4</sub>

l 2 Column5 Column6 1 1 Column7 1.00000000 0.32722331 0.22118759 0.12756570 0.01482936 1 1 1 2 Column5 Column6 Column7 1.00000000 0.32633413 0.21988113 0.12696182 0.01608455 Regional frequency distribution: gev Parameters: xi alpha k 0.71141382 0.43682973 -0.07821653 50061800 50063800 50064200 50065500 50067000 50071000 6398.095 8112.093 7121.176 10115.880 4392.258 8766.939 Discordancy measures (critical value 1.65) 0.63 0.96 1.07 1.46 1.09 0.79 Heterogeneity measures (based on 1000 simulations) 1.30 1.36 0.92 Goodness-of-fit measures (based on 1000 simulations) glo gev gno pe3 gpa 3.01 1.69 1.26 0.42 -1.47 0.8 0.9 0.96 0.98 0.99 1.406616 1.786263 2.298939 2.704607 3.129966 0.8 0.9 0.96 0.98 0.99 50061800 8999.660 11428.679 14708.83 17304.33 20025.82 50063800 11410.596 14490.330 18649.21 21940.02 25390.58 50064200 10016.757 12720.292 16371.15 19259.98 22289.04 50065500 14229.154 18069.621 23255.79 27359.48 31662.36 50067000 6178.218 7845.727 10097.53 11879.33 13747.62 50071000 12331.713 15660.057 20154.66 23711.12 27440.22

# Cluster C<sub>a4</sub>

```
1 1
              1 2 Column5 Column6 Column7
1.0000000 0.4425629 0.3302840 0.1823672 0.1015092
      1 1 1 2 Column5 Column6 Column7
1.0000000 0.4440802 0.3363856 0.1897202 0.1084583
Regional frequency distribution: gev
Parameters:
       xi
                 alpha
                                  k
 0.5727163 0.4873727 -0.2351710
 50028000 50028400 50031200 50034000 50035000 50038100 50038320 50039500 5569.020 4486.484 13010.820 5380.914 26879.660 37825.100 8318.000 9118.269
 50043000 50047850 50055000 50057000
18647.350 9964.897 25410.590 22066.000
Discordancy measures (critical value 2.76)
1.61 2.09 0.13 1.10 0.19 0.82 1.56 0.81 0.37 1.59 0.76 0.97
Heterogeneity measures (based on 1000 simulations)
4.54 3.80 2.45
Goodness-of-fit measures (based on 1000 simulations)
 glo gev gno pe3 gpa
 3.18 2.09 0.96 -1.02 -1.08
    0.8
             0.9 0.96 0.98
                                          0.99
1.449276 2.018455 2.897281 3.688250 4.614033
               0.8 0.9 0.96
                                              0.98
                                                         0.99
500280008071.04511240.81716135.0220539.9425695.64500284006502.1529055.76712998.6116547.2720700.785003120018856.26526261.75737696.0047987.1560032.35
50034000 7798.428 10861.134 15590.02 19846.16 24827.71
50035000 38956.038 54255.388 77877.93 99138.90 124023.63
50038100 54818.998 76348.268 109589.95 139508.42 174526.25
50038320 12055.075 16789.510 24099.59 30678.86 38379.53
50039500 13214.886 18404.817 26418.19 33630.45 42071.99
50043000 27025.151 37638.840 54026.62 68776.09 86039.49
50047850 14441.883 20113.698 28871.11 36753.03 45978.36
50055000 36826.950 51290.136 73621.63 93720.60 117245.30
50057000 31979.718 44539.231 63931.41 81384.92 101813.25
```

## Cluster D<sub>a4</sub>

1 1 1 2 Column5 Column6 Column7 1.0000000 0.4465239 0.4289497 0.3271842 0.1964339 1 1 1 2 Column5 Column6 Column7 1.000000 0.4581269 0.4367300 0.3316957 0.1996111 Regional frequency distribution: gev Parameters: xi alpha 0.5454715 0.3987028 -0.3673280 50138000 50144000 50147800 12319.72 24100.42 24749.53 Discordancy measures (critical value 3.00) 1.00 1.00 1.00 Heterogeneity measures (based on 1000 simulations) 5.31 4.74 3.12 Goodness-of-fit measures (based on 1000 simulations) glo gev gno pe3 gpa -0.70 -0.90 -1.38 -2.21 -1.66 0.8 0.9 0.96 0.98 0.99 1.343180 1.940865 2.974470 4.010627 5.341045 0.8 0.9 0.96 0.98 0.99 50138000 16547.60 23910.91 36644.64 49409.81 65800.18 50144000 32371.20 46775.66 71685.98 96657.80 128721.44 50147800 33243.07 48035.50 73616.73 99261.14 132188.36

#### Approach *b*: Two Regions

#### Cluster A<sub>b2</sub>

```
1 2 Column5 Column6 Column7
      1 1
1.0000000 0.3957002 0.3347058 0.2215854 0.1171007
    1 1 1 2 Column5 Column6 Column7
1.000000 0.399638 0.338693 0.221202 0.114994
Regional frequency distribution: gev
Parameters:
       xi
               alpha
                               k
0.6168512 0.4320202 -0.2412818
 50028000 50028400 50034000 50038320 50050900 50051310 50056400 50061800
5569.0204486.4845380.9148318.0008055.4556562.06111524.0006398.0955006380050064200500655005006700050071000500905005009200050112500
8112.093 7121.176 10115.880 4392.258 8766.939 3719.061 6217.364 2119.511
50115000 50124200
3291.971 4747.267
Discordancy measures (critical value 3.00)
1.56 0.27 1.26 1.20 1.51 1.15 0.14 0.39 0.80 0.58 1.81 2.00 0.99 0.52 0.21 0.65
1.13 1.85
Heterogeneity measures (based on 1000 simulations)
4.32 5.78 4.31
Goodness-of-fit measures (based on 1000 simulations)
 glo gev gno pe3 gpa
1.16 0.14 -0.97 -2.90 -2.87
    0.8 0.9 0.96 0.98
                                      0.99
1.397639 1.908024 2.700203 3.416760 4.259046
              0.8 0.9
                                0.96 0.98
                                                    0.99
50028000 7783.478 10625.824 15037.487 19028.00 23718.711
50028400 6270.484 8560.319 12114.420 15329.24 19108.140
50034000 7520.574 10266.913 14529.563 18385.29 22917.559
50038320 11625.559 15870.944 22460.293 28420.61 35426.742
50050900 11258.615 15370.002 21751.368 27523.55 34308.551
50051310 9171.390 12520.570 17718.900 22420.98 27948.118
50056400 16106.388 21988.069 31117.145 39374.74 49081.242
50061800 8942.225 12207.719 17276.158 21860.75 27249.779
50063800 11337.775 15478.068 21904.302 27717.07 34549.775
50064200 9952.831 13587.375 19228.624 24331.35 30329.414
50065500 14138.345 19301.342 27314.934 34563.53 43083.995
50067000 6138.790 8380.534 11859.990 15007.29 18706.827
50071000 12253.013 16727.530 23672.519 29954.52 37338.794
50090500 5197.904 7096.058 10042.221 12707.14 15839.651
50092000 8689.628 11862.880 16788.148 21243.24 26480.037
50112500 2962.311 4044.078 5723.111 7241.86 9027.094
501150004600.9866281.1608888.99211247.8714020.655501242006634.9649057.90012818.58716220.2720218.827
```

## Cluster B<sub>b2</sub>

Discordancy measures (critical value 2.63) 0.14 0.05 0.67 0.83 0.27 0.58 1.06 0.81 2.28 2.20 2.11 Heterogeneity measures (based on 1000 simulations) 7.19 5.06 4.28 Goodness-of-fit measures (based on 1000 simulations) glo gev gno pe3 gpa 2.58 1.80 0.66 -1.33 -0.68 0.8 0.9 0.96 0.98 0.99 1.425514 2.035182 3.022193 3.951586 5.082365 0.96 0.8 0.9 0.98 0.99 50031200 18547.11 26479.39 39321.21 51413.38 66125.74 50035000 38317.33 54705.00 81235.52 106217.30 136612.25 50038100 53920.21 76980.97 114314.75 149469.15 192240.97 50039500 12998.22 18557.34 27557.17 36031.63 46342.37 50043000 26582.06 37950.75 56355.89 73686.61 94772.64 50047850 14205.10 20280.38 30115.84 39377.15 50645.24 50055000 36223.15 51715.18 76795.71 100412.14 129145.89 50057000 31455.39 44908.33 66687.71 87195.70 112147.47 50138000 17561.93 25072.87 37232.57 48682.44 62613.31 50144000 34355.49 49048.74 72836.12 95234.89 122487.13 50147800 35280.80 50369.80 74797.85 97799.90 125786.15

#### Approach b: Four Regions

### Cluster A<sub>b4</sub>

1 1 12 Column5 Column6 Column7 1.0000000 0.4061591 0.3423380 0.2276285 0.1253744 1 1 1 2 Column5 Column6 Column7 1.0000000 0.4127310 0.3456598 0.2239698 0.1206823 Regional frequency distribution: gev Parameters: xi alpha 0.6047995 0.4368115 -0.2517863 50028000 50028400 50034000 50038320 50051310 50056400 50061800 50071000 5569.020 4486.484 5380.914 8318.000 6562.061 11524.000 6398.095 8766.939 50092000 50112500 50115000 50124200 6217.364 2119.511 3291.971 4747.267 Discordancy measures (critical value 2.76) 1.50 0.69 1.12 1.42 1.11 0.15 0.41 1.67 0.23 0.85 1.41 1.46 Heterogeneity measures (based on 1000 simulations) 3.38 4.29 3.00 Goodness-of-fit measures (based on 1000 simulations) glo gev gno pe3 gpa 0.93 0.13 -0.81 -2.43 -2.29 0.8 0.9 0.96 0.98 0.99 1.400876 1.927251 2.751630 3.503742 4.394355 0.8 0.9 0.96 0.98 0.99 50028000 7801.505 10732.900 15323.882 19512.412 24472.250 50028400 6285.007 8646.581 12345.144 15719.485 19715.203 50034000 7537.992 10370.372 14806.284 18853.337 23645.645 50038320 11652.485 16030.875 22888.058 29144.130 36552.243 50051310 9192.632 12646.739 18056.363 22991.772 28836.024 50056400 16143.693 22209.642 31709.783 40377.128 50640.545 50061800 8962.936 12330.736 17605.190 22417.277 28115.500 50071000 12281.393 16896.093 24123.372 30717.097 38525.041 50092000 8709.755 11982.422 17107.885 21784.042 27321.303 50112500 2969.172 4084.830 5832.110 7426.221 9313.883 50115000 4611.643 6344.455 9058.286 11534.219 14466.089 50124200 6650.331 9149.176 13062.722 16633.201 20861.176

# Cluster B<sub>b4</sub>

l 2 Column5 Column6 Column7 1 1 1.00000000 0.37201677 0.31742319 0.20790141 0.09836562 1 1 1 2 Column5 Column6 Column7 1.0000000 0.3734521 0.3247595 0.2156665 0.1036175 Regional frequency distribution: gev Parameters: xi alpha 0.6439550 0.4199373 -0.2172917 50050900 50063800 50064200 50065500 50067000 50090500 8055.455 8112.093 7121.176 10115.880 4392.258 3719.061 Discordancy measures (critical value 1.65) 1.17 0.80 0.35 1.60 1.19 0.89 Heterogeneity measures (based on 1000 simulations) 2.88 4.10 3.50 Goodness-of-fit measures (based on 1000 simulations) glo gev gno pe3 gpa 0.90 0.24 -0.38 -1.46 -1.65 0.8 0.9 0.96 0.98 0.99 1.388605 1.862771 2.583778 3.223281 3.962465

0.80.90.960.980.995005090011185.84815005.46520813.50625965.0031919.465006380011264.49615110.96820959.84626147.5632143.88500642009888.50313265.11718399.53622953.5528217.415006550014046.96518843.56426137.18632606.3340083.82500670006099.1138181.76911348.61914157.4817404.17500905005164.3086927.7579609.22711987.5814736.65

## Cluster C<sub>b4</sub>

l\_1 l\_2 t\_3 t\_4 t\_5 1.0000000 0.42321821 0.29945388 0.15747254 0.07053799 l\_1 l\_2 t\_3 t\_4 t\_5 1.0000000 0.42790019 0.31162856 0.17107216 0.08329827 Regional frequency distribution: gev Parameters: xi alpha k 0.6001912 0.4940580 -0.1920423 50031200 50039500 50043000 50047850 50055000 50057000 50147800 13010.820 9118.269 18647.350 9964.897 25410.590 22066.000 24749.530 Discordancy measures (critical value 1.92) 0.15 0.84 0.28 1.48 1.14 1.27 1.84

Heterogeneity measures (based on 1000 simulations) 7.08 5.07 3.47

Goodness-of-fit measures (based on 1000 simulations) glo gev gno pe3 gpa 2.97 1.99 1.22 -0.14 -0.70 0.8 0.9 0.96 0.98 0.99 1.459011 1.990942 2.782513 3.470235 4.251214 0.8 0.9 0.96 0.98 0.99 50031200 18982.93 25903.79 36202.78 45150.61 55311.78 50039500 13303.65 18153.95 25371.70 31642.54 38763.71 50043000 27206.69 37125.80 51886.49 64710.70 79273.88 50047850 14538.89 19839.53 27727.46 34580.54 42362.91 50055000 37074.33 50591.02 70705.30 88180.73 108025.86 50057000 32194.53 43932.13 61398.93 76574.22 93807.29 50147800 36109.83 49274.89 68865.89 85886.70 105215.55

# Cluster D<sub>b4</sub>

1 1 12 t3 t4 t 5 1.0000000 0.5268037 0.4833716 0.2882961 0.1722818 l\_1 l\_2 t\_3 t\_4 t\_5 1.0000000 0.5396063 0.5006915 0.3039932 0.1851802 Regional frequency distribution: gev Parameters: xi alpha k 0.4530612 0.4119288 -0.4367087 50035000 50038100 50138000 50144000 26879.66 37825.10 12319.72 24100.42 Discordancy measures (critical value 3.00) 1.00 1.00 1.00 1.00 Heterogeneity measures (based on 1000 simulations) 1.82 2.34 2.12 Goodness-of-fit measures (based on 1000 simulations) glo gev gno pe3 gpa 0.98 0.78 0.02 -1.27 -0.12 0.8 0.9 0.96 0.98 0.99 1.325776 2.030006 3.322786 4.693895 6.542071 0.99 0.8 0.9 0.96 0.98 50035000 35636.40 54565.86 89315.37 126170.30 175848.65 50038100 50147.60 76785.17 125684.73 177547.05 247454.50 5013800016333.1925009.1040935.8057827.4780596.495014400031951.7548923.9980080.55113124.84157666.67

# C. Selected regions for further analysis

						Region 1							
id	d Site number		Mean	L-CV	L- skewness	L- kurtosis		Lat	Long	CDA	MAR	CS	Di
	number	n	L1	t	t3	t4	t5		2000	(mi <sup>2</sup> )	(in)	(ft/mi)	
1	50028000	51	5569.02	0.296	0.208	0.251	0.215	18.301	66.783	18	72.32	139.7	2.0
2	50028400	31	4486.484	0.379	0.334	0.186	0.086	18.414	66.714	22.2	63.11	102.4	1.4
4	50034000	35	5380.914	0.548	0.528	0.321	0.171	18.236	66.455	16.7	74.61	89.62	1.1
11	50050900	33	8055.455	0.469	0.611	0.470	0.336	18.119	65.989	5.99	72.76	334.4	1.4
22	50090500	33	3719.061	0.489	0.505	0.359	0.208	18.027	65.940	5.29	66.10	225.9	0.2
23	50092000	44	6217.364	0.451	0.443	0.313	0.203	18.034	66.033	18.4	60.94	227	0.1
24	50112500	47	2119.511	0.489	0.577	0.375	0.272	18.086	66.563	9.68	60.51	476.2	0.7
25	50115000	34	3291.971	0.503	0.573	0.442	0.309	18.079	66.634	8.8	58.15	280	0.7
26	50124200	30	4747.267	0.540	0.516	0.224	0.032	18.044	66.798	18.9	50.83	238.6	1.4

Table 14. L-moments and site characteristics of Region 1

Table 15. L-moments and site characteristics of Region 2

	Region 2												
id	Site	n	Mean	L-CV	L- skewness	L- kurtosis	t5	Lat	Long	CDA	MAR	CS	Di
	number		L1	t	t3	t4				(mi²)	(in)	(ft/mi)	
12	50051310	33	6562.061	0.367	0.134	0.033	-0.026	18.158	65.958	10.2	77.56	55.37	0.9
14	50056400	40	11524.000	0.379	0.256	0.147	0.063	18.216	65.926	16.4	69.17	126	0.6
16	50061800	42	6398.095	0.372	0.214	0.100	-0.002	18.319	65.889	10.2	66.97	332.4	0.3
17	50063800	43	8112.093	0.333	0.182	0.050	-0.005	18.360	65.814	8.64	64.80	364.5	0.9
18	50064200	34	7121.176	0.389	0.369	0.191	0.004	18.345	65.842	7.34	65.59	449.6	1.1
19	50065500	34	10115.880	0.279	0.034	0.100	0.051	18.329	65.751	6.8	62.60	492.2	1.9
20	50067000	31	4392.258	0.282	0.249	0.125	0.028	18.331	65.731	3.91	60.28	406.1	1.4
21	50071000	49	8766.939	0.303	0.272	0.197	0.022	18.299	65.695	14.8	52.95	281.2	0.9

						Region 3	3						
id	d Site number		Mean	L-CV	L- skewness	L- kurtosis	t5	Lat	Long	CDA	MAR	CS	D.
Iu	number	n	L1	t	t3	t4		Lat	Long	( <b>mi</b> <sup>2</sup> )	(in)	(ft/mi)	$\boldsymbol{\nu}_{i}$
3	50031200	44	13010.82	0.470	0.374	0.172	0.034	18.296	66.413	55.2	68.50	86.25	0.4
5	50035000	59	26879.66	0.492	0.427	0.239	0.118	18.324	66.460	134	68.19	76.76	0.2
6	50038100	49	37825.1	0.476	0.285	0.081	0.024	18.431	66.527	165	59.76	54.57	1.2
8	50039500	52	9118.269	0.498	0.407	0.130	0.024	18.448	66.375	81.6	59.33	73.6	1.2
9	50043000	34	18647.35	0.526	0.440	0.192	0.040	18.160	66.229	63.2	59.37	69.37	0.5
10	50047850	29	9964.897	0.493	0.472	0.372	0.339	18.336	66.137	41.7	59.33	59.79	1.7
13	50055000	51	25410.59	0.356	0.193	0.187	0.127	18.243	66.009	89.6	62.60	105	2.1
15	50057000	50	22066	0.469	0.275	0.046	0.020	18.258	65.968	60.1	66.34	171.3	0.8

Table 16. *L*-moments and site characteristics of Region 3

Table 17. L-moments and site characteristics of Region 4

	Region 4												
id	Site number n	n	Mean	L-CV	L- skewness	L- kurtosis	t5	Lat	Long	CDA	MAR	CS	D:
	number		L1	t	t3	t4		1	20118	( <b>mi</b> <sup>2</sup> )	(in)	(ft/mi)	
27	50138000	36	12,320	0.695	0.693	0.477	0.316	18.143	67.149	120	62.36	75.35	1
28	50144000	48	24,100	0.495	0.598	0.420	0.282	18.285	67.051	134	74.84	81.67	1
29	50147800	43	24,750	0.184	0.020	0.098	-0.00002	18.362	67.093	71.3	65.91	53.41	1

# C.1 L-moment Ratio Diagrams for regions defined

## **Example R Input**

library(lmom)
library(lmomRFA)
#Draw the L-moment Diagram for the clusters identified.
lmrd (regdata, xaxs="i", yaxs="i",las=1, twopar=FALSE)

# **R** Outputs



Figure 10. L-moment ratio diagram Region 1



Figure 11. L-moment ratio diagram Region 1



Figure 12. L-moment ratio diagram Region 3



Figure 13. L-moment ratio diagram Region 4

# C.2 Relative Regional RMSE

# **Example R Input**

```
library(lmom)
library(lmomRFA)
rfit <- regfit(regdata_cluster, "gev")
evplot(rfit) # Plot the regional growth curve
# Compute error bounds for quantile estimates. We will
# (optimistically) generate bounds for a homogeneous region
# with the same frequency distribution as the one fitted to
# the regdata_cluster4 data.
fval <- seq(.2, .99, by=.01) # A lot of quantiles
simq <- regsimq(rfit$qfunc, nrec=regdata_clusterb$n, nrep=100, f=fval,
fit=rfit$dist)
# Regional growth curve, and bounds
rbounds <- regquantbounds(simq, rfit)
evplot(rfit, rbounds)
```

F	<b>q(F)</b>	Relative RMSE	rel.bound. 0.05	rel.bound. 0.95	F	<b>q(F)</b>	Relative RMSE	rel.bound. 0.05	rel.bound. 0.95
0.2	0.370	0.024	0.323	0.415	0.6	0.818	0.049	0.727	0.855
0.21	0.379	0.025	0.332	0.423	0.61	0.834	0.049	0.743	0.871
0.22	0.389	0.025	0.340	0.432	0.62	0.851	0.050	0.759	0.887
0.23	0.398	0.026	0.348	0.440	0.63	0.868	0.050	0.776	0.904
0.24	0.408	0.027	0.356	0.450	0.64	0.886	0.050	0.793	0.921
0.25	0.417	0.027	0.365	0.458	0.65	0.904	0.051	0.812	0.939
0.26	0.426	0.028	0.373	0.466	0.66	0.923	0.051	0.830	0.958
0.27	0.436	0.029	0.381	0.476	0.67	0.943	0.051	0.850	0.977
0.28	0.445	0.030	0.389	0.485	0.68	0.964	0.051	0.869	0.997
0.29	0.454	0.030	0.397	0.495	0.69	0.985	0.052	0.890	1.018
0.3	0.464	0.031	0.406	0.505	0.7	1.008	0.052	0.913	1.039
0.31	0.473	0.032	0.415	0.514	0.71	1.031	0.052	0.937	1.062
0.32	0.483	0.033	0.423	0.524	0.72	1.055	0.052	0.963	1.085
0.33	0.493	0.033	0.432	0.533	0.73	1.081	0.052	0.989	1.110
0.34	0.502	0.034	0.441	0.543	0.74	1.107	0.052	1.017	1.135
0.35	0.512	0.035	0.449	0.553	0.75	1.136	0.051	1.045	1.162
0.36	0.522	0.035	0.458	0.563	0.76	1.165	0.051	1.076	1.191
0.37	0.532	0.036	0.467	0.573	0.77	1.197	0.051	1.108	1.223
0.38	0.542	0.037	0.476	0.583	0.78	1.230	0.050	1.141	1.256
0.39	0.553	0.037	0.486	0.593	0.79	1.265	0.049	1.177	1.291
0.40	0.563	0.038	0.496	0.604	0.8	1.302	0.048	1.216	1.328
0.41	0.574	0.039	0.506	0.614	0.81	1.343	0.047	1.259	1.368
0.42	0.584	0.039	0.516	0.625	0.82	1.386	0.046	1.302	1.410
0.43	0.595	0.040	0.526	0.636	0.83	1.432	0.044	1.351	1.456
0.44	0.606	0.040	0.537	0.647	0.84	1.482	0.043	1.404	1.506
0.45	0.618	0.041	0.547	0.658	0.85	1.537	0.040	1.462	1.560
0.46	0.629	0.042	0.558	0.669	0.86	1.597	0.038	1.526	1.619
0.47	0.641	0.042	0.568	0.681	0.87	1.663	0.035	1.597	1.684
0.48	0.653	0.043	0.579	0.693	0.88	1.736	0.032	1.676	1.761
0.49	0.665	0.043	0.590	0.705	0.89	1.819	0.028	1.759	1.841
0.50	0.677	0.044	0.601	0.717	0.9	1.912	0.026	1.848	1.945
0.51	0.690	0.044	0.612	0.729	0.91	2.019	0.025	1.955	2.062
0.52	0.702	0.045	0.623	0.742	0.92	2.144	0.029	2.088	2.211
0.53	0.716	0.046	0.635	0.755	0.93	2.293	0.038	2.231	2.389
0.54	0.729	0.046	0.647	0.768	0.94	2.475	0.055	2.401	2.613
0.55	0.743	0.047	0.659	0.782	0.95	2.706	0.081	2.602	2.907
0.56	0.757	0.047	0.672	0.796	0.96	3.011	0.120	2.888	3.298
0.57	0.772	0.048	0.685	0.810	0.97	3.448	0.183	3.273	3.887
0.58	0.786	0.048	0.698	0.824	0.98	4.158	0.301	3.894	4.855
0.59	0.802	0.048	0.712	0.839	0.99	5.681	0.601	5.198	7.138

Outputs Table 18. Regional relative RMSE and error bounds at 90% confidence level Region 1

F	<b>q(F)</b>	Relative RMSE	rel.bound. 0.05	rel.bound. 0.95	F	q(F)	Relative RMSE	rel.bound. 0.05	rel.bound . 0.95
0.2	0.470	0.016	0.443	0.506	0.6	1.022	0.014	0.990	1.048
0.21	0.483	0.016	0.454	0.519	0.61	1.039	0.013	1.009	1.064
0.22	0.496	0.016	0.466	0.532	0.62	1.056	0.013	1.028	1.081
0.23	0.509	0.016	0.479	0.547	0.63	1.074	0.013	1.048	1.097
0.24	0.522	0.016	0.491	0.561	0.64	1.092	0.012	1.067	1.114
0.25	0.535	0.017	0.504	0.575	0.65	1.110	0.012	1.085	1.131
0.26	0.548	0.017	0.516	0.589	0.66	1.129	0.011	1.105	1.149
0.27	0.561	0.017	0.529	0.601	0.67	1.148	0.011	1.124	1.169
0.28	0.574	0.017	0.541	0.614	0.68	1.167	0.010	1.144	1.188
0.29	0.586	0.017	0.553	0.628	0.69	1.188	0.010	1.165	1.209
0.3	0.599	0.017	0.565	0.642	0.7	1.208	0.010	1.186	1.229
0.31	0.612	0.017	0.577	0.654	0.71	1.229	0.009	1.208	1.248
0.32	0.625	0.017	0.589	0.667	0.72	1.251	0.009	1.231	1.269
0.33	0.637	0.017	0.601	0.679	0.73	1.274	0.009	1.254	1.291
0.34	0.650	0.017	0.614	0.691	0.74	1.297	0.009	1.278	1.313
0.35	0.663	0.017	0.626	0.704	0.75	1.321	0.009	1.303	1.337
0.36	0.676	0.017	0.639	0.716	0.76	1.345	0.009	1.328	1.362
0.37	0.689	0.017	0.651	0.729	0.77	1.371	0.009	1.353	1.389
0.38	0.702	0.017	0.664	0.742	0.78	1.398	0.009	1.378	1.417
0.39	0.715	0.017	0.677	0.755	0.79	1.425	0.010	1.403	1.444
0.4	0.728	0.017	0.690	0.768	0.8	1.454	0.010	1.429	1.473
0.41	0.742	0.017	0.703	0.781	0.81	1.484	0.011	1.456	1.503
0.42	0.755	0.017	0.716	0.794	0.82	1.516	0.012	1.487	1.537
0.43	0.769	0.017	0.729	0.806	0.83	1.549	0.014	1.516	1.573
0.44	0.782	0.017	0.743	0.819	0.84	1.583	0.015	1.547	1.611
0.45	0.796	0.017	0.756	0.832	0.85	1.620	0.017	1.581	1.651
0.46	0.810	0.017	0.770	0.845	0.86	1.659	0.019	1.619	1.693
0.47	0.824	0.016	0.785	0.859	0.87	1.701	0.021	1.657	1.741
0.48	0.838	0.016	0.799	0.872	0.88	1.746	0.023	1.697	1.792
0.49	0.852	0.016	0.814	0.885	0.89	1.794	0.026	1.738	1.847
0.5	0.866	0.016	0.829	0.899	0.9	1.846	0.029	1.781	1.906
0.51	0.881	0.016	0.844	0.914	0.91	1.904	0.033	1.832	1.971
0.52	0.896	0.016	0.859	0.928	0.92	1.968	0.037	1.888	2.044
0.53	0.911	0.015	0.874	0.942	0.93	2.039	0.041	1.948	2.125
0.54	0.926	0.015	0.890	0.957	0.94	2.121	0.046	2.016	2.221
0.55	0.941	0.015	0.906	0.972	0.95	2.217	0.053	2.099	2.333
0.56	0.957	0.015	0.922	0.986	0.96	2.334	0.061	2.200	2.470
0.57	0.973	0.015	0.938	1.002	0.97	2.482	0.071	2.328	2.646
0.58	0.989	0.014	0.955	1.017	0.98	2.687	0.087	2.505	2.893
0.59	1.005	0.014	0.972	1.032	0.99	3.032	0.114	2.795	3.312

Table 19. Regional relative RMSE and error bounds at 90% confidence levelRegion 2

F	<b>q(F)</b>	Relative RMSE	rel.bound . 0.05	rel.bound . 0.95	F	<b>q(F)</b>	Relative RMSE	rel.bound . 0.05	rel.bound . 0.95
0.2	0.274	0.014	0.247	0.314	0.6	0.904	0.025	0.857	0.956
0.21	0.285	0.015	0.257	0.328	0.61	0.928	0.024	0.882	0.978
0.22	0.296	0.016	0.266	0.342	0.62	0.952	0.024	0.907	1.002
0.23	0.307	0.016	0.276	0.354	0.63	0.977	0.023	0.933	1.026
0.24	0.318	0.017	0.286	0.367	0.64	1.003	0.023	0.960	1.050
0.25	0.330	0.018	0.296	0.380	0.65	1.030	0.022	0.987	1.075
0.26	0.342	0.018	0.307	0.393	0.66	1.057	0.022	1.016	1.101
0.27	0.354	0.019	0.317	0.406	0.67	1.085	0.021	1.045	1.128
0.28	0.366	0.020	0.328	0.420	0.68	1.114	0.020	1.076	1.155
0.29	0.378	0.020	0.339	0.433	0.69	1.144	0.019	1.107	1.184
0.3	0.391	0.021	0.351	0.446	0.7	1.175	0.018	1.140	1.213
0.31	0.404	0.021	0.363	0.460	0.71	1.208	0.018	1.174	1.243
0.32	0.417	0.022	0.375	0.474	0.72	1.241	0.017	1.210	1.275
0.33	0.430	0.022	0.387	0.488	0.73	1.275	0.015	1.246	1.307
0.34	0.443	0.023	0.400	0.502	0.74	1.311	0.014	1.284	1.342
0.35	0.457	0.023	0.413	0.516	0.75	1.349	0.013	1.324	1.377
0.36	0.471	0.024	0.426	0.532	0.76	1.387	0.012	1.365	1.414
0.37	0.485	0.024	0.440	0.547	0.77	1.428	0.011	1.407	1.451
0.38	0.500	0.024	0.454	0.563	0.78	1.470	0.010	1.450	1.491
0.39	0.515	0.025	0.468	0.577	0.79	1.515	0.009	1.496	1.534
0.4	0.530	0.025	0.483	0.592	0.8	1.562	0.008	1.542	1.579
0.41	0.545	0.025	0.497	0.607	0.81	1.611	0.008	1.590	1.628
0.42	0.560	0.026	0.512	0.622	0.82	1.662	0.009	1.641	1.680
0.43	0.576	0.026	0.528	0.637	0.83	1.717	0.011	1.691	1.739
0.44	0.592	0.026	0.544	0.653	0.84	1.776	0.013	1.743	1.803
0.45	0.609	0.026	0.560	0.669	0.85	1.838	0.016	1.798	1.870
0.46	0.626	0.026	0.576	0.686	0.86	1.904	0.019	1.856	1.943
0.47	0.643	0.026	0.593	0.703	0.87	1.975	0.023	1.919	2.023
0.48	0.661	0.026	0.611	0.720	0.88	2.053	0.027	1.987	2.110
0.49	0.678	0.026	0.628	0.737	0.89	2.137	0.031	2.061	2.204
0.5	0.697	0.027	0.646	0.755	0.9	2.229	0.037	2.142	2.308
0.51	0.715	0.026	0.665	0.773	0.91	2.331	0.043	2.231	2.423
0.52	0.734	0.026	0.684	0.791	0.92	2.445	0.050	2.330	2.553
0.53	0.754	0.026	0.704	0.810	0.93	2.574	0.058	2.443	2.701
0.54	0.774	0.026	0.724	0.829	0.94	2.724	0.068	2.573	2.872
0.55	0.794	0.026	0.745	0.849	0.95	2.902	0.080	2.722	3.075
0.56	0.815	0.026	0.766	0.869	0.96	3.119	0.095	2.906	3.326
0.57	0.837	0.026	0.788	0.890	0.97	3.400	0.116	3.148	3.651
0.58	0.859	0.025	0.811	0.911	0.98	3.796	0.145	3.491	4.112
0.59	0.881	0.025	0.834	0.933	0.99	4.476	0.198	4.075	4.903

Table 20. Regional relative RMSE and error bounds at 90% confidence levelRegion 3

F	<b>q(F)</b>	Relative RMSE	Rel.Bound. 0.05	Rel.Bound. 0.95	F	<b>q(F)</b>	Relative RMSE	Rel.Bound. 0.05	Rel.Bound. 0.95
0.2	0.415	0.049	0.361	0.621	0.6	0.828	0.059	0.709	0.900
0.21	0.424	0.049	0.368	0.628	0.61	0.844	0.060	0.725	0.918
0.22	0.433	0.049	0.375	0.634	0.62	0.861	0.061	0.741	0.936
0.23	0.442	0.048	0.381	0.639	0.63	0.879	0.063	0.754	0.953
0.24	0.450	0.048	0.387	0.644	0.64	0.897	0.064	0.768	0.970
0.25	0.458	0.048	0.393	0.647	0.65	0.915	0.065	0.785	0.989
0.26	0.467	0.047	0.399	0.649	0.66	0.935	0.067	0.803	1.012
0.27	0.475	0.047	0.406	0.651	0.67	0.955	0.068	0.821	1.036
0.28	0.483	0.046	0.412	0.653	0.68	0.976	0.070	0.841	1.059
0.29	0.491	0.046	0.417	0.658	0.69	0.998	0.071	0.863	1.085
0.3	0.499	0.046	0.425	0.662	0.7	1.021	0.072	0.885	1.110
0.31	0.507	0.045	0.432	0.665	0.71	1.045	0.074	0.908	1.131
0.32	0.515	0.045	0.439	0.667	0.72	1.070	0.075	0.932	1.157
0.33	0.523	0.045	0.446	0.670	0.73	1.096	0.076	0.956	1.185
0.34	0.532	0.045	0.455	0.673	0.74	1.124	0.078	0.979	1.215
0.35	0.540	0.045	0.464	0.676	0.75	1.153	0.079	1.002	1.247
0.36	0.549	0.045	0.470	0.679	0.76	1.184	0.080	1.028	1.280
0.37	0.558	0.045	0.477	0.682	0.77	1.216	0.081	1.059	1.318
0.38	0.567	0.045	0.484	0.686	0.78	1.251	0.082	1.095	1.362
0.39	0.576	0.045	0.490	0.689	0.79	1.287	0.083	1.129	1.406
0.4	0.585	0.045	0.497	0.694	0.8	1.326	0.084	1.165	1.448
0.41	0.595	0.045	0.503	0.699	0.81	1.368	0.085	1.203	1.492
0.42	0.604	0.045	0.510	0.705	0.82	1.412	0.085	1.245	1.543
0.43	0.614	0.046	0.518	0.711	0.83	1.460	0.086	1.287	1.600
0.44	0.624	0.046	0.528	0.717	0.84	1.512	0.087	1.336	1.660
0.45	0.635	0.047	0.538	0.723	0.85	1.569	0.087	1.390	1.729
0.46	0.645	0.048	0.548	0.733	0.86	1.630	0.087	1.448	1.803
0.47	0.656	0.048	0.558	0.738	0.87	1.698	0.087	1.505	1.877
0.48	0.667	0.049	0.567	0.744	0.88	1.773	0.086	1.583	1.956
0.49	0.679	0.050	0.577	0.756	0.89	1.857	0.086	1.661	2.047
0.5	0.691	0.051	0.588	0.762	0.9	1.952	0.086	1.753	2.148
0.51	0.703	0.051	0.598	0.768	0.91	2.060	0.087	1.866	2.258
0.52	0.715	0.052	0.608	0.777	0.92	2.186	0.090	1.987	2.402
0.53	0.728	0.053	0.619	0.791	0.93	2.334	0.096	2.096	2.562
0.54	0.741	0.054	0.630	0.805	0.94	2.514	0.110	2.273	2.808
0.55	0.754	0.054	0.641	0.820	0.95	2.738	0.136	2.497	3.091
0.56	0.768	0.055	0.653	0.835	0.96	3.032	0.179	2.768	3.489
0.57	0.782	0.056	0.665	0.850	0.97	3.445	0.254	3.142	4.154
0.58	0.797	0.057	0.679	0.866	0.98	4.096	0.398	3.756	5.270
0.59	0.812	0.058	0.693	0.883	0.99	5.433	0.790	4.649	7.912

Table 21. Regional relative RMSE and error bounds at 90% confidence levelRegion 4

# C.3 At-Site quantiles

				Quantile (cfs	)	
Region	Site	5	10	25	50	100
		0.8	0.9	0.96	0.98	0.99
	50028000	7,253.27	10,647.97	16,769.53	23,153.72	31,635.21
	50028400	5,843.34	8,578.16	13,509.78	18,652.97	25,485.79
	50034000	7,008.27	10,288.32	16,203.10	22,371.65	30,566.66
	50050900	10,491.68	15,402.04	24,256.73	33,491.30	45,759.58
Region 1	50090500	4,843.82	7,110.85	11,198.90	15,462.34	21,126.39
	50092000	8,097.69	11,887.61	18,721.84	25,849.27	35,318.17
	50112500	2,760.52	4,052.51	6,382.31	8,812.06	12,040.03
	50115000	4,287.57	6,294.25	9,912.84	13,686.68	18,700.27
	50124200	6,182.99	9,076.78	14,295.06	19,737.20	26,967.18
	50051310	9,340.18	11,900.09	15,335.58	18,037.68	20,856.49
	50056400	16,402.81	20,898.42	26,931.66	31,676.98	36,627.24
	50061800	9,106.80	11,602.75	14,952.39	17,586.98	20,335.35
Region 2	50063800	11,546.44	14,711.03	18,958.01	22,298.38	25,783.03
Region 2	50064200	10,136.00	12,914.03	16,642.23	19,574.57	22,633.55
	50065500	14,398.55	18,344.84	23,640.87	27,806.36	32,151.75
	50067000	6,251.77	7,965.22	10,264.73	12,073.36	13,960.11
	50071000	12,478.52	15,898.57	20,488.39	24,098.41	27,864.35
	50031200	18,917.95	26,869.91	39,444.98	51,024.16	64,844.64
	50035000	39,083.47	55,511.81	81,491.23	105,413.19	133,965.56
	50038100	54,998.32	78,116.31	114,674.59	148,337.61	188,516.55
Pagion 2	50039500	13,258.11	18,831.03	27,643.91	35,758.86	45,444.55
Region 5	50043000	27,113.55	38,510.46	56,533.29	73,128.78	92,936.55
	50047850	14,489.12	20,579.48	30,210.64	39,079.05	49,664.06
	50055000	36,947.42	52,477.89	77,037.44	99,651.99	126,643.87
	50057000	32,084.33	45,570.65	66,897.63	86,535.60	109,974.76
	50138000	16,336.89	24,047.07	37,357.93	50,465.45	66,938.45
Region 4	50144000	31,958.99	47,042.02	73,081.36	98,722.91	130,948.17
	50147800	32,819.76	48,309.03	75,049.70	101,381.87	134,475.07

Table 22. Site quantiles estimated from regional quantiles

				Quantile (cfs	)	
Region	Site	5	10	25	50	100
		0.8	0.9	0.96	0.98	0.99
	50028000	7,166.83	8,152.25	10,039.50	12,425.41	16,236.97
	50028400	6,555.87	8,914.71	12,036.68	14,401.21	16,768.20
	50034000	7,164.08	11,490.99	19,287.60	27,291.89	37,724.10
	50050900	8,949.71	13,610.59	23,381.32	35,115.51	52,730.70
Region 1	50090500	4,731.70	7,149.31	11,690.70	16,612.62	23,372.31
	50092000	8,269.70	12,029.03	18,629.61	25,342.15	34,068.24
	50112500	2,473.27	3,795.96	6,475.48	9,592.38	14,143.57
	50115000	3,877.78	5,997.22	10,272.70	15,226.51	22,435.15
	50124200	6,088.15	9,480.04	15,929.88	23,000.82	32,808.78
	50051310	9,898.84	12,360.84	15,324.64	17,422.10	19,422.51
	50056400	16,658.97	21,930.42	29,367.99	35,507.42	42,181.16
	50061800	9,387.40	12,128.41	15,798.38	18,678.43	21,677.12
Pagion 2	50063800	11,659.00	14,635.37	18,454.11	21,329.58	24,220.14
Region 2	50064200	9,675.50	13,314.99	19,195.03	24,721.04	31,433.22
	50065500	14,365.67	16,745.65	19,234.04	20,767.32	22,068.36
	50067000	5,866.96	7,350.07	9,423.15	11,119.05	12,948.44
	50071000	11,800.88	15,053.87	19,733.88	23,670.17	28,018.06
	50031200	18,577.42	26,634.25	39,726.06	52,097.12	67,194.39
	50035000	37,112.55	54,809.46	85,347.08	115,897.18	155,053.79
	50038100	57,914.03	80,263.77	112,943.97	140,862.71	172,114.20
Decien 2	50039500	12,871.06	18,922.96	29,134.21	39,134.60	51,719.73
Region 5	50043000	25,894.01	39,038.02	62,049.36	85,386.96	115,653.55
	50047850	13,160.68	19,739.36	31,668.21	44,173.83	60,862.63
	50055000	37,105.86	47,226.15	60,405.78	70,474.56	80,721.38
	50057000	33,830.84	46,559.21	64,944.42	80,467.32	97,669.58
	50138000	12,531.39	22,817.50	45,664.25	74,353.94	118,979.26
Region 4	50144000	28,408.28	45,017.60	77,888.30	114,841.33	166,987.58
	50147800	31,864.22	35,690.53	39,184.13	40,997.82	42,312.83

Table 23. At-site quantiles estimated from the GEV distribution

Region	Site	Quantile				
		5	10	25	50	100
		0.8	0.9	0.96	0.98	0.99
Region 1	50028000	1%	31%	67%	86%	95%
	50028400	-11%	-4%	12%	30%	52%
	50034000	-2%	-10%	-16%	-18%	-19%
	50050900	17%	13%	4%	-5%	-13%
	50090500	2%	-1%	-4%	-7%	-10%
	50092000	-2%	-1%	0%	2%	4%
	50112500	12%	7%	-1%	-8%	-15%
	50115000	11%	5%	-4%	-10%	-17%
	50124200	2%	-4%	-10%	-14%	-18%
Region 2	50051310	-4%	-2%	0%	1%	2%
	50056400	1%	-3%	-8%	-13%	-17%
	50061800	-1%	-3%	-5%	-8%	-10%
	50063800	1%	2%	3%	2%	2%
	50064200	7%	-1%	-13%	-23%	-31%
	50065500	2%	12%	23%	31%	39%
	50067000	9%	10%	9%	6%	3%
	50071000	8%	8%	4%	0%	-5%
Region 3	50031200	9%	9%	2%	-5%	-13%
	50035000	13%	9%	-2%	-12%	-22%
	50038100	2%	5%	4%	2%	-2%
	50039500	11%	7%	-2%	-12%	-21%
	50043000	12%	6%	-6%	-17%	-28%
	50047850	18%	13%	-2%	-14%	-27%
	50055000	7%	20%	31%	37%	41%
	50057000	2%	6%	6%	4%	1%
Region 4	50138000	30%	5%	-18%	-32%	-44%
	50144000	12%	4%	-6%	-14%	-22%
	50147800	3%	35%	92%	147%	218%

Table 24. Percent errors between at-site quantile and regional estimated sitequantile