# Sensitivity analysis in the minimization of energy cost by combining the PREPA supply with a photovoltaic system

by

Melitza Colón Vázquez

A thesis submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE
in
INDUSTRIAL ENGINEERING

UNIVERSITY OF PUERTO RICO
MAYAGÜEZ CAMPUS
2011

Approved by:

______________________________ __________________
Mayra Méndez, Ph. D Date
President, Graduate Committee

______________________________ __________________
Pedro Resto, Ph. D Date
Member, Graduate Committee

______________________________ __________________
Lionel Orama, Ph. D Date
Member, Graduate Committee

______________________________ __________________
Viviana Cesaní, Ph. D Date
Department Head

______________________________ __________________
Ricky Valentín, Ph. D Date
Representatives of Graduate Studies

## Abstract

The manufacturing companies, the major consumers of energy, are constantly seeking alternatives to reduce energy consumption in their operations. Among the alternatives available to address this situation are industrial appliances that consume the energy efficiently and the use of renewable energy systems, such as photovoltaic systems. In order to justify the economic feasibility of such alternatives optimization models were developed. The objective of the models was to maximize the economic benefits generated by the alternatives. The results obtained show how the successful use of optimization models identified $78, 483 in economics savings in three years in a particular case. In addition, it was demonstrated that there are important parameters that affect the analysis of the alternatives and should be considered in the models. The most significant parameters are: economic analysis method, implementation cost, and efficiency of the solar panels. The variation of these parameters demonstrated a difference in economic savings of up to $21,333.

**Resumen**

Los principales consumidores de energía, las compañías de manufactura, buscan alternativas para reducir su consumo de energía. Entre las alternativas disponibles para atender esta situación se encuentran equipos industriales que consumen energía eficientemente y sistemas de energía renovable, como lo son los sistemas fotovoltaicos. A través de esta investigación se desarrollaron modelos de optimización para analizar la viabilidad económica de las alternativas antes mencionadas. Los objetivos de estos modelos son maximizar los beneficios económicos generados por estas alternativas. Los resultados obtenidos demostraron el uso exitoso de estos modelos identificando ahorros económicos de $78,483 en tres años para un caso en particular. También, se demostró que existen parámetros que afectan y deben ser considerados en el análisis de las alternativas, destacándose: método financiero, costos de implantación de los sistemas y eficiencia de los paneles solares. La variación de estos parámetros demostró una diferencia en ahorros económicos de hasta $21,333.

*This work is dedicated to my grandfather Alfredo Colón. I would have liked to see your face showing how proud you were. Although, I know that all the time you were with me and that right know you are in heaven celebrating with me this important accomplishment. I miss you!*

## Acknowledgements

This work is my most important accomplishment. It is a representation of my dedication, effort, and desire to be a complete professional. I want to express my gratitude to God, for giving me the talent, the patience and the health to get here. Thanks for all the blessings that you give me in special by putting in my way angels that helped me in this process.

The success of this work is not only mine. It is the success of all the persons that helped me to develop this research and I want to thanks and recognize these persons. I am deeply thankful to my advisor Dr. Mayra Méndez. Thank you for believing in my talent, for supporting me in my idea and helping me to turn it into a successful research. I appreciate the time that you dedicated to listening to me, the knowledge that you shared with me, and the advices that you gave me. I know that this process was a learning process for both of us and I have great memories of them. It is a pleasure and an honor for me to be her first graduate student. In addition, I would like to thank Dr. Pedro Resto and Dr. Lionel Orama for accepting my invitation to be part of my graduate committee and for helping me in the development of this research.

I want to thank my parents, René Colón and M. Haydeé Vázquez, and my sister Belitza Colón for supporting me all the time. I appreciate your understanding and the patience that you have had waiting for me all this time. I know that I sacrificed a lot of time that I should have spent with you, but this is my reward for you. Thanks for being my motivation. I express my gratitude to Cristina Alicea and Milessa Ortiz for being my unconditional friends, for helping me with their positive vibrates and being part of the best weekends of my life to release the stress.

I sincerely appreciate the support of my ININ partners. Oscar Herrera, thanks for being my first graduate partner and for all the knowledge that you humbly shared with me. Alma Sagredo and Berenice Rodríguez thanks for teaching me new and international words. Magaly González thanks for being my academic and travel partner. Karla M. Iglesias thanks for being my friend in the bachelor and master degrees and I am sure that you will be my friend for the rest of my life. Thanks for all the great moments, for accompanying me in the long nights and for making the best oatmeal that I have eaten. I will never forget these times.

Last but no less important, I want to thank my best friends, Celimar Reinés and Yamil Santiago. I do not have word to express my gratitude for you. You began with me this process, from filling out the application up to here. I appreciate your attention for my work. Thanks for staying here all the time, for having the best words in the right moment, for showing me that best friends do exist. You know that I want you to be here and I know that you want to be here too, but you are always with me no matter the distance. Our connection goes beyond the physical presence; you will always be in my heart.

# Table of Contents

## List of Tables



## List of Figures

## 1. Introduction

Currently many government and manufacturing companies are trying to address the main problems related to energy consumption: the increment in the emission of carbon dioxide ($CO_2$) and the instability in the petroleum oil market. Studies about the emission of $CO_2$ reveal that since the Industrial Revolution in the 1700s the concentration of $CO_2$ in the atmosphere has increased by 35% (Climate change - Greenhouse gas emissions, 2010). Another factor that has an impact in the increment of the emission of $CO_2$ and in the instability of the petroleum oil market is the increment in the world population (La inminente escasez de petróleo mundial, 2009). As mentioned above one common factor between the emission of $CO_2$ and the instability in the petroleum market is the energy consumption. In Puerto Rico, for 2009 it was consumed 1.07 thousand barrels of oil per day and generated 20.92 billion kilowatts hours (Statistics on Puerto Rico , 2010 ). In addition, for 2007, the Puerto Rico Electric Power Authority (PREPA) has 1,315,345 residential customers, 130,082 commercial customers, and 1,618 industrial customers. However the industrial sector consumed 21% of the energy generated in Puerto Rico (Flores, 2007). This has forced manufacturing companies to explore alternatives to reduce energy consumption.

To seek alternatives to reduce the energy consumption it is common that manufacturing companies hire a consulting agency to evaluate alternate energy sources and projects that may allow companies to save on energy consumption. This process represents an overall investment for companies and the benefits they can generate are uncertain. The analyses of the alternatives presented by the consultants are based on the investment cost, economic benefits, and the period to recover the investment cost. These variables are related to energy costs, the equipment necessary for the alternative being evaluated, and the cost of the equipment. Considering the factors previously mentioned, it is possible that alternatives that are not feasible at the moment could be feasible in the future. To work with this situation, manufacturing companies have the option of hiring the services of consulting agencies periodically, incurring in a considerable investment. On the other hand, companies also have the option to internally identify energy-saving alternatives by using systems and tools that monitor the energy performance of their equipment and allow them to evaluate viable alternatives for energy saving.

The objective of this study is to analyze the use of optimization models in the process of selecting feasible economics alternatives to reduce energy consumption. For this purpose, an optimization model was developed to identify the equipment that could be replaced for equipment that consumes less energy. The feasibility of the alternatives was evaluated considering the payback method and the internal rate of return method (IRR). The payback analysis is used to calculate how long it takes to recover the investment while the internal rate of return analysis is used to calculate the break even interest rate that equates the present worth. The significant difference between these methods is that the internal rate of return method considers the cash flow during the lifetime of the project and the payback method ignores that aspect. Both methods were considered and compared in this research. In addition, throughout this study five systems were identified as the major consumers of energy in manufacturing companies (air conditioner, compressed air, illumination, exhaust, and machines). However, the optimization model focuses on evaluating alternatives for the first three of the five systems: air conditioner system, compressed air system, and illumination because these systems represent the major part of the energy consumed by the companies. The objective of this optimization model is to maximize the net economic benefits by saving on energy consumption after implementing the evaluated alternative. Constraints associated with the systems in the analyzed areas (i.e. offices, production, warehouse, and exterior) and established limits of energy demands are considered in the model.

Although there exist, alternatives to reduce the energy consumption using energy efficient equipment, it is necessary to explore alternatives that may represent independence from the energy generated by fuel oil. The ideal scenario is when companies can optimize the energy they consume. If the companies achieve this goal, finding alternatives to continue reducing the energy consumption is very challenging. In this case, the efforts to reduce the energy consumption are limited by the advantages of the products available in the market and made for this purpose. Considering the factors mentioned above in this study an optimization model was developed to identify the size of the photovoltaic system that can be funded with the economic savings generated by the equipment that consumes energy efficiently. The optimization model has the objective of maximizing the kilowatts generated by the photovoltaic system. In the model, it was considered the space available to place the photovoltaic system, the benefits that could be received for the implementation of the system, and the cost of the investment. These optimization models represent an effective option for many companies to analyze and study alternatives to reduce the energy consumption without investing in consulting services. In the next sections of this document, the application of the optimization algorithms into different cases is discussed and the results analyzed.

The organization of this document is as follow: in Section Two a review of relevant literature is presented, in Section Three the methodology used including the optimization models is discussed. Section Four presents the application of the algorithms and the analysis of the results; Section Five includes the adjusted models; Section Six presents the sensitivity analysis; Section Seven discusses a comparison between payback and internal rate of return methods. The summary of the results for the first part of this thesis is presented in Section Eight. The second part of this thesis starts with section nine where the consideration of a photovoltaic system is discussed. Section ten presents the summary of the results for the second part of this thesis and section eleven presents the conclusions and future works.

# 2. Literature Review

## 2.1 Optimization model to reduce the energy consumption

The optimization models seeks to find the values of the decisional variables that maximize or minimize an objective function among the set of all possible values for the decision variables that satisfy the given constraints (Winston, 2004). The components of the optimization models are: objective function, decisional variables, and constraints. The restriction on the values of decision variables are known as constraint. While the variables, whose values are under control and influence the performance of the systems, are the decisional variable, and the function that is to be maximized or minimized is the objective function (Winston, 2004).

Studies related to energy consumption show that the results of implementing energy optimization models are an effective tool to reduce the emission of $CO_2$ and to reduce energy consumption in some industries. For example, G. Ordorica-García (2008), in collaboration with other colleagues developed an energy optimization model with $CO_2$ emission constraints for the Canadian oil sands industries. In this study, they developed an optimization model that determines optimal combinations of power and hydrogen plants to satisfy energy demand of oil sands operations at minimal cost while reducing $CO_2$ emissions. This study demonstrated that this model has the potential to reduce the energy consumption cost by 2-7% and to reduce the $CO_2$ emissions by up to 30%, (Ordorica-García, 2008). Moreover, a non-linear programming model was developed to identify the load management strategy that minimizes the total operating cost of a typical factory. The results obtained from the model show the optimal values and times for the electrical energy consumption based on doing activities and operations in a factory (Ortadis, 2007). The model considered the energy consumed by the factory per period, the maximum value of energy consumption determined through energy suppliers, the minimum needed value of energy consumption, the value of energy consumption in excess, and the penalty associated with deviations of in excess. But the optimization models are not only applied to analysis of the operations that consume energy in different industries to reduce the energy consumption. R. Baños (2011) presented in his work *Optimization methods applied to renewable and sustainable energy: A review* that the number of research papers that use optimization methods to solve renewable energy problems has increased dramatically in recent years, especially for wind and solar energy systems. His research presents the review of more than two hundred papers from major referenced journals in the fields of renewable energy and computational optimization.

### 2.2 Payback method and internal rate of return method

The decisions that a firm make about their capital budgeting have an impact in the success of the firms for several reasons (Cooper, Morgan, Redman, & Smith, 2001). According to William D. Cooper (2001) some of the reasons are: the capital expenditures typically require large outlays of funds, firms must ascertain the best way to raise and repay these funds, and most capital budgeting decisions require a long-term commitment. The analysis presented in the article *Capital budgeting models: theory vs. practice* (2001) showed that the 57% of the companies in the study used internal rate of return as the primary method to analysis their project and 20% of the companies used payback method (Cooper, Morgan, Redman, & Smith, 2001).

Internal rate of return is a method used to find the break-even interest rate that equates the present worth of a project's cash flows to the present worth of its cash inflows (Park, 2007). This rate is compared with the capital cost to decide if the project is feasible (Fares 2008). However, the payback is defined as the period it takes the cash inflows from an investment project to equal the cash outflows. Usually this period is expressed in years (Aidan Berry, 2006). The main limitation of the payback method is that this method does not consider the cash flows throughout the lifespan of the project (Fares 2008).

Studies that analyzed the economic feasibility of the photovoltaic systems used either method in their analysis. For example, Hillmon P. Ladner-Garcia (2009) used the payback method to analyze the economic feasibility of a grid-tied photovoltaic system in Puerto Rico. However, Eyad Ali Fares (2008) used the internal rate of return method to analyze the economic feasibility of installing a photovoltaic system in a residence in Puerto Rico.

### 2.3 Application of the photovoltaic systems

Renewable energy or clean energy is the energy that is generated by a natural method, using an inexhaustible resource. Some of these methods are known as: solar, wind, hydraulic, and geothermic energy (Kutz, 2007). The solar system transforms the radiant energy transmitted by the sun into electric power. There is more than one technique for converting solar flux into energy (Kutz, 2007). Their applications depend on and vary with season, location, time of day, and surface orientation. The main technology used to convert the solar flux in electric power is the photovoltaic solar energy systems. The principal components in the system are the photovoltaic cells. These cells are composed of thin layers of semiconductor material that are exposed to the solar flux. In addition to cells, the photovoltaic systems contain an electrical storage and a control system (Kutz, 2007).

There exist different types of photovoltaic systems. Some of these are: grid-tied and off-grid (stand-alone) system. The grid-tied photovoltaic system is directly connected to the electric distribution network and do not require battery storage (Ladner 2009). However, the stand-alone photovoltaic system requires batteries. In this system, the power generated by the solar modules is either consumed immediately or directed to batteries for storage and later use (Hren, 2010). Stephen Hren (2010) showed in his book *A solar buyer's guide for the home and office* a comparison between the grid-tied system and stand-alone system. The analysis of the comparison demonstrated that the grid-tied system is more cost effective versus the stand alone system. Moreover, in the research *Photovoltaic based distributed generation as a demand response strategy in Puerto Rico*, Hillmon P. Ladner-Garcia (2009) mentioned the following benefits of the grid-tied photovoltaic system:

- Smaller photovoltaic system arrays can supply the same load reliably.
- Less balance of system components are needed.
- Comparable emission reduction potential taking advantage of existing infrastructure.
- Eliminates the need for energy storage and the costs associated to substituting and recycling batteries. Storage can be included if desired to enhance reliability for the client.
- Takes advantage of the existing infrastructure.

Puerto Rico is located in the Caribbean region and it has the following characteristics: a hot and humid climate with little variation, its day length has little variation, and it has abundant solar resource. Because of these characteristics the photovoltaic systems are recommended in Puerto Rico. Many research efforts have been devoted to demonstrate that this system is feasible for the generation of energy for residential and commercial buildings. For example, the Hillmon P. Ladner-Garcia thesis (2009) demonstrated that the use of grid-tied photovoltaic systems in Puerto Rico is feasible. This result is supported by the geographic localization and the little variation in the weather throughout the year that Puerto Rico has. However, Ladner also demonstrated that the photovoltaic systems have a high investment cost, which does not make it proportional to the cost of saving energy in the first year (Ladner 2009). The results show that the system begins to generate profit after half of the projects life. A conclusion similar to this can be seen in the work of Eyad Ali Fares. He demonstrated through his thesis that by May 2008 it was not yet economically feasible to use photovoltaic systems in residences (Fares 2008). The factors that support his conclusion were the high investment cost of the system and the small incentives from the government. As it can be seen, the photovoltaic system is a good method to generate energy in Puerto Rico but the investment cost of the system limits its use.

# 3. Methodology

## 3.1 Optimization algorithms considering the payback and the internal rate of return methods

This study presents algorithms that considered the payback period and the internal rate of return methods to analyze the feasibility of the alternatives to reduce the energy consumed by a manufacturing company. The objective of the algorithms is to select a cost-effective alternative to replace the current equipment by an alternative that reduces the energy consumed by a manufacturing company, maximizing the net economic benefits. The model evaluates the alternatives in the different areas and maximizes the benefits while complying with the constraints. To simplify the process of interpreting the results, the areas have been divided into offices, production areas, warehouse, and exterior. However, it is necessary to consider that some of these areas can be found in more than one building. In this model, this situation is considered and the areas are subdivided. The following general notation is used throughout the models: $i$ is used as an index to identify the facilities areas and $a$ is used as an index for the subdivisions per area.

Each area has various alternatives to reduce energy consumption and particular constraints that must be considered. In general, the total energy savings per day are limited due to the minimum consumption required by Puerto Rico Electric Power Authority (PREPA). The minimum energy that the manufacturing companies should be consuming is established in the contract that the company have with the PREPA agency. Also, there are costs associated with exceeding the maximum limits of energy consumption or not consuming the minimum energy consumption limits. In addition, each algorithm has a particular constraint associated with the method considered. The assumptions for the models are the following:

**<u>General assumptions</u>**

- ✓ The kilowatts per hour (kwh) that are consumed by all analyzed equipment are known.
- ✓ The hours that the equipments operate per day are given.
- ✓ The cost of kilowatts per hour of PREPA is known.
- ✓ The cost of purchasing and installing systems for energy reduction are given.
- ✓ All equipments analyzed were used every day of work.

**<u>Particular assumption in the payback method model</u>**

- ✓ The time the company wants to recover the investment is known.

**<u>Particular assumptions in the internal rate of return method model</u>**

- ✓ The minimum acceptable rate of return is known.
- ✓ The interest generated by the alternatives fluctuates between -100% and 100%
- ✓ Regular and irregular weeks were considered depending on work days and holidays.

## 3.2 Systems considered in the algorithms to reduce the energy consumed

### 3.2.1 Air conditioner system

In order to reduce the energy consumed by the air conditioner systems, it should be analyzed replacing them. It is possible that same areas have more than one air conditioner system with different operating characteristics. Therefore, this model considers evaluating each air conditioning system separately. To identify the air conditioner system per area $i$ and subdivision $a$ we include an additional notation $k$ which represents a number that identifies the air conditioner system. The cost of replacing the air conditioner system must be offset within the desired recovery time period.

### 3.2.2 Compressed air system

In order to reduce the energy that is consumed by the compressed air system, an efficient system should be analyzed for replacement. The amount of compressed air systems in companies depend on the capacity of the compressed air units and the compressed air required. However, companies usually divide these requirements in two or more systems to prevent companies from running out of compressed air if one system fails. These systems can have different operating characteristics. Therefore, this model considers evaluating each compressed air system separately. To identify the compressed air system per area $i$, an additional notation $n$ is included. It is assumed by the analyst that the equipment to replace the existent compressed air system has the capacity to satisfy the compressed air requirements. The cost of replacing the compressed air system must be offset within the desired time period.

### 3.2.3 Illumination system

In order to reduce the energy consumption of the illumination systems, changing current bulbs with energy efficient bulbs is considered. All bulbs per area should be replaced with others that consume less kilowatts-hour (kwh) than the existing ones in the different areas. The costs associated with replacing the bulbs must be recovered within the desired time period. It is assumed by the analyst that the energy efficient bulbs will be capable of maintaining the amount of required lumens in the areas. Also, it is assumed that the number of bulbs per area is known.

## 3.3 Algorithms considering the payback method

The main constraint of the algorithms considering the payback method is that the investment should be recovered in the pre-established time period. This time period is also known as the payback period. It is established by the manufacturing company, but commonly, two years is the desired time period to recover the investment cost. The following equations represent the algorithm to evaluate the feasibility of the alternatives by systems considering the payback method:

### 3.3.1 Air conditioner system

The areas where it is feasible to replace the air conditioner system are given by:

$$AC_{iak} * \left((KACA_{iak} - KACR_{iak}) * HR_1 * Kc\right) = SAC_{iak}, \ \forall i \ \forall a \ \forall k \quad (1)$$

$$SAC_{iak} * Dys \geq AC_{iak} * C_{1iak}, \ \forall i \ \forall a \ \forall k \quad (2)$$

$$@bin\ (AC_{iak}), \ \forall i \ \forall a \ \forall k \quad (3)$$

where $AC_{iak}$ represents the decision variable that takes value of one when an air conditioner system $k$ in area $i$ subdivision $a$ should be replaced for a more efficient air conditioner system, and take value of zero when it is not feasible to replace the equipment. $KACA_{iak}$ is the kilowatts-hour consumed by the air conditioner system $k$ in area $i$ subdivision $a$, $KACR_{iak}$ is the kilowatts-hour consumed by the recommended air conditioner system $k$ in area $i$ subdivision $a$, $HR_1$ are the hours that the air conditioner system operates per day, $Kc$ is the cost of energy (cost per kilowatts-hour) by PREPA, $SAC_{iak}$ is the savings per day ($) of the air conditioner system $k$ by replacing them in area $i$ subdivision $a$, $Dys$ are the labor days to recover the investment, and $C_{1iak}$ is the cost ($) of purchasing and installing the new air conditioner system to replace the air conditioner system $k$ in area $i$ subdivision $a$.

Equation (1) is used to calculate the savings ($) of each air conditioner system per area $i$ and subdivision $a$. Equation (2) represents the payback investment constraints. Equation (3) defines the decision variable $AC_{iak}$ as a binary variable. Through these equations the model evaluates the areas where the energy saving of the air conditioner system could be implemented if the investment can be recovered in the desired time. All constraints and equations are subject to the decision variable $AC_{iak}$.

### 3.3.2 Compressed air system

The areas where it is feasible to replace the compressed air system are given by:

$$CA_{in} * \big((KCAA_{in} - KCAR_{in}) * HR_2 * Kc\big) = SCA_{in},\ \forall i\ \forall n \tag{4}$$

$$SCA_{in} * Dys >= CA_{in} * C_{2in},\ \forall i\ \forall n \tag{5}$$

$$@bin\ (CA_{in})\ ,\forall i\ \forall n \tag{6}$$

where $CA_{in}$ represents the decision variable that takes value of one when a compressed air system $n$ in area $i$ should be replaced with a more efficient compressed air system and takes value of zero when it is not feasible to replace the equipment. $KCAA_{in}$ is the kilowatts hours consumed by compressed air system $n$ in area $i$, $KCAR_{in}$ is the kilowatts-hour consumed by the recommended compressed air system $n$ in area $i$, $HR_2$ are the hours that the compressed air system operates per day, $Kc$ is the cost of energy (cost per kilowatts-hour) by PREPA, $SCA_{in}$ is the savings per day ($) of the compressed air system $n$ by replacing it in area $i$, $Dys$ are the labor days in which the investment should be recovered, and $C_{2in}$ is the cost ($) of purchasing and installing the recommended compressed air system to replace the compressed air system $n$ by replacing it in area $i$.

Equation (4) allows the calculation of the savings ($) of the compressed air system per area $i$. Equation (5) represents the payback investment constraints. Equation (6) defines the decision variable $CA_{in}$ as a binary variable. Through these equations the model evaluates the areas where the energy saving of the compressed air system could be implemented if the investment can be recovered in the desired time. All constraints and equations are subject to the decision variable $CA_{in}$.

### 3.3.3 Illumination system

The areas where it is feasible to replace the existent bulbs for others that consume less energy are given by:

$$I_{ia} * ((BA_{ia} * (KBA_{ia} - KBR_{ia}) * HR_3 * Kc) = SI_{ia}, \ \forall i \ \forall a \tag{7}$$

$$SI_{ia} * Dys \geq I_{ia} * (BA_{ia} * C_{3ia}), \ \forall i \ \forall a \tag{8}$$

$$@bin\ (I_{ia}), \ \forall a \ \forall i \tag{9}$$

where $I_{ia}$ represents the decision variable that takes value of one when the bulb of area $i$ subdivision $a$ should be replaced and takes value of zero when it is not feasible to replace the bulbs. $BA_{ia}$ is the current number of bulbs in area $i$ subdivision $a$, $KBA_{ia}$ is the kilowatts-hour consumed by each bulb in area $i$ and subdivision $a$, $KBR_{ia}$ is the kilowatts-hour consumed by each bulb that is replaced in area $i$ subdivision $a$, $HR_3$ are the bulbs operating hours per day for each bulb, $Kc$ is the cost of energy (cost per kilowatts-hour) by PREPA, $SI_{ia}$ is the savings per day ($) that would be achieved in the illumination system if light bulbs are replaced in area $i$ subdivision $a$, $Dys$ are the labor days to recover the investment, and $C_{3ia}$ is the cost ($) of purchasing and installing the new bulbs to replace the bulbs in area $i$ subdivision $a$ .

Equation (7) calculates the savings ($) in the illumination system per area $i$. Equation (8) represents the payback investment constraints. Equation (9) defines the decision variable $I_{ia}$ as a binary variable. Through these equations the model evaluates the areas where the energy saving of the illumination system could be implemented if the investment can be recovered in the desired time. All constraints and equations are subject to the decision variable $I_{ia}$.

### 3.3.4 Objective function

The objective function of this model is to maximize the economic benefits per year through the implementation of viable alternatives that contribute to reduce energy consumption by replacing existing equipment with more efficient equipment in the areas identified. This equation shows the overall sum of the daily savings per area (*ST)* and multiplies it by the working days per year (*Dys*) to obtain net benefits. The investments of the recommended alternatives are subtracted from the resulting net benefits to achieve the net savings. The maximum net savings per year, if the recommended alternatives are implemented, is given by:

$$Max\ \ Sav = (\,ST) * Dys - \left( \sum_{i=1}^{z} \sum_{a=1}^{m} \sum_{k=1}^{x} AC_{iak} * C_{1iak} \ + \sum_{i=1}^{z} \sum_{n=1}^{y} CA_{\ in} * C_{2in} + \sum_{i=1}^{z} \sum_{a=1}^{m} I_{ia} * \ C_{3ia} \right) \tag{10}$$

where

$$ST = \left( \sum_{i=1}^{z} \sum_{a=1}^{m} \sum_{k=1}^{x} SAC_{iak} + \sum_{i=1}^{z} \sum_{n=1}^{y} SCA_{\ in} + \sum_{i=1}^{z} \sum_{a=1}^{m} SI_{ia} \right) \tag{11}$$

### 3.4 Algorithms considering the internal rate or return method

The main constraint of these algorithms is that the interest generated by the benefits of the alternatives during the lifetime of the equipments should be greater than or equal to the minimum acceptable rate of return (MARR). This means that the benefits will generate the same or more economic savings compared to the benefits of depositing the investment cost of the alternative in a bank account with the same interest rate. In January 2001 the *Business Forum* published a study by Morgan and other colleagues about the capital budgeting decisions. This study demonstrated that for 1990, 57% of the companies in the study used internal rate of return as the first method to evaluate their projects. In addition, most companies in the study, 53%, used between 10% and 15 % as their MARR (Cooper, Morgan, Redman, & Smith, 2001).

For this reason, during this research, 15% was used as the MARR decision criteria. Since the MARR is an annual measurement, it is converted to an effective weekly rate to be used on the algorithms. The following expression is the effective interest equation used in the algorithm to convert the MARR from annually to weekly.

$$(1 + IMARR)^{W} - 1 - MARR = 0 \tag{12}$$

where *IMARR* represents the weekly minimum acceptable rate of return, *W* represents the weeks of work during the pre-established period of time and *MARR* represents the minimum acceptable rate of return select for the analysis.

The following equations represent the algorithm to evaluate the feasibility of the alternatives by system considering the internal rate of return method:

### 3.4.1 Air conditioner system

The areas where it is feasible to replace the air conditioner system are given by:

$$((KACA_{iak} - KACR_{iak}) * HR_1 * Kc) = SAC_{iak}, \ \forall i \ \forall a \ \forall k \tag{13}$$

$$\frac{(RW * DRW * SAC_{iak}) + (IW * DIW * SAC_{iak})}{WY} = SWAC_{iak}, \ \forall i \ \forall a \ \forall k \tag{14}$$

$$(@fpa(IAC_{iak}, LE_{iak}) * SWAC_{iak}) - C_{1iak} = 0, \ \forall i \ \forall a \ \forall k \tag{15}$$

$$AC_{iak} = @if(\ IAC_{iak} \ \#GE\# \ IMARR, 1,0\ ), \ \forall i \ \forall a \ \forall k \tag{16}$$

$$\left(SWAC_{iak} * \left(\frac{(1+IMARR)^{W} - 1}{IMARR}\right)\right) = STAC_{iak}, \ \forall i \ \forall a \ \forall k \tag{17}$$

$$@bnd\ (-1, IAC_{iak}, 1), \ \forall i \ \forall a \ \forall k \tag{18}$$

where $KACA_{iak}$ is the kilowatts-hour (kwh) consumed by the current air conditioner system *k* in area *i* subdivision *a;* $KACR_{iak}$ is the kilowatts-hour (kwh) consumed by the recommended air conditioner system *k* in area *i* subdivision; $HR_1$ are the hours that the air conditioner system operates per day; *Kc* is the cost of energy (cost per kilowatts-hour) by PREPA; $SAC_{iak}$ is the savings per day ($) of the air conditioner system *k* by replacing them in area *i* subdivision *a*. In addition, *RW* represents the amount of regular weeks; *DRW* represents the amount of days in regular weeks; *IW* represents the amount of irregular weeks; *DIW* represents the amount of days in irregular weeks; *WY* represents the amount of working weeks in a year. $SWAC_{iak}$ represents the weighted average of the weekly economic savings generated by the air conditioner system *k* in area *i* subdivision *a*. $IAC_{iak}$ represents the weakly interest generated by the economic savings if the air conditioner system *k* in area *i* subdivision *a* is replaced during the lifetime of the equipment. The lifetime of the air conditioner *k* in area *i* subdivision *a* is represented by $LE_{iak}$ (in weeks) and $C_{1iak}$ is the cost ($) of purchasing and installing the new air conditioner system in area *i* subdivision *a*. $AC_{iak}$ is the decision variable that takes value of one if the interest generated by the economic savings is greater than or equal to the *IMARR* and takes value of zero otherwise. $STAC_{iak}$ represents the benefits generated by the air conditioner *k* in area *i* subdivision *a* after *W* weeks of work.

Equation (13) is used to calculate the savings ($) per day of the air conditioner system per area and subdivision. Equation (14) allows calculating the weighted average of the economic weekly savings. The interest generated by the air conditioner alternative is calculated by equation (15). Equation (16) defines the decision variables of the air conditioner alternatives as a binary variables and equation (17) is used to calculate the economic saving of the air conditioner system alternative after *W* weeks of work. Equation (18) establishes the range of the interest generated by the air conditioner alternatives. This range allows the negative interests to be identified.

### 3.4.2 Compressed air system

The areas where it is feasible to replace the compressed air system are given by:

$$(KCAA_{in} - KCAR_{in}) * HR_2 * Kc) = SCA_{in}, \ \forall i \ \forall n \tag{19}$$

$$\frac{(RW*DRW*SCA_{in}) + (IW*DIW*SCA_{in})}{WY} = SWCA_{in}, \ \forall i \ \forall n \tag{20}$$

$$(@fpa(ICA_{in}, LE_{in}) * SWCA_{in}) - C_{2in} = 0, \ \forall i \ \forall n \tag{21}$$

$$CA_{in} = @if(\ ICA_{in}\ \#GE\#\ IMARR, 1,0\ ), \ \forall i \ \forall n \tag{22}$$

$$\left(SWCA_{in} * \left(\frac{(1+IMARR)^{W} - 1}{IMARR}\right)\right) = STCA_{in}, \ \forall i \ \forall n \tag{23}$$

$$@bnd\ (-1, ICA_{in}, 1), \ \forall i \ \forall n \tag{24}$$

where $KCAA_{in}$ is the kilowatt-hours (kwh) consumed by the current compressed air system *n* in area *i*, $KCAR_{in}$ is the kilowatts-hour (kwh) consumed by the suggested compressed air system *n* in area *i*, $HR_2$ are the hours that the compressed air system operates per day; *Kc* is the cost of energy (cost per kilowatts-hour) by PREPA; $SCA_{in}$ is the savings per day ($) in the compressed air system *n* by replacing them in area *i*. In addition, *RW* represents the amount of regular weeks; *DRW* represents the amount of days in regular weeks; *IW* represents the amount of irregular weeks; *DIW* represents the amount of days in irregular weeks; *WY* represents the amount of working weeks in a year. $SWCA_{in}$ represents the weighted average of the weekly economic savings generated by the compressed air system *n* in area *i*. $ICA_{in}$ represent the weekly interest generated by the economic savings if the compressed air system *n* in area *i* is replaced during the lifetime of the equipment. The lifetime of the compressed air *n* in area *i* is represented by $LE_{in}$ (in weeks) and $C_{2in}$ is the cost ($) of purchasing and installing the new compressed air system in area *i*. $CA_{in}$ is the decision variable that takes value of one if the interest generated by the economic savings is greater than or equal to the *IMARR* and takes value of zero otherwise. $STCA_{in}$ represents the benefits generated by the compressed air *n* in area *i* after *W* weeks of work.

Equation (19) is used to calculate the savings ($) per day of the compressed air system per area. Equation (20) allows calculating the weighted average of the weekly economic savings. The interest generated by the compressed air alternative is calculated by equation (21). Equation (22) defines the decision variables of the compressed air alternatives as binary variables and equation (23) is used to calculate the economic saving of the compressed air system alternative after *W* weeks of work. Equation (24) establishes the range of the interest generated by the compressed air alternatives. This range allows the negative interests to be identified.

### 3.4.3 Illumination system

The areas where it is feasible to replace the existent bulbs for others that consume less energy are given by:

$$(KIA_{ia} - KIR_{ia}) * HR_3 * Kc) = SI_{ia}, \ \forall i \ \forall a \tag{25}$$

$$\frac{(RW*DRW*SI_{ia})+(IW*DIW*SI_{ia})}{WY} = SWI_{ia}, \forall i \ \forall a \tag{26}$$

$$(@fpa(II_{ia}, LE_{ia}) * SWI_{ia}) - C_{3ia} = 0, \ \forall i \ \forall a \tag{27}$$

$$I_{ia} = @if(\, II_{ia} \, \#GE\# \, IMARR, 1,0\,), \ \forall i \, \forall a \tag{28}$$

$$\left(SWI_{ia} * \left(\frac{(1+IMARR)^{W}-1}{IMARR}\right)\right) = STI_{ia}, \ \forall i \ \forall a \tag{29}$$

$$@bnd\,(-1, II_{ia}, 1), \ \forall i \, \forall a \tag{30}$$

where $KIA_{ia}$ is the kilowatt-hours (kwh) consumed by the current illumination system in area *i* subdivision *a*, $KIR_{ia}$ is the kilowatts-hour (kwh) consumed by the suggested illumination system in area *i* subdivision *a*, $HR_3$ are the hours that the illumination system operates per day, *Kc* is the cost of energy (cost per kilowatts-hour) by PREPA, $SI_{ia}$ is the savings per day ($) in the illumination system by replacing the current bulbs in area *i* subdivision *a*. In addition, *RW* represents the amount of regular weeks; *DRW* represents the amount of days in regular weeks;

*IW* represents the amount of irregular weeks; *DIW* represents the amount of days in irregular weeks; *WY* represents the amount of working weeks in a year. $SWI_{ia}$ represents the weighted average of the weekly economic savings generated by the illumination system in area *i* subdivision $a$. $II_{ia}$ represents the weekly interest generated by the economic savings if the current bulbs in area *i* subdivision *a* are replaced during the lifetime of the equipment. The lifetime of the bulbs in area *i* subdivision *a* is represent by $LE_{ia}$ (in weeks) and $C_{3ia}$ is the cost ($) of purchasing and installing the new illumination system in area *i* subdivision $a$. $I_{ia}$ is the decision variable that takes value of one if the interest generated by the economic savings is greater than or equal to the *IMARR* and takes value of zero otherwise. $STI_{ia}$ represents the benefits generated by the illumination system in area *i* subdivision *a* after *W* weeks of work.

Equation (25) is used to calculate the savings ($) per day of the illumination system per area. Equation (26) allows calculating the weighted average of the weekly economic savings. The interest generated by the illumination system alternative is calculated by equation (27). Equation (28) defines the decision variable of the illumination system alternatives as binary variables and equation (29) is used to calculate the economic saving by the illumination system alternative after *W* weeks of work. Equation (30) establishes the range of the interest generated by the illumination system alternatives. This range allows that the negative interests to be identified.

### 3.4.4 Objective function

The objective function of this model is to maximize the economic benefits per year through the implementation of viable alternatives that contribute to reduce energy consumption by replacing existing equipment with more efficient equipment in the areas identified. This equation shows the overall sum of future savings per area multiplied by the decision variable of the alternative. In this case the investment cost of the alternatives is not subtracted from the benefits. If the alternative was identified as feasible, it means that the benefits generated by the alternative during the lifetime of the equipment guarantee the recovery of the initial inversion. Most of this kind of equipment has between five and ten years of lifetime. Considering this, it is recommended to the analyst to use between one and two years as the time to get the benefits. The maximum net savings per year, if the recommended alternatives are implemented, is given by:

$$Max\ Sav = \left( \sum_{i=1}^{z} \sum_{a=1}^{m} \sum_{k=1}^{x} AC_{iak} * STAC_{iak} + \sum_{i=1}^{z} \sum_{n=1}^{y} CA_{in} * STCA_{in} + \sum_{i=1}^{z} \sum_{a=1}^{m} I_{ia} * STI_{ia} \right) \tag{31}$$

### 3.5 Adjusted models were developed to consider the load contracted by the manufacturing companies

In addition to the constraints associated with the methods considered in the algorithms presented in section 3.3 and 3.4, the adjusted models have a general constraint that considers the parameters related with the energy load contracted from Puerto Rico Power Authority (PREPA). The purpose of this constraint is to prevent a penalty cost in the energy bill, by consuming less energy than the one established in the contract with PREPA. Since the objective function of the models is to maximize the net savings, adding this constraint will ensure that the recommended alternatives are those that generate the highest savings per year.

The energy savings in the model are calculated in watts and the load contracted in PREPA is invoiced in voltage amperes (KVA). A conversion is made by adding a variable to represent the power factor of the system (pf) in the constraints. To simplify the analysis, the total kilowatts energy savings ($KWS_j$) per system $j$ are calculated in equations (32), (33), and (34). In addition, the energy savings were multiplied by the number of working days per month (*MDys*) to convert the daily savings in kilowatts per hour to monthly savings in kilowatts per hour (35) because the contract with PREPA establishes monthly consumption. To evaluate the feasibility of the alternatives versus the load contracted to PREPA, considering the payback method, it is necessary to add to the algorithm presented in section 3.3 the following equations:

$$KWS_1 = \left( \sum_{i=1}^{z} \sum_{a=1}^{m} \sum_{k=1}^{x} AC_{iak} * MDys\,(KACA_{iak} - KACR_{iak}) \right) \tag{32}$$

$$KWS_2 = \left( \sum_{i=1}^{z} \sum_{n=1}^{y} CA_{in} * MDys\,(KCAA_{in} - KCAR_{in}) \right) \tag{33}$$

$$KWS_3 = \left( \left( \sum_{i=1}^{z} \sum_{a=1}^{m} I_{ia} * MDys\,(BA_{ia} * (KBA_{ia} - KBR_{ia})) \right) \right) \tag{34}$$

$$LC - (LC * LB) \leq \frac{KW - (KWS_1 + KWS_2 + KWS_3)}{pf} \tag{35}$$

where *LC* represents the load contracted to PREPA, *LB* is the constant that establishes the lower bound of the contracted load, and *KW* is the kilowatts hour per month currently consumed by the company. $AC_{iak}$, $KACA_{iak}$, $KACR_{iak}$, $CA_{in}$, $KCAA_{in}$, $KCAR_{in}$, $I_{ia}$, $BA_{ia}$, $KBA_{ia}$, and $KBR_{ia}$ are the same variables that were mentioned and explained in sections 3.3.1,3.3.2, and 3.3.3.

In the algorithm where the IRR method is considered to evaluate the feasibility of the alternatives versus the load contracted to PREPA, it is necessary to add a binary decision variable by alternative and to modify the decision variables defined in equations (16), (22), and

(28), and the load contracted constraints presented in equations (32), (33), and (34). The following equations represent the modifications necessary in the algorithm considering the internal rate of return method to consider the load contracted constraints:

$$AC_{iak} = \left(@if(\ IAC_{iak}\ \#GE\#\ IMARR, 1{,}0\ )\right) * DAC_{iak},\ \ \forall i\ \forall a\ \forall k \tag{16.a}$$

$$CA_{in} = \left(@if(\ ICA_{in}\ \#GE\#\ IMARR, 1{,}0\ )\right) * DCA_{in},\ \ \forall i\ \forall n \tag{22.a}$$

$$I_{ia} = \left(@if(\ II_{ia}\ \#GE\#\ IMARR, 1{,}0\ )\right) * DI_{ia},\ \ \forall i\ \forall a \tag{28.a}$$

$$@bin\ (DAC_{iak}),\ \ \forall i\ \forall a\ \forall k \tag{36}$$

$$@bin\ (DCA_{in}),\ \ \forall i\ \forall n \tag{37}$$

$$@bin\ (DI_{ia}),\ \ \forall i\ \forall a \tag{38}$$

$$KWS_1 = \left(\sum_{i=1}^{z}\sum_{a=1}^{m}\sum_{k=1}^{x}\ (AC_{iak}\ * MDys\ (KACA_{iak} - \ KACR_{iak})) * DAC_{iak}\right), \forall i\ \forall a\ \forall k \tag{32.a}$$

$$KWS_2 = \left(\sum_{i=1}^{z}\sum_{n=1}^{y}\ (CA_{in}\ * MDys\ (KCAA_{in} - \ KCAR_{in})) * DCA_{in}\right), \forall i\ \forall a\ \forall k \tag{33.a}$$

$$KWS_3 = \left(\sum_{i=1}^{z}\sum_{a=1}^{m}(I_{ia}\ * MDys\ (\ BA_{ia} * (KBA_{ia} - KBR_{ia})) * DI_{ia}\right), \forall i\ \forall a\ \forall k \tag{34.a}$$

Equations (16.a), (22.a), and (28.a) represents the modification of the equation (16), (22), and (28) that defines the value of the decision variable of the alternatives. The modification consists in multiplying the original equation the by new decision variables (*$DCA_{iak},$ $DCA_{in}$ or $DI_{ia}$)*. In this case, the decision variable of the alternative takes the value of one if the interest generated by the alternative is greater than or equal to IMARR, and if the kilowatts saving generated by the alternative do not affect the kilowatts contracted to PREPA. Equations (36), (37), and (38) represent the example of the binary decision variables definition. In this case, it is important to mention that it is necessary to define a binary decision variable for each alternative. Equations (32.a) to (34.a) represent the total kilowatts energy savings ($KWS_j$) per system $j$ with the corresponding modifications.

### 3.6 Model variation

Currently on the market there are many equipment manufactured to consume less amount of energy. For that reason, it is probable that there are more than one alternative to replace the equipment that consume too much energy. For this reason, the model was adapted to attend the possibility of having more than one alternative to replace the current equipment. The adaptation of the model consists of two steps: adding a notation to identify the alternatives for the same equipment, and adding a constraint to select only one of these alternatives. For the identification of the alternative, the notation $c$ was used; this index can have any value between $a$ and $z$. Adding the new index the decisional variables are: $AC_{iakc}$, $CA_{inc}$, and $I_{iac}$. The new constraint shows that the sum of the binary variables of the alternatives to replace the same equipment, must add to one.

For example, if we have two alternatives to replace the same air conditioner equipment, the decision variables would be: $AC_{iaka}$ and $AC_{iakb}$, where $i$ is the index of the facilities area, $a$ is the index of the subdivision area, and $k$ is the index to identify the air conditioner system. The constraints needed to be added in the model for this example is:

$$AC_{iaka} + AC_{iakb} = 1 \tag{39}$$

It is important to mention that the addition of the notation *c* and the constraint presented in equation (39) are necessary in the analyses only if the equipment have more than one alternative to replace it.

# 4. Application of the algorithms and results analysis

In order to evaluate the performance of the optimization models, the software Lingo 12.0 was used. The data used to evaluate the algorithm correspond to an electronic manufacturing company. The following sections present the Lingo's solutions for the analysis of the electronic manufacturing company case study. The models' validations, sensitivity analyses, and comparison of results will be discussed in subsequent sections.

## 4.1 Performance evaluation of the algorithms to identify feasible economic alternatives

The software Lingo is recognized as an effective tool to solve problems regarding optimization models. This software generates a solver status review about the models. In the case of the algorithm considering the payback period, the software shows that the model to select the cost-effective alternatives to reduce energy consumption is classified as Mixed Integer Linear Program (MILP). This classification demonstrates that all expressions in the model are linear, and a subset of the variables is restricted to integer values. The solution of the algorithm considering the internal rate of return to select the cost-effective alternatives to reduce energy consumption shows that the model is classified as Nonlinear program (NLP). This means that at least one of the relationships in the model is nonlinear with respect to the variables.

With respect to the method used to solve the algorithms, the solution review shows that the software uses a branch-and-bound specialized solver. Lingo employs this strategy in order to solve models with integer restrictions. Branch-and-bound is a systematic method to implicitly enumerate all possible combinations of the integer variables. This method begins by solving the linear program (LP) relaxation of the integer program (IP). If all the decision variables assume integer values at the optimal solution to the LP relaxation, then that solution will be the optimal solution to the IP (Winston, 2004). The solution, by this method, is generated through the followings steps: determine a fathoming solution, apply the simplex algorithm, select the variable to branch, calculate the upper bound for each node, evaluate nodes and determine which ones should be eliminated, observe if there is a node to branch. The optimal solution is found when all nodes are eliminated and the optimal solution is equal or greater than the upper bound for maximization problems and equal or less than the lower bound for minimization problems (Taha, 2007). The following schematic (refer to Figure 1) illustrates a typical branch and bound tree.

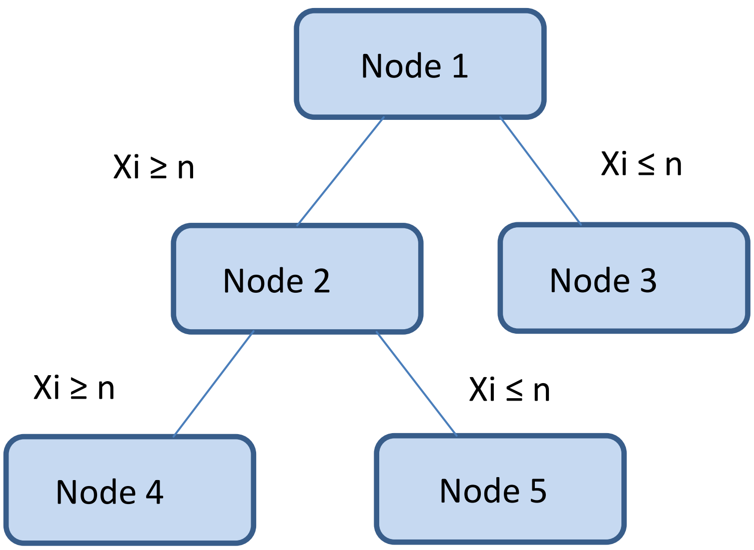


Figure 1. Branch and bound algorithm tree

It is important to mention that this schematic only illustrates five nodes, but the model can have more than that. The amount of nodes in the system is established by the amount of variables the problem has.

## 4.2 Electronic manufacturing case study analysis

In March 29, 2009, an electronic manufacturing company conducted an activity with the objective of identifying alternatives to reduce the consumption of energy in its plant. Through this activity, many alternatives were suggested but, and it was decided to consider three of them for this research: replacing three (3) air conditioner systems, replacing one (1) compressed air system and, replacing current bulbs in the warehouse area.  These alternatives were evaluated considering that the desired period to recover the investment is one year which is equivalent to 255 days of work. Tables 1 and 2 summarize the facility plant's distributions of the company and the data of the alternatives studied.

Table 1. Manufacturing facilities distribution

| | Areas | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Offices | | Production | | Warehouse | | Exterior | |
| **Subareas** | 1 | Principal Off. | 1 | Side Mounting | 1 | Shipping | 1 | Parking |
| | 2 | Material Off. | 2 | Back Frame | | Documents Control | | |
| | 3 | Palomar | 3 | Basico | 2 | Receiving | | |
| | | | 4 | Machine Center | | | | |
| | | | 5 | Malvinas | | | | |
| | | | 6 | PBII Small | | | | |
| | | | 7 | PBII Large | | | | |
| | | | 8 | Global & QMR | | | | |
| | | | 9 | Press | | | | |
| | | | 10 | VMC (Monarch) | | | | |
| | | | 11 | VMC (Okuma) | | | | |
| | | | 12 | Welding | | | | |
| | | | | Paint | | | | |
| | | | 13 | Weid | | | | |
| | | | 14 | Maintenance | | | | |
| | | | 15 | Dojo | | | | |
| | | | 16 | New building | | | | |
| | | | 17 | Welder | | | | |
| | | | 18 | M/S | | | | |
| | | | 19 | B/S | | | | |
| | | | 20 | Programadores | | | | |
| | | | 21 | Lathe | | | | |
| | | | 22 | Drilling | | | | |
| | | | 23 | Aut. Press | | | | |
| | | | 24 | Saw | | | | |

Table 2. Data of the alternatives identified

| **Alternative** | **Area** | **Actual kwh consumed by equipment** | **kwh consumed by recommended equipment** | **kwh savings per working day** | **Saving ($) per year** | **Investment ($)** | **Payback period (year)** | **Feasible alternative** |
|---|---|---|---|---|---|---|---|---|
| Replace the actual air conditioner system | Production 15 | 3.0 | 1.4 | 19.2 | 1,028 | 1,119 | 1.1 | NO |
| | Production 12 | 5.2 | 2.6 | 31.3 | 1,677 | 2,990 | 1.8 | NO |
| | Warehouse 1 | 9.8 | 4.7 | 61.2 | 3,277 | 2,068 | 0.6 | YES |
| Replace the actual compressed air system | Production | 78.2 | 55 | 278.4 | 14,908 | 16,689 | 1.1 | NO |
| Replace the actual bulb | Warehouse 2 | 0.58 | 0.31 | 2.3 | 122 | 150 | 1.2 | NO |

The electronic manufacturing company has the financial policy of approving only projects that generate the necessary income to recover the investment in one year. Taking this into consideration, from all the alternatives mentioned in Table 2 described, there is only one that has characteristics to be a feasible alternative for implementation.

### 4.3 Applications of the models

Taking into account the financial policy of the electronic manufacturing company, the results of the algorithm considering the payback method were compared with the results of the manufacturing company analysis to validate it. The algorithm considering the IRR method was not validated; however, it was used to analyze the case study with the intention of comparing the payback method results with the results from the IRR method.

#### 4.3.1 Validations of the algorithm considering the payback method

To observe the application of the model considering the payback method, the algorithm was used to solve and analyze the electronic manufacturing company case study. This company is not considering the quantity of energy (load) that should be consumed, as established through a contract with PREPA, as a constraint when analyzing their alternatives. With the objective of comparing the results, the load constraint (equations 32-35) were not used to solve the case study at this moment. As mentioned before, the Lingo 12.0 software was used to solve the model. The software generated the results shown in Table 3.

Table 3. Results of the algorithm considering the payback method

| **Objective value** | 1209.26 |
|---|---|

| **Decisional variables** | **Value** |
|---|---|
| $AC_{1,15,1}$ | 0 |
| $AC_{1,12,1}$ | 0 |
| $AC_{3,2,1}$ | 1 |
| $CA_{1,1}$ | 0 |
| $I_{3,2}$ | 0 |

| **Variables** | **Value** |
|---|---|
| $SAC_{1,15,1}$ | 0 |
| $SAC_{1,12,1}$ | 0 |
| $SAC_{3,2,1}$ | 12.852 |
| $SCA_{1,1}$ | 0 |
| $SI_{3,2}$ | 0 |

The Lingo solver status shows that the model has eleven (11) variables and twelve (12) constraints, five of those are integer variables (refer to Appendix 8). These integer variables are binary integer variables; this means that they can only take values of 1 or 0. From these five decision variables, only one has a value of one. Therefore, only one of the five alternatives is considered by the model as feasible to implement. The decision variable with value of 1 is $AC_{321}$. This variable represents the alternative of replacing the air conditioner system 1 in the documents control area of the warehouse. The implementation of this alternative will generate savings of 102 kilowatts hours per month. This will represent economical savings of $12.85 per working day generating $1,209.26 of net profit a year. The Lingo solver status in addition shows that this result is a global optimal solution. This means that it is the solution that gives the best results for the objective function (Jorge Nocedal, 1999). Referring to Table 2 and comparing it to these results, we observe that the results are the same. This demonstrates that the optimization model can be used to analyze and find economic feasible alternatives to replace the current systems by systems that consume less energy.

### 4.3.2 Application of the algorithm considering internal rate of return method

To observe the application of the model considering the internal rate of return method, it was used in this study to solve and analyze the electronic manufacturing company case study. To validate the results obtained by the algorithm the Microsoft Excel program was used to verify the computations, then the value of the decision variables were corroborated. To compare these results with the results obtained using the payback method, the load constraints were not included in this analysis. Table 4 shows a summary of the results obtained by the algorithm that consider the internal rate of return method when the case study was analyzed.

Table 4. Summary of the results of the algorithm considering the internal rate of return method

| **Objective value** | 22,542.87 |
|---|---|

| **Decisional Variables** | **Value** |
|---|---|
| $AC_{1,15,1}$ | 1 |
| $AC_{1,12,1}$ | 1 |
| $AC_{3,2,1}$ | 1 |
| $CA_{1,1}$ | 1 |
| $I_{3,2}$ | 1 |

| **Variables** | **Value** |
|---|---|
| IMARR | .0027 |
| $IAC_{1,15,1}$ | .0175 |
| $STAC_{1,15,1}$ | 1,096.42 |
| $IAC_{1,12,1}$ | .0107 |
| $STAC_{1,12,1}$ | 1,799.78 |
| $IAC_{3,2,1}$ | .0304 |
| $STAC_{3,2,1}$ | 3,516.80 |
| $ICA_{1,1}$ | .0172 |
| $STCA_{1,1}$ | 15,998.00 |
| $II_{3,2}$ | .0155 |
| $STI_{3,2}$ | 131.88 |

The Lingo solver status shows that the model has sixteen (16) variables and seventeen (17) constraints (refer to Appendix 9). In this case, all decision variables take value of one. This means that the benefits generated by the alternatives over the lifespan of the equipment, will be greater than or equal to the benefits to be gained by placing the money need for the investment of the alternative in a bank account at the MARR. After one year of implementing the alternative the benefits are $22,542 representing a saving of 712.8 kilowatts hours monthly. These results showed significant discrepancy with the results of the payback method. The difference between the benefits that can be obtained in one year is $21,332. The Lingo solver status also shows that this solution is a locally optimal solution. This means that this solution is better than other possible solutions but it is not necessarily the best solution (Jorge Nocedal, 1999). However, the Lingo software uses methods to find and report the best local optimal point ( Local Optima vs. Global Optima), making the results presented in Table 4 the best local optimum solution.

## 5. Application of the models considering the load contracted constraints

The PREPA offers to manufacturing companies different tariffs to paid the energy consumed. This tariff is established in the contract that the manufacturing companies have with the agency. There are other specifications are included in these contracts. The difference between the contracts is the characteristics of the service and the price for that. All contracts consider a limit in the demand contracted. In addition, these contracts cannot be changed in one year (Tarifas para el servicio de electricidad, 2000). Considering these factors, it is important to analyze how the reduction in the energy consumed affects the contract that the company has with the PREPA. The next part shows the addition of the load contracted constraints to the models.

### 5.1 Optimization model for electronic manufacturing company case study adding constraints of demand load contracted in PREPA in the model considering the payback method

When the constraint related to the demand load contracted in PREPA is added to the algorithm, the model maintains the same amount of variables. This means that this constraint does not complicate the analysis since additional equations are not needed. The demand load constraint was added to the model and the electronic manufacturing company case study was analyzed again using the same data that was used in the previous analyses. The results of this model were similar to the results of the model without the demand load constraint. This means that the kilowatts hour savings per month by implementing the alternative of replacing the air conditioner system one (1) in the documents control area of the warehouse, do not affect the demand load that the company contracted in PREPA. For this reason this alternative remains as a viable alternative.

To verify that this constraint have an effect in the model and limits the feasible region the model was solved varying the actual kilowatts hour consumed by the air conditioner system one (1) in the documents control area of the warehouse. For example assume that the actual air conditioner system consumes 5774.7 kilowatts hour and the new equipment consumes 4.7 kilowatts hour the alternative to replace the system becomes not feasible. The variable that is changed affects the kilowatts hour savings. This is the variation that causes the economic benefits to increase. Considering this increment the payback restriction did not affect the feasibility of the alternative. However, if the kilowatts hour savings increase, the energy demand required for the company decreases, affecting the load contracted demand. This demonstrates that the constraint that limits the alternative is the demand load contracted constraint. If the kilowatts hour consumed by the actual air conditioner system is 5774.7 kilowatts hour, and the load contracted constraint is not considered, the alternative is feasible and generate 115400 kilowatts hour per month. This result indicates that the maximum kilowatts hour savings that the company can generate without adding a penalty cost in the energy bill by consuming less load than the one contracted is little less than 5774.7 kilowatts hour or 115390 kilowatts hour per month.

## 5.2 Optimization model for electronic manufacturing company case study adding constraints of demand load contracted from PREPA in the model that considered the internal rate of return method

When the load contracted constraints were added to this model the results show that all alternatives are feasible. This result demonstrates that the load contracted constraints do not have an effect in the model. If all the alternatives are implemented energy savings will be 712.8 kilowatts hours monthly. Considering that the manufacturing company can save as most 115390 kilowatts hour per month without receiving a penalty (refer to section 5.1), the manufacturing company has the opportunity to identify other alternatives that can generate 114678 kilowatts hour per month. This means that the manufacturing company has a wide range to reduce the current energy consumption without changing the actual load contracted.

## 6. Sensitivity analyses

Analyzing the algorithms we can see that the time desired to recover the investment in the payback method algorithm is an important variable to define the feasibility of the alternatives. In the case of the algorithm that considers the internal rate of return method the time desired to get the benefits do not affect the feasibility of the alternatives. To deepen the analysis of the payback method algorithm, the time desired to recover the investment cost was varied to two years, which is equivalent to 510 days of work for the case study. In the next section, the results and the analysis of the scenario studied are shown.

### 6.1 Using two years to recover the investment in the payback method algorithm

In this occasion, the electronic manufacturing company case study applying the optimization model but using 510 days of work as the period of time to recover the investment was analyzed. For this analysis, the constraint of the load contracted was not considered. Using the Lingo 12.0 software, the following results were obtained (refer to Table 5).

Table 5. Results of the algorithm considering the payback method and two years as the period to recover the investment cost

| **Objective value** | 5871.15 |
|---|---|

| **Decisional variables** | **Value** |
|---|---|
| $AC_{1,15,1}$ | 1 |
| $AC_{1,12,1}$ | 1 |
| $AC_{3,2,1}$ | 1 |
| $CA_{1,1}$ | 0 |
| $I_{3,2}$ | 1 |

| **Variables** | **Value** |
|---|---|
| $SAC_{1,15,1}$ | 4.007 |
| $SAC_{1,12,1}$ | 6.577 |
| $SAC_{3,2,1}$ | 12.852 |
| $SCA_{1,1}$ | 0 |
| $SI_{3,2}$ | .4819 |

In this case it can be observed that more alternatives are feasible comparing them with the original electronic manufacturing company case study (refer to Table 3). Using 510 days of work to recover the investment, the three alternatives for replacing the air conditioner systems are feasible alternatives. In addition, the alternative of replacing the current bulbs in the receiving warehouse area is also a feasible solution. This demonstrates that in this case when considering two years as the period to recover the investment cost, more benefits are generated than when considering one year.

### 6.2 Time to get the maximum economic benefits

In addition, to reduce the energy consumption in the manufacturing company generated benefits of the implemented alternatives could be used for other investments. Considering this, it is recommended to wait the time necessary to get the maximum benefits. To establish the time to get the maximum benefits it is necessary to identify the time required to recover all the investments in the alternatives and consider the lifespan of the equipment. For this, all the alternatives were considered as one project.  Assuming this, it is necessary to identify the equipment with the minimum lifespan and the maximum time to recover the total investment. In this case the lifespan of the equipments are known and the time to recover the investment of the alternatives can be calculated with the net present value equation:

$$(@fpa(IMARR, n) * SWAC_{iak}) - C_{1iak} \; = \; 0, \;\; \forall i \;\; \forall a \, \forall k \qquad (40)$$

*The variables correspond to the air conditioner alternative

where *IMARR* represents the minimum acceptable rate of return, *n* is the unknown variable that represents the time necessary to recover the investment cost, $SWAC_{iak}$ represents the weighted average of the weekly economic savings generated by the air conditioner system *k* in area *i* subdivision *a*, and $C_{1iak}$ is the cost ($) of purchasing and installing the new air conditioner system

Using the case study shown in this research, the minimum lifespan of the equipment is five (5) years, corresponding to the illumination system, and the maximum time to recover the total investment is two (2) years. Considering this, the maximum benefit will be achieved during three (3) years. Waiting three (3) years to get the maximum benefits represents a total of $78,483.91 in savings. These are the benefits that the manufacturing company can obtain when replacing one of equiptments and ensuring recovery of the total investment.

## 7. Payback method versus internal rate of return method

Through the models developed in this research it was demonstrated that there exists significant differences between the payback method and the internal rate of return method. The algorithm that considers the payback method has fewer variables, twenty-nine versus thirteen, and requires less computational power in comparison with the algorithm that considers the internal rate of return method. Morgan established that the method of payback period have been used more frequently because it is simple to calculate and easy to understand (Cooper, Morgan, Redman, & Smith, 2001). However, the simplicity may be equivalent to lost benefits.

The results of the algorithms show that more alternatives are identified as feasible alternatives when analyzed with the internal rate of return method. For that reason more economics benefits could be obtained. Even though the algorithm that considers the internal rate of return method is more extensive, the time required to obtain the results is the same as the time required to analyze the algorithm that considers the payback method. Considering these results, this research demonstrates that the internal rate of return method is a dependable and trusted one economic analysis method used to analyze alternatives without spending too much time in the analysis.

## 8. Summary of results of the algorithms to identify feasible economic alternatives

Currently, there exist many studies that demonstrate that optimization models can be used to optimize the energy consumed by manufacturing processes in companies but not to reduce the entire plant energy consumption. Through this research it was demonstrated that financial methods, such as the internal rate of return and the payback, can be used in Mixed Integer Linear Program optimization models to identify economic feasible alternatives to replace the current equipment with other equipment that consume less energy. This model was applied to the electronic manufacturing company case study and the solution was generated with the Lingo 12.0 program. These results were validated or proven as correct.

In the payback method model, the main constraints are the time period to recover the investment and the limits of the demand load contracted from the PREPA. In the internal rate of return method the main constraints are the comparison between the breakeven interest of the alternative and the minimum acceptable rate of return, and the limits of the demand load contracted from the PREPA. In both models the variables that influence the feasibility of the alternatives are: the kilowatts saving by the equipment, the cost to replace the equipment, and the kilowatts demand contracted from the PREPA. The amount of variables in the models are between (11) and (16) variables and between (12) and (17) constraints. However, the complexity of the model can increase if the amount of alternatives per area increases.

The results between the algorithm that considers the payback method and the algorithm that considers the internal rate of return method demonstrated that the payback method analysis is more simple and easy to understand. However, the use of the internal rate of return method guarantees a complete analysis through the lifespan of the projects, identifying more viable alternatives than the other method.

This study focused in the analysis of alternatives to reduce energy consumption of the air conditioner, compressed air, and illumination systems. For future studies, it will be interesting to evaluate alternatives for exhaust systems and inefficient motors. In addition, this study only evaluates the alternative of replacing existing systems. Therefore, analyses that evaluate replacing alternatives versus other alternatives that allow energy reduction in those systems could be a significant contribution to the subject.

## 9. Considering investment in a photovoltaic system

The manufacturing companies at the end of the fiscal year analyze the costs incurred and benefits received during the period. This analysis is used to decide which benefits will be used for investment in the company and/or which benefits will be granted to the owners. This activity is known as a change in equity statements (Canada, 2005). A similar analysis can be done when projects are finished.

In this part of the research, it is analyzed how the benefits generated by the alternatives that reduce the energy consumption can be used for investment in other projects that support the activities to reduce the energy consumption. The previous analyses (in sections 3.3.1 and 3.3.2) demonstrated that the IRR method is more appropriate to make a decision about the feasibility of the projects in comparison to the payback period method for the followings reasons: the payback method do not consider the lifespan of the project; it gives an approximation of the time to recover the initial costs; and it is not recommended to compare more than one alternative with different lifespans (Capital budgeting decisions).  Considering this, in the following sections the results obtained in the analysis using the IRR method were used.

In order to support the reduction in energy consumption, there exist in the market equipment or systems that help to reduce it without replacing the currently used equipment. Some of these systems are control systems operations and renewable energy systems. This research focuses in evaluating the uses of the photovoltaic systems; a type of renewable energy. The success of the photovoltaic system implementation to reduce the energy consumption generated by fuel oil was demonstrated in previous research and statistics (O'Grady, 2011). However, the initial high investment cost that this system requires, limits its implementation.

Although there exists alternatives to reduce the energy consumption using energy efficient equipment, it is necessary to explore alternatives that may represent independence from the energy generated by fuel oil. The ideal scenario is when companies can optimize the energy they consume. If the companies achieve this goal, finding alternatives to continue reducing the energy consumption is very challenging. In this case, the efforts to reduce the energy consumption are limited by the advantages of the products in the market made for this purpose. Considering the factors mentioned above in the following sections, the development of a photovoltaic system to identify the size of a photovoltaic system that can be funded with the economic savings generated by the equipment that consume energy efficiently is shown. An optimization model was developed to maximize the kilowatts generated by a photovoltaic system.

### 9.1 Optimization algorithm for the photovoltaic system

The objective of this model is to determine the maximum amount of solar panels that can be purchased using the economic benefits generated with the alternatives that reduced energy consumption. In this model, the following information was considered: the area available to install the photovoltaic system, the implementation cost of the photovoltaic system, and the technical specifications of the solar panels (the kilowatts generated by solar panels and its efficiency). In addition to the amount of solar panels, the computations of the kilowatts generated by the photovoltaic system per hour and per month and the percentage of the energy reduced by the photovoltaic system were included in the model. The general assumptions for the model are the following:

**General assumptions**

- ✓ The dimensions of the space available for placement of the photovoltaic system are known.
- ✓ The dimensions of the photovoltaic panels are known.
- ✓ The watts produced by the photovoltaic panels, its efficiency, and its technical information are known and considered the standards test conditions (STC)[1].
- ✓ The photovoltaic panels operate in Maximum Point Power Tracker (MPPT)[2].
- ✓ The costs associated with purchasing and installing the photovoltaic system are known.

The size of the photovoltaic system than can be funded with the previous economic benefits is given by:

$$(HP * LP) + \big((HP + 1) * SP\big) \leq SH \tag{41}$$

$$(VP * WP) + \big((VP + 1) * SP\big) \leq SV \tag{42}$$

$$HP * VP = FP \tag{43}$$

$$(LBC * FP * WPP) + CINV + (CR * FP) = OC \tag{44}$$

$$FP * FPC \; + \; OC \; - \; SUB \; \leq SAV \tag{45}$$

$$FP * EFF * WPP = WPS \tag{46}$$

$$@gin(HP) \tag{47}$$

$$@gin(VP) \tag{48}$$

---

[1] A set of reference photovoltaic device measurement conditions consisting of irradiance of 1 kW/m$^2$, AM 1.5, and 25 °C cell temperature (standard test conditions (STC)).

[2] Maximum Power Point Tracking is an electronic system that operates the Photovoltaic (PV) modules in a manner that allows the modules to produce all the power they are capable of. MPPT is a fully electronic system that varies the electrical operating point of the modules so that the modules are able to deliver maximum available power (Cullen).

where *HP* is the amount of solar panels located throughout the length in inches of the system; *LP* is the length of the solar panels; *SP* is the space between solar panels; *SH* is the length of the space available to locate the photovoltaic system; *VP* represent the amount of solar panels located across the width of the system; *WP* is the width of the solar panels; *SV* is the width of the space available to locate the photovoltaic system. In addition, *FP* represents the total amount of solar panels in the photovoltaic system; *LBC* is the cost of labor to install the solar panels; *WPP* is the watts generated by each photovoltaic panel considering the efficiency of the panels; *CINV* is the cost of the inverter for the system; *CR* is the cost of the solar panels racks; *OC* represents the additional cost associated with the implementation of the photovoltaic system. *FPC* represents the solar panels cost; *SUB* represent the economic benefit that the company receives for the implementation of the photovoltaic system; *SAV* represents the sum of the maximum economic benefits generated by the alternatives that reduce the energy consumption (calculated in the equations 10 and 31); *EFF* is the efficiency of the system; and *WPS* represents the watts generated by the photovoltaic system.

Equations (41) and equation (42) represent the available space constraints. These equations limit the number of solar panels to the space available to place the system. Equation (43) is used to calculate the number of solar panels for the photovoltaic system. Equation (44) allows the calculation of the total cost ($) to implement the photovoltaic system. Equation (45) represents the investment constraint. Through this equation the model identify the maximum number of solar panels that can be purchased with the available funds. Equation (46) calculates the watts generated by the system per hour. Equations (47) and (48) establish the variables *HP* and *VP* as integer, respectively.

### 9.2 Objective function of the algorithm for the photovoltaic system

The objective of the model is to maximize the numbers of solar panels for the photovoltaic systems. This function also allows maximizing the watts generated by the photovoltaic system. The maximum numbers of solar panels that can be purchased with the previously calculated economic savings is given by:

$$Max = FP \tag{49}$$

### 9.3 Application of the algorithm to establish the size of the photovoltaic system

In order to be consistent, this model was also solved in the software Lingo 12. To show the execution and the application of the algorithm to establish the size of the photovoltaic system, data provided by a firm that works with implementation of photovoltaic systems was used. In the following sections, the performance of the model and the results of the model application are discussed.

#### 9.3.1 Performance evaluation of the algorithm to establish the size of a photovoltaic system

To develop and analyze the execution of the model that establishes the size of the photovoltaic system that can be funded with previously calculated economic savings, the software Lingo 12 was used. The review of the software status report shows that this algorithm is classified as an Integer Nonlinear Program (MINLP). This means that at least one equation in the model is not linear and the variables have integer values. The method that the software used to solve the algorithm is the *branch-and-bound method*, the same method that the software used to solve the previous algorithms (section 3.1).

### 9.3.2 Application of the algorithm

To observe the application of the algorithm, data provided by a firm that installs photovoltaic systems was considered. Table 6 shows the information provided by the firm.

Table 6. Summary of information used for the application of the algorithm that establishes the size of the photovoltaic system

| | |
|---|---|
| Solar panels | Dimension: 39.6" x 62" |
| | Cost per panel: $600.00 |
| | Watts generated per panels: 230 w |
| | Efficiency of the panels: 15% |
| Additional cost associated with the system | Racks per panels: $167.00 |
| | Inverter: $2,500.00 |
| Labor cost | Cost per watt of the system: $3.75 |

Regarding the dimensions available in the manufacturing company to place the solar panels, the following dimensions were used: 1872" x 768". In the additional costs associated with the system, batteries to store energy were not considered. The photovoltaic system suggested in this analysis is a grid-type photovoltaic system. This type of system is connected directly coupled with the electric distribution network and do not require a battery for storage. The addition of a storage system for photovoltaic systems represents other costs such as materials, infrastructure, and management of equipment; for that reason this system was not considered in this research. In addition, it was assumed that the manufacturing company will not receive any benefits from the government for the installation of photovoltaic systems. Currently in Puerto Rico, the incentives to promote the development of renewable energy projects are dispersed in various laws. Moreover, the fact of not having a unified scheme of incentives for encouraging and developing green energy projects has had the effect of precluding the development of alternatives that reduce the costs of energy production (Ley de Incentivos de Energía Verde de Puerto Rico y enmienda la Ley Núm. 70 de 1978; Ley de Desperdicios Sólidos y la Ley Núm. 120 de 1994; Código de Rentas Internas, 2010). To show consistency in the analysis, the funds available considered for investing in a photovoltaic system were the benefits generated by the implementation of the alternatives identified in previous sections (sections 3.3.1, 3.3.2, 6.1 and, 6.2). This amounted to $78,483.

The results generated by the Lingo software are shown in Table 7.

Table 7. Results obtained from Lingo software for the algorithm that establishes the size of the photovoltaic system

| **Objective value** | 46 |
|---|---|

| **Variables** | **Value** |
|---|---|
| hp | 23 |
| vp | 2 |
| oc | 49,857 |
| wps | 8,959 |

The Lingo Solver Status showed that the model has five (5) variables and eight (8) constraints, where two of these variables are integer (refer to Appendix 12). The results show that forty-six (46) solar panels can be used in the photovoltaic system. The arrangement of the system is two (2) lines of twenty-three (23) solar panels (refer to Figure 2). The cost associated with the photovoltaic system; is $49,857 and the solar panels would be generating 8.9 kilowatts per hour.

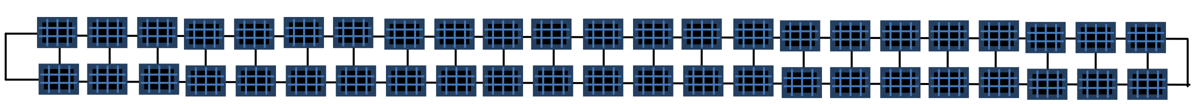

Figure 2. Arrangement of the photovoltaic system with 46 solar panels (not to scale)

### 9.4 Analysis of the results generated by the algorithm

To explain the impact of the photovoltaic system established by the algorithm it is necessary to calculate the kilowatts per day that the system can generate and convert this energy reduction into economics savings. To do this calculation it is necessary to consider the efficient sun hours per day. Table 8 shows the solar resource measurements for San Juan, Puerto Rico at different tilt angles (Ladner 2009).

Table 8. Mean daily solar resource per month in metro area at different tilt angles (kWh/m2)

| | 0° | 5° | 10° | 15° | 18.4° | 20° | 25° | 30° | 35° | 40° | 45° |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Jan. | 4.28 | 4.51 | 4.72 | 4.90 | 5.00 | 5.05 | 5.17 | 5.27 | 5.33 | 5.36 | 5.36 |
| Feb. | 4.91 | 5.09 | 5.25 | 5.38 | 5.45 | 5.47 | 5.54 | 5.57 | 5.58 | 5.55 | 5.49 |
| Mar. | 5.72 | 5.83 | 5.92 | 5.96 | 5.98 | 5.98 | 5.96 | 5.91 | 5.82 | 5.71 | 5.56 |
| Apr. | 6.10 | 6.11 | 6.09 | 6.04 | 5.99 | 5.96 | 5.84 | 5.69 | 5.51 | 5.30 | 5.06 |
| May | 5.78 | 5.73 | 5.64 | 5.52 | 5.43 | 5.38 | 5.21 | 5.01 | 4.79 | 4.54 | 4.29 |
| June | 6.05 | 5.95 | 5.83 | 5.67 | 5.55 | 5.49 | 5.28 | 5.04 | 4.77 | 4.50 | 4.21 |
| July | 6.09 | 6.01 | 5.90 | 5.76 | 5.64 | 5.58 | 5.38 | 5.15 | 4.89 | 4.62 | 4.34 |
| Aug. | 5.96 | 5.94 | 5.89 | 5.81 | 5.73 | 5.69 | 5.55 | 5.38 | 5.18 | 4.95 | 4.70 |
| Sep. | 5.53 | 5.60 | 5.63 | 5.64 | 5.63 | 5.61 | 5.56 | 5.47 | 5.36 | 5.21 | 5.04 |
| Oct. | 4.92 | 5.07 | 5.19 | 5.29 | 5.33 | 5.35 | 5.38 | 5.39 | 5.36 | 5.31 | 5.22 |
| Nov. | 4.31 | 4.52 | 4.70 | 4.86 | 4.95 | 4.99 | 5.09 | 5.17 | 5.21 | 5.22 | 5.21 |
| Dec. | 3.97 | 4.18 | 4.37 | 4.54 | 4.64 | 4.68 | 4.80 | 4.88 | 4.94 | 4.97 | 4.97 |
| Ave. | 5.30 | 5.38 | 5.43 | 5.45 | 5.44 | 5.44 | 5.40 | 5.33 | 5.23 | 5.10 | 4.95 |

Considering the information presented on Table 8, the annual average of efficient sun hours per day in San Juan is 5.3 hours. It is important mention that the efficient hours per day can change per region. In this case the efficient hours per day of San Juan were used. Multiplying the kilowatts hour that the system generates by the efficient solar hours per day a total of 47.5 kilowatts per day can be generated. This represents a total of 1,424 kilowatts per month considering that the system operates every day (30 days per month) and the energy is used by the manufacturing company all the time.  In economic savings, this amount of energy represents $299 monthly and $3,589 per year.

If the impact of the photovoltaic system is measured comparing the investment in the system with the calculated benefits, it can be concluded that the photovoltaic system does not have a significant impact in the reduction of energy expenses. Considering that the manufacturing company of the case study implemented the alternative identified and had a 712.4 kilowatts hour savings per month (refer to section 4.3.2), the average kilowatts consumed by the company is 191,627 kilowatts hour per month. The energy reduced with the photovoltaic system represents a 0.14% of the energy (kilowatts hour) that the company uses per month. This is not a significant amount of energy.

One important and limiting factor of the solar panels is its efficiency. This percentage defines the amount of energy that the solar panels can generate. Typically, the efficiency of the solar panels will range between 6 and 18% depending on the cell technology and the materials used in the solar panels (Ladner 2009). The importance of improving the efficiency of the solar panels is recognized by experts and industries that work with this issue (Solar panels, 2011). In February, Amonix, a company that design and manufacture utility-scale solar power systems, announced a new solar panel that has a 41.6% of efficiency (Scanlon, 2011). Spectrolab also announced the manufacturing of photovoltaic cells that have a 40.7% of efficiency (Thomas). To improve the efficiency of the solar panels, the Advanced Technology & Research Corp., with economic support of Maryland Energy Administration, developed pole solar trackers that use global positioning system (GPS) technology for the solar panels following the sunlight. This technology helps the solar panels to achieve a 30% of efficiency (Sthepanie, 2011). These equipments have a cost then fluctuate of $760 and $1,760 per solar panel.

To analyze the effect of improving the efficiency of the solar panels on the results obtained by the application of the algorithm, it was considered that the polar solar track has an average price of $1,270. Replacing the racks of the solar panels for the solar pole trackers, the algorithm generated the results shown in Table 9.

Table 9. Results obtained from Lingo software for the algorithm that establishes the size of the photovoltaic system considering solar panels with 30% of efficiency

| **Objective value** | 27 |
|---|---|

| **Variables** | **Value** |
|---|---|
| hp | 27 |
| vp | 1 |
| oc | 60,077 |
| wps | 10,517 |

The results show that twenty-seven (27) solar panels can be used in the photovoltaic system. The arrangement of the system is one (1) line of twenty-seven (27) solar panels (refer to Figure 3). The cost associated with the system is $60,007 and the solar panels could generate 10.5 kilowatts per hour. Considering 5.3 effective solar hours, this photovoltaic system will generate 55.7 kilowatts per day.  This represents a total of 1,672 kilowatts per month (30 days per month). In economic savings, this amount of energy represents $351 monthly and $4,214 per year. Considering that the manufacturing company of the case study implemented the alternative identified and had a 712.4 kilowatts hour savings per month (refer to section 4.3.2), the average kilowatts consumed by the company is 191,627 kilowatts hour per month. The energy reduced with the photovoltaic system represents a 0.16% of the energy that the company uses hourly per month. Table 10 shows the comparison of the results generated by the algorithm in the different analyzed cases.

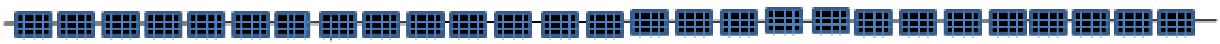

Figure 3. Arrangement of the photovoltaic system with 27 solar panels (not to scale)

Table 10. Comparison of results generated by the algorithm considering solar panels with 15% and 30% of efficiency

| | Solar panels with 15% efficiency | Solar panels with 30% efficiency | Difference |
|---|---|---|---|
| Number of solar panels in the photovoltaic system | 46 | 27 | 19 |
| Kilowatts generated by the photovoltaic system per month | 1,424 | 1,672 | 248 |
| Monthly economic savings generated by the photovoltaic system | $299 | $351 | $52 |
| Percentage in energy reduction per month | 0.14% | 0.16% | 0.02% |

Doubling the efficiency of the solar panels using the solar pole trackers represents an increment of 0.02% in energy reduction compared to the energy reduced considering the solar panels with 15% of efficiency. To decide if this increment is acceptable or not will depend on, the personal interpretation of the analyst. These results demonstrate that the efficiency of the solar panels have an effect in energy reduction. However, in this case there exists a relation between the costs of the investment in the photovoltaic system and the improvement of the efficiency of the system.  The best scenario for the viability of the photovoltaic system is achieved by improving the efficiency of the system while minimizing its investment cost (Advantages and disadvantages of photovoltaics).

### 9.5 Photovoltaic systems in the future

The efficiency of the photovoltaic system to generate energy has been proved. The US Department of Energy (DOE) is developing investigations and projects to improve the photovoltaic systems. *Sunshot* is an initiative to reduce the total cost of the photovoltaic system by about 75% before 2020. The efforts are oriented to make the costs of solar energy technologies competitive with other forms of energy without using subsidies. The 75% reduction represents a reduction in the cost for utility of about $1 per watt, which is similar to approximately 6 cents per kilowatts hour (Sunshot initiative, 2011). Considering that this project could be achieved, the case study analyzed in this research was evaluated using this scenario.

If the manufacturing company decides to wait for the DOE results to invest in a photovoltaic system, they need to decide what to do with the economic benefits generated by the implement energy reduction alternatives. In this section, the alternatives to use a savings account or a certificate of deposit are analyzed to save the economic benefits while waiting to invest in a photovoltaic system. For the analysis, it was considered that the manufacturing company could achieve the maximum economic benefits in 2015 and then wait five (5) years (until 2020) to invest in a photovoltaic system. The following sections show the analyses of the alternatives.

#### 9.5.1 Savings account

The objective of this analysis is to know the equivalence of the benefits at the end of year 2020 if a savings account is used to save the economic benefits. These benefits are to be invested in a photovoltaic system considering the expected improvements of the photovoltaic system by DOE. The period of time used for the analyses is five (5) years. Using the data available by the Federal Deposit Insurance Corporation (FDIC), the rate of interest for the savings account during the past year is known. Figure 4 shows the behavior of the rate of interest during this period.

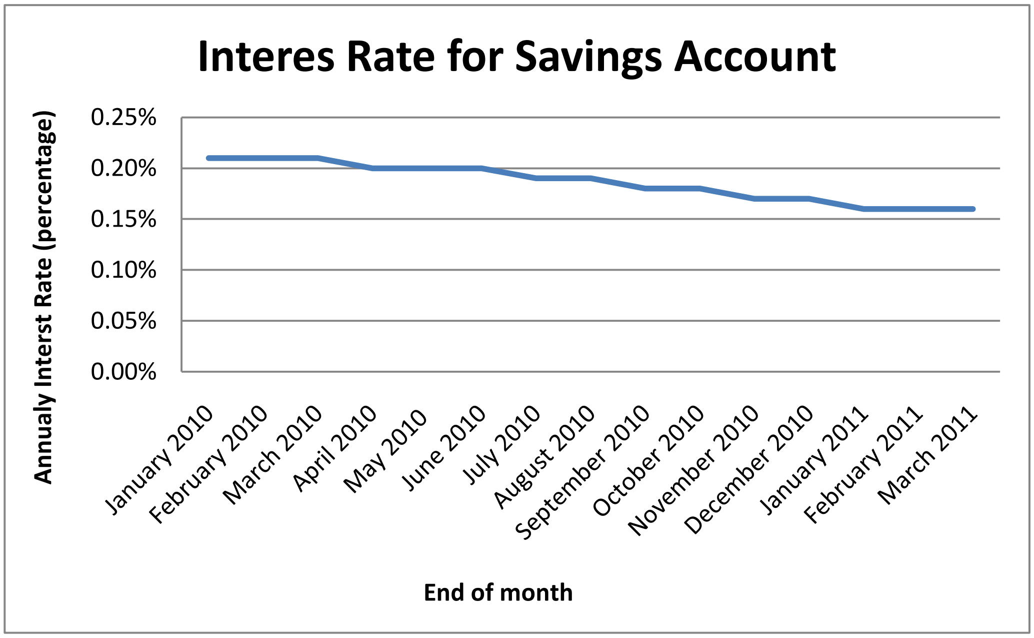


Figure 4. Behavior of the savings account rate of interest (January 2010 to date)

The graph shows a decrease in the rate of interest during the period studied. However in the last three (3) months the rate of interest is constant at 0.16% annually. Considering this, the rate of interest used in this analysis was 0.16% annually.

To calculate the benefits at the end of year 2020 the future value equation was used, considering a period of five (5) years and a rate of interest of 0.16% annually. The result of this calculation was $79,113. Considering the original data used to analyze the photovoltaic system and the money available in the savings account and assuming that the DOE can make the solar panels to have a value of $1 per watt, the manufacturing company could implement a system of sixty (60) solar panels, which could generate 11.7 kilowatts per hour at that time. This represents a saving of 62 kilowatts per day (considering 5.3 hours of sun) and 1,858 kilowatts per month. This energy saving can be converted to $390 monthly economic savings and $4,682 yearly economic savings. The energy generated by this system is equivalent to 0.18% of the average energy consumed hourly by the manufacturing company of the case study per month.

### 9.5.2 Certificate of deposit

The objective of this analysis is to know the equivalence of the benefits at the end of year 2020 if a certificate of deposit (CD) is used to save the economic benefits. These benefits are to be invested in a photovoltaic system considering the expected improvements of the photovoltaic system by DOE. The period of time used for the analysis is five (5) years. Using the data available from the Federal Deposit Insurance Corporation (FDIC), the rate of interest for the certificate of deposit, during the past year is known. Figure 5 shows the behavior of the rate of interest during this period.

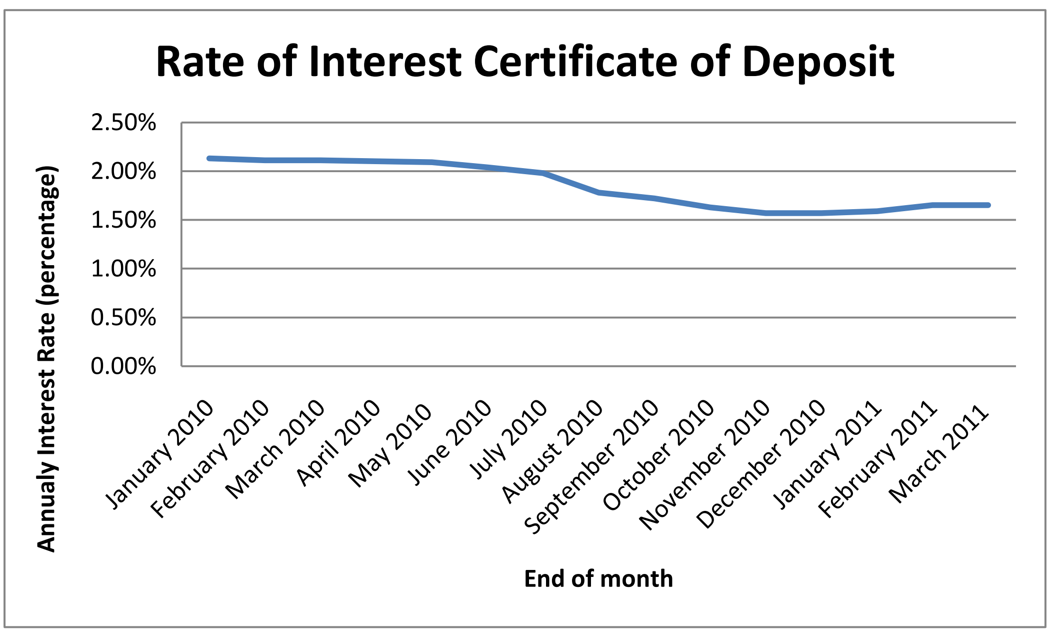


Figure 5. Behavior of the certificate of deposit rate of interest (January 2010 to date)

The graph shows a decrease in the rate of interest until December 2010 when the rate of interest began to slowly increase to 1.65%. The last two (2) months the rate of interest is constant at 1.65% annually. Considering this, the rate of interest used in this analysis was 1.65% annually.

To calculate the benefits at the end of year 2020 the future value equation was used, considering a period of five (5) years and a rate of interest of 1.65% annually. The result of this calculation was $85,175. Considering the original data used to analyze the photovoltaic system and the money available in the saving account; assuming that the DOE can make the solar panels to have a value of $1 per watt, the manufacturing company could implement a system of sixty-five (65) solar panels, which could generate 12.6 kilowatts per hour. This represents a saving of 67 kilowatts per day (considering 5.3 hours of sun) and 2,013 kilowatts per month. This energy saving can be converted to $422 monthly economic savings and $5,072 yearly economic savings. The energy generated by this system is equivalent to 0.20% of the average energy consumed by the manufacturing company of the case study per month.

### 9.5.3 Saving account versus certificate of deposit

In the previous analysis it can be observed that using the certificate of deposit to save the funds generate more benefits than using a savings account. The difference between the rates of interest is 1.49%. This represents a difference of $6,062 at the end of five years. However, using the money that generates a savings account versus using the money that generates a certificate of deposit to invest in a photovoltaic system with solar panels that have a cost of $1 per watt, represent a difference of 155 kilowatts of monthly energy savings. This represents a difference of $32 monthly economic savings. It can be observed that there exists a significant difference between the money generated in a certificate of deposit versus a savings account. However, for these differences to have a significant impact in the investment of a photovoltaic system, it is necessary to improve the efficiency of the system along with the reduction of cost.

## 10. Summary of results of the algorithm to establish the size of a photovoltaic system

Many companies had the commitment to reduce the energy consumption. The photovoltaic system is an effective instrument to reduce the dependence on the energy produced by fuel oil. The initial high cost of this system limits its implementation. Through this research it could be seen that there exists a relation between the system cost and its efficiency. Experts are working to improve photovoltaic systems, minimizing its cost, and increasing its efficiency. The amount of money that the company has to invest in the system is another important factor. This research shows that the initial cost of the system, the efficiency of the system, and the amount of money that the company has to invest for the system can be analyzed together in an optimization model.

The optimization model developed in this research can be used successfully to identify the size of the photovoltaic system that can be implemented considering the available space in the manufacturing company facility and the amount of money available for investment. In addition to identifying the amount of solar panels, the optimization model gives the optimal array to place the solar panels, maximizing the amount of solar panels that can be placed in the space available.

Another important factor that the manufacturing companies should consider is the way to manage their savings, especially if these savings will be used for future investment. Good management of the savings at the moment represents more benefits in the future. In this case, it could be seen that it is better to choose a certificate of deposit to save the money versus a savings account. However, it is important to mention that in this research only the rate of interest is considered to evaluate the savings method. Other aspects of the savings method should be considered before making a decision.

# 11. Conclusions and future work

## 11.1 Conclusions

Through this research it was demonstrated that optimization models are successful tools to analyze feasible economic alternatives to reduce energy consumption in different areas of a manufacturing company and to identify the size and optimal array of a photovoltaic system for a manufacturing company considering different parameters and constraints associated with the decisional variables. The application of the models demonstrated that the analysis of the models require few minutes to generate the results. In addition, it was demonstrated that the models can be successfully used to do sensitivity analysis with the intention of choosing the best alternatives. With respect to the economic analysis method that should be used to analyze the economic feasibility of the alternatives, the following conclusions were found:

- The analyses considering the payback method requires less variables, calculation parameters, and constraints when compared with the analysis considering the internal rate of return method.
- The alternatives that are not found feasible considering the payback method can be feasible alternatives when considering the internal rate of return method.
- The time requires to obtain the results of the optimization model that consider the payback method and to obtain the results of the optimization model that consider the internal rate of return method are similar.

However the feasibility of the alternatives is not only affected by the method used to analyze the alternatives. In addition to the economic analysis methods, the feasibility of the alternatives is mainly affected by the following factors: the kilowatts hour saving for the alternative, the lower limit of the demand contracted by the manufacturing company, the cost of the investment of the alternative, and the price of the energy consumed. In the case of the photovoltaic systems, their economic feasibility are affected by the following actors: the funds available to invest in the system, the space available to place the system, the cost of the system, and the economic benefits that the manufacturing company receives by investing in an renewable energy system. In addition, the energy reductions that the manufacturing company achieves by implementing a photovoltaic system are affected by the following factors: the effective hours of sun in the region to place the system, and the efficiency of the solar panels. The analysis in this research demonstrated that there exists a direct relationship between the cost of the solar panels and their efficiency. To improve the economic feasibility of the photovoltaic system it is necessary for the cost of the system to decrease and the efficiency to increase. Some energy agencies and companies are considering this factor and are working to improve the photovoltaic system. If the manufacturing company decides to save money to wait for the advances in the photovoltaic system, the manufacturing company should be considering a certificate of deposit to save the funds in order to obtain more economic benefits.

Some advantages that the manufacturing company have if the optimization models showed in this research to identify alternatives to reduce energy consumption are used, are the following:

- The manufacturing companies can identify alternatives to improve the energy consumed at their facilities without hiring an external firm.
- The manufacturing companies can continuously monitor the alternative to reduce the energy consumption considering the continued advances in the market to achieve the energy consumption reduction.

- The optimizations models allow the manufacturing company to prevent the penalty cost by not meeting the lower limit of the demand contracted if the alternatives to reduce the energy consumption are implemented.
- The manufacturing companies have the flexibility to analyze different scenarios to obtain the best results and make informed decisions considering all relevant factors.

## 11.2 Future work

In this research, the analysis to find feasible alternatives to reduce energy consumption was only considering alternatives to replace the current systems by systems that consume energy efficiently. To extend this research more alternatives to reduce energy consumption can be considered, such as an implementation of a system that controls the energy consumption. In addition, investing in different alternatives from the same supplier could be considered when analyzing the economic savings to reduce the investment costs. This optimization model could be applied to other systems of the manufacturing companies such as an exhausted system and production machines. In the same way, the models showed in this research could be applied to other type of manufacturing companies like pharmaceutical and medical devices companies. To deepen the implementation of renewable energy systems to reduce energy consumption, other renewable energy systems can be considered such as wind systems and water systems. In addition, analyzing the effect of implementing the photovoltaic system in different regions of Puerto Rico should be interesting considering the difference in efficient hours of sun per day by region.

# Appendices

## Appendix 1. Notations

- The following notations are used to index the models variables:

$i$ index for facilities areas, $i$ = {1-offices, 2-production, 3-warehouse, 4-exterior}

$j$ index for systems, $j$ = {1-air conditioner, 2-air compressed, 3-illumination}

$a$ index for subdivision areas i, $a = \{1,2,3\ldots m\}$

$k$ index to identify air conditioner system in area $i$ subdivision $a$, $k = \{1,2,3\ldots m\}$

$n$ index to identify compressed air system in area $i$ $n = \{1,2,3\ldots m\}$

- Parameters with a single data value:

$BA_{ia}$ current number of bulbs in area $i$ subdivision $a$

$C_{2in}$ cost ($) of purchasing and installing compressed air $n$ in area $i$

$C_{1iak}$ cost ($) of purchasing and installing air conditioner $k$ in area $i$ subdivision $a$

$C_{3ia}$ cost ($) of purchasing and installing bulbs in area $i$ subdivision $a$

$CINV$ cost of the inverter ($)

$CR$ cost of the racks for the solar panels ($)

$DIW$ amount of days in irregular weeks

$DRW$ amount of days in regular weeks

$Dys$ labor days to recover the inversion

$EFF$ efficiency of the system

$FPC$ cost of the solar panels ($)

$HR_j$ operation hours (hr) per day of work of system $j$

| | |
|---|---|
| *IMAAR* | weekly minimum acceptable rate of return |
| *IW* | Amount of irregulars weeks |
| $KACA_{iak}$ | kwh consumed by air conditioner *k* in area *i* subdivision *a* |
| $KACR_{iak}$ | kwh consumed by air conditioner to replace air conditioner *k* in area *i* subdivision *a* |
| $KBA_{ia}$ | kwh consumed by each current bulb in area *i* subdivision *a* |
| $KBR_{ia}$ | kwh consumed by each bulb replaced in area *i* subdivision *a* |
| *Kc* | kwh cost by Puerto Rico Electric Power Authority |
| $KCAA_{in}$ | kwh consumed by compressed air *n* in area *i* |
| $KCAR_{in}$ | kwh consumed by compressed air to replace compressed air *n* in area *i* |
| *KW* | average energy (KW) consumed hourly by the company per month |
| $KWS_j$ | total energy saving (kwh) by system *j* per month |
| *LB* | constant that establishes the lower bound of the contracted load (kwh) per month by PREPA |
| *LBC* | labor cost to implement the photovoltaic system ($) |
| *LC* | load contracted to PREPA per month |
| *LE* | lifespan of the equipment |
| *LP* | length of the solar panels (inches) |
| *MARR* | minimum acceptable rate of return |
| *MDys* | labor days per month |
| *pf* | power factor of the company energy system |
| *RW* | amount of regulars weeks |
| *SAV* | funds available to invest in the photovoltaic system ($) |
| *SH* | length of the space available in the manufacturing company to place the photovoltaic system (inches) |
| *SP* | space between the solar panels (inches) |
| *SUB* | benefits obtained by implementing a photovoltaic system ($) |

| | |
|---|---|
| *SV* | width of the space available in the manufacturing company to place the photovoltaic system (inches) |
| *W* | weeks of work in the pre-established time period |
| *WP* | width of the solar panels (inches) |
| *WPP* | watts of the solar panels |
| *WY* | weeks in one year of work |

- Computed parameters

| | |
|---|---|
| *FP* | amount of solar panels in a photovoltaic system |
| $IAC_{iak}$ | weekly interest generated by the benefits of the air conditioner system *k* in area i subdivision *a* |
| $ICA_{in}$ | weekly interest generated by the benefits of the compressed air system *n* in area *i* |
| $II_{ia}$ | weekly interest generated by the benefits of the illumination system in area *i* subdivision *a* |
| *OC* | cost of the additional material associated with the implemen- tation of the photovoltaic system ($) |
| $SAC_{iak}$ | savings ($ per day) in the air conditioner system *k* in area *i* subdivision *a* |
| $SCA_{in}$ | savings ($ per day) in the compressed air system *n* in area *i* |
| $SI_{ia}$ | savings ($ per day) in the illumination system in area *i* subdivision *a* |
| *ST* | total savings ($ per day) |
| $STAC_{iak}$ | total saving generated by the air conditioner system *k* in area *i* subdivision *a* |
| $STCA_{in}$ | total savings generated by compressed air system *n* in area *i* |
| $STI_{ia}$ | total savings generated by illumination system in area *i* subdivision *a* |
| $SWAC_{iak}$ | weighted average of the benefits generated for the efficient air conditioner system *k* in area *i* subdivision *a* |

$SWCA_{in}$ weighted average of the benefits generated for the efficient compressed air system $n$ in area $i$

$SWI_{ia}$ weighted average of the benefits generated for the efficient illumination system in area $i$ subdivision $a$

$WPS$ watts generated by the photovoltaic system per hour

- The decision variables are defined as follows:

$$AC_{iak}\begin{cases}1 = \text{ if air conditioner system } k \text{ of area } i \text{ subdivision } a \text{ should be replaced}\\ 0 = \text{ otherwise}\end{cases}$$

$$CA_{in}\begin{cases}1 = \text{ if compressed air system } n \text{ of area } i \text{ should be replaced}\\ 0 = \text{ otherwise}\end{cases}$$

$$DAC_{iak}\begin{cases}1 = \text{ if the kw saving generated by the air conditioner system } k \text{ of area } i \text{ subdivision } a\\ \qquad \text{do not affect the lower limit of the demand contracted}\\ 0 = \text{ otherwise}\end{cases}$$

$$DCA_{in}\begin{cases}1 = \text{ if the kw saving generated by the compressed air conditioner system } n \text{ of area } i\\ \qquad \text{do not affect the lower limit of the demand contracted}\\ 0 = \text{ otherwise}\end{cases}$$

$$DI_{ia}\begin{cases}1 = \text{ if the kw saving generated by the illumination system of area } i \text{ subdivision } a\\ \qquad \text{do not affec the lower limit of the demand contracted}\\ 0 = \text{ otherwise}\end{cases}$$

$HP$ amount of solar panels throughout the length of the photovoltaic system

$$I_{ia}\begin{cases}1 = \text{ if illumination system of area } i \text{ subdivision } a \text{ should be replaced}\\ 0 = \text{ otherwise}\end{cases}$$

$VP$ amount of solar panels across the width of the photovoltaic system

## Appendix 2. Justification of selected areas to reduce the energy consumption in a manufacturing company

To select the areas that would be analyzed in the optimization model of energy consumption, a literature review was conducted. In the process of conducting the literature review we could not find a study that showed the distribution of the energy consumed by a company or a study that showed the areas that represent the largest percentage of energy consumed. For this reason we utilized the data available about the energy consumption by an electronic manufacturing company's equipment to analyze the distribution of the energy consumed by companies. Analysis of this data demonstrates that the air conditioner, air compressor, illumination, and exhausted system are the systems that consume more energy by month. The reason why these systems consume large amounts of energy is because these systems are used most of time and consume more kilowatts per hour, in contrast to other equipment. As we can see the variables that make this equipment consume a considerable amount of energy is the kilowatt hours consumed by the equipment and the hours the equipment is used. The kilowatt hours consumed by the equipment is a variable that will not change regardless of the company where they use the equipment. And the variable of hours the equipments are used can vary by company, but this change should not be considerable.

In addition, General Electric Company has a computerized system that monitors and records the data on energy consumption per equipment per hour in different plants. This data also shows that air conditioning systems, compressed air, lighting, and air exhausted are the systems that consume most of the energy per hour. Given this analysis it was decided to select these systems to be analyzed in the optimization model. Table 11 shows the data provided by the electronic manufacturing company.

Table 11. Analysis of the distribution of the energy consumed by an electronics manufacturing company

| Systems | Equipment | Quantity | Hours (monthly) | Kilowatts (monthly) | Cost (monthly) |
|---|---|---|---|---|---|
| Air conditioner system | A/C condenser | 2 | 520 | 66,690.00 | S79,892.80 |
| | A/C condenser | 4 | 520 | 13,202.80 | |
| Air compressor system | Air compressor | 1 | 520 | 21,902.40 | $32,687.20 |
| | Air compressor | 1 | 520 | 10,784.80 | |
| Lighting system | Outside lighting | 45 | 364 | 18,629.52 | $77,244.44 |
| | Outside poles | 9 | 364 | 2,915.64 | |
| | Fluorescent lamp T-12 8’ | 118 | 728 | 13,759.20 | |
| | Fluorescent lamp T-12 8’ | 110 | 728 | 12,827.36 | |
| | Fluorescent lamp T-12 4’ | 160 | 728 | 11,196.64 | |
| | Fluorescent lamp T-8 4’ | 94 | 728 | 6,581.12 | |
| | Fluorescent lamp T-8 8’ | 77 | 728 | 5,387.20 | |
| | Fluorescent lamp T-12 4’ | 70 | 728 | 4,899.44 | |
| | Fluorescent lamp T-8 4’ | 15 | 728 | 1,048.32 | |
| Exhaust system | Exhausts (roof) | 1 | 728 | 2,336.88 | $18,070.06 |
| | Exhausts #1 | 1 | 728 | 2,802.80 | |
| | Exhausts #2 | 1 | 728 | 2,802.80 | |
| | Exhausts #3 | 1 | 728 | 2,802.80 | |
| | Exhausts #4 | 1 | 728 | 2,336.88 | |
| | Exhausts #5 | 1 | 728 | 1,164.80 | |
| | Exhausts #6 | 1 | 728 | 1,164.80 | |
| | Exhausts #7 | 1 | 728 | 1,164.80 | |
| | Deck vacuum | 2 | 515 | 1,493.50 | |
| Production machine | Wave solder #1 | 1 | 347 | 2,470.64 | $14,362.33 |
| | Wave solder #2 | 1 | 347 | 2,880.10 | |
| | Wave solder #3 | 1 | 347 | 2,234.68 | |
| | Wave solder #4 | 1 | 347 | 3,705.96 | |
| | Wave solder #5 | 1 | 347 | 3,070.95 | |
| Manual product equipment | Production equipment | 26 | 347 | 1,852.98 | $6,992.05 |
| | Production equipment | 1 | 347 | 69.40 | |
| | Hand solder | 61 | 347 | 4,070.31 | |
| | Hand solder | 18 | 347 | 999.36 | |
| Offices equipment | Computers | 45 | 174 | 66.12 | $110.72 |
| | Computers | 16 | 174 | 24.36 | |
| | Copies | 2 | 88 | 20.24 | |

## Appendix 3. Optimization model considering the payback method

```
max= (ST)*255 - (ac1151*1119 + ac1121*2990 + ac321*2068  + ca11*36890 + i32*150 );

!air conditioner system;

@bin(ac1151);
@bin(ac1121);
@bin(ac321);

ac1151* ((2.99-1.4)*12*.21) = sac1151;
ac1121* ((5.21-2.6)*12*.21) = sac1121;
ac321* ((9.8-4.7)*12*.21) = sac321;

sac1151*255 >= ac1151*1119;
sac1121*255 >= ac1121*2990;
sac321*255 >= ac321*2068;

!air compressed system;

@bin(ca11);

ca11* ((78.2-55)*12*.21) = sca11;
sca11*255 >= ca11*16689;

!ilumination system;

@bin(i32);

i32* (18*(.032-.017)*8.5*.21) = si32;
si32*255 >= i32*150;


ST = sac1151 + sac1121 + sac321 + sca11 + si32;

end
```

## Appendix 4. Optimization model considering the internal rate of return method

```
max=ac1151*stac1151+ac1121*stac1121+ac321*stac321+i32*sti32+ca11*stca11;

((((1+imarr)^52)-1)-.15)=0;

!air conditioner system;
@bnd(-1,iac1151,1);
((2.99-1.4)*12*.21) = sac1151;
swac1151= ((47*5*sac1151)+(5*4*sac1151))/52;
(@fpa(iac1151,520)*swac1151)-1119=0;
ac1151=@if(iac1151#GT#imarr,1,0);
stac1151=(swac1151*((((1+imarr)^52)-1)/imarr));

@bnd(-1,iac1121,1);
((5.21-2.6)*12*.21) = sac1121;
swac1121= ((47*5*sac1121)+(5*4*sac1121))/52;
(@fpa(iac1121,520)*swac1121)-2990=0;
ac1121=@if(iac1151#GT#imarr,1,0);
stac1121=(swac1121*((((1+imarr)^52)-1)/imarr));

@bnd(-1,iac321,1);
((9.8-4.7)*12*.21) = sac321;
swac321= ((47*5*sac321)+(5*4*sac321))/52;
(@fpa(iac321,520)*swac321)-2068=0;
ac321=@if(iac321#GT#imarr,1,0);
stac321=(swac321*((((1+imarr)^52)-1)/imarr));

!compressed air system;
@bnd(-1,ica11,1);
((78.2-55)*12*.21) = sca11;
swca11= ((47*5*sca11)+(5*4*sca11))/52;
(@fpa(ica11,520)*swca11)-16689=0;
ca11=@if(ica11#GT#imarr,1,0);
stca11=(swca11*((((1+imarr)^52)-1)/imarr));

!illumination system;
@bnd(-1,ii32,1);
(18*(.032-.017)*8.5*.21) = si32;
swi32= ((47*5*si32)+(5*4*si32))/52;
(@fpa(ii32,470.58)*swi32)-150=0;
i32=@if(ii32#GT#imarr,1,0);
sti32=(swi32*((((1+imarr)^52)-1)/imarr));
end
```

**Appendix 5. Optimization model considering the payback method and adding constraints of demand load contracted in PREPA**

```
max= (ST)*255 - (ac1151*1119 + ac1121*2990 + ac321*2068  + ca11*36890 + i32*150 );

!air conditioner system;

@bin(ac1151);
@bin(ac1121);
@bin(ac321);

ac1151* ((2.99-1.4)*12*.21) = sac1151;
ac1121* ((5.21-2.6)*12*.21) = sac1121;
ac321* ((5774.7-4.7)*12*.21) = sac321;

sac1151*255 >= ac1151*1119;
sac1121*255 >= ac1121*2990;
sac321*255 >= ac321*2068;

!air compressed system;

@bin(ca11);

ca11* ((78.2-55)*12*.21) = sca11;
sca11*255 >= ca11*16689;

!ilumination system;

@bin(i32);

i32* (18*(.032-.017)*8.5*.21) = si32;
si32*255 >= i32*150;

!kwh saving per month by systems;
kws1= (ac1151*(2.99-1.4)*20) + (ac1121*(5.21-2.6)*20)+ (ac321*(5774.7-4.7)*20);
kws2= ca11*((78.2-52.13)*20);
kws3= i32*(18*(.032-.017)*20);

!demand load contracted constrian;
(209100-(209100*.6)) <= (192340-(kws1+ kws2+kws3))/.92;

ST = sac1151 + sac1121 + sac321 + sca11 + si32;

end
```

## Appendix 6. Optimization model considering the internal rate of return method and adding constraints of demand load contracted in PREPA

```
max=ac1151*stac1151+ac1121*stac1121+ac321*stac321+i32*sti32+ca11*stca11;

imarr=((((1+imarr)^52)-1)-.15);

!air conditioner system;
((2.99-1.4)*12*.21) = sac1151;
swac1151= ((47*5*sac1151)+(5*4*sac1151))/52;
(@fpa(iac1151,520)*swac1151)-1119=0;
ac1151=(@if(iac1151#GT#imarr,1,0))*dac1151;
stac1151=(swac1151*((((1+imarr)^52)-1)/imarr));
@bnd(-1,iac1151,1);
@bin(dac1151);

((5.21-2.6)*12*.21) = sac1121;
swac1121= ((47*5*sac1121)+(5*4*sac1121))/52;
(@fpa(iac1121,520)*swac1121)-2990=0;
ac1121=(@if(iac1151#GT#imarr,1,0))*dac1121;
stac1121=(swac1121*((((1+imarr)^52)-1)/imarr));
@bnd(-1,iac1121,1);
@bin(dac1121);

((9.8-4.7)*12*.21) = sac321;
swac321= ((47*5*sac321)+(5*4*sac321))/52;
(@fpa(iac321,520)*swac321)-2068=0;
ac321=(@if(iac321#GT#imarr,1,0))*dac321;
stac321=(swac321*((((1+imarr)^52)-1)/imarr));
@bnd(-1,iac321,1);
@bin(dac321);

!compressed air system;
((78.2-55)*12*.21) = sca11;
swca11= ((47*5*sca11)+(5*4*sca11))/52;
(@fpa(ica11,520)*swca11)-16689=0;
ca11=(@if(ica11#GT#imarr,1,0))*dca11;
stca11=(swca11*((((1+imarr)^52)-1)/imarr));
@bnd(-1,ica11,1);
@bin(dca11);

!illumination system;
!vida util de las bombillas 20,000 hrs. 20,000hrs/(8.5*5) = 470 semanas;
(18*(.032-.017)*8.5*.21) = si32;
swi32= ((47*5*si32)+(5*4*si32))/52;
(@fpa(ii32,470.58)*swi32)-150=0;
i32=(@if(ii32#GT#imarr,1,0))*di32;
sti32=(swi32*((((1+imarr)^52)-1)/imarr));
@bnd(-1,ii32,1);
@bin(di32);

!kwh saving per month by systems;
kws1= (ac1151*(2.99-1.4)*20*dac1151) + (ac1121*(5.21-2.6)*20*dac1121)+ (ac321*(9.8-4.7)*20*dac321);
kws2= ca11*((78.2-52.13)*20)*dca11;
kws3= i32*(18*(.032-.017)*20)*di32;

!demand load contracted constrian;
(209100-(209100*.6)) <= (192340-(kws1+ kws2+kws3))/.92;

end
```

## Appendix 7. Optimization model to establish the size of the photovoltaic system

```
max=fp;

sp=18; !inches;

sh=1872; !inches;
sv= 768; !inches;
lp= 39.6; ! inches;
wp= 64.8; ! inches;
fpc=600; !fotovoltaic panels cost;
sub= 0;
lbc=3.75;!labor cost per watts;
invc=2500;!inverter cost;
cr=167;!adicional material cost;
sav=78483;


wpp=230; !in watts;

hp*lp+((hp+1)*(sp))<= sh;

vp*wp+((vp+1)*(sp))<=sv;

fp=hp*vp;

oc=(lbc*fp*wpp)+invc+(cr*fp);

fp*fpc + oc - sub <= sav;

wfs= fp*.15*1527*.85;

@gin(hp);
@gin(vp);

end
```

**Appendix 8. Results generated by Lingo 12 software of algorithm considering the payback method**

```
Global optimal solution found.
Objective value:                              1209.260
Objective bound:                              1209.260
Infeasibilities:                              0.000000
Extended solver steps:                               0
Total solver iterations:                             0

Model Class:                                      MILP

Total variables:                     11
Nonlinear variables:                  0
Integer variables:                    5

Total constraints:                   12
Nonlinear constraints:                0

Total nonzeros:                      32
Nonlinear nonzeros:                   0

                              Variable           Value        Reduced Cost
                                    ST        12.85200            0.000000
                                AC1151        0.000000            97.26600
                                AC1121        0.000000            1312.814
                                 AC321        1.000000           -1209.260
                                  CA11        0.000000            21981.68
                                   I32        0.000000            27.10275
                               SAC1151        0.000000            0.000000
                               SAC1121        0.000000            0.000000
                                SAC321        12.85200            0.000000
                                 SCA11        0.000000            0.000000
                                  SI32        0.000000            0.000000
```

**Appendix 9. Results generated by Lingo 12 software of algorithm considering the internal rate of return method**

```
Local optimal solution found.
Objective value:                              22542.87
Infeasibilities:                            0.1673470E-07
Total solver iterations:                             8

Model Class:                                       NLP

Total variables:                     16
Nonlinear variables:                 16
Integer variables:                    0

Total constraints:                   17
Nonlinear constraints:               17

Total nonzeros:                      41
Nonlinear nonzeros:                  31
```

| Variable | Value | Reduced Cost |
|---|---|---|
| AC1151 | 1.000000 | 0.000000 |
| STAC1151 | 1096.415 | 0.000000 |
| AC1121 | 1.000000 | 0.000000 |
| STAC1121 | 1799.775 | 0.000000 |
| AC321 | 1.000000 | 0.000000 |
| STAC321 | 3516.802 | 0.000000 |
| I32 | 1.000000 | 0.000000 |
| STI32 | 131.8801 | 0.000000 |
| CA11 | 1.000000 | 0.000000 |
| STCA11 | 15998.00 | 0.000000 |
| IMARR | 0.2737187E-02 | 0.000000 |
| IAC1151 | 0.1755713E-01 | 0.000000 |
| SAC1151 | 4.006800 | 0.000000 |
| SWAC1151 | 19.64873 | 0.000000 |
| IAC1121 | 0.1074554E-01 | 0.000000 |
| SAC1121 | 6.577200 | 0.000000 |
| SWAC1121 | 32.25358 | 0.000000 |
| IAC321 | 0.3047593E-01 | 0.000000 |
| SAC321 | 12.85200 | 0.000000 |
| SWAC321 | 63.02423 | 0.000000 |
| ICA11 | 0.1717644E-01 | 0.000000 |
| SCA11 | 58.46400 | 0.000000 |
| SWCA11 | 286.6985 | 0.000000 |
| II32 | 0.1574595E-01 | 0.000000 |
| SI32 | 0.4819500 | 0.000000 |
| SWI32 | 2.363409 | 0.000000 |

**Appendix 10. Results generated by Lingo 12 software of algorithm considering the payback method and adding constraints of demand load contracted in PREPA**

```
Global optimal solution found.
Objective value:                              1209.260
Objective bound:                              1209.260
Infeasibilities:                              0.000000
Extended solver steps:                               0
Total solver iterations:                             0

Model Class:                                      MILP

Total variables:                    14
Nonlinear variables:                 0
Integer variables:                   5

Total constraints:                  16
Nonlinear constraints:               0

Total nonzeros:                     43
Nonlinear nonzeros:                  0

                              Variable           Value        Reduced Cost
                                    ST        12.85200            0.000000
                                AC1151        0.000000            97.26600
                                AC1121        0.000000            1312.814
                                 AC321        1.000000           -1209.260
                                  CA11        0.000000            21981.68
                                   I32        0.000000            27.10275
                               SAC1151        0.000000            0.000000
                               SAC1121        0.000000            0.000000
                                SAC321        12.85200            0.000000
                                 SCA11        0.000000            0.000000
                                  SI32        0.000000            0.000000
                                  KWS1        102.0000            0.000000
                                  KWS2        0.000000            0.000000
                                  KWS3        0.000000            0.000000
```

**Appendix 11. Results generated by Lingo 12 software of algorithm considering the internal rate of return method and adding constraints of demand load contracted in PREPA**

```
Local optimal solution found.
Objective value:                                22542.87
Infeasibilities:                                0.1673470E-07
Total solver iterations:                               8

Model Class:                                         NLP

Total variables:                     16
Nonlinear variables:                 16
Integer variables:                    0

Total constraints:                   17
Nonlinear constraints:               17

Total nonzeros:                      41
Nonlinear nonzeros:                  31
```

| Variable | Value | Reduced Cost |
|---|---|---|
| AC1151 | 1.000000 | 0.000000 |
| STAC1151 | 1096.415 | 0.000000 |
| AC1121 | 1.000000 | 0.000000 |
| STAC1121 | 1799.775 | 0.000000 |
| AC321 | 1.000000 | 0.000000 |
| STAC321 | 3516.802 | 0.000000 |
| I32 | 1.000000 | 0.000000 |
| STI32 | 131.8801 | 0.000000 |
| CA11 | 1.000000 | 0.000000 |
| STCA11 | 15998.00 | 0.000000 |
| IMARR | 0.2737187E-02 | 0.000000 |
| IAC1151 | 0.1755713E-01 | 0.000000 |
| SAC1151 | 4.006800 | 0.000000 |
| SWAC1151 | 19.64873 | 0.000000 |
| IAC1121 | 0.1074554E-01 | 0.000000 |
| SAC1121 | 6.577200 | 0.000000 |
| SWAC1121 | 32.25358 | 0.000000 |
| IAC321 | 0.3047593E-01 | 0.000000 |
| SAC321 | 12.85200 | 0.000000 |
| SWAC321 | 63.02423 | 0.000000 |
| ICA11 | 0.1717644E-01 | 0.000000 |
| SCA11 | 58.46400 | 0.000000 |
| SWCA11 | 286.6985 | 0.000000 |
| II32 | 0.1574595E-01 | 0.000000 |

**Appendix 12. Results generated by Lingo 12 software of algorithm that establish the size of the photovoltaic system**

```
Global optimal solution found.
Objective value:                              46.00000
Objective bound:                              46.00000
Infeasibilities:                              0.000000
Extended solver steps:                               1
Total solver iterations:                           285

Model Class:                                     MINLP

Total variables:                      5
Nonlinear variables:                  2
Integer variables:                    2

Total constraints:                    7
Nonlinear constraints:                1

Total nonzeros:                      12
Nonlinear nonzeros:                   2

                             Variable           Value        Reduced Cost
                                   FP        46.00000            0.000000
                                   SP        18.00000            0.000000
                                   SH        1872.000            0.000000
                                   SV        768.0000            0.000000
                                   LP        39.60000            0.000000
                                   WP        64.80000            0.000000
                                  FPC        600.0000            0.000000
                                  SUB        0.000000            0.000000
                                  LBC        3.750000            0.000000
                                 INVC        2500.000            0.000000
                                   CR        167.0000            0.000000
                                  SAV        78483.00            0.000000
                                  WPP        230.0000            0.000000
                                   HP        23.00000           -2.000001
                                   VP        2.000000           -23.00000
                                   OC        49857.00            0.000000
                                  WFS        8955.855            0.000000
```

## References


*Advantages and disadvantages of photovoltaics*. (n.d.). Retrieved March 25, 2011, from CET Online: http://www.cetonline.org/Renewables/PV_pro_con.php

Aidan Berry, R. J. (2006). *Accounting in a business context.* Thomson.

Baños, R. (2011). Optimization methods applied to renewable and sustainable energy: A review. *Renewable and Sustainable Energy Reviews*, 1753-1766.

Canada, J. R. (2005). *Capital Investment Analysis for Engineering and Management.* Prentice Hall.

*Capital budgeting decisions*. (n.d.). Retrieved March 27, 2011, from Accounting for management : http://www.accountingformanagement.com/pay_back_method_of_capital_budgeting_decisions.htm

*Climate change - Greenhouse gas emissions*. (2010, March 03). Retrieved March 28, 2011, from U.S. Enviromental protection agency : http://www.epa.gov/climatechange/emissions/co2.html

Cooper, W. D., Morgan, R. G., Redman, A., & Smith, M. (2001). *Library: Business Publications*. Retrieved 2 15, 2011, from Business Publications: http://findarticles.com/p/articles/

Cullen, R.A. (n.d.). What is Maximum Power Point Tracking (MPPT) and How does it work? Retrieved 3 21, 2011, from http://www.blueskyenergyinc.com/uploads/pdf/BSE_What_is_MPPT.pdf

Fares, A. E (2008) *Análisis económico para la implementación de la tecnología fotovoltaica a residencias en el área sur y suroeste de Puerto Rico.* Tesis sometida en cumplimiento parcial de los requerimientos para el grado de Maestro en Administración de Empresa, Universidad de Puerto Rico, Recinto Universitario de Mayagüez. Puerto Rico.

Flores, J. F. (2007). Industria Eléctrica de Puerto Rico: Desarrollo innovador e integración al mercado eléctrico mundial . *COPIMERA* . Perú.

Hren, S. a. (2010). *A solar buyer's guide for the home and office.* Vermont: Chelsea Green Publishing.

Jorge Nocedal, S. J. (1999). *Numerical Optimization* . New York: Springer Science Business Media, Inc.

Kutz, M. (2007). *Enviromental Conscious:Alternative Energy Production.* New Jersey, USA.

Ladner, H.P (2009) *Photovoltaic based distributed generation as a demand response strategy in Puerto Rico.* Tesis sometida en cumplimiento parcial de los requerimientos para el grado de Maestro en Ciencias Ingeniería Eléctrica, Universidad de Puerto Rico, Recinto Universitario de Mayagüez. Puerto Rico.

*La inminente escasez de petroleo mundial*. (2009, April 13). Retrieved March 28, 2011, from www. pasajero.com: http://www.pajareo.com/2834-La-inminente-escasez-del-petroleo-mundial.html(n.d.).

*Ley de Incentivos de Energía Verde de Puerto Rico y enmienda la Ley Núm. 70 de 1978; Ley de Desperdicios Sólidos y la Ley Núm. 120 de 1994; Código de Rentas Internas.* (2010, July19). Retrieved March 25,2011, from LexJuris Puerto Rico: http://www.lexjuris.com/lexlex/Leyes210/lex12010083.htm

Local Optima vs. Global Optima. (n.d.). *LINGO 12 online users manual*.

O'Grady, P. (2011). Arizona 4th in photovoltaic installations . *Phoenix Business Journal*

Ordorica-García, G. (2008). Energy Optimization Model with $CO_2$ Emission Constraint for the Canadian Oil Sands Industry. *Energy & Fuels ,* 2660-2670

Ortadis, B. (2007). A non-linear programming model for optimization of the electrical energy consumption in typical factory. *Applied Mathematics and Computation*, 944-950.

Park, C. S. (2007). *Contemporary engineering economics.* New Jersey : Prentice Hall.

*Puerto Rico Energy profile*. (n.d.). Retrieved December 27, 2009, from U.S Energy Information Administration Independent Statistics and Analysis: http://tonto.eia.doe.gov/country/country_time_series.cfm?fips=RQ#co2

Scanlon, B. (2011, February 16). *Feature News.* Retrieved March 9, 2011, from NREL: National Renawable Energy Laboratory: http://www.nrel.gov/features/20110216_low-cost_solar.html

*Solar panels.* (2011,March 9). Retrieved March 25,2011 , from getsolar.com: http://www.getsolar.com/News/Colorado/Solar-Paneles/Colorado-State-University-Researchers-Work-to-Improve -Solar-Panels-Efficiency---800453457

*standard test conditions (STC)*. (n.d.). Retrieved April 2, 2011, from The encyclopedia of alternative energy and sustainable living: http://www.daviddarling.info/encyclopedia/S/AE_standard_test_conditions.html

*Statistics on Puerto Rico.* (2010, July 14). Retrieved March 26, 2011, from U.S. Energy Information Administration : http://www.eia.gov/countries/country-data.cfm?fips=RQ#underfined

Sthepanie. (2011, February 24). *Solar panels.* Retrieved March 9, 2011, from Solar power - PV system: http://solarpowerpanels.ws/solar-panels/solar-panel-efficiency-increased-with-solar-pole-tracker

*Sunshot inititive.* (2011, March 3). Retrieved March 25, 2011, from U.S. Department of energy: http://wwwl.eere.energy.gov/solar/sunshot/

Taha, H. A. (2007). *Operations research: an introduction.* Upper Sandle River, NJ: Pearson Education, Inc.

Thomas, J. (n.d.). *Science & technologie.* Retrieved March 9, 2011, from Treehugger: http://www.treehugger.com/files/2007/04/stateoftheart_m.php

Winston, W. L. (2004). *Operations research application and algorithms* . California : Thomson Learning.

(2000, June.). Retrieved March 28, 2011, from Tarifas para el servicio de electricidad: http://www.aeepr.com/DOCS/manuales/LibroTarifas02.pdf