TOWARDS OPTIMIZATION BY SIMILARITY: FINDING WINDOWS OF MAXIMAL SIMILARITY

by

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Abstract

This work proposes a method which uses a Window of Maximum Similarity (WMS) to find a region of similarity between two responses, one of them with known and desirable characteristics. The WMS method is one of minimization of squared errors and can be used to explore experimentally or pseudo-experimentally generated data to find at least a WMS. This method is a viable element that will serve for the future development of the Optimization by Similarity method. The progressive development of the WMS method and a series of examples are presented to show its feasibility and capability for generating a two-dimensional WMS. Data from real time series served as a basis to generate a one-dimensional WMS. Given that this work corresponds to the initial development of the proposed method, we believe that the results obtained signals to a useful tool for data exploration of interest to detect zones with distinctive patterns.

Resumen

Este trabajo propone un método que usa una Ventana de Máxima Similaridad (WMS por sus siglas en inglés) para encontrar una región de similaridad entre dos respuestas, una de ellas con características conocidas y deseables. El método de WMS es uno de minimización de errores cuadrados que puede ser usado para explorar datos generados seudo o experimentalmente para encontrar al menos una WMS. Este método es un elemento viable que servirá para el futuro desarrollo del método de Optimización por Similaridad. El desarrollo progresivo del método de WMS y una serie de ejemplos son presentados para mostrar su factibilidad y capacidad generando una WMS de dos dimensiones. Datos provenientes de series de tiempo reales sirvieron como base para generar una WMS de una dimensión. Dado que este trabajo corresponde al desarrollo inicial del método propuesto, creemos que los resultados obtenidos apuntan a una herramienta útil para exploración de datos de interés para detectar zonas con distintos patrones.

To my family.

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Chapter 1

1.1 Introduction

Within the area of modeling, it is possible to think about two basic ideas: when available data is used to generate models (fit a model) or when simulated data from a model is used to generate a metamodel (fit a metamodel). A Metamodel is, indeed, a model fitted to data generated with another model (simulation, mostly). These models consist of an approximating function, and commonly, they require experimental designs. Metamodeling techniques require finding the model parameters that result in the most competitive fit to the available experimental dataset, typically minimizing an error function.

On the other hand, it could be of interest to find a region of data such that matches a model with desirable characteristics. This idea leads to finding the experimental region where a model with desirable characteristics is a good descriptor of data. The premise of finding the experimental region where a model fits best - Inverse Metamodeling – was reported in (Rivera-Nazario and Cabrera-Ríos, 2013).

The method proposed in this thesis presents the use of windows of maximum similarity to find the region with maximal similarity between two responses. This method includes techniques like inverse metamodeling and optimization. These techniques combined with the similarity concept under our proposed approach will allow the generation of a similarity region between two functions. The WMS method proposed here has been developed as a viable element for future Optimization by Similarity. The Optimization by Similarity method proposes that a characterization of how an optimal solution should look like can be used to explore experimentally or pseudo-experimentally generated data to find at least a local optimum. Also, the WMS method can be useful as data exploration tool.

1.2 Problem Description

The problem to be addressed in this thesis is to find the region of maximum similarity through the use of WMS. It entails matching one function with desirable characteristics to another function (or data) that is not well characterized in such terms. The aim is to find the region where the uncharacterized function resembles the well-characterized one. This aim, if fulfilled, will reveal areas of potential maximum similarity that were previously unknown.

The concept can be generalized to matching one function with well-established properties of interest to another that is uncharacterized in such terms.

1.3 Motivation

The main motivation of this work is to develop a new and original method that helps to define a region of similarity delimited by a Window of Maximum Similarity (WMS). Basically, the WMS represents a region of interest where a collection of data might resemble a function with desirable characteristics. The proposed method can be used to explore experimentally or pseudo-experimentally generated data to find at least one WMS match. The WMS method will serve as a basis for future development of the Optimization by Similarity method.

1.4 General Objective

The objective of this thesis is to find the experimental region where a model with desirable characteristics is a good descriptor of the data at hand. Initially, the ‘data at hand’ comes from another model or mathematical function. In particular, the use of a WMS to find regions where a model is a good descriptor will be demonstrated.

1.5 Thesis Organization

To show the development of this work, the thesis is organized as follows:

In Chapter 1, the introduction was presented.

In Chapter 2, the literature review of the important issues related to this work is discussed.

In Chapter 3, the proposed method is described. The subsections of this chapter include the description of three initial cases of interest, a description of the development of the proposed approach using the initial cases of interest, and an explanation of the proposed WMS method.

Chapter 4 presents the evaluations of the method using the function *AOG 1*, an original development.

In Chapter 5, the use of windows of maximum similarity to find regions of interest in time series is presented. This chapter includes an extensive set of evaluations obtained from the use of windows of maximum similarity a) with the minimum size predefined, b) with a window where the size was automatically generated, and c) with windows from a 3D projection of the series. The results for all these cases are discussed.

Chapter 6 shows an application of the proposed method using unconstrained global optimization test functions and their respective results. These instances prepare the field for the future testing of optimization by similarity.

Finally, Chapter 7 describes the limitations of the method, the conclusions, the future work, the contribution of the thesis, and the potential application in other areas.

Additionally, the appendices are included at the end of this thesis.

Chapter 2

In this chapter, the literature review of the thesis is presented. Information about metamodeling techniques and similarity applications are presented as well.

2.1 Literature Review

Many studies to fit empirical models can be found in the literature. According to Santos and Santos (2009) a metamodel replaces the simulation model by a simplified input-output relationship, frequently a mathematical function with customized parameters. In addition, building the metamodel is based on a set of design points resulting from a simulation. Many metamodels are commonly used in practice, such as in the use of linear regression. For information related to regression models, it is highly suggested to consult Montgomery (2010). For more extensive information about metamodels in general, we suggest consulting (Barton, 1998; Shin et al., 2002; Cheng and Currie, 2004; Villarreal, 2007; Gunes et al., 2008; Villarreal et al., 2008; Barton, 2009; Liu and Staum, 2009; Santos and Santos, 2010).

Comparative studies of metamodeling techniques have been reported in the literature. For example, in Jin et al. (2000) a comparative study of various metamodel techniques based on multiple performance measures was carried out. The metamodels included polynomial regression, radial basis functions, Kriging models and multivariate adaptive regression splines (MARS). Also, the authors used test function problems and a real problem in engineering such as the vehicle design, to evaluate the performance of the metamodels. The authors recommended that the polynomial regression should be implemented first to determine if a reasonable fit can be generated. In Li et al. (2010) the authors compared polynomial regression with MARS, Kriging, Artificial Neural Networks, Radial Basis Functions and Support Vector Regression (SVR) for simulation systems optimization to aid the decision making process. SVR was the best method in terms of prediction accuracy and robustness, according to the results of comparison using simulation optimization problems. The authors then proposed a general optimization framework GA-META which integrates Genetic Algorithms and any metamodels to improve the efficiency of the decision making process illustrating in the Decision Support System a job shop simulation problem.

Among the most recent studies of metamodeling, we can find Cancelliere et al. (2013). The authors developed a neural network-based prediction model of the energy produced by the wind turbines of a wind farm. The authors compared the performances of perceptron neural network, trained and tested with the classical backpropagation algorithm and with the Extreme Learning Machine (ELM) method. Better performances were obtained by using the ELM method.

Lin et al. (2011) used a combination of Taguchi methods and experimental design method (DOE) to find the optimum parameters and architecture of a neural network. The first method is used to find the important factors. The second method is used to find the precise combination of the values of important design parameters. The application they presented focused in distinguishing the types of vibration modes during engagement in the derailleur chain system on a bicycle.

Peralta-Donate et al. (2013) proposed a scheme of adaptive neural network for time series forecasting (ADANN). This scheme builds ensembles of networks from which the best design network (architecture) is obtained. This choice is based on the minimization of Mean Squared Error (MSE) and the cross-validation method. Experimentation is carried out on six real series (monthly sales of paper, monthly airline passengers, ozone concentration, monthly air temperature, Dow-Jones index monthly closings, and daily IBM closing stock prices). The forecast performance is evaluated with Symmetric Mean Absolute Percentage Error (SMAPE) forecasting from 1 to 19 periods in the future and they compare the proposed approach with the Holts-Winters method finding competitive results with their approach. They conclude that their proposed scheme of the n-fold artificial neural network ensemble provides accurate forecasts for their intended applications.

An important technique in the universe of metamodeling is the Response Surface Methodology (RSM). This method consists of a collection of mathematical and statistical techniques useful for the modeling and analysis of problems in which a response of interest is influenced by several variables and the objective is to optimize this response (Montgomery, 2010). Recently, many works have been reported on the use of this technique. Experimental designs in RSM (Dette and Grigoriev, 2014), RSM in the surface roughness analysis (Jayswal and Taufik, 2011), RSM in the optimization of manufacturing turning process (Saini and Parkash, 2014) are just a few.

From a system point of view, in RSM, known input variables undergo a process, and then, an expected response (output) is represented by the response surface. That metamodeling task gives as output a response surface that depends on a set of independent variables. The method proposed in this thesis takes a different idea. The WMS method is developed under the concept of inverse metamodeling in which a region is generated as an output. A comparison of both metamodeling techniques will be show in Appendix E.

In summary, from the literature review about metamodeling studies, including the comparative studies, these studies have worked under the traditional focus of adjusting of a model to available data, sample data from a DOE, or simulated data. In addition these works include techniques such as simulation and optimization. In this thesis, metamodeling works in a different manner.

Inverse metamodeling finds experimental regions where a model fits best (Rivera-Nazario and Cabrera-Ríos, 2013). In addition, this thesis will use polynomial regression and test functions to illustrate the proposed methodology.

Rivera-Nazario and Cabrera-Ríos (2013) reported the premise that an inverse metamodel is the experimental area where a metamodel fits best. Their study presented an application of a common problem in the modeling of polymers: the relationship between deformation and viscosity. The problem of the study was to find the region of the rheological data where a first order linear regression fits well. Data was analyzed with three methods: 1) Baseline method, 2) Fixed-starting-point linear regression, and 3) Moving-starting-point linear regression. The importance of this work lies in providing one of the first recognizable attempts to systematically and objectively find a data region where a model best explains the results through local adjustments, as opposed to a global fit. The methods relied heavily on enumeration, which shall be avoided in this work.

Barton (2005) described a combined forward and inverse metamodeling strategy. Metamodels are often used to identify system design parameters that result in the target measure of performance, i.e., from design parameters towards the performance (forward metamodels). The inverse metamodel idea is a mapping from system performance specifications (customer-driven technical specifications) to process design parameters. Under this approach, a desired performance (or multiple performance measures) will help the design parameters. Also, the strategy reported by this author included the use of optimal experimental designs for fitting metamodels. Aungst et al. (2001) cited in Barton (2005) presented an integrated design method that uses qualitative information and statistical models to map customer needs to technical requirements in the product/process design. From Barton’s work it is possible to identify that the inverse metamodeling approach differs from the inverse modeling used in this thesis, because his research work described an inverse methodology mapping, (from the target performance or multiple performance measures that the customers need to technical requirements of the design parameters), that depends on the *invertibility* of the simulation mapping using the Jacobian Matrix. In this thesis, under the premise of inverse metamodeling where data is explored to find the experimental regions where a model fits best (premise reported in Rivera-Nazario and Cabrera-Ríos, 2013), we propose a method that uses WMS to determine the experimental region with maximal similarity between two responses (available data) or functions. Optimization problems are modeled to determine the experimental region of maximum similarity. These optimization problems can automatically generate the windows, or the window size can be controlled by the user.

Using the customer-driven design of systems or products approach (the identification of system design parameters that produces a target performance vector), Barton (2006) discussed a maxi-min algorithm to find experimental designs using forward and inverse metamodels. The author mentioned the idea of generating experimental designs that allow simultaneous fitting of both forward and inverse metamodels. To build the inverse metamodels, the author incorporated the maxi-min experiment designs. This maxi-min strategy maximizes the minimum distance between any two points in the experiment design space. One of the examples used to apply the maxi-min strategy was a Semiconductor Freescale model. The author mentioned that this example would allow to explore the cost/on-time-delivery performance space, and to choose a Pareto-optimal operating condition. Having as design parameters the die inventory level and the reduction in front-end lead time, the inverse metamodel provided the values of these variables to achieve their performance and cost objectives. In his work, it was mentioned that issues such as the *invertibility* of map on performance measures and experimental regions were detailed in Barton (2005). The premise of inverse metamodels used in this thesis consists in finding experimental areas where a model is a good descriptor. Also, the proposed method in this thesis adds the novelty that data regions of our interest to be detected are delimited by a window of maximum similarity (WMS). An optimization model conveniently coded in a spreadsheet finds the WMS. This optimization model can generate a WMS with a minimum size user-defined by the assignation of size constraints in the model, or a size automatically generated by the formulation of a composite objective function.

Couckuyt et al. (2010) proposed a method to inverse surrogate modeling. The focus of the forward problem and inverse problem is described. Typically, in a forward problem it is of interest to find the optimal performance characteristics of the simulation system (output) given the input parameters. In an inverse problem the focus is on exploring the input parameters: given the known desired performance to find the associated design parameters. In the work of these authors it was mentioned that given that the intricacies stated in Barton (2005) the inverse metamodel is often reduced to the task of finding one (or more) input parameter combinations for a certain output characteristic. Solving inverse problems by the identification of the regions in the input space that corresponds to the desired output value (or values) was focused in the Couckuyt et al.' work. Their method is a sequential design step in surrogate modeling that efficiently samples the input regions in a quasi-uniform way. To show their work, the authors used the Gaussian Process based on Kriging and the Generalized Probability of Improvement (gPoI), an extension of Probability of Improvement statistical infill criteria used to measure how interesting a data point is in the input space. Besides gPoI criterion, the authors used the Euclidean distance (maximin distance criterion) in their strategy. In addition, the implementation of the strategy was carried out in Matlab SUrrogate MOdeling (SUMO) Toolbox and they illustrated their method in Branin function. The inverse metamodeling approach in Couckuyt et al. is similar to the inverse modeling premise handled in this thesis. However, we can distinguish that our proposed WMS method attempts to be applicable to detect zones/regions with a pattern of interest in different kinds of data. Conveniently, the proposed method in this thesis has been coded in a spreadsheet, keeping low computational resources. Also, the proposed method does not consider the use of probability density function (PDF); Gaussian PDF was used in Couckuyt et al. (2010). The method proposed in this thesis gives the flexibility to control the minimum WMS size by the optimization problem formulation or automatically generate the WMS size as well. We believe that the WMS method for future optimization by similarity will serve as a useful tool for data exploration.

For our research work, an important concept is similarity. Most papers in the literature focus on studying the similarity in applications that use electronic images in their assessments, recognize patterns in geographic data, and identify genetic similarity (Sharma M. et. al, 2014), to name a few.

Winter (2000) used the location and location-based similarity as a reference frame to measure similarity. Motivated on the similarity concept in terms of space and geographic information systems (GIS), he assumed that spatial entities in databases (regions) are models of real world objects. In addition, the author hypothesized that in the comparison of the location of two regions from different data sets, both are modeling the same object. The author talked of *similarity* as common location and *dissimilarity* as distinct location. Basically, the systematic investigation of Winter's work was applied to measures of similarity of two discrete regions from different datasets. The location of regions, a function of coordinates in a given geometry, was the only aspect considered as a similarity measure. Criteria for the location-based similarity measures included the symmetry, normalization, and freedom dimension. Winter's work was carried out as follows: he introduced the concept of location as a reference frame to the similarity of the regions; next, he included region intersections and defined preconditioned ratios as similarity measures; finally, tests were performed to establish the similarities between the two regions. In his work, Winter also characterized different similarity measures and specific behavior. Although Winter's work incorporated some concepts that can be similar in our proposed method, we can distinguish that Winter described his research based on topological relations in intersection sets, and considered similar and dissimilar location-based similarity measures. In this thesis, we approached similarity between two responses found through an optimization model that minimizes an error function. Here, the similarity region is delimited by a window of maximum similarity whose size can be automatically generated or user-defined manually.

In Galal et. al (2012) they developed learnable hyperspectral measurements from a static similarity threshold with which they recognize if the spectrum is similar or different. These hyperspectral measures help capture the degree of similarity between two spectra. Some of these measures are: spectral correlation, information divergence, Euclidean distance, and Pearson correlation coefficient, among others. The authors also proposed two patterns of similarity which are classified using the support vector machine (SVM). For analysis, an airborne visible infrared imaging spectrometer (AVIRIS) image (area of mixed agriculture and forestry) was the dataset used. The experiments are conducted under two classification approach features and an additional combined approach the authors proposed. As a conclusion, the authors indicated that their proposed method, in its different versions, is able to capture the specific aspect of similarity appropriate for each spectral region. In the work of these authors, they focused on similarity measuring of hyperspectral images using spectra information. In this thesis, the focus is on data generated from functions or available data that do not depend on spectra information. Our proposed method attempts to be an exploratory tool of data that can be applicable to different kinds of data.

Hsu et al. (2012) proposed an approach to estimate the road region from images captured by a vehicle-mounted monocular camera. In this approach, road region candidates of intensity similarity image (ISI) using the statistical feature analysis (SFA) and breadth-first search (BFS) algorithm are generated. Then, spatial and texture information of these region candidates are used to classify road regions using similarity measures such as media, standard deviation, and entropy of a square neighborhood around each pixel. The road regions are identified by voting scores for these similarity measures. In addition, a metric derived from the Bhattacharyya distance (a metric to measure similarity of two discrete or continuous probability distributions) is used to express the similarity between a road model obtained by drivers selected and the road region candidates. The authors mentioned that, according to their results, the proposed approach can detect road regions in road scenes. From Hsu et al., the statistical similarity measures were used to calculate similarity distance for each pixel from image. In this thesis, data generated from functions or available data is used to find the region with the maximum similarity between two responses that will be generated by a window and that do not depend on the images' information.

Luo et al. (2012) proposed a region-based image fusion method. The method consists of three steps: correlation analysis where similarity maps are generated, region generation (regional segmentation) for partitioning the similarity maps into regions, and fusion. The segmentation process is operated on the similarity characteristics of source images. The similarity characteristics of source images included luminance, contrast, and structure characteristic comparisons as part of the structural similarity index (SSIM) method. The authors compared their method with six image fusion methods. According to experimental results, the method proposed achieved better results over previous image fusion methods. In the work of these authors information from source images was used to evaluate similarity characteristics. These characteristics of similarity are different from the similarity concept used in this thesis. The proposed method uses windows of maximum similarity to find regions where a model fits best using least squares.

In the field of computer vision task, Liu and Zeng (2012) proposed two new image descriptors based on Local Self-Similarity (LSS) texture feature and Cartesian location grid. These two new interest region descriptors are used for image matching tasks. Through extensive experiments on the INRIA Oxford Affine dataset using structured images with different geometric and photometric transformations, they studied the performance of these descriptors. The Euclidean distance was one measure used to measure the results of matching experiments. These results indicated that the proposed descriptors can yield more stable and robust results. From this work, the authors defined the similarity for image matching. This approach is different from our proposed method because the WMS method focuses on the use of windows with maximum similarity to find regions in experimentally or pseudo-experimentally generated data where a model fits best. The WMS method is not oriented in the construction of image descriptors.

In Marcello et al. (2007), region matching techniques were used in the estimation of the ocean surface motion. Metric variants of the sum of absolute differences, sum of square differences, and cross- correlation were used in the region matching techniques. The authors approached the maximum similitude by using images in their evaluations. That approach is different to the similarity concept handled in this thesis. The proposed WMS method focuses on generating a region of experimental data where a model fits best considering an optimization approach.

Cubillos-Buitrago (2012) reported the use of a method of windows of maximum similarity in the comparison of maps. The evaluations included simulations of the effects of forest loss and the land cover change in the population dynamics of the *Panthera Onca* based on cartographic material. The windows method that the author reported was focused on land cover maps of simulated scenarios. The windows were defined in terms of pixels and were used to compare the similarity in the maps. The method reported for the author had a different approach to the WMS method in this thesis. The proposed WMS method in this thesis uses windows of maximum similarity to find experimental data regions with patterns of interest considering an optimization approach. Comparisons of maps were not used in the evaluations of this thesis. Optimization test functions, polynomial regression, and time series data has been used in the evaluations of this thesis.

In the area of digital image processing, Coronado-Chacón (2001) used sensor images in the identification of secondary forest cover land. Within the method of supervised classification, homogenous samples (training areas) are identified in the image under study. Information categories of interest are identified from samples. One of the methods of supervised classification that the author considered was the method of maximum similitude. This method determines probability functions to the spectral signature and classifies the pixels included in the training areas based on the nearness to the spectral signature. This approach compared to the WMS method in this thesis is distinct because sensor images or probabilities were not elements considered. This thesis approaches similarity between two responses found through an optimization model that minimizes an error function.

2.2 Two techniques used in this thesis

Least squares

The least squares method is used typically to estimate the coefficients in a model regression. Coefficients of regression are selected by the minimization of sum of squared errors (SSE). SSE is given by equation 1. In this equation, (Y_i) is the real response and (Z_i) is the predicted response of the metamodel (Montgomery, 2010). Detailed information about regression can be found in the same reference. For this thesis, Y_i is the response of the function to approximate (data) that needs to be optimally addressed and Z_i is the response of the model or function to superimpose which has known and desirable characteristics (well-established optimality properties).

$$SSE = \sum_i (Y_i - Z_i)^2 \tag{1}$$

Multiple starting points

In order to increase the chance of finding an attractive solution close to the global optimum, the multiple starting points technique, a heuristic method, is frequently used. When a local optimization method is used, this method is executed many times considering different starting points. In this manner, it is possible to increase the chance to converge to a very competitive solution (Frontline Systems Inc., 2015a).

Chapter 3

This chapter presents the progressive construction of the method, the development of the proposed approach using initial cases of interest, and, finally, presents the step-by-step proposed method to this point.

3.1 Three initial cases of interest

In this section, three instances are shown to explain the progressive construction of the method. Each trial considered two functions, a fixed function (of interest) and another function not optimally characterized (the function to superimpose).

Trial 1. *Function Y sampled at specific points and function G is a fixed function*. The experimental points in *Y* are matched with the fixed function *G* to determine where they are similar. This trial will show us if the method can generate a window of maximum similarity (WMS) considering a fixed function across fixed data.

Equations 2 and 3 show the form of functions *Y* and *G* respectively. There are a total of 121 points in the experimental region (discretizing in this manner) of function *Y* in a square grid configuration, with each variable ranging from -5 to 5.

$$Y = 10x_1^2 + 10x_2^2 \tag{2}$$

$$G = 10x_1 + 10x_2 \tag{3}$$

Trial 2. *Function Y sampled at specific points and function H is movable.* This trial follows from Trial 1 except that the function to superimpose, function *H*, has the coefficients as variables to be determined. Function *H* (in the form of a hyperplane) is given by equation 4. The function to approximate is the same as Trial 1 in equation 2. There is a total of 121 points in the experimental region.

$$H = \beta_0 + \beta_1 x_1 + \beta_2 x_2 \tag{4}$$

This trial will show us if the method can generate the WMS considering a movable function across fixed data.

Trial 3. *Function Y sampled at specific points and has a target region, and function I is movable.* The experimental region is given by the approximated function in equation 2, in a range of [-5, 5], adding a cutting plane given by equation 5, which contains 30 of the 121 total points in the experimental region. The cutting plane is delimited on x_1 to fall in the range [0, 5] and x_2 in the range [0, 4]. Function *I*, in the form of a hyperplane, is the function to superimpose, and is expressed mathematically by equation 6. There is also a total of 121 points in the experimental region of the function to superimpose *I*.

$$Y_{cuttingplane} = 40x_1 + 40x_2 \tag{5}$$

$$I = \beta_0 + \beta_1 x_1 + \beta_2 x_2 \tag{6}$$

The idea in Trial 3 is to verify that the method finds the region of the cutting plane (equation 5), as the movable function (equation 6) has the shape of a hyperplane.

3.2 Development of the proposed approach using the initial cases of interest

This strategy was developed incrementally, first experimenting with formulations with one objective function and then moving on to establish three-objective functions in a composite function. When using one objective, the minimum size of the WMS is defined by the user. The subjectivity of this approach is alleviated by the three-objective formulation, which automatically determines the size of the WMS. In the next sections, both approaches will be illustrated through the three Trials detailed previously.

3.2.1 One objective approach

The one objective approach consists on determining the size of the WMS with minimum dimensions set by the user. The x_1^U, x_1^L, x_2^U and x_2^L are (decision) variable bounds that define the size of the WMS. These bounds are found through the minimization of the sum of squared errors (SSE) as a means to maximize similarity. The three trials detailed above are used next with this approach.

Trial 1. Function Y sampled at specific points and function G is a fixed function

The objective of this evaluation is to minimize the SSE of the two regions by finding the limits of the WMS.

As the first step (i), we identify the function to approximate, *function Y* (equation 1). Next, (ii) we define the function to superimpose, *function G* (equation 3), as specified in section 3.1.

(iii) We consider a total of 121 points in the experimental region (grid) for both *Y* and *G* functions. The experimental region serves as the space where the WMS will adjust its size.

In step (iv), a False-True logic test is created to assign these values for each combination of the variables x_1 and x_2 in the experimental region.

In step (v), SSE values are obtained for each combination in the grid and (vi) another False-True logic test is created to filter the SSE values for the WMS. If the False-True logic test in (iv) is true, a value of 1 will multiply the SSE value obtained in (v). If false, a value of 0 will multiply the SSE value.

Optimization problem (7) was formulated for Trial 1 as step (vii), to find the value of the variables where a region of similarity of two functions exists. The optimization problem was solved using Excel Solver, a local optimizer included in MS Excel. The default Solver parameters were used. MS Solver uses the Generalized Reduced Gradient (GRG) algorithm to solve non-linear optimization problems, and it uses the Branch and Bound method to solve mixed-integer and constraint programming problems (Frontline Systems Inc., 2015a and 2015b).

The objective of the optimization problem is to minimize the SSE considering the bounds for x_1 and x_2 as continuous variables to delimit a WMS, where x_1^L and x_1^U are lower and upper bounds for x_1, while x_2^L and x_2^U are lower and upper bounds for x_2, corresponding to the dimensions of the WMS.

In addition, a *range* of values was defined as well as an *epsilon* value. The ranges for x_1 and x_2 were both [-5, 5]. The epsilon value will be the distance between lower and upper bounds, for each variable. Also, the epsilon values represent the minimum size of the WMS and they are user-defined. For this instance, the values of ε_1 and ε_2 take a value of 2. The difference between lower and upper bounds was restricted to be greater than or equal to epsilon.

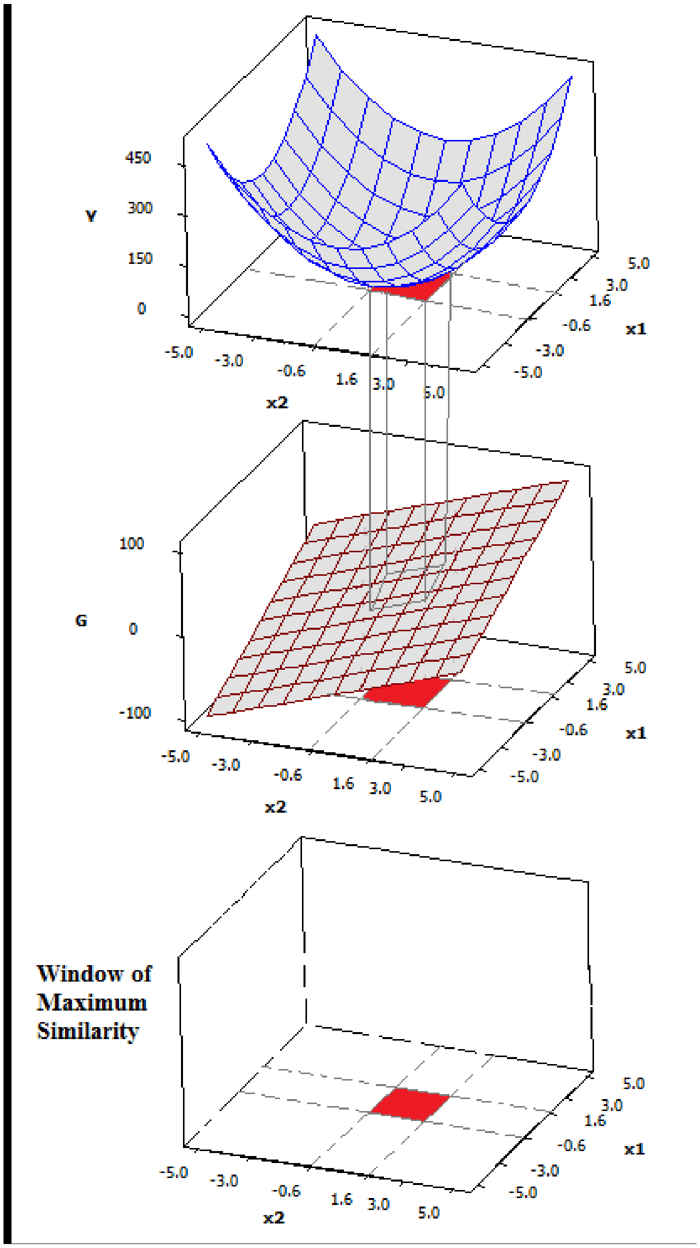
In step (viii) the optimization problem is solved. To increase the probability of finding a competitive solution, the initialization was carried out using multiple starting points, a total of ten in this instance.

Finally, in step (ix), the results are reported (see Table 1). Table 1 is organized as follows: first, the best objective value (minimum SSE) is presented, followed by the best solution (boundary values that delimit the WMS), the fixed parameters previously defined and finally, the dimensions of the WMS. The functions *Y*, *G*, and the WMS of the region of similarity between both functions are shown in Figure 1. The dotted line delimits the region of similarity. The WMS is the result of the minimum SSE between *Y* and *G* responses.

Best objective value (Minimize *SSE*)	Best solution				Parameters			WMS dimensions	
	x_1^L	x_1^U	x_2^L	x_2^U	ε_1	ε_2	*Beta values*	$x_1^U - x_1^L$	$x_2^U - x_2^L$
0.0000	-0.6	1.6	-0.6	1.6	2	2	(0,10,10)	2.20	2.20

Table 1. Best result for Trial 1 under one objective approach.

The WMS for Trial 1 includes four points (0, 0), (0, 1), (1, 0) and (1, 1) of the 121 total points of the experimental region. The dimensions of the WMS are 2.2 by 2.2.



$$
\begin{aligned}
&\textit{Find} \quad x_1^L, x_1^U, x_2^L, x_2^U \quad \textit{for} \\
&\textit{Minimize SSE} \\
&\textit{St.} \\
&-5 \leq x_1^L \leq 5 \\
&-5 \leq x_1^U \leq 5 \\
&-5 \leq x_2^L \leq 5 \\
&-5 \leq x_2^U \leq 5 \\
&x_1^U - x_1^L \geq \varepsilon_1 \\
&x_2^U - x_2^L \geq \varepsilon_2 \\
&\varepsilon_1,\ \varepsilon_2 \geq 0
\end{aligned}
\tag{7}
$$

Figure 1. Functions and optimization problem for Trial 1.

Trial 2. Function Y sampled at specific points and function H is movable.

WMS is generated from a region of similarity between both functions, which results in the minimum SSE for trial 2, heuristically speaking. In steps (i) and (ii) of the method, function *Y*, the function to approximate is fixed and, function H, the function to superimpose is movable. From step (iii) to step (vi) the path of the method is similar to that in Trial 1.

In step (vii), the optimization problem (8) of this trial is similar to the optimization problem modeled in (7), but adds the betas of the function H as variables. The ranges of x_1 and x_2 and epsilon values are the same as in model (7). Twelve random starting points were used. Under these conditions, the model was optimized according to step (viii).

Table 2 shows the best results (ix) for Trial 2. This table is organized, from left to right, as follows: the minimum SSE, the best solutions for the seven variables (four bounds for the WMS, and three betas of the function H), parameter values, and WMS dimensions found for the method. Figure 2 shows function Y and function H graphically, as well as the associated WMS.

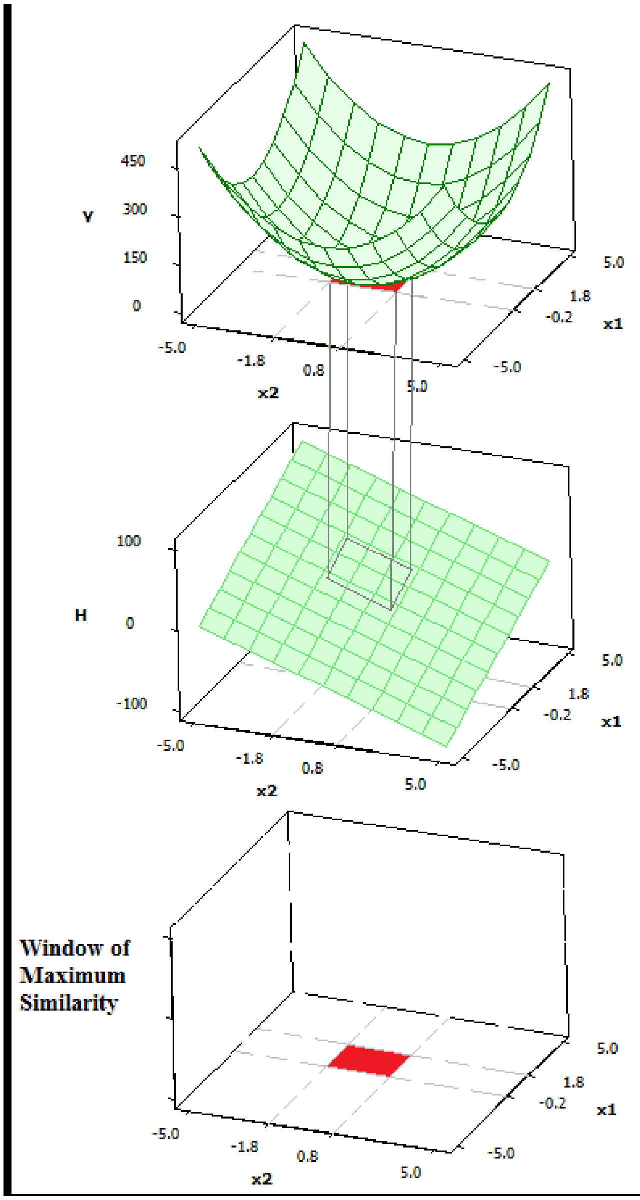
Best objective value (Minimize *SSE*)	Best solution							Parameters		WMS dimensions	
	x_1^L	x_1^U	x_2^L	x_2^U	β_0	β_1	β_2	ε_1	ε_2	$x_1^U - x_1^L$	$x_2^U - x_2^L$
0.0000	-0.20	1.80	-1.8	0.8	0.0000	10.0000	-10.0000	2	2	2.00	2.60

Table 2. Best result for Trial 2 under one objective approach.

According to Table 2 and Figure 2, we can see the following:

- The WMS contains the points (0, 0), (0, -1), (1, 0) and (1, -1), i.e. four points of the 121 total points of the experimental region.
- WMS dimensions are 2.2 by 2.6.

The rest of starting points provide other solutions with a SSE of or close to 0, but these solutions are not listed here.



$$
\begin{aligned}
&\textit{Find}\quad x_1^L, x_1^U, x_2^L, x_2^U, \beta_0, \beta_1, \beta_2 \ \textit{for}\\
&\textit{Minimize SSE}\\
&\textit{St.}\\
&-5 \le x_1^L \le 5\\
&-5 \le x_1^U \le 5\\
&-5 \le x_2^L \le 5\\
&-5 \le x_2^U \le 5\\
&x_1^U - x_1^L \ge \varepsilon_1\\
&x_2^U - x_2^L \ge \varepsilon_2\\
&\varepsilon_1, \varepsilon_2 \ge 0
\end{aligned}
\tag{8}
$$

Figure 2. Functions and optimization problem for Trial 2.

Trial 3. Function Y sampled at specific points and has a target region, and function I is movable.

In steps (i) and (ii), we define the function to approximate - function *Y* (fixed) - and the function to superimpose - function *I* (movable) -, respectively. From step (iii) to step (vi) the method follows similarly to Trials 1 and 2.

In step (vii) and (viii), 12 random starting points were used. The remaining parameters (ranges of variables, epsilons, etc.) and the modeled optimization problem are the same in Trial 2, as presented in (8).

Finally, for step (ix) the results are shows in Table 3. From left to right: the minimum SSE, the best solutions for boundary values and beta values for function *I*, parameter values previously defined, and the WMS dimensions found for the method. Figure 3 shows, in the form of a graph: the composed function (Y & $Y_{cutting\ plane}$), function *I*, and the WMS resulting from the minimum SSE of the two responses for this trial.

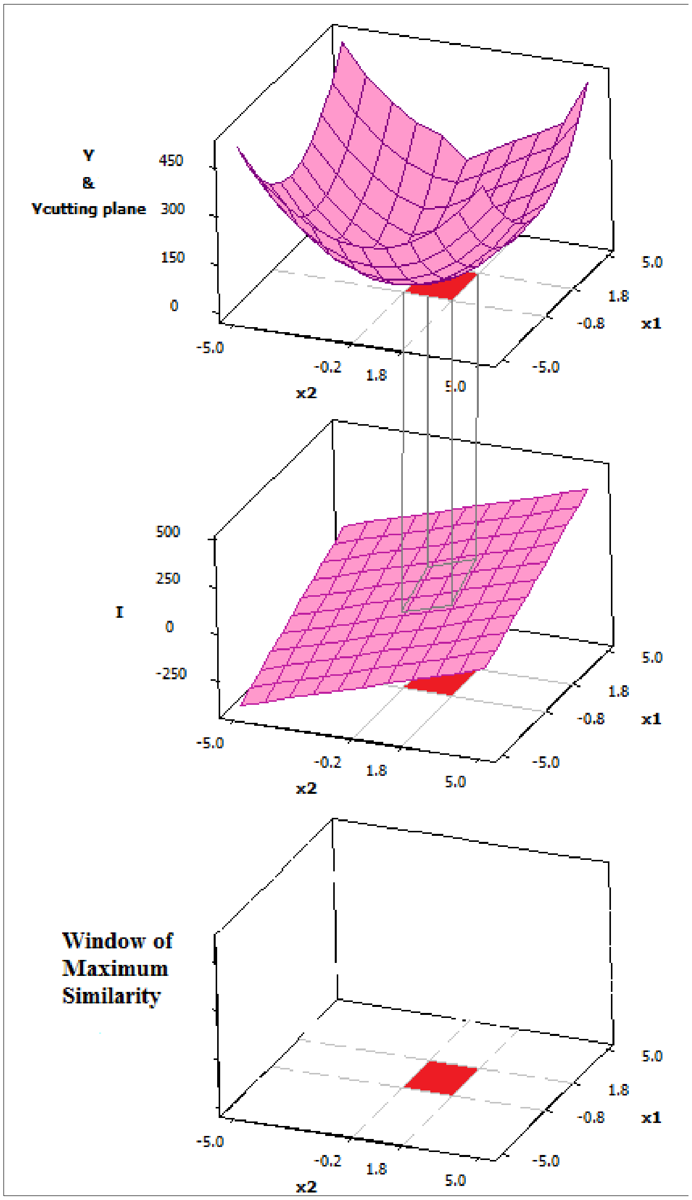
Best objective value (Minimize *SSE)*	Best solution							Parameters		WMS dimensions	
	x_1^L	x_1^U	x_2^L	x_2^U	β_0	β_1	β_2	ε_1	ε_2	$x_1^U - x_1^L$	$x_2^U - x_2^L$
0.0000	-0.8	1.8	-0.20	1.80	0.0000	40.0000	40.0000	2	2	2.60	2.00

Table 3. Best result for Trial 3 under one objective approach.

According to Table 3, we can conclude the following:

- The WMS contains the points (0, 0), (1, 0), (0, 1) and (1, 1), i.e. four of the 121 points of the whole experimental region.
- WMS dimensions are 2.6 by 2.0.

Additionally, other solutions for the 12 starting points used provided a SSE close to 0 which made sense, but these solutions are not shown here.



$$
\begin{aligned}
&\textit{Find} \quad x_1^L, x_1^U, x_2^L, x_2^U, ß_0, ß_1, ß_2\\
&\textit{for}\\
&\textit{Minimize SSE}\\
&\textit{St.}\\
&-5 \le x_1^L \le 5\\
&-5 \le x_1^U \le 5\\
&-5 \le x_2^L \le 5 \qquad\qquad\qquad\qquad (8)\\
&-5 \le x_2^U \le 5\\
&x_1^U - x_1^L \ge \varepsilon_1\\
&x_2^U - x_2^L \ge \varepsilon_2\\
&\varepsilon_1, \varepsilon_2 \ge 0
\end{aligned}
$$

Figure 3. Functions and optimization problem for Trial 3.

Evaluations using multiple starting points

The three cases previously mentioned were carried out manually with multiple starting points, i.e. we provided an enumeration of initial solutions on each case according to uniform probability distributions. As a better alternative, for each trial case, a set of runs was executed using automatic multiple starting points in the Excel Solver. The parameters defined in the Excel Solver that were used for our evaluations include:

- The use of multiple starting points using a population size of 50.
- A constraint precision of $1x10^{-7}$.
- A convergence of $1x10^{-4}$.
- Bounded variables.
- Continuous and integer variable cases. For the integer variable cases, add the restriction: x_1^L, x_1^U, x_2^L, x_2^U = integer to models.
- For Trial 1 and Trial 3, we changed the epsilons values to 5 for ε_1 and 4 for ε_2.
- Other parameters are the same as those previously defined for the three trials in the manual initializations.

Table 4 shows the best results obtained for the three trials using Excel Solver as previously defined. The table includes the following from left to right: the type of variable, continuous (C) or integer (I) variables; the number of runs, corresponding to number of the times that the solver was executed for a particular case, being two runs for Trial 1 and four runs for Trial 2 and Trial 3; the best objective value (BOV), that is the minimum SSE for each run of a particular trial; the best solution, bounds of variables that delimit the WMS, and beta values for the function to superpose in each trial, and; if not applicable, N/A is presented; and, finally, the WMS dimensions (the differences between bound for x_1 and x_2). The SSE values in bold letter represent the minimum value between all runs.

Run number 4 for integer variables in Trial 3 was generated using the starting points exactly where the region of $Y_{cutting\ plane}$ function is defined. This is: on x_1 within [0, 5] and x_2 within [0, 4]. Here, the method found the WMS where $Y_{cutting\ plane}$ was defined.

Essentially, the three cases describe the construction of the method. The evaluations consider the bounds as continuous and integer variables.

In summary, Trial 1 presented the graphic description (Figure 1), the optimization problem, and the solution for this case. The results showed that when using continuous or integer boundary variables, the objective values are the same within the sets of runs for each case.

In addition to the graphic description (Figure 2), the optimization problem, and the solution, Trial 2 presented an analysis of the proposed method. From Table 4, it is possible to observe that, for cases where the bounds take continuous values, the best objective value coincides in the four runs. However, the boundary values and beta values for function *H* are different. The explanation for this is that function *H* (function in the form of a plane) can be adjusted around function *Y*, because the function *Y* is a fixed bowl. The result of these adjustments will display the same values as the objective values, although the variable values are different. The case in which the bounds take integer values (a more complex problem than using continuous variables) results in two similar objective values. In Trial 2, the method should find multiple global solutions with the same value for the objective function.

To validate the method, Trial 3 was used (Figure 3). The composed function of Trial 3 included a target region which is a plane. This region is delimited by $Y_{cutting\ plane}$. The method should find x_1^L, x_1^U, x_2^L, and x_2^U within the region. If this occurs, the method is being effective. In addition, Table 4 shows the objective value for four runs, considering continuous variables. This objective value of 0 coincides for the four runs.

Type of variable	Run number	BOV (min)	Best solution							WMS dimensions	
		SSE	x_1^L	x_1^U	x_2^L	x_2^U	β_0	β_1	β_2	$x_1^U - x_1^L$	$x_2^U - x_2^L$
Trial 1											
C	1	**29600**	-1.5958	3.7999	-1.6072	2.3928	N/A	N/A	N/A	5.3957	4.0000
C	2	29600	-1.9153	3.0847	-1.7263	2.7563	N/A	N/A	N/A	5.0000	4.4826
I	1	98400	-2	3	-2	2	N/A	N/A	N/A	5	4
I	2	**98400**	-2	3	-1	3	N/A	N/A	N/A	5	4
Trial 2											
C	1	0.0000	-0.6951	1.3049	1.5947	3.6753	-60.0000	9.9999	50.0000	2.0000	2.0807
C	2	0.0000	-2.0265	-0.0265	-1.7502	0.4943	-20.0000	-30.0000	-10.0000	2.0000	2.2445
C	3	0.0000	-1.3734	0.6266	-1.4252	0.6853	-0.0001	-10.0000	-10.0000	2.0000	2.1105
C	4	**0.0000**	-1.2276	0.7723	-2.6449	-0.4488	-20.0000	-9.9999	-30.0000	1.9999	2.1961
I	1	3199.9857	2	4	0	2	-120.0001	70.0000	10.0001	2	2
I	2	1399.9964	-2	0	1	3	-33.3334	-10.0000	40.0000	2	2
I	3	**400.0000**	-2	0	-5	-3	-156.6731	-20.0002	-80.0015	2	2
I	4	1399.9959	-1	1	1	3	-33.3333	9.9999	40.0000	2	2
Trial 3											
C	1	0.0000	-0.3676	4.6324	-0.0216	4.2206	0.0001	39.9999	40.0000	5.0000	4.2422
C	2	0.0000	-0.8572	4.1428	-0.5456	4.2558	0.0002	40.0001	39.9999	5.0000	4.8014
C	3	0.0000	-0.6372	4.3628	-0.9594	3.1223	0.0001	40.0000	40.0000	5.0000	4.0817
C	4	**0.0000**	-0.0541	4.9458	-0.0940	4.4147	0	39.9999	39.9999	4.9999	4.5087
I	1	3542.8571	-1	4	0	4	11.4289	35.7142	39.9999	5	4
I	2	3810.7143	-1	5	0	4	10.7143	36.7857	40.0000	6	4
I	3	1119.2585	0	5	0	4	10.7143	36.7857	40.0000	5	4
I	4	**0**	0	5	0	4	0	40	40	5	4

Table 4. Best solutions obtained for three trials using Solver setting.

3.2.2 Three-objective approach

The three-objective approach was developed to automatically determine the size of the WMS through optimization. The purpose of this approach is to eliminate the dependence of epsilon values for the WMS. This approach consists of placing the differences of the variables in the objective function, thereby formulating a composite objective function. The composite objective function includes an objective for the SSE and two additional objectives for the WMS size (differences of the bounds) of two dimensions. Each difference was weighted. Weights μ_1, μ_2, and μ_3 take values of 1/3 in order to sum 1. Given that the SSE will always be a positive value and many times will result in large positive values, μ_2 and μ_3 are preceded by minus signs. The objective is to minimize the composite objective function.

Under this three-objective approach, the three trials previously defined and used were evaluated. A new optimization problem was then created for Trial 1, as given by (9).

$$
\begin{aligned}
&\text{Find} \quad x_1^L, x_1^U, x_2^L, x_2^U \quad \text{for} \\
&\text{Minimize} \\
&\qquad \mu_1[f(SSE)] - \mu_2[x_1^U - x_1^L] - \mu_3[x_2^U - x_2^L] \\
&\text{St.} \\
&\quad -5 \le x_1^L \le 5 \\
&\quad -5 \le x_1^U \le 5 \\
&\quad -5 \le x_2^L \le 5 \\
&\quad -5 \le x_2^U \le 5 \\
&\quad x_1^U - x_1^L \ge 0 \\
&\quad x_2^U - x_2^L \ge 0
\end{aligned} \tag{9}
$$

The resulting WMS, however, was not effective as it contained a single point. This was due to the distance between the lower and upper bounds (or the difference) of each variable taking a value of 0 during execution. One way to correct this problem was to make the differences between bounds take a sufficiently small value. A value of $1x10^{-06}$ was assigned. Considering this correction (correction 1), the optimization problem is given by (10.1) and (10.2). Model (10.1) approaches Trial 1, while (10.2) describes the optimization problems for Trials 2 and 3.

$$
\begin{aligned}
&\text{Find} \quad x_1^L, x_1^U, x_2^L, x_2^U \quad \text{for} \\
&\text{Minimize} \\
&\quad \mu_1[f(SSE)] - \mu_2[x_1^U - x_1^L] - \mu_3[x_2^U - x_2^L] \\
&\text{St.} \\
&\quad -5 \le x_1^L \le 5 \\
&\quad -5 \le x_1^U \le 5 \\
&\quad -5 \le x_2^L \le 5 \\
&\quad -5 \le x_2^U \le 5 \\
&\quad x_1^U - x_1^L \ge 1x10^{-06} \\
&\quad x_2^U - x_2^L \ge 1x10^{-06}
\end{aligned} \tag{10.1}
$$

$$
\begin{aligned}
&\textit{Find} \quad x_1^L,\, x_1^U,\, x_2^L,\, x_2^U,\, \beta_0,\, \beta_1,\, \beta_2 \quad \textit{for} \\
&\textit{Minimize} \\
&\qquad \mu_1[f(SSE)] - \mu_2[x_1^U - x_1^L] - \mu_3[x_2^U - x_2^L] \\
&\textit{St.} \\
&\quad -5 \le x_1^L \le 5 \\
&\quad -5 \le x_1^U \le 5 \\
&\quad -5 \le x_2^L \le 5 \\
&\quad -5 \le x_2^U \le 5 \\
&\quad x_1^U - x_1^L \ge 1x10^{-06} \\
&\quad x_2^U - x_2^L \ge 1x10^{-06} \\
&\quad -1000 \le \beta_0 \le 1000 \\
&\quad -1000 \le \beta_1 \le 1000 \\
&\quad -1000 \le \beta_2 \le 1000
\end{aligned}
\tag{10.2}
$$

The parameters defined in the Excel Solver for these evaluations include:

- Multiple starting points using a population size of 50 for Trial 2, a population size of 100 for Trials 1 and 3.
- A constraint precision of $1x10^{-7}$.
- A convergence of $1x10^{-4}$.
- Required bounds on variables for Trial 2 and Trial 3.
- The setting of integer tolerance to 0%.

During the execution of models (10.1) and (10.2), when the bounds x_1^L, x_1^U, x_2^L, x_2^U were considered continuous variables and were also truncated at four decimals, the obtained results shown a reduced region for the WMS. This was due to the distance between lower and upper bounds (or the difference) of each variable taking a value very close to 0. For the case when the bounds were considered integer variables, the beta values were considered continuous variables. Beta values were truncated at four decimals. The solutions of all the evaluations, using the optimization problems given by (10.1) and (10.2), will be presented later in Table 5.

Transformation of the composite objective function

The optimization problem developed under the three-objective approach showed difficulties due to the dimensionality present in the objective values of the distances. In order to solve this, a new optimization problem was formulated. The composite objective function was transformed, incorporating the logarithm base 10 in each term of the objective function (to differentiate the transformed model, we will name the objective function in (10.1) and (10.2) *untransformed composite objective function*). Additionally, a sufficiently small value ($1x10^{-6}$) was fixed to avoid errors in the logarithm function of Excel, as this function, by definition, does not take negative or zero values. Correction 2 for the three-objective optimization problem of Trial 1 is given by (11).

$$
\begin{aligned}
&\text{Find} \quad x_1^L, x_1^U, x_2^L, x_2^U \quad \text{for} \\
&\text{Minimize} \\
&\quad \mu_1[logf(SSE)] - \mu_2[log(x_1^U - x_1^L)] - \mu_3[log(x_2^U - x_2^L)] \\
&\text{Subject to} \\
&\quad -5 \le x_1^L \le 5 \\
&\quad -5 \le x_1^U \le 5 \\
&\quad -5 \le x_2^L \le 5 \\
&\quad -5 \le x_2^U \le 5 \\
&\quad x_1^U - x_1^L \ge 1x10^{-6} \\
&\quad x_2^U - x_2^L \ge 1x10^{-6}
\end{aligned}
\tag{11}
$$

Because there were still execution errors in (11), it was decided to create a correction (third correction), which involves defining each variable in (11) as a difference of two nonnegative variables, giving rise to model (12).

$$
\begin{aligned}
&Find\ \ x_1^{+L}, x_1^{-L}, x_1^{+U}, x_1^{-U}, x_2^{+L}, x_2^{-L}, x_2^{+U}, x_2^{-U}\ \ for\\
&Minimize\\
&\quad \mu_1[logf(SSE)] - \mu_2[log(x_1^U - x_1^L)] - \mu_3[log(x_2^U - x_2^L)]\\
&St.\\
&x_1^{+L} - x_1^{-L} = x_1^L\\
&x_1^{+U} - x_1^{-U} = x_1^U\\
&x_2^{+L} - x_2^{-L} = x_2^L\\
&x_2^{+U} - x_2^{-U} = x_2^U\\
&-5 \le x_1^L \le 5\\
&-5 \le x_1^U \le 5\\
&-5 \le x_2^L \le 5\\
&-5 \le x_2^U \le 5\\
&x_1^U - x_1^L \ge 1x10^{-6}\\
&x_2^U - x_2^L \ge 1x10^{-6}\\
&0 \le x_1^{+L}, x_1^{-L}, x_1^{+U}, x_1^{-U}, x_2^{+L}, x_2^{-L}, x_2^{+U}, x_2^{-U} \le 1000
\end{aligned}
\tag{12}
$$

For this model, the solver execution continued indicating an error of solution. We supposed that this execution error was due to the internal initialization of the multiple starting points of the solver. Given that it is possible to use random numbers for the multiple starting points, these random numbers could take negative or zero values. Under this premise, a transformation was generated by adding +1 to the SSE value, as well as adding absolute value and +1 to the differences within logarithm function. The reference model for this transformation was model (11). If the SSE value takes a 0 value, the +1 would avoid having an error. If the differences (x_1^U - x_1^L) and (x_2^U - x_2^L) take a zero or negative value due to the initializations, the +1 and the absolute value will help prevent the error.

The new correction (correction 4) was applied to the optimization problems of each trial. The optimization problem for Trial 1 is described by (13.1). For Trial 2 and Trial 3, the model given by (13.2) was formulated. Following equation 1, to compute the SSE in model (13.1), Y represents the function to approximate (data) that needs to be optimally addressed and G represents the function to superimpose with known and desirable characteristics, this is, for example, $SSE = \sum_i (Y_i - G_i)^2$ for all i point of the experimental region. In model (13.2), to compute the SSE, Y represents the function to approximate (data) and function H is the function to superimpose, $SSE = \sum_i (Y_i - H_i)^2$ for all i point of the experimental region for Trial 2; similarly, for Trial 3, Y & $Y_{cutting\ plane}$ represents the function (data) that needs be optimally addressed and function I is the function with desirable characteristics to superimpose.

$$
\begin{aligned}
&\textit{Find} \quad x_1^L, x_1^U, x_2^L, x_2^U \quad \textit{for} \\
&\textit{Minimize} \\
&\mu_1[log(SSE+1)] - \mu_2[log(|x_1^U - x_1^L| + 1)] - \mu_3[log(|x_2^U - x_2^L| + 1)] \\
&\textit{St.} \\
&\quad -5 \le x_1^L \le 5 \\
&\quad -5 \le x_1^U \le 5 \\
&\quad -5 \le x_2^L \le 5 \\
&\quad -5 \le x_2^U \le 5 \\
&\quad x_1^U - x_1^L \ge 1x10^{-6} \\
&\quad x_2^U - x_2^L \ge 1x10^{-6}
\end{aligned}
\tag{13.1}
$$

$$
\begin{aligned}
&\textit{Find} \quad x_1^L, x_1^U, x_2^L, x_2^U, ß_0, ß_1, ß_2 \quad \textit{for} \\
&\textit{Minimize} \\
&\mu_1[log(SSE+1)] - \mu_2[log(|x_1^U - x_1^L| + 1)] - \mu_3[log(|x_2^U - x_2^L| + 1)] \\
&\textit{St.} \\
&\quad -5 \le x_1^L \le 5 \\
&\quad -5 \le x_1^U \le 5 \\
&\quad -5 \le x_2^L \le 5 \\
&\quad -5 \le x_2^U \le 5 \\
&\quad x_1^U - x_1^L \ge 1x10^{-6} \\
&\quad x_2^U - x_2^L \ge 1x10^{-6} \\
&\quad -1000 \le ß_0 \le 1000 \\
&\quad -1000 \le ß_1 \le 1000 \\
&\quad -1000 \le ß_2 \le 1000
\end{aligned}
\tag{13.2}
$$

For the integer variable cases, the restriction: x_1^L, x_1^U, x_2^L, x_2^U are *integer* was added. The parameters for Excel Solver for these evaluations include:

- The use of multiple starting points using a population size of 50 for Trial 2, and a population size of 100 for Trial 1 and 3.
- A constraint precision of $1x10^{-7}$ and a convergence of $1x10^{-4}$.
- Required bounds on variables for Trial 2 and Trial 3.

- An integer tolerance configured to 0%.

Finally, with models (13.1) and (13.2), the solver execution was carried out successfully. The best solutions obtained by models (10.1), (10.2), (13.1), and (13.2) are presented in Table 5. This table shows the following from left to right: the number of trials followed by the composite objective function and the type of variable, continuous (C) or integer (I); the best objective value (BOV) that includes the SSE and the WMS dimensions found for a particular trial; and, finally, the best solution integrated by the bounds of variables that delimit the WMS and beta values for the function to superpose in each trial. If not applicable, N/A is presented.

The evaluations consider the bounds (x_1^L, x_1^U, x_2^L, x_2^U) as continuous (C) and integer (I) variables. The values obtained for continuous variables, bounds and beta variables, were truncated to four decimal places. For the case when the bounds were considered integer variables, the beta values were considered continuous variables. Beta values were truncated at four decimal places. Additionally, results for the transformed (T) (for models (13.1) and (13.2)) and untransformed (U) (for models (10.1) and (10.2)) are presented in these evaluations.

Trial	Composite objective function	Type of variables	BOV found	BOV found *WMS dimensions*		*Best solution*						
			SSE	$x_1^U - x_1^L$	$x_2^U - x_2^L$	x_1^L	x_1^U	x_2^L	x_2^U	β_0	β_1	β_2
1	U	C	0	0.1726	6.1444	-4.434	-4.262	-3.366	2.778	N/A	N/A	N/A
		I	289,064.00	7	1	-4	3	-4	-3	N/A	N/A	N/A
	T	C	0	5.4416	0.0401	-1.169	4.2726	-4.175	-4.135	N/A	N/A	N/A
		I	1,453,600.00	9	1	-5	4	4	5	N/A	N/A	N/A
2	U	C	0	9.6047	0.3906	-5	4.6047	-3.63	-3.24	-554	-104.4455	-436
		I	68,049,330.90	10	1	-5	5	4	5	-501.5755	551.7378	221.9010
	T	C	0	8.9972	0.5584	-5	3.9972	3.1113	3.6697	856	-202.8417	344
		I	358,997,581.76	7	1	-5	2	4	5	-316.9269	747.7585	-590.3982
3	U	C	0.0110	5.2224	5.0481	-0.832	4.3903	-0.0482	4.9999	0	39.9998	39.9915
		I	0.0020	4	2	1	5	0	2	0	40	40
	T	C	0	8.4233	0.7970	-3.737	4.6864	2.1492	2.9462	653	-164.3144	624.4082
		I	20,583,087.32	1	10	-3	-2	-5	5	245	-276	179

Table 5. Best solutions obtained for three trials using three-objective approach.

3.3. Method Proposed for future Optimization through Similarity

In order to find the WMS, the following steps of the method are provided:

i. *Identify the function to approximate*. This function and its parameters are previously known. The function to approximate is fixed and represented by samplings as opposed to having the actual function. In this step, the function (that can be represented by data) is uncharacterized in terms of the optimality.

ii. *Define the function to superimpose*. This function has characteristics of our interest as well. The function to superimpose should be adjustable to a range where the properties of interest are kept. In this thesis's interest, the characteristics are optimality properties. Here, the main idea is to identify the function with well-established properties of interest.

iii. *Define the experimental region*. The experimental region of interest is where the WMS will be generated. The WMS will adjust its size on this space.

iv. *Create False-True logic tests*. That is, for example, for two variables case, if $x_1^L \leq x_1 \leq x_1^U$ and if $x_2^L \leq x_2 \leq x_2^U$, true. If not, false. For each combination of the variables x_1 and x_2, a True or False will be assigned.

v. *Obtain the sum of squared errors (SSE).* For each combination of the variables x_1 and x_2, a SSE value from both functions' responses will be generated.

vi. *Filter the SSE values for the window of maximum similarity (WMS)*. This filter will help to determine the SSE values of the WMS through another false-true logic test. If the False-True logic test in (iv) is true, a value of 1 will multiply the SSE value obtained in (v). If not, a value of 0 will multiply the SSE value. Therefore, the SSE of the WMS will be generated by the region where the filter works.

vii. *Model an optimization problem*. Develop an optimization problem to find bounds x_1^L, x_1^U, x_2^L, x_2^U that delimit the WMS, as well as other variables of interest.

viii. *Optimize the model*. Here, the solver execution is required. The solver parameters (convergence, integer tolerance, number of iterations, etc.) are user-defined. Basically, a matching of the functions described in steps (i) and (ii) defines the window with maximum similarity through least squares.

ix. *Report the results*. The bounds that delimit the WMS represent the similarity region of our interest.

At this point, with the steps previously described, it is possible to generate the WMS between two responses. These steps are the basis to the future development of the Optimization through Similarity method.

Chapter 4

In this chapter the evaluation of the proposed method using a function designed in our research group, which we named *AOG 1*, is presented. This function is of our interest because it has a flat region and a curve region. Also, it is of our interest to find the maximum similarity between function *AOG 1* and a quadratic function with the form of a bowl because it is reasonable to understand that, potentially, the resulting WMS will match the curve region of the function *AOG 1* with the quadratic function. If the WMS method finds this region, the results will be reasonable.

4.1. Method evaluation using *Function AOG 1*

Once it was possible to automatically determine the size of the WMS, a case to match two functions that have a similar quadratic region was developed. Considering the method proposed for future optimization by similarity described in section 3.3, the strategy is as follows:

(i). Identify the function to approximate: For this case, a function designed in our research group, which we named *AOG 1*. Mathematically, this function has the form of a plane with a bowl in the central experimental region. The ranges x_1 and x_2 are [-5, 5]. The experimental region contains 121 points and is divided in two regions. The quadratic function ($f_1(x) = 5x_1^2 + 5x_2^2$) contains 16 points and is defined by region [-2, 1] for x_1, and [-3, 0] for x_2. The remaining 105 points follow the linear function, ($f_2(x) = 500 - 5x_1 + 5x_2$).

(ii). Define the function to superimpose: the quadratic function $f(x_1, x_2)$, the function to superimpose, is mathematically expressed by equation 14. The ranges x_1 and x_2 are [-5, 5]. This experimental region contains all 121 points.

$$f(x_1, x_2) = 5x_1^2 + 5x_2^2 \tag{14}$$

(iii). Define the experimental region: The experimental region contains 121 points as explained in steps (i) and (ii).

In *steps (iv) to (vi)* of the method, the True-False logic tests are generated until obtaining the filtered SSE values for the WMS.

(vii). Model an optimization problem: The optimization problems used for this case are given by (10.1), (13.1), and (15). Model 15 is the same as the optimization problem described in (7) taking into consideration that model 15 has epsilon values (ε_1 and ε_2) of $1x10^{-06}$.

The global minimum for function *AOG 1* is given by point (0, 0) and its objective value is 0. Essentially, the maximum similarity will be found if the WMS is adjusted within the quadratic region of both functions.

$$
\begin{array}{l}
\textit{Find} \quad x_1^L, x_1^U, x_2^L, x_2^U \quad \textit{for} \\
\textit{Minimize} \\
\quad SSE \\
\textit{St.} \\
\quad -5 \leq x_1^L \leq 5 \\
\quad -5 \leq x_1^U \leq 5 \\
\quad -5 \leq x_2^L \leq 5 \\
\quad -5 \leq x_2^U \leq 5 \\
\quad x_1^U - x_1^L \geq 1x10^{-06} \\
\quad x_2^U - x_2^L \geq 1x10^{-06}
\end{array}
\tag{15}
$$

For the integer variable cases, add the restriction: x_1^L, x_1^U, x_2^L, x_2^U = *integer* to the models.

The model (13.1) was developed to automatically generate the WMS size. The WMS size is found through the minimization of a composite objective function. This composite objective function includes an objective for the SSE and two objectives for the WMS size (the differences of the bound variables that delimit the WMS) of two dimensions. In addition, each objective of the composite objective function was weighted. Weights μ_1, μ_2, and μ_3 take values of 1/3 for the sum to be 1 (similar to a structure of preferences). Given that, the problem is of minimization, with the minus sign in the differences of the bounds, these differences will tend to take a higher value for the WMS size.

In order to keep in the same order of magnitude the SSE and the differences of the bounds, the logarithm base 10 was incorporated in each term of the composite objective function. The logarithm function is not defined for zero or negative values.

During the preliminary development of the mathematical formulation of the optimization problem for the three initial cases of interest, the execution of the solver indicated error of solution. We supposed that this execution error occurred due to the internal initialization of the multiple starting points of the solver setting. Given that it is possible to use random numbers for the multiple starting points, these random numbers could take negative or zero values; or the difference of the bound variables could take negative or zero values. Under this premise, it was added +1 to the SSE value, as well as were added the absolute value and +1 to the differences within logarithm function. With this formulation of the composite objective function, the solver execution was carried out successfully.

(viii). Optimize the model: The parameters defined in the Solver setting that were used for these evaluations include:

- The use of multiple starting points using a population size of 100.
- A constraint precision of $1x10^{-7}$.
- A convergence of $1x10^{-4}$.
- The evaluations consider continuous and integer variable cases.
- The transformed or untransformed composite objective function was evaluated. One objective function was evaluated too.

- An integer tolerance of 0% was defined in solver settings when integer variables were used.

(ix). Report the results: Table 6 shows the best solutions for function *AOG 1*. In the table, the results where the objective function to be minimized has one objective or three objectives are reported. The composite objective function (COF) (three-objective) was evaluated for transformed (using logarithm) and untransformed (not including the logarithm) objective function. In addition, the type of variables (ToV), the best solution, the best objective value found - composite objective function value (COFv) and SSE - and WMS dimensions, for each case, are reported.

Objective function (minimize)	COF	ToV	Best solution				Best objective value found				WMS size
			x_1^L	x_1^U	x_2^L	x_2^U	COFv	SSE	x_1^U- x_1^L	x_2^U- x_2^L	
One objective: SSE	N/A	C	-1.8	0.8	-2	1	N/A	500,000	N/A	N/A	2.60 x 3.00
		I	-1	0	-1	0	N/A	0	N/A	N/A	1 x 1
Three objective:	U	C	-1.7229	0.3532	-3.3889	0.8156	-2.0935	0	2.0761	4.2045	2.0761 x 4.2045
		I	-2	1	-3	0	-2	0	3	3	3 x 3
	T	C	-2.1729	1.1091	-3.7277	0.35	-0.4458	0	3.2820	4.0777	3.2820 x 4.0777
		I	-2	1	-3	0	-0.4014	0	3	3	3 x 3

Table 6. Solutions obtained for function *AOG 1*.

The function *AOG 1*, the quadratic function $f(x_1, x_2)$ to superimpose, and the WMS of the transformed composite objective function for the integer case are presented in Figure 4.

In all cases, the stationary point (0, 0) is within the WMS. According to the results, it is possible to conclude that the method demonstrated potential to find regions of similarity between two responses, where optimality can be a pattern of interest.

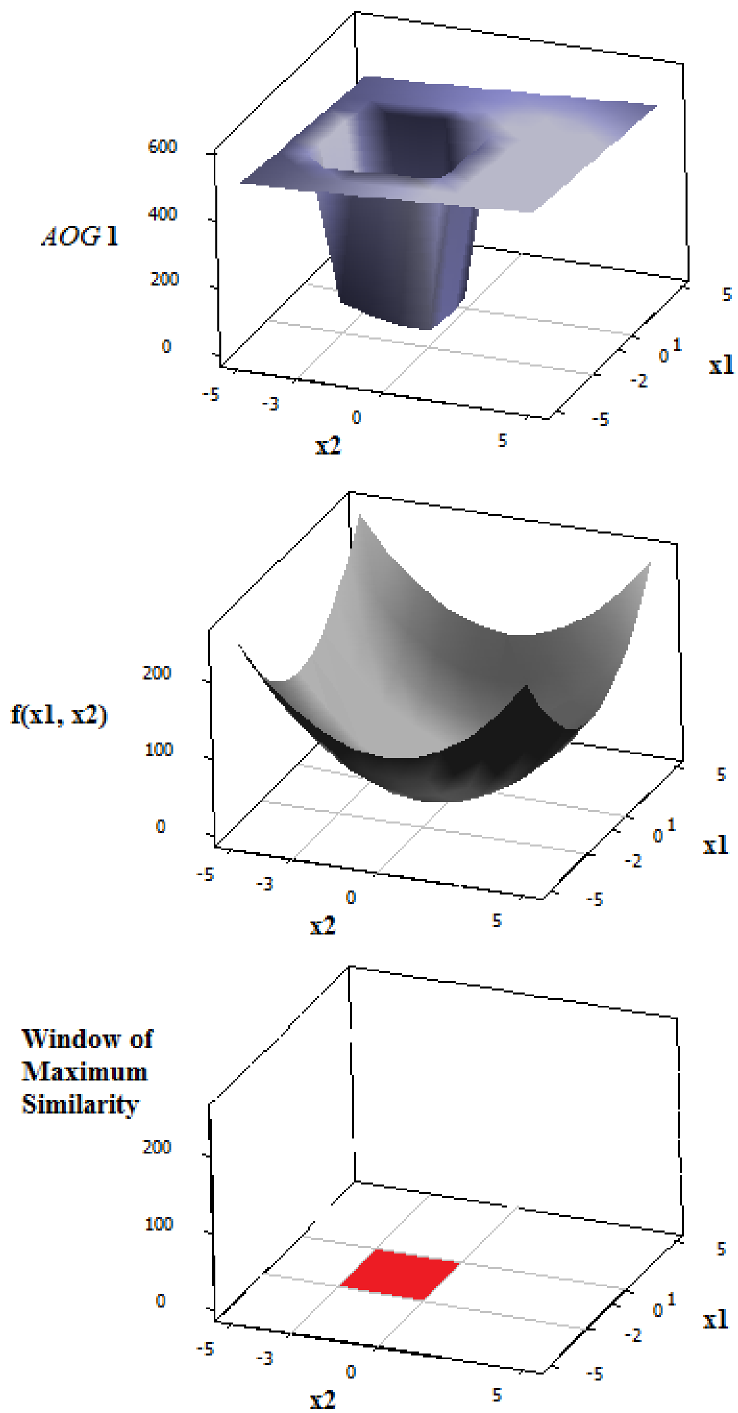


Figure 4. Function *AOG 1* for Optimization by similarity case.

Additional evaluations using a DOE 2^4

New evaluations incorporating an experimental design as initialization points were executed, obtaining the results reported in Appendix A. Tables A.1 to A8 contain the detailed results. The number of initializations was defined by the number of combinations resulting in a factorial *DOE* 2^4, that is, 16 combinations. The initializations considered the variables ranged to [-1, 1] and [-5, 5]. In addition, the evaluations considered continuous variables truncated to four decimal places.

For these evaluations, the parameters defined in the Solver setting include:

- Initial points are carried out manually and using multiple starting points with a population size of 100.
- The constraint precision was considered in two levels: $1x10^{-3}$ and $1x10^{-9}$.
- A convergence of 1×10^{-4}.
- The transformed (automatic window size) composite objective function was evaluated.

Figures 5.a to 5.d present the best solution obtained for each evaluation of function *AOG 1* using a DOE 2^4 ranging the initial points of the variables to [-1, 1]. Figures 6.a to 6.d present the best solution obtained for each evaluation of function *AOG 1* using a DOE 2^4 ranging the initial points of the variables to [-5, 5]. With an objective value of -0.4456, the best solution found by the method was obtained using a level precision of $1x10^{-9}$ and with the automatic multiple starting points. Figure 5.c presents the WMS obtained by this solution. In addition, with an objective value of -0.4504, the best solution found by the method was obtained using a level precision of $1x10^{-3}$ and with the automatic multiple starting points. Figure 6.a presents the WMS obtained by this solution.

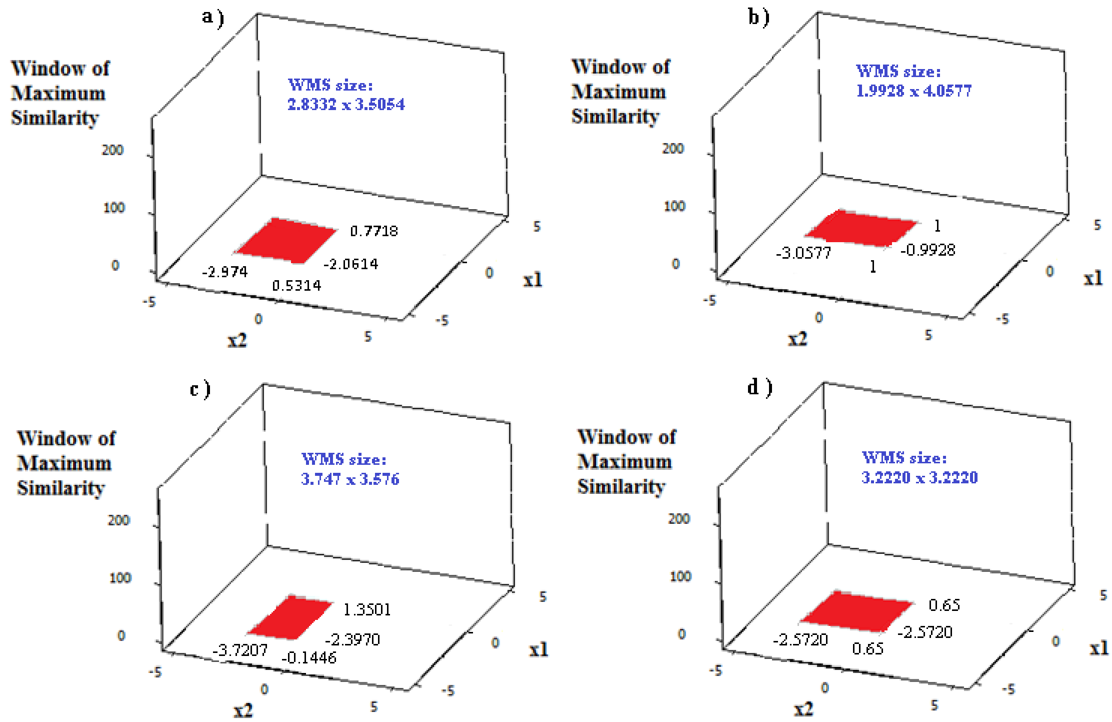


Figure 5. Best solution found by the method for function *AOG 1* where variables were ranged to [-1, 1].

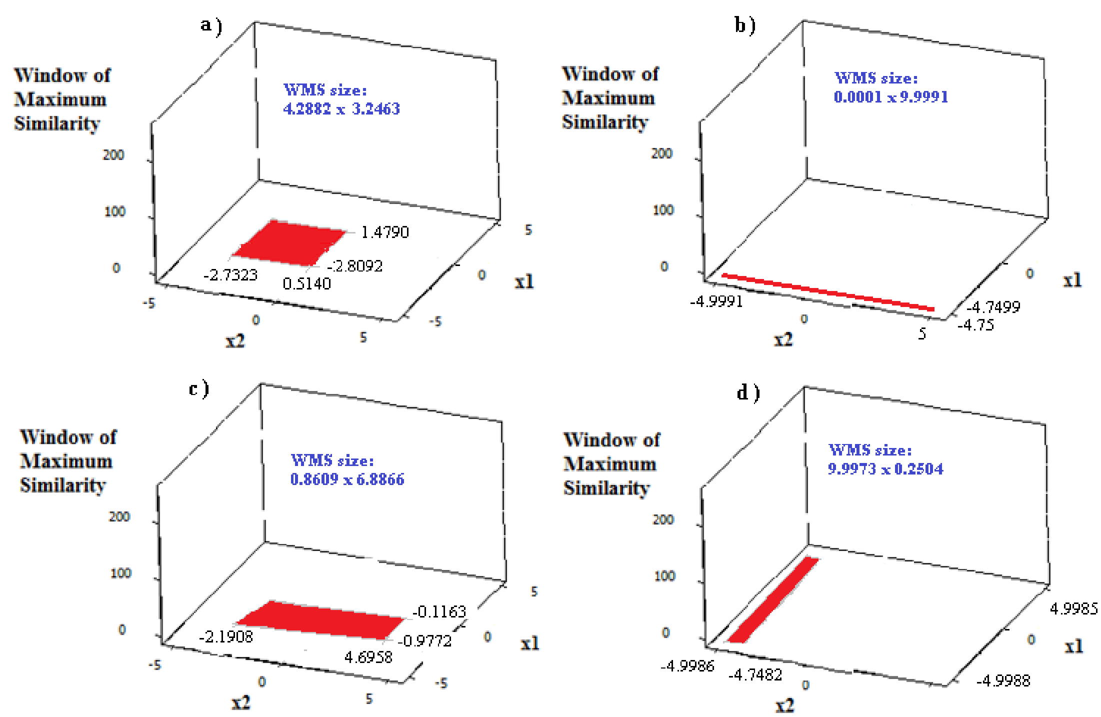


Figure 6. Best solution found by the method for function *AOG 1* where variables were ranged to [-5, 5].

Chapter 5

In this chapter, the use of windows of maximum similarity is demonstrated. Real time series were used to find a region where a lineal model fits best by determining a one-dimensional WMS.

5.1 The use of windows of maximum similarity to find regions where a model is a good descriptor

The proposed method for future optimization by similarity was applied to a set of 18 real time series in order to evaluate the method's capabilities and to find the one-dimensional WMS. These series were obtained from the Mexican National Institute of Statistics and Geography (INEGI) website (http://www.inegi.org.mx/), in the section of the Bank of Economic Information (BIE) (http://www.inegi.org.mx/sistemas/bie/), and from the Secretary of Agriculture, Livestock, Rural Development, Fishing and Food (SAGARPA) (www.sagarpa.gob.mx). Plots of these time series are presented in Figures 7.a and 7.b.

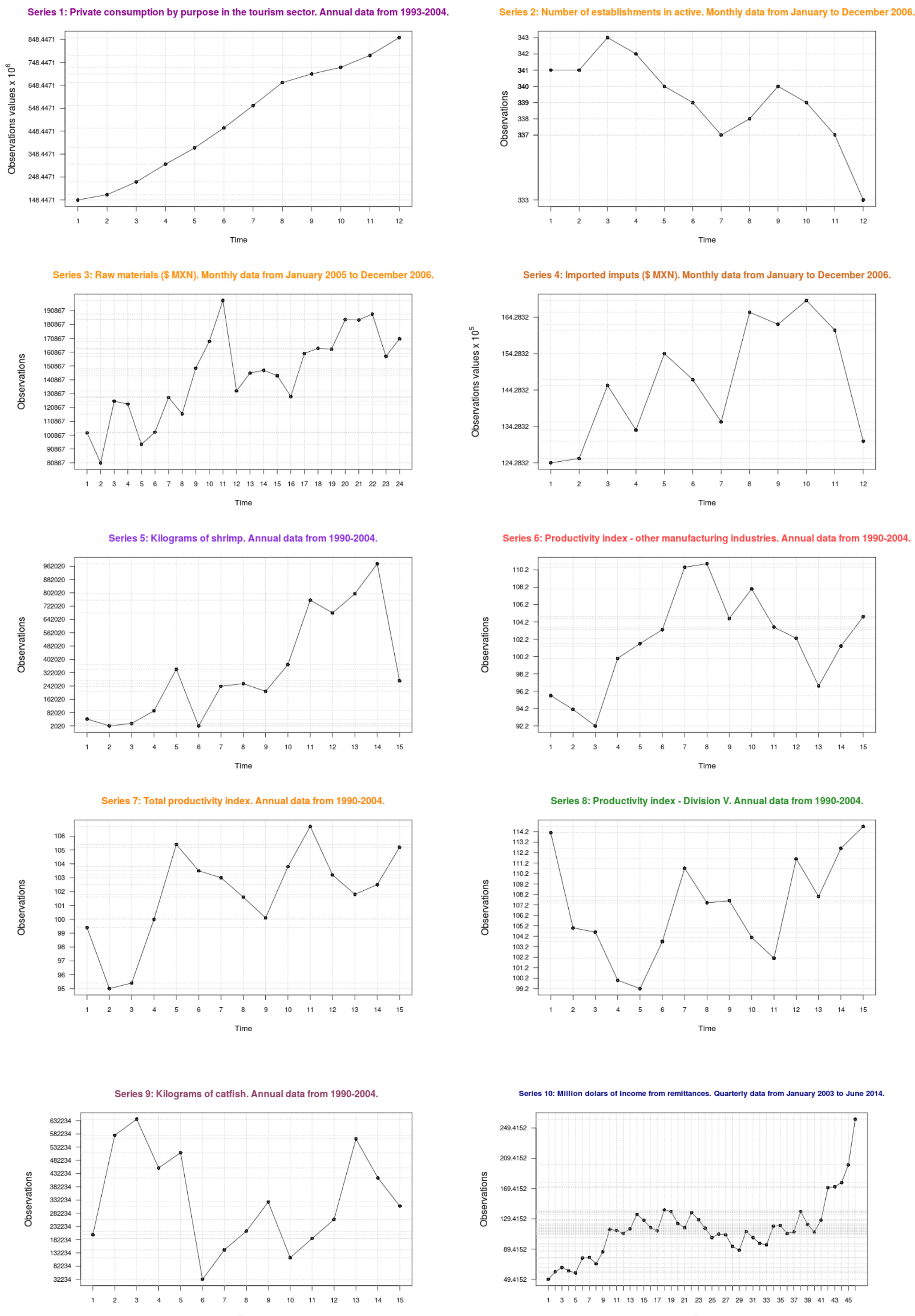


Figure 7.a. Time series plots of real data obtained from INEGI and SAGARPA (México). Series 1-10.

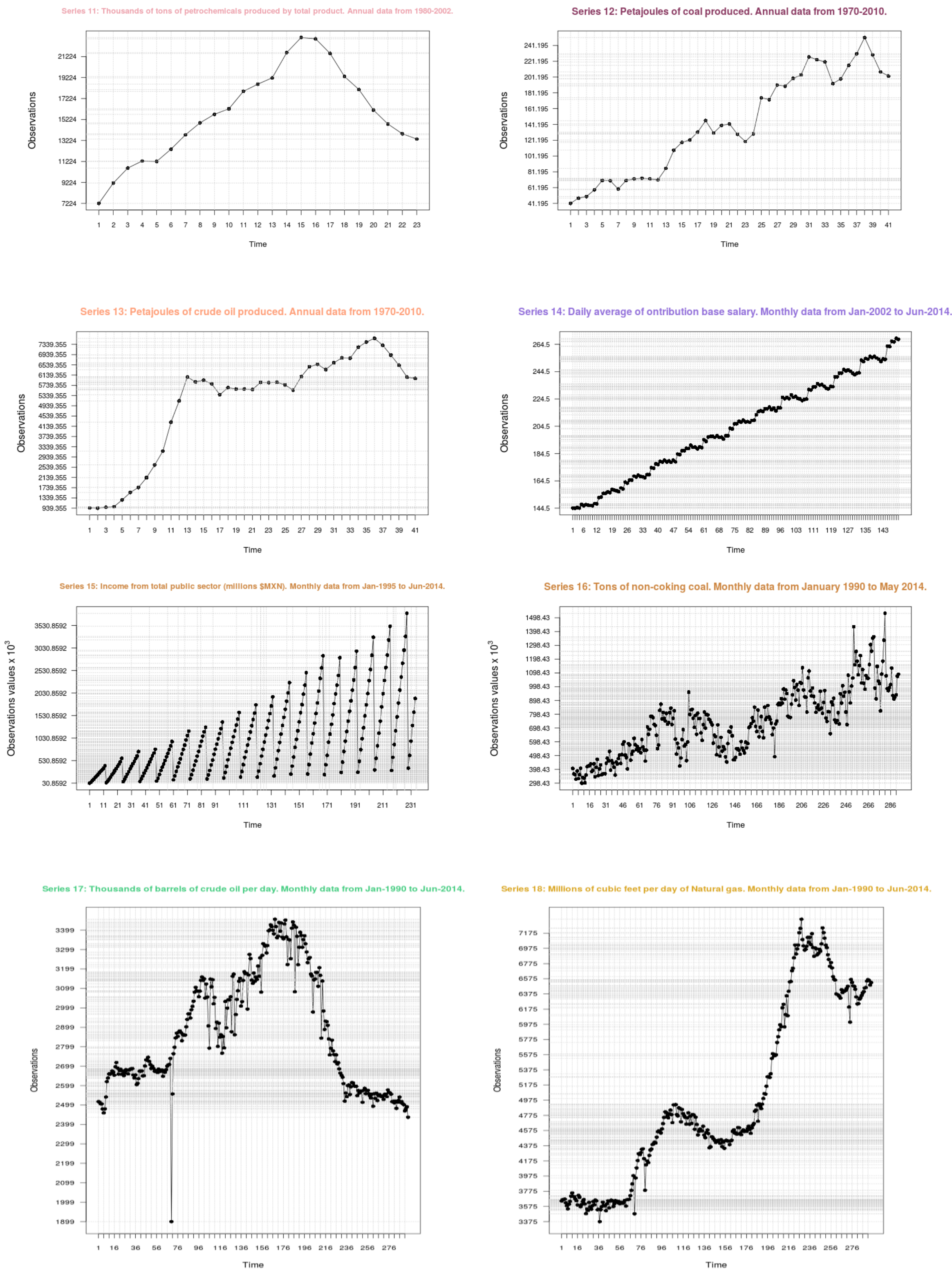


Figure 7.b. Time series plots of real data obtained from INEGI (México). Series 11-18.

The objective of the application of the method to time-series-data is to define a one-dimensional WMS. Since data from the time series are fixed observations that only depend on the time variable, in our case the one-dimensional WMS will indicate a period of time. Specifically, it is our interest to find the WMS where an increasing linear pattern exists. In practice, for example, if we talk about socks sales, this objective can be translated into defining periods in time where an upwards trend is apparent.

Considering the proposed method described in section 3.3, the development of this application is as follows:

(i). Identify the function to approximate: Each of the time series represents the data where the WMS of interest will be adjusted to detect increasing linear behavior.

(ii). Define the function to superimpose:

The increasing behavior trend in a time series can be characterized by an adjusted straight line obtained from a linear regression (as Figure 8 shows). Figure 8 presents the four observations from Series 3, points 8 – 11. The increasing behavior is defined for these four points and it is characterized by an adjusted straight line. The resulting equation is $f(t) = 26{,}545.1t - 94{,}075.7$. In addition, the four observations were scaled to fall within [-1, 1] for their use in other evaluations. Figure 9 presents the observations and its adjusted straight line $f(t_e) = 0.4524t - 3.9816$.

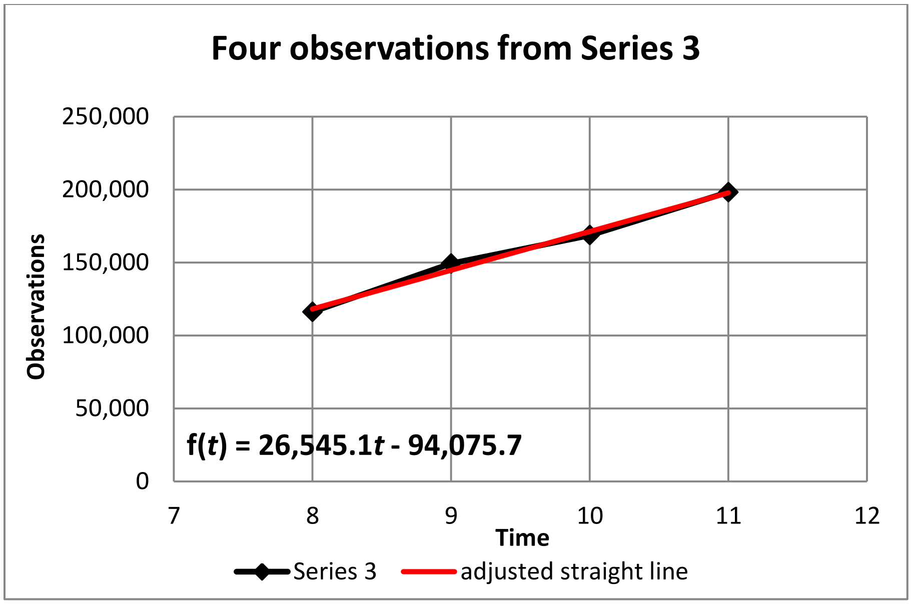


Figure 8. Increasing behavior given for four points and characterized by an adjusted straight line.

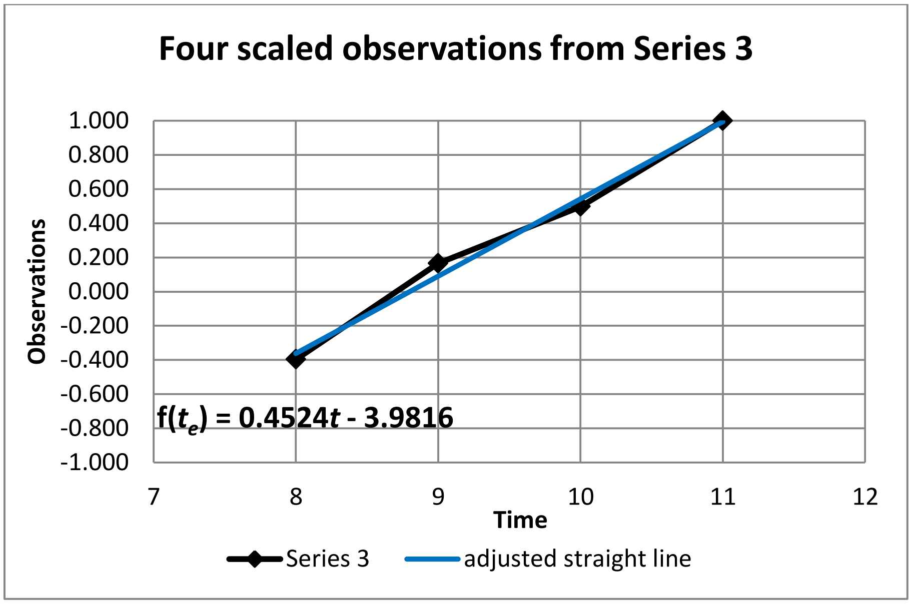


Figure 9. Increasing behavior given for four points from Series 3 and characterized by an adjusted straight line using scaled data.

Translated to the WMS method, the increasing behavior, illustrated in Figure 8, represents the function with known and well-characterized properties.

(iii). Define the experimental region: the region in time.

(vii). Model an optimization problem and (viii) optimize the model: the models will be presented in each particular evaluation.

Finally, (ix) the WMS size will be reported.

From eighteen time series, Series 3 was used as the first evaluation.

5.1.1 Series 3: original data & positive slope.

The problem in this case consisted in finding a one-dimensional WMS and beta values of the straight-line equation with a positive slope, and to determine the size and location of the WMS. To this end, the optimization problem presented in (16) was formulated. In this case, the model (16) was developed to automatically generate the WMS size.

The automatically generated WMS size was found through the minimization of a *composite objective function*. This composite objective function includes an objective for the SSE and an objective for the difference of the bound variables that delimit the one-dimensional WMS size (t^L and t^U). In addition, each objective of the composite objective function was weighted. Weights were assigned with values of 0.1 for SSE and 0.9 for the difference between t^L and t^U (WMS size), similar to a structure of preferences. The largest of the two weights (0.9) was assigned to the difference between t^L and t^U because it will allow for the WMS size to be the first objective to be resolved. The objective that contains the SSE was weighted with 0.1 because it complements the weight 0.9 for the sum to be 1. Given that, the problem is of minimization, with the minus sign in the difference of the bounds, this difference will tend to take a higher value for the WMS size.

In order to keep in the same order of magnitude the SSE and the difference of the bounds, the logarithm base 10 was incorporated in each term of the composite objective function. The logarithm function is not defined for zero or negative values.

During the preliminary development of the mathematical formulation of the optimization problem for the three initial cases of interest, the execution of the solver indicated error of solution. We supposed that this execution error occurred due to the internal initialization of the multiple starting points of the solver setting. Given that it is possible to use random numbers for the multiple starting points, these random numbers could take negative or zero values; or the difference of the bound variables could take negative or zero values. Under this premise, it was added +1 to the SSE value, as well as were added the absolute value and +1 to the difference within logarithm function.

Time series 3 was evaluated using its original data. Given that Series 3 has 24 observations, the ranges for t^L and t^U variables where the WMS will be adjusted are [1, 24].

$$\begin{aligned}
&\textit{Find} \quad t^L, t^U, \beta_0, \beta_1 \quad \textit{for} \\
&\textit{Minimize} \quad 0.1[\log(SSE + 1)] - 0.9[\log(|t^U - t^L| + 1)] \\
&\textit{St.} \\
&1 \le t^L \le 24 \\
&1 \le t^U \le 24 \\
&t^U - t^L \ge 1x10^{-6} \\
&\beta_1 \ge 0 \\
&\beta_0 \ \textit{unrestricted}
\end{aligned} \tag{16}$$

For the integer variable cases the restriction: $t^L, t^U = integer$ was added.

The evaluations considered the following parameters defined in the solver:

- Use of non-truncated continuous, truncated continuous, and integer variables.
- Initializations using multiple starting points with a population size of 100.
- Two levels of constraint precision: $1x10^{-3}$ and $1x10^{-9}$.
- A convergence of 1×10^{-4}.

Results

Appendix B shows the results of these evaluations, Tables B1 to B6. Under the conditions described above, the method generated WMS adjusted to data in different matches. The intercept and the slope took rather large values considering both integer and continuous variables. For the cases considering integer variables in Table B5, the best objective value (0.5269) (best WMS match) out of the 10 initializations on each level of precision was obtained under a precision level of $1x10^{-9}$ in run number 5. In addition, for the integer cases, the WMS included two points (observations) in the time series. Given that the obtained intercept and slope values took rather large values, Figure 10 presents the best WMS match for integer case (and where it only is possible to plot).

Based on the previous findings, it was decided to evaluate the method restricting the slope value. In first instance, the slope was restricted to the ranges [20,000, 30,000] and then, to the ranges [24,545.1, 28,545.1]. Sections 5.1.2 and 5.1.3 will explain these evaluations, respectively.

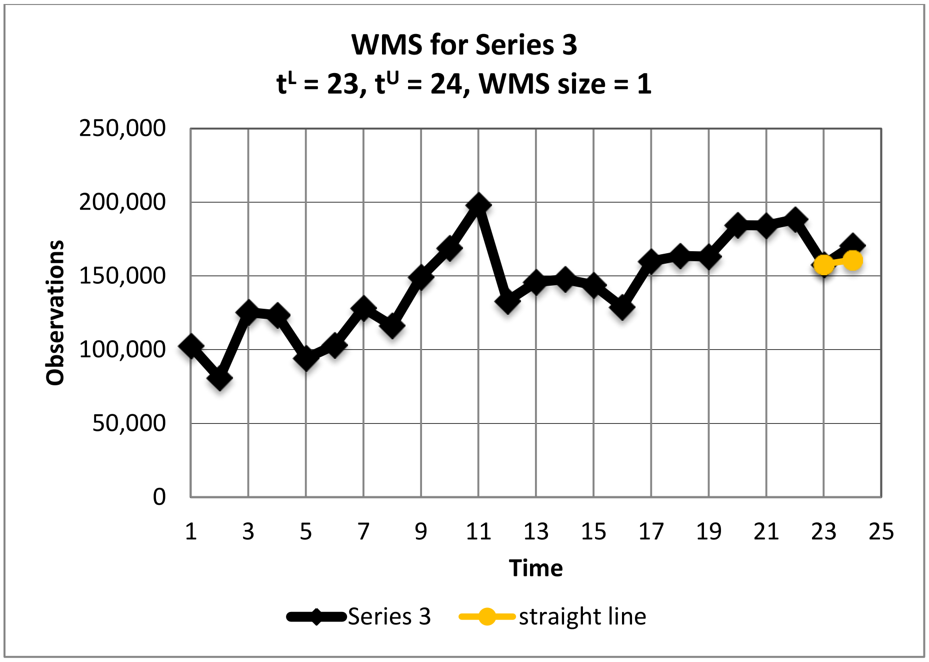


Figure 10. Series 3 and its best WMS match obtained using model (16), to a level of precision of 1x0^{-9}.

5.1.2 Series 3: original data & slope restricted to [20,000, 30,000]

Based on the results discussed in section 5.1.1, new evaluations were executed considering a slope restricted to range [20,000, 30,000]. The problem of this case consisted in finding a one-dimensional WMS and beta values of the straight-line equation with a slope ($ß_1$) restricted to [20,000, 30,000], and determining the size and where WMS follows the model. This range was arbitrarily selected. The optimization problem formulated for this case is presented in (17). The weight values assigned to the composite objective function were of 0.1 for SSE and 0.9 for the difference between lower and upper *t* value. An explanation of the composite objective function formulation was presented in section 5.1.1. The ranges for t^L and t^U variables were [1, 24]. Time Series 3 was evaluated using its original data.

$$
\begin{aligned}
&\textit{Find} \quad t^L, t^U, \beta_0, \beta_1 \quad \textit{for}\\
&\textit{Minimize} \quad 0.1[log\ (SSE + 1)] - 0.9[log\ (|t^U - t^L| + 1)]\\
&\textit{St.}\\
&1 \le t^L \le 24\\
&1 \le t^U \le 24\\
&t^U - t^L \ge 1x10^{-6}\\
&20{,}000 \le \beta_1 \le 30{,}000\\
&\beta_0 \ \textit{unrestricted}
\end{aligned} \tag{17}
$$

For the integer variable cases the restriction: $t^L, t^U = integer$ was added.

The evaluations were considered under the following parameters:

- Use of non-truncated continuous, truncated continuous, and integer decision variables of time.
- Initializations using multiple starting points with a population size of 100.
- Two levels of constraint precision: $1x10^{-3}$ and $1x10^{-9}$.
- A convergence of 1×10^{-4}.

Results

Tables B7 to B12, included in Appendix B, show the detailed results for all evaluations generated in this section. In many of the evaluations in this section, the intercept value took large values and the WMS included different matches. Figure 11 presents the WMS matches for integer case, evaluated to a level precision of $1x10^{-9}$ where it was possible to plot. In this figure, each line is adjusted to a range of time according to its respective WMS. For example, the line Y2_P1 (given by the run number 2 and the precision level of $1x10^{-9}$) represents the straight line found by the method for the match of a WMS that included all the points (*t* ranged to [1, 24]). The straight line equation obtained in this case was Y2_P1 = 20,000 t + $5.11x10^{-6}$; Y2_P1 mean the run number 2 (Y2) using a level of precision of $1x10^{-9}$ (precision level 1 or P1), Y4_P1 mean the run number 4 (Y4) using a level of precision of $1x10^{-9}$ (P1), and so on. Appendix B presents these results in more detail.

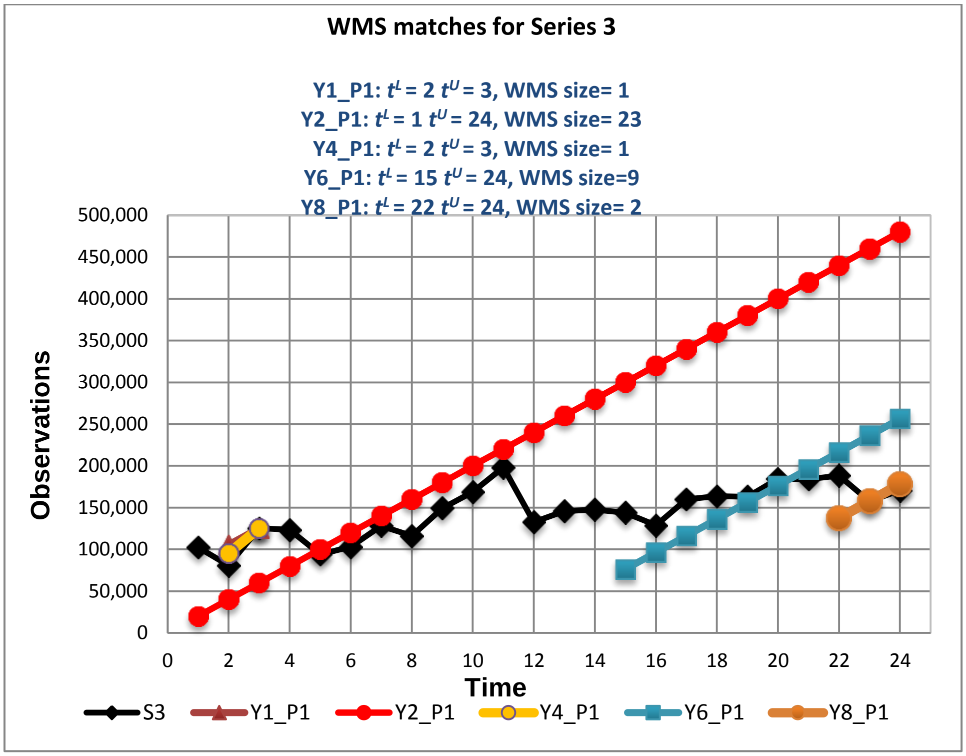


Figure 11. WMS matches for the time Series 3 using model (17), to a level of precision of $1x0^{-9}$.

5.1.3 Series 3: original data & slope restricted to [24,545.1, 28,545.1]

The problem of this case consisted in finding a one-dimensional WMS and beta values of the straight equation with a slope ($ß_1$) restricted to [24,545.1, 28,545.1], and to determine the size of the WMS. This selected range was computed by adding and subtracting 2,000 to the slope value (26,545.1 ± 2,000) of the adjusted straight line shown in Figure 8. The optimization problem formulated for this case is presented in (18). The weight values and ranges of t^L and t^U variables were [1, 24]. Time Series 3 was evaluated using its original data.

$$\begin{aligned}
&\textit{Find} \quad t^L, t^U, \beta_0, \beta_1 \quad \textit{for} \\
&\textit{Minimize} \quad 0.1[\log(SSE+1)] - 0.9[\log(|t^U - t^L| + 1)] \\
&\textit{St.} \\
&1 \le t^L \le 24 \\
&1 \le t^U \le 24 \\
&t^U - t^L \ge 1x10^{-6} \\
&24{,}545.1 \le \beta_1 \le 28{,}545.1 \\
&\beta_0 \textit{ unrestricted}
\end{aligned} \tag{18}$$

For the integer variable cases, the restriction: $t^L, t^U = integer$ was added.

The evaluations were considered under the following parameters:

- Use of non-truncated continuous, truncated continuous, and integer variables.
- Initializations using multiple starting points with a population size of 100.
- Two levels of constraint precision: $1x10^{-3}$ and $1x10^{-9}$.
- A convergence of 1×10^{-4}.

For the case where the intercept was fixed to -94,075.7 (the intercept value from straight line as shown in Figure 8), the model (19) was formulated.

$$\begin{aligned}
&\textit{Find} \quad t^L, t^U, \beta_1 \quad \textit{for} \\
&\textit{Minimize} \quad 0.1[\log(SSE+1)] - 0.9[\log(|t^U - t^L| + 1)] \\
&\textit{St.} \\
&1 \le t^L \le 24 \\
&1 \le t^U \le 24 \\
&t^U - t^L \ge 1x10^{-6} \\
&24{,}545.1 \le \beta_1 \le 28{,}545.1 \\
&t^L, t^U = \textit{integer}
\end{aligned} \tag{19}$$

The evaluations fixing the intercept included:

- Initializations using multiple starting points with a population size of 100.
- Two levels of constraint precision: $1x10^{-3}$ and $1x10^{-9}$.
- A convergence of 1×10^{-4}.

Results

Tables B13 to B20, included in Appendix B, show the detailed results for all evaluations generated in this section. Tables B13 to B18 present the results using model (18). Using model (18), the intercept ($ß_0$) value took large values and the WMS included different matches. Figure 12 illustrates some matches of WMS for integer case evaluated to a level precision of $1x10^{-9}$ (Table B17). The adjusted line Y1_P1 represents the best solution found by the method. This WMS included all the points of the time series. The straight line equation obtained in this case was Y1_P1 = 24,545.10 t -161,894.30. From Figure 12, Y1_P1 means run number 1 (or Y1) using a level of precision of $1x10^{-9}$ (P1).

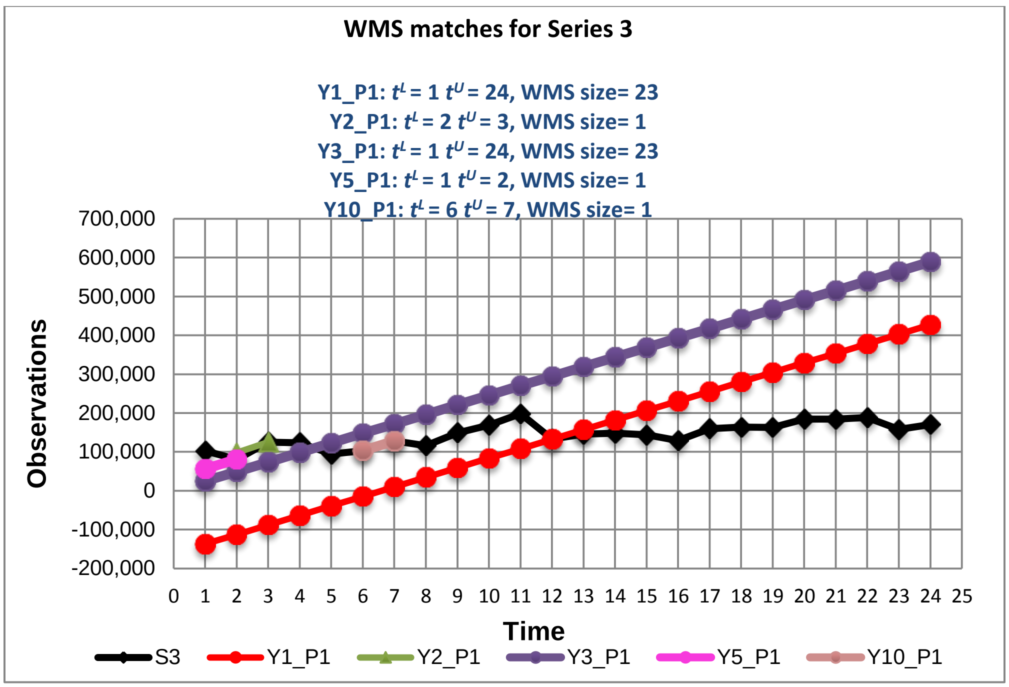


Figure 12.WMS matches for the time Series 3 using model (18), to a level of precision of $1x0^{-9}$.

For the case in which the intercept ($ß_0$) was fixed to -94,075.7, the decision variables of the model were reduced to three: slope ($ß_1$), t^L and t^U, (refer to model 19). Tables B19 and B20 present the detailed results using model (19). Figures 13 and 14 show the WMS matches found by the method. In Figure 13, the best solution of WMS can be observed in Y6_P1. In Figure 14, the best solution of WMS found corresponding to the best objective value (0.36773) can be observed in Y10_P2. This result originated the fact that the WMS included three associated points at time, where the increasing behavior was more evident. The WMS included points 8 to 10 of the time series (line Y10_P2). However, although this looks attractive, the objective of the evaluation of the method must consider the intercept and slope as decision variables to be more generally applicable.

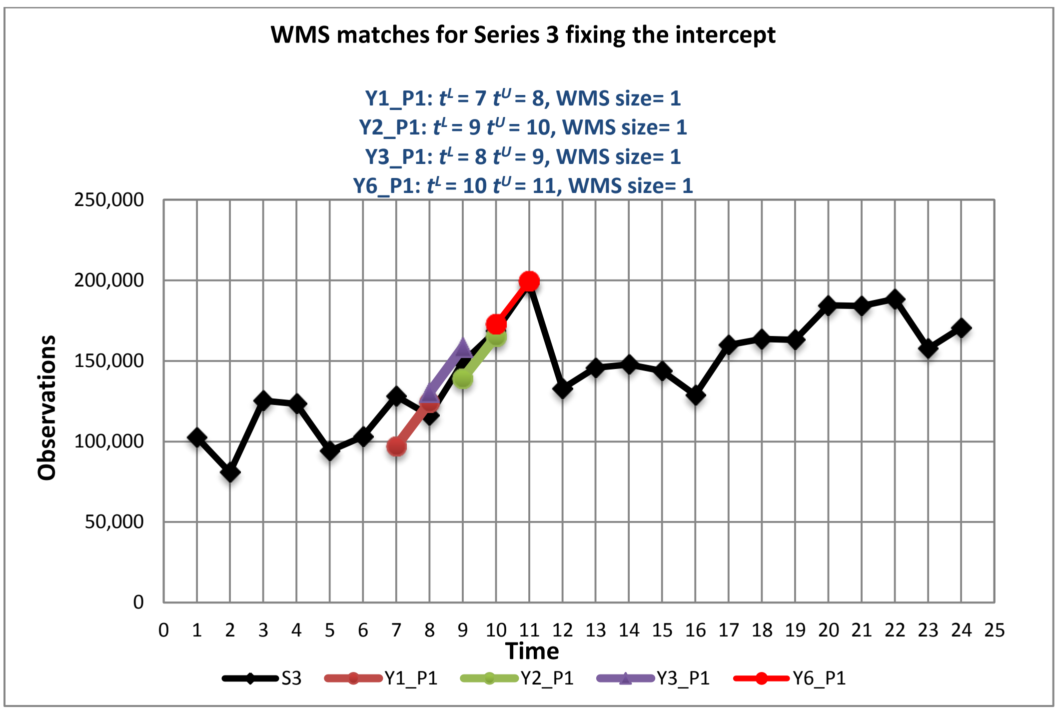


Figure 13.WMS matches for the time Series 3 using model (19), considering fixed intercept, and a level of constraint precision of $1x0^{-9}$.

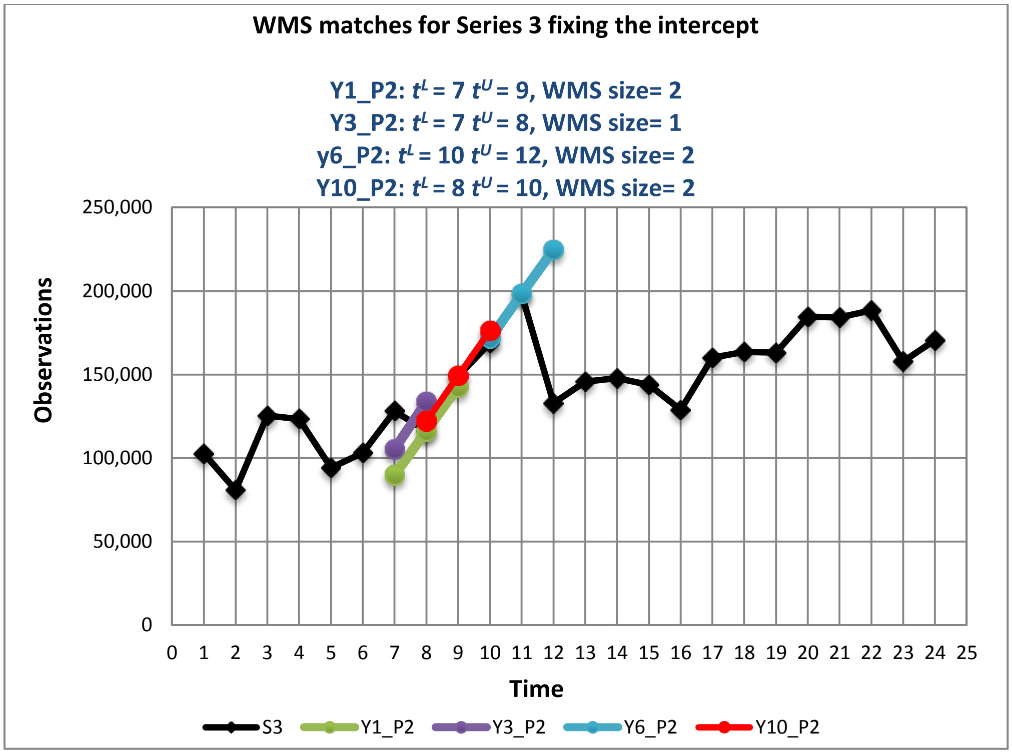


Figure 14.WMS matches for the time Series 3 using model (19), considering fixed intercept, and a level of precision of 1x0^{-3}.

With these results, we inferred that it possible to improve the output if the time series are scaled. Given that the results obtained for evaluations in sections 5.1.1, 5.1.2 and 5.1.3 provide WMS which included all the observations but did not necessarily show an increasing behavior, this motivated the development of new evaluations using some parameters as in the previous evaluations but scaling the time series data to range [-1, 1].

5.1.4 Series 3: scaled data, slope more than 0, & slope restricted to [0.0024, 0.9024]

At this stage, new evaluations using scaled data of Time Series 3 were generated. The data was scaled in a range of [-1, 1]. Basically, the problem of the evaluation presented in this section consisted in finding a one-dimensional WMS and beta values of the straight-line equation with a positive slope, and to determine the size of WMS. A first instance using weights of 0.1 for SSE and the 0.9 for WMS size in the composite objective function was performed. In a second instance, the weights were varied to three additional levels. An explanation of the composite objective function formulation was presented in section 5.1.1.

First instance. The optimization problem presented in (16) was applied to the case where the slope was restricted to be positive and considering integer variables, and the optimization problem in (20) was applied to the integer case where the slope variable ($ß_1$) was restricted to [0.0024, 0.9024]. Weights of 0.1 for SSE and the 0.9 for WMS size were assigned. The range for the time variables t^L and t^U were of [1, 24] and the selected ranges for slope ($ß_1$) was computed adding and subtracting 0.45 to the slope value (0.4524 ± 0.45) of the adjusted straight line to the four points shown in Figure 9.

$$\begin{aligned}
&\text{Find} \quad t^L, t^U, ß_0, ß_1 \quad \text{for} \\
&\text{Minimize} \quad 0.1[\log(SSE + 1)] - 0.9[\log(|t^U - t^L| + 1)] \\
&\text{St.} \\
&1 \le t^L \le 24 \\
&1 \le t^U \le 24 \\
&t^U - t^L \ge 1x10^{-6} \\
&0.0024 \le ß_1 \le 0.9024 \\
&ß_0 \text{ unrestricted} \\
&t^L, t^U = \text{integer}
\end{aligned} \qquad (20)$$

The evaluations were considered under the following parameters:

- Initializations using multiple starting points with a population size of 100.
- The constraint precision was considered in two levels: $1x10^{-3}$ and $1x10^{-9}$.
- A convergence of $1 \text{ x } 10^{-4}$.

Second instance. The optimization problems for diverse weights values in the composite objective value were generalized and presented in (21.1) and (21.2). Weights of 1 for SSE (α_1=1) combined with levels of 90, 900, and 9,000 for the WMS size (α_2), were assigned. $ß_1$ was restricted to more than 0 (model (21.1)) and to [0.0024, 0.9024] (model (21.2)). The time variables were restricted to the same ranges as was set for previous evaluations.

$$\begin{aligned}
&Find \quad t^L, t^U, ß_0, ß_1 \quad for\\
&Minimize \quad \alpha_1 \left[log\ (SSE + 1)\right] - \alpha_2 \left[log\ (|t^U - t^L| + 1)\right]\\
&St.\\
&1 \le t^L \le 24\\
&1 \le t^U \le 24\\
&t^U - t^L \ge 1x10^{-6}\\
&ß_1 \ge 0\\
&ß_0 \ unrestricted\\
&t^L, t^U = integer
\end{aligned} \tag{21.1}$$

$$\begin{aligned}
&Find \quad t^L, t^U, ß_0, ß_1 \quad for\\
&Minimize \quad \alpha_1 \left[log\ (SSE + 1)\right] - \alpha_2 \left[log\ (|t^U - t^L| + 1)\right]\\
&St.\\
&1 \le t^L \le 24\\
&1 \le t^U \le 24\\
&t^U - t^L \ge 1x10^{-6}\\
&0.0024 \le ß_1 \le 0.9024\\
&ß_0 \ unrestricted\\
&t^L, t^U = integer
\end{aligned} \tag{21.2}$$

The evaluations were considered under the following parameters:

- Three levels of weights for the objective composite function.
- Initializations using multiple starting points with a population size of 100.
- The constraint precision was considered in two levels: $1x10^{-3}$ and $1x10^{-9}$.
- A convergence of 1×10^{-4}.

An additional evaluation fixing the intercept ($ß_0$) to -3.9816 (the intercept value from straight line shows in Figure 9) was evaluated for the integer case where the slope variable ($ß_1$) was restricted to [0.0024, 0.9024]; in this evaluation case, the set variables were reduced to three decision variables: t^L, t^U, and $ß_1$.

Results

Tables B21 to B44, included in Appendix B, show the detailed results for all evaluations generated in this section.

Figure 15 illustrates the best solution matches of WMS for the first instance, when the slope was restricted to be positive, in the two precision levels using the weights combination of 0.1 and 0.9. The results obtained using the weights combination of 0.1 and 0.9, were practically the same because the values were very closed for the initializations. The best solutions were generated in Y7_P1 (run number 7 and constraint precision of $1x10^{-9}$) and Y2_P2 (run number 2 and constraint precision of $1x10^{-3}$). Tables B21 and B22 present these results.

The best solution of WMS matches for the second instance, when the slope was restricted to be positive, using the weights combination of 1 and 90 were provided in Y1_P1 and Y6_P1, Y6_P2, Y9_P2, and Y10_P2, which corresponded to the lowest values of all initializations in each case (Tables B23 and B24).

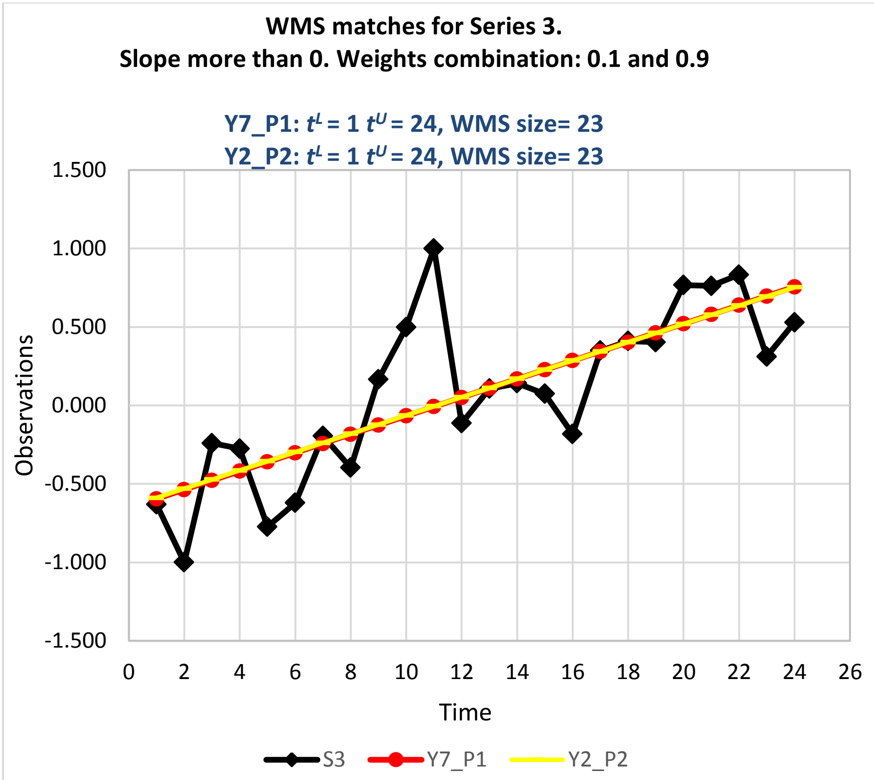


Figure 15.Solutions of WMS matches for scaled Time Series 3 for the weights combination of 0.1 and 0.9.

From the resulting WMS matches when the slope was restricted to be positive, using the weights combination of 1 and 900, the best objective value was generated in Y2_P1 and Y4_P2 (Tables B25 and B26). In addition, when the results obtained considered the level precision 2 (P2), the straight-line equations were different among them (see Table B26). For the weights combination of 1 and 9000, the best solution corresponded to matches Y5_P1 (see Table B27) and Y4_P2 (Table B28). Figure 16 shows the solutions of three runs with a SSE lower (include the best solution Y4_P2).

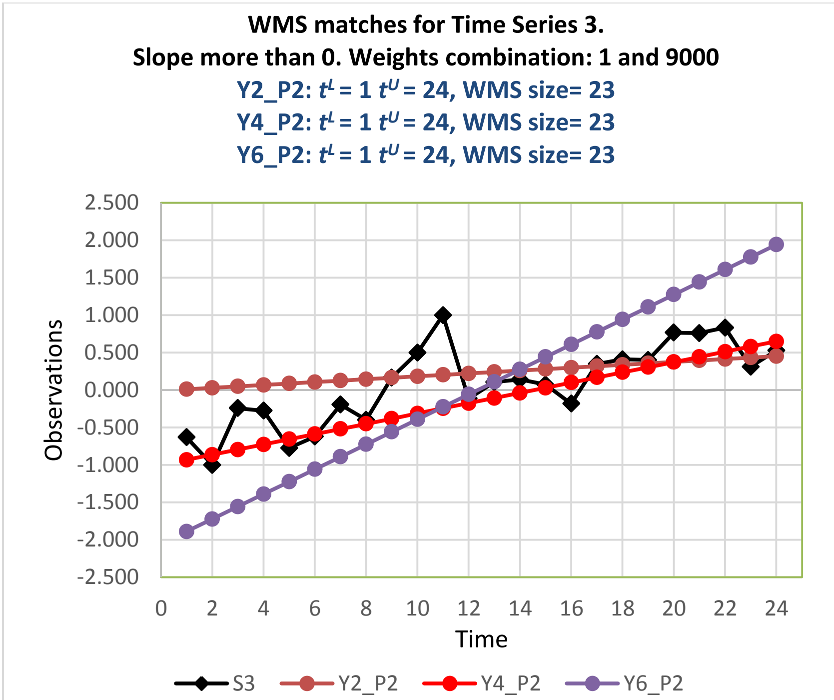


Figure 16. Solutions of WMS matches for scaled Time Series 3 for the weights combination of 1 and 9000. Level of precision of $1x10^{-3}$.

For the weights combination of 0.1 and 0.9, when the slope was restricted to [0.0024, 0.9024], the best solutions were provided in Y2_P1 and Y4_P2 (and Y3_P2) (see Tables B29 and B30). Figure 17 presents the matches up to where they are possible to see with clarity. Under the level of precision of $1x10^{-9}$, the WMS sizes obtained were: in Y2_P1 of 23, Y4_P1 and Y5_P1 of 22.

When weights of 1 and 90 were assigned, when the slope was restricted to [0.0024, 0.9024], the best matches resulted in Y8_P1 and Y3_P2 (see Tables B31 and B32). Figure 18 shows some matches for the results obtained in this case using a level of precision of $1x10^{-9}$.This figure presents the different matches up to where they are possible to see with clarity.

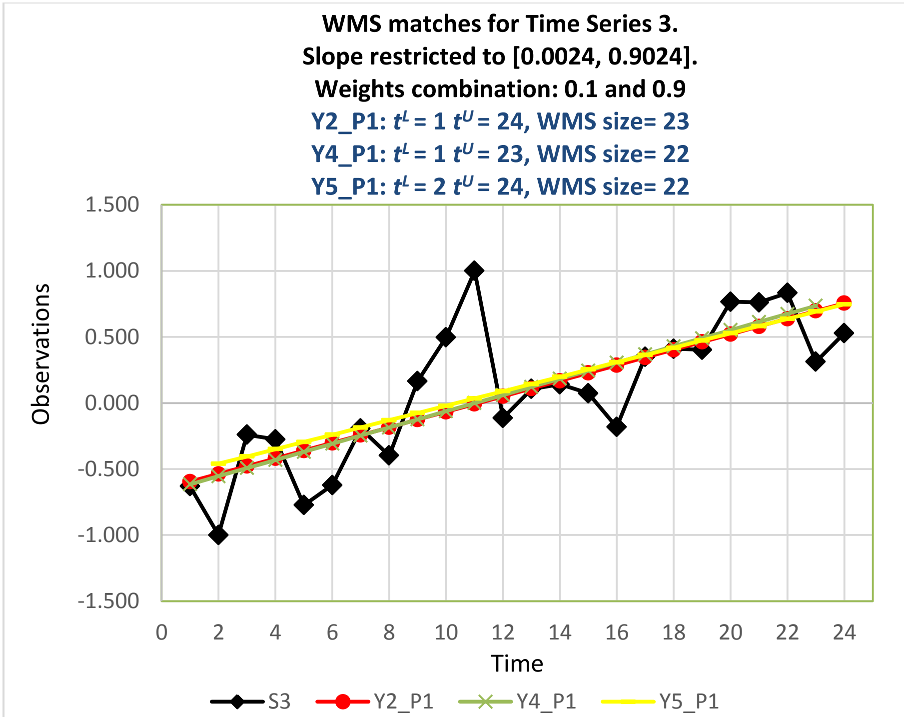


Figure 17. Solutions of WMS matches for scaled Time Series 3 for the weights combination of 0.1 and 0.9. Level of precision of $1x10^{-9}$.

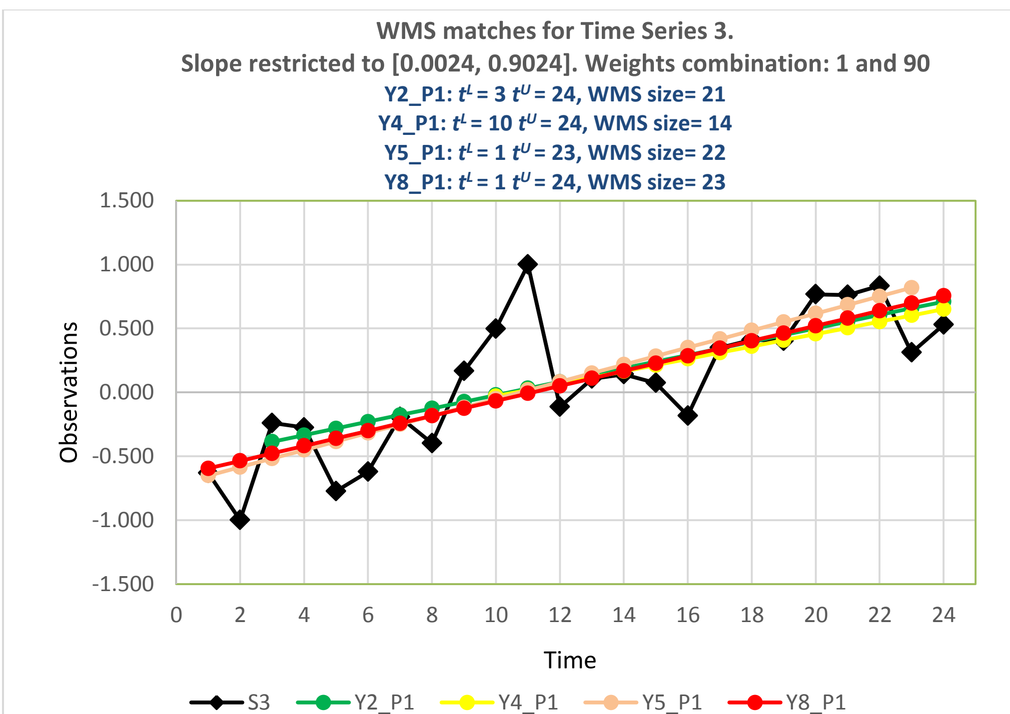


Figure 18. Solutions of WMS matches for scaled Time Series 3 for the weights combination of 1 and 90. Level of precision of $1x10^{-9}$.

Using the weights of 1 and 900, when the slope was restricted to [0.0024, 0.9024], the best solutions of WMS match for each case were obtained in Y4_P1 (run number 4 and level of precision of $1x10^{-9}$) and Y2_P2 (run number 2 and level of precision of $1x10^{-3}$). When weights combination of 1 and 9000 were assigned, the best solutions were obtained in Y5_P1, Y7_P1, Y10_P1, and Y4_P2. Tables B33, B34, B35, and B36 present the detailed results. In the cases when the weights combination of 1 and 900 and 1 and 9000 were assigned, the size of the windows of maximum similarity shows similar patterns including all observations of the time series (WMS size = 23).

From the evaluations fixing the intercept ($ß_0$) to -3.9816, the best solutions of WMS matches were generated in Y3_P1 and Y2_P2 (weights of 0.1 and 0.9), Y1_P1 and Y7_P2 (weights of 1 and 90), Y1_P1, Y10_P1 and Y3_P2 (weights of 1 and 900), and Y3_P1 and Y3_P2 (weights of 1 and 9000). These solutions included all points of the time series (WMS size = 23). Figure 19 shows some of the solution matches when the weights combination of 0.1 and 0.9 considering the fixed intercept was evaluated. Tables B37 to B44 present the results considering the fixed intercept.

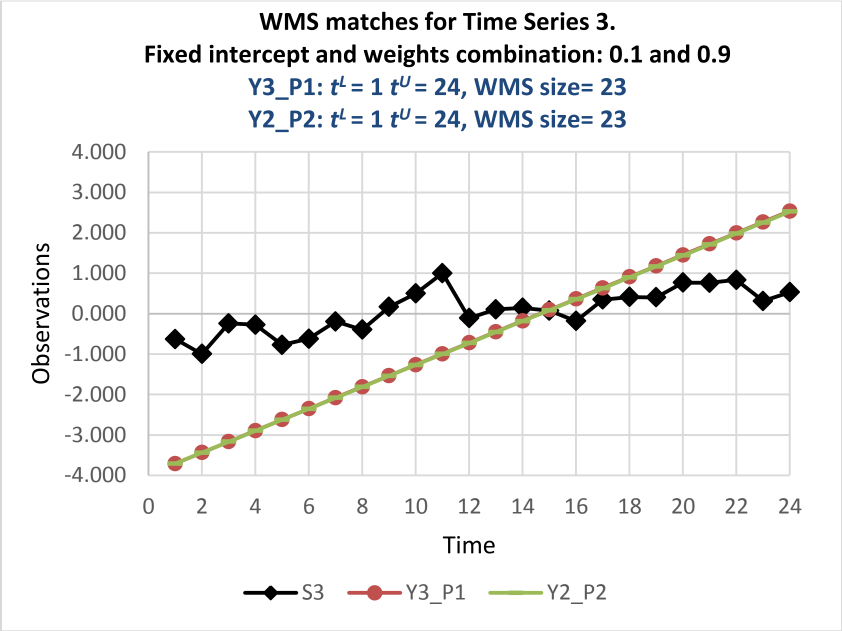


Figure 19. Solutions of WMS matches for scaled Time Series 3 fixing the intercept for the weights combination of 0.1 and 0.9.

In general, for the cases that considered a positive slope and weights combinations of 0.1 and 0.9 and weights of 1 and 90, the results indicated the following: the WMS included all points of the time series and the SSE values was consistent along the evaluations. For the weights combinations of 1 and 900, and of 1 and 9000, the WMS included all points and the constraint precision to a level of $1x10^{-3}$ provided SSE values pretty high.

For the cases that considered the slope restricted to [0.0024, 0.9024] and weight combinations of 0.1 and 0.9, the results indicated that the WMS provided evidence of different matches. Using weights of 1 and 90, the results indicated different WMS matches, but the WMS matches generated using weights of 0.1 and 0.9 were more. For the weight combinations of 1 and 900, and weights of 1 and 9000, the WMS matches included all points of the time series. The evaluations using a precision level of $1x10^{-3}$ provided high values of SSE for both cases of weights, and gave large values of the intercept.

In the cases that considered a fixed intercept and a slope restricted to [0.0024, 0.9024] for the four weight combinations, the WMS included all points.

In summary, with the level of precision of $1x10^{-9}$ the incidence was greater in obtaining better solution values (smaller values) of the SSE. With the weights combination of 0.1 and 0.9 and restricting the slope to [0.0024, 0.9024] there was a better tend of the method to generate a WMS that did not included all observations of the time series. For this reason, it was considered to continue the method application at the remaining seventeen time series using weights of 0.1 for SSE and 0.9 for the window size and the slope restricted to [0.0024, 0.9024].

5.1.5 Evaluations along all series

As was mentioned in section 5.1.4, the remaining seventeen series (time series 1 and 2 as well as those from 4 to 18) were evaluated using the optimization model generalized in (22) because the method tended to provide different WMS matches. In this model, t_{max} represents the number of total observations of the time series. The parameters and conditions were considered as following:

- Weights combination (for the composite objective function) of 0.1 (for SSE) and 0.9 (for the window size).
- Scaled time series to range [-1, 1].
- Integer time decision variables.
- Initializations using multiple starting points with a population size of 100.
- Two levels of precision: $1x10^{-3}$ and $1x10^{-9}$.
- A convergence of 1×10^{-4}.

Find $t^L, t^U, ß_0, ß_1$ *for*

Minimize $0.1\ [log\ (SSE + 1)] - 0.9\ [log\ (\left| t^U - t^L \right| + 1)]$

St.

$$
\begin{aligned}
&1 \leq t^L \leq t_{max} \\
&1 \leq t^U \leq t_{max} \\
&t^U - t^L \geq 1x10^{-6} \\
&0.0024 \leq ß_1 \leq 0.9024 \\
&ß_0 \text{ unrestricted} \\
&t^L, t^U = \text{integer}
\end{aligned}
\qquad (22)
$$

Results

Identifying two general patterns in the time series and grouping the series in a) the time series that show a global linear trend in their data (times series 1, 12 and 14) and b) in the time series that include evident zones of increasing behavior and zones with decreasing behavior in their data (series 2, 5 to 11, as well as series 13, and 15 to 18), it was selected some of them to illustrate the results of the evaluations in this section. Time series 5, 11, 12, and 18 are illustrated.

Figures 20, 21, 22 show some matches for time series 5, 11, and 12, respectively, evaluated to a constraint precision of $1x10^{-9}$. Figure 23 shows some matches for time series 18 evaluated to a constraint precision of $1x10^{-3}$. Also, each figure indicates the WMS size associated to the match. The four figures illustrated here show different WMS matches. However, the best solution in each case, the WMS included all points of the time series. From Appendix B, Tables B45 to B78 show the detailed results of each time series.

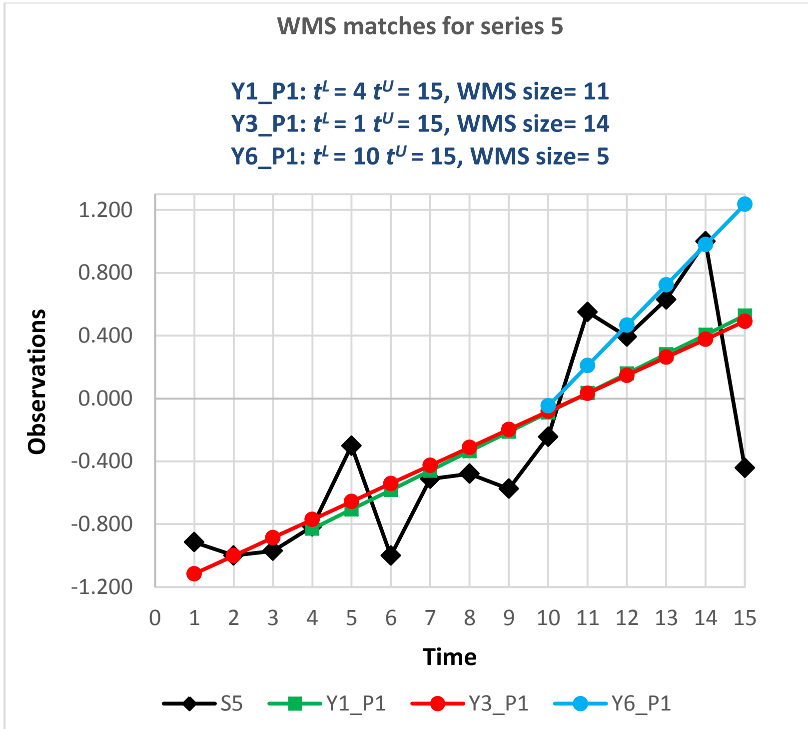


Figure 20. Solutions of WMS matches for scaled Time Series 5. Level of precision of $1x10^{-9}$.

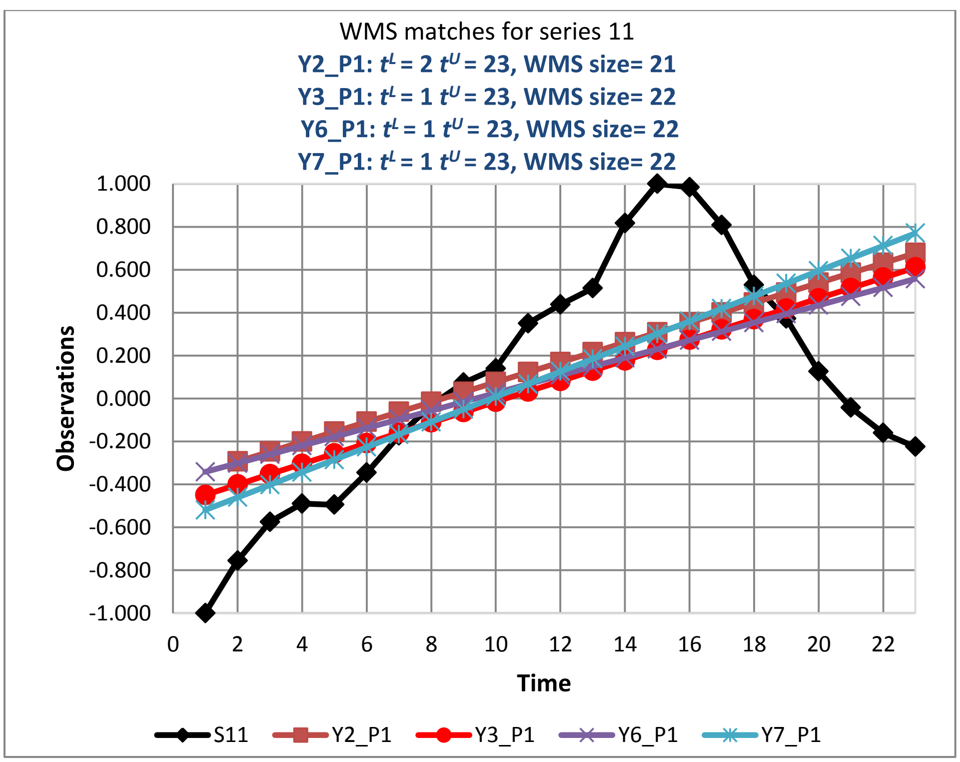


Figure 21. Solutions of WMS matches for scaled Time Series 11. Level of precision of $1x10^{-9}$.

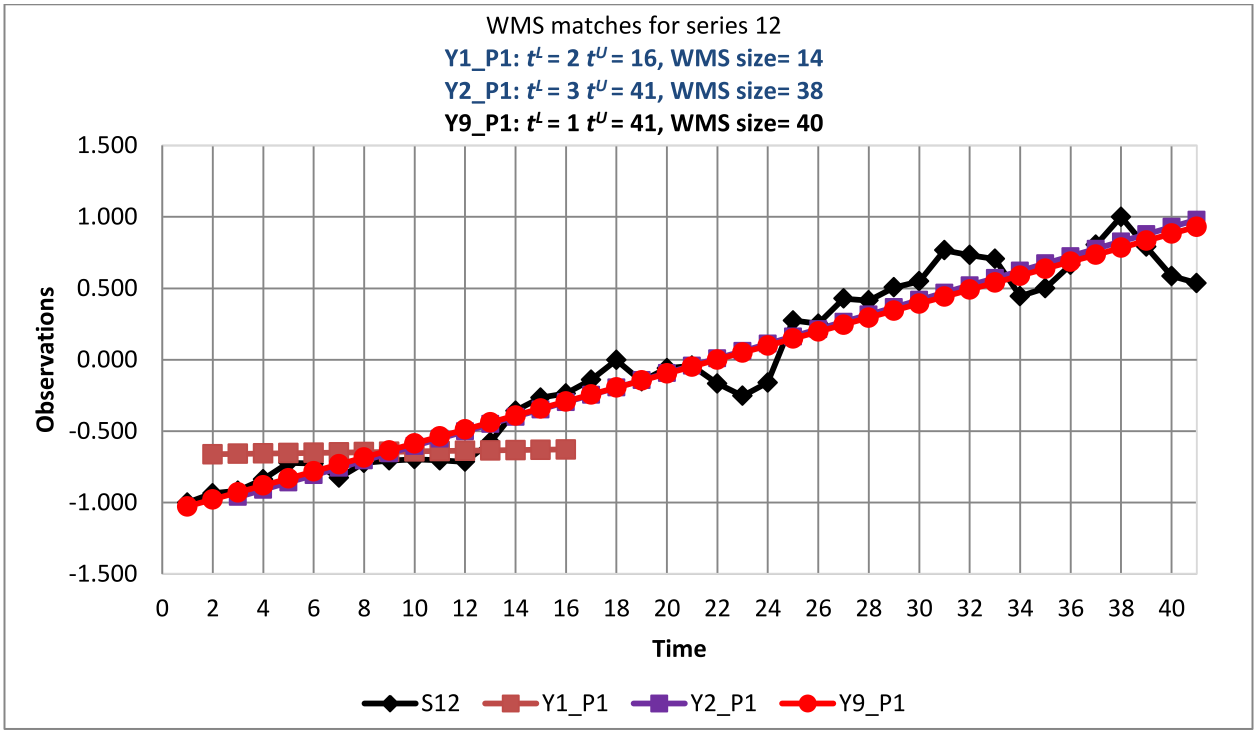


Figure 22. Solutions of WMS matches for scaled Time Series 12. Level of precision of $1x10^{-9}$.

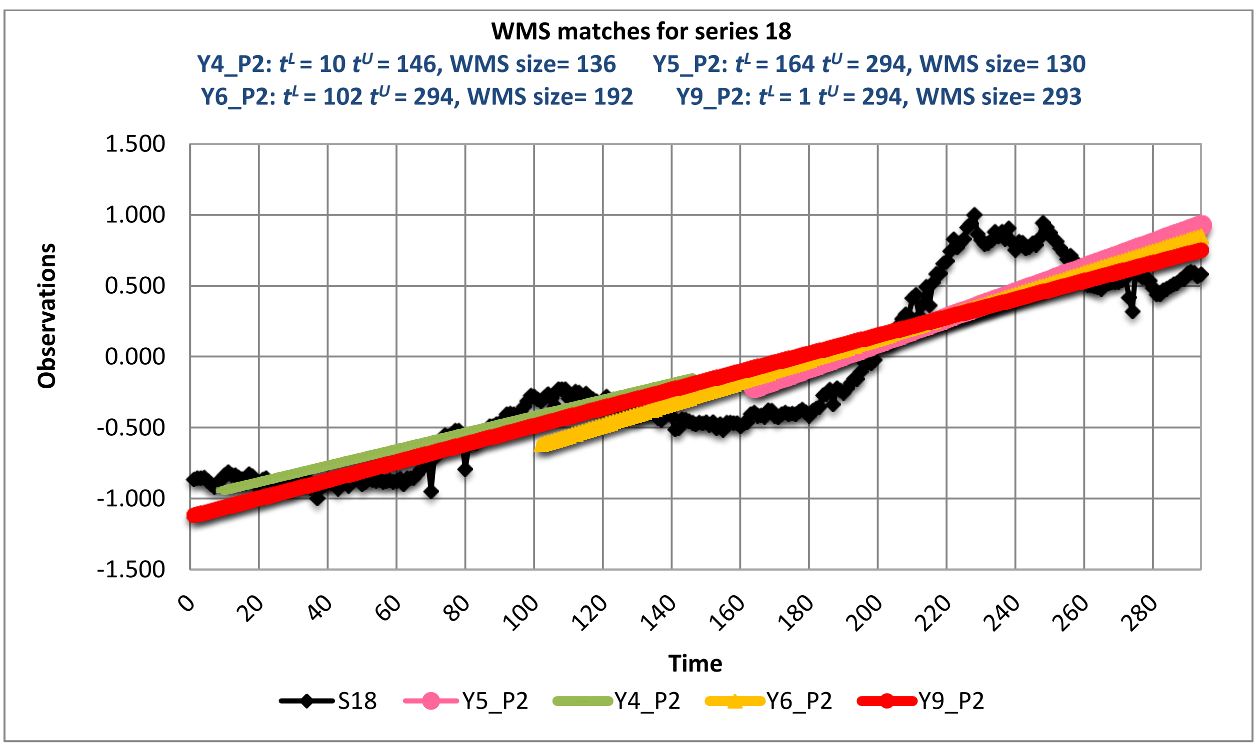


Figure 23. Solutions of WMS matches for scaled Time Series 18. Level of precision of $1x10^{-3}$.

Although the results of these evaluations provided different matches for each time series, a tendency prevailed in that the best WMS match included all points of the time series for the most of the series. Therefore, it was decided to evaluate the method considering the model with the one-objective approach (minimizing SSE).

5.1.6 Exploration of the time series under the one-objective approach

In order to explore if the method generated different matches of WMS with a better discernment, the scaled time series were evaluated under one-objective approach. For each scaled series, a predefined minimum WMS size (epsilon value) was used. Table 7 presents the epsilon values for each time series. The problem formulated in (23) was used for this exploration. In model (23), t_{max} represents the number of total observations of the time series. The parameters and conditions were considered as following:

- Initializations using multiple starting points with a population size of 100.
- A level of precision of $1\text{x}10^{-9}$.
- A convergence of 1×10^{-4}.

$$
\begin{aligned}
&\textit{Find} \quad t^L, t^U, ß_0, ß_1 \quad \textit{for} \\
&\textit{Minimize} \quad SSE \\
&\textit{St.} \\
&1 \leq t^L \leq t_{max} \\
&1 \leq t^U \leq t_{max} \\
&0.0024 \leq ß_1 \leq 0.9024 \\
&ß_0 \textit{ unconstrained} \\
&t^L, t^U = \textit{integer} \\
&t^U - t^L \geq \textit{Epsilon for Time Series 1, 4, 5, 6, ..., 18.} \\
&t^U - t^L = \textit{Epsilon for Time Series 2 and 3.}
\end{aligned}
\tag{23}
$$

Time Series	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18
Epsilon value	3	2	3	1	1	1	2	1	2	1	5	4	4	20	12	20	10	25

Table 7. Epsilon values of the optimization problem (23) for each Series.

Results

The objective of reconsidering the formulation made under one-objective approach was to explore if the one-dimensional WMS matches can be determined by the method with a notable enough discernment. Although the results of this exploration provided matches that are limited to the epsilon values of each series for the most part, there are some cases where different matches can be observed. Figures 24 to 41 show some WMS matches for each time series. These matches correspond to the lowest cases of error. The zone of the WMS where the linear model is a good descriptor of data is highlighted in the figure. In the case where the same match were obtained in more than two executed initializations, only one of these coincidences was presented in the graph. For the case where the same result coincided in the ten executed initializations, the graph mentioned it as “the lowest case of SSE”. For example, in series 5 all solutions coincided with the same match ($t^L = 2$, $t^U = 3$) and the linear model ($Y = -1.0584 + 0.0297t$) and the Figure 28 mentions “the lowest case of SSE”. Another example corresponds to series 6, where the results only were two different matches and the remaining solutions coincided with Y5 or Y7. Figure 29 exhibits “the two lowest case of SSE” given these two matches. Detailed results are included in Appendix B, Tables B79 to B96.

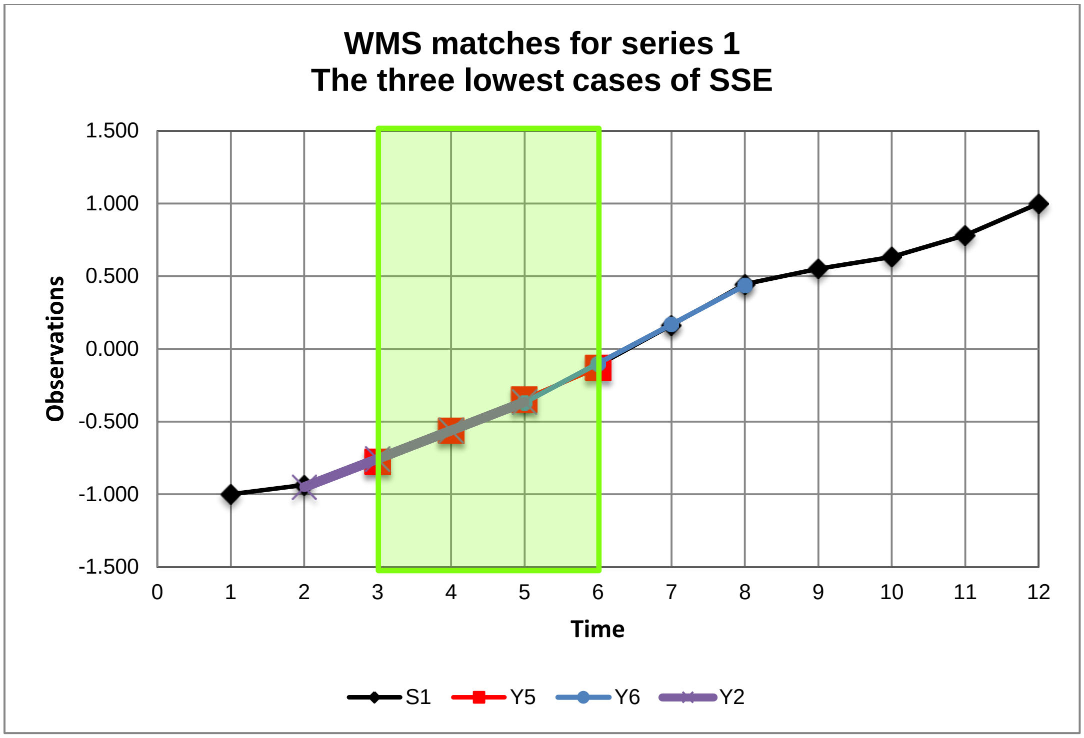


Figure 24. The lowest cases of SSE, WMS matches for scaled Time Series 1.

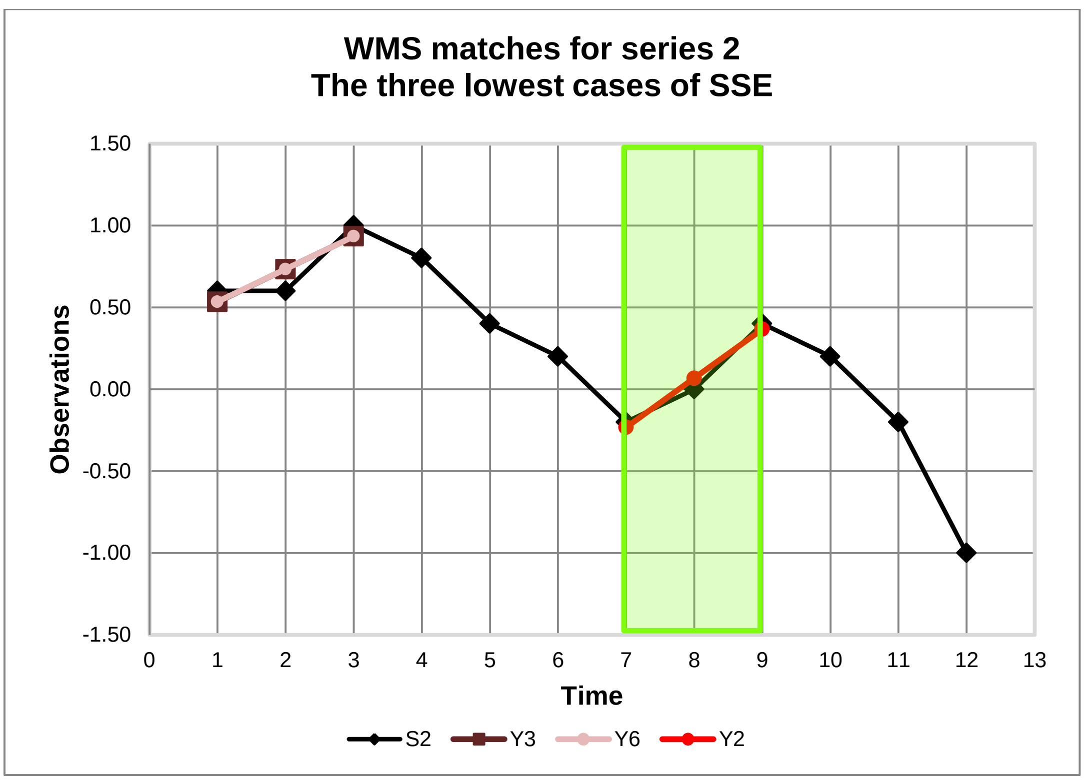


Figure 25. The lowest cases of SSE, WMS matches for scaled Time Series 2.

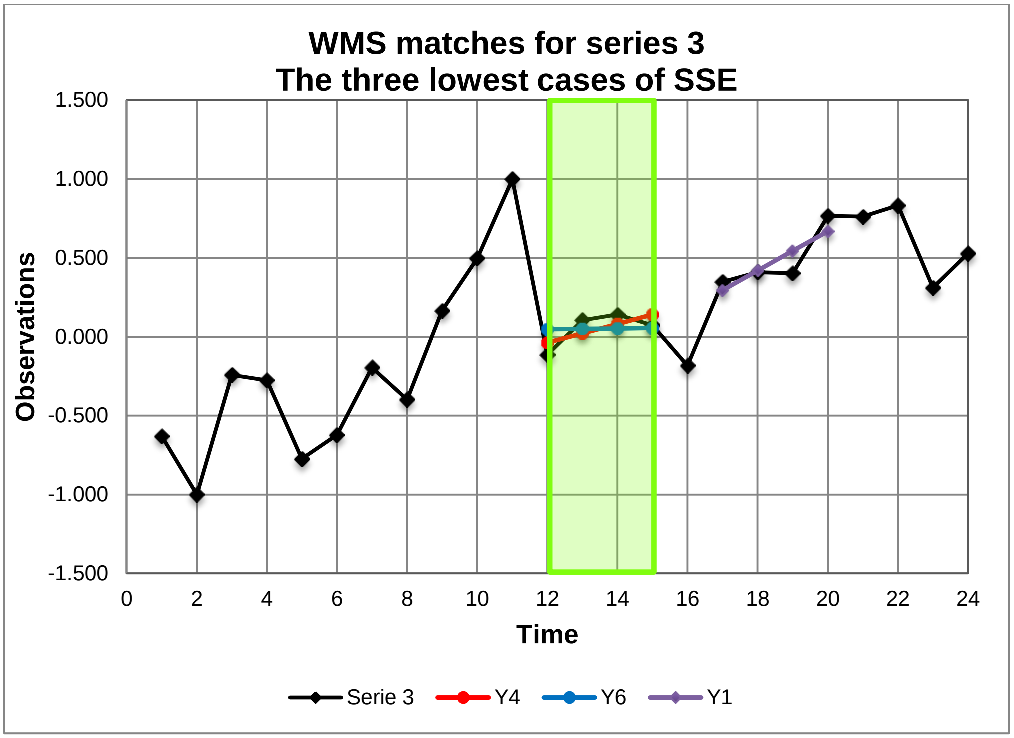


Figure 26. The lowest cases of SSE, WMS matches for scaled Time Series 3.

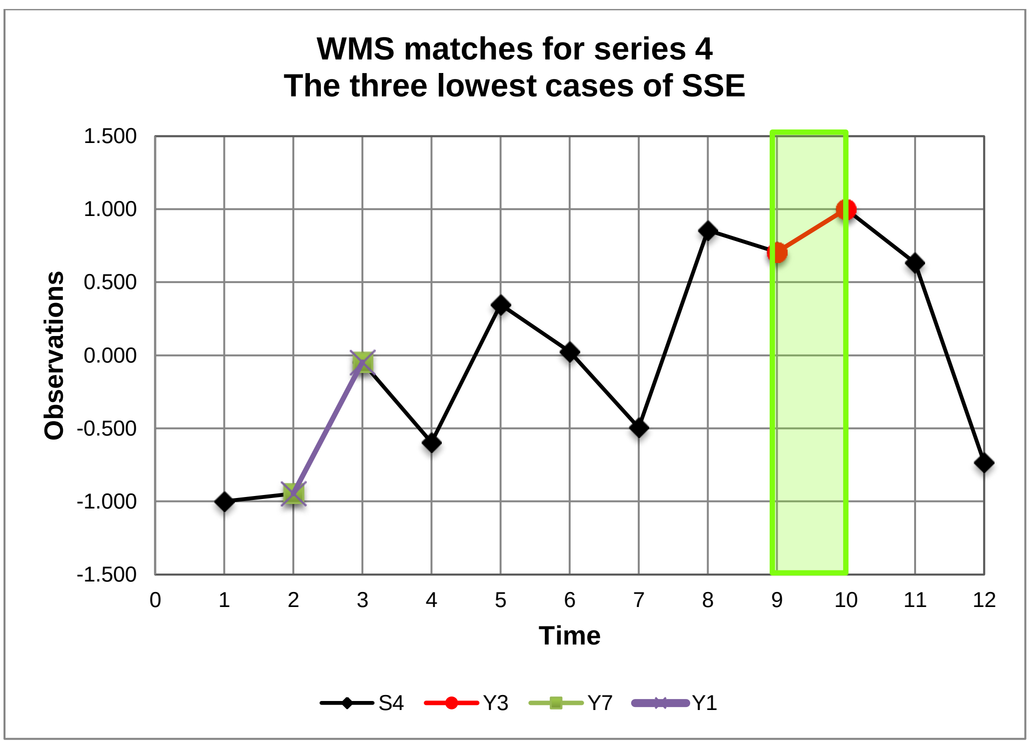


Figure 27. The lowest cases of SSE, WMS matches for scaled Time Series 4.

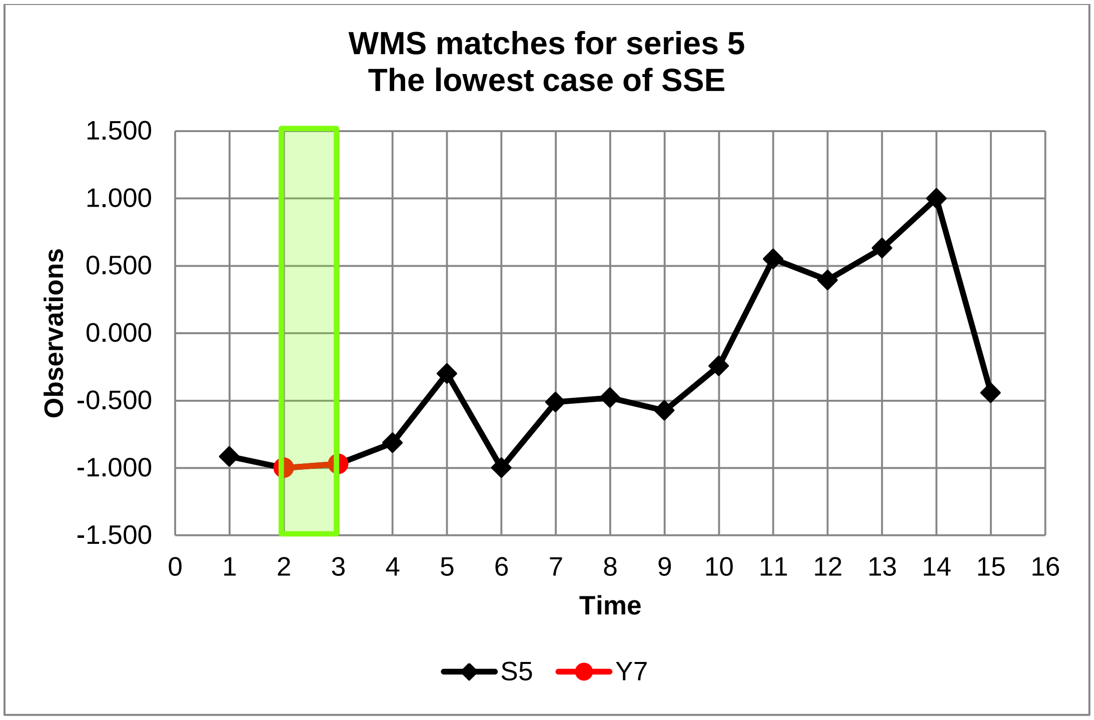


Figure 28. The lowest cases of SSE, WMS matches for scaled Time Series 5.

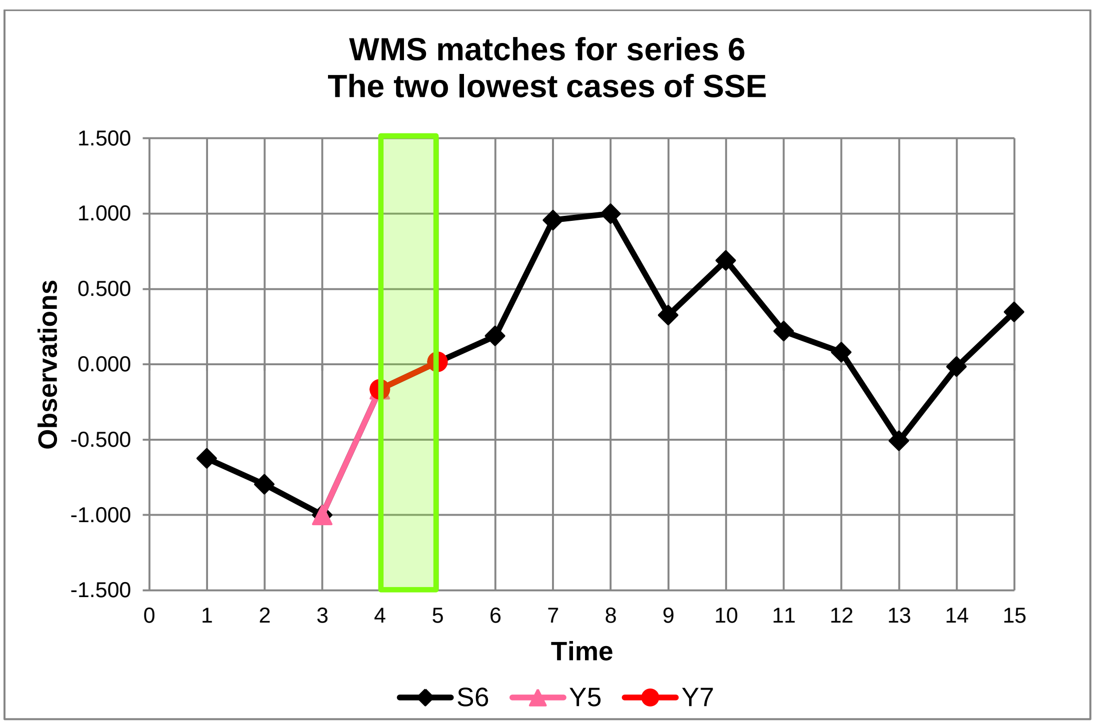


Figure 29. The lowest cases of SSE, WMS matches for scaled Time Series 6.

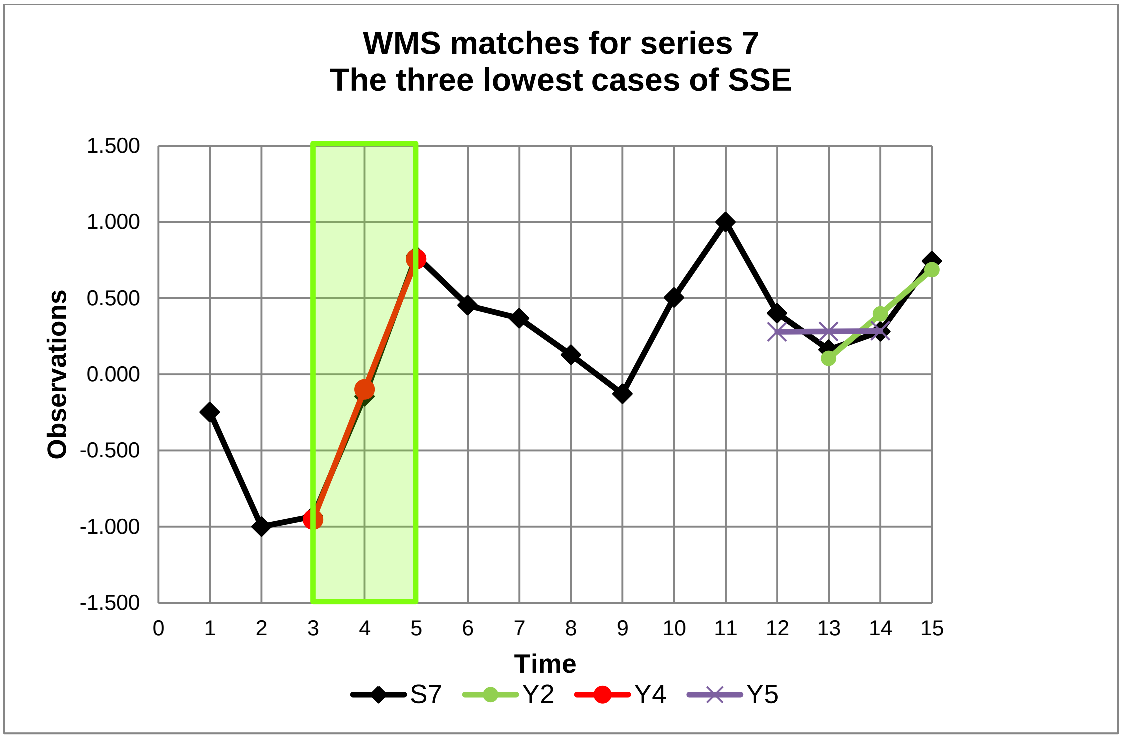


Figure 30. The lowest cases of SSE, WMS matches for scaled Time Series 7.

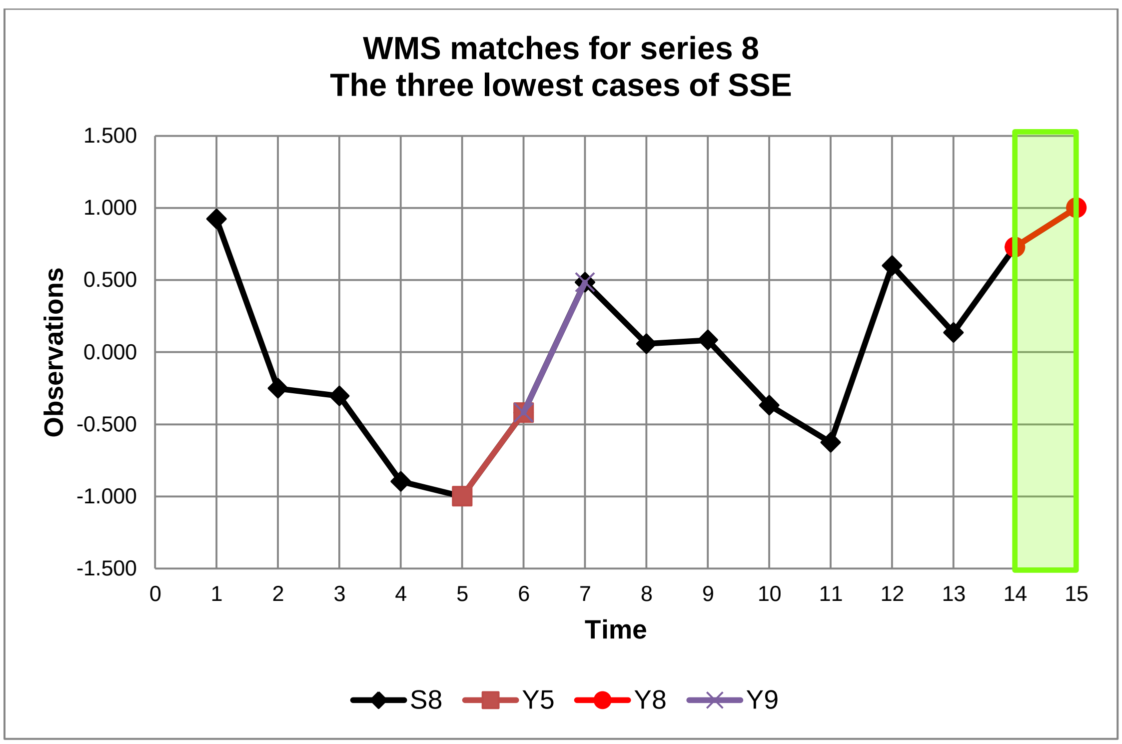


Figure 31. The lowest cases of SSE, WMS matches for scaled Time Series 8.

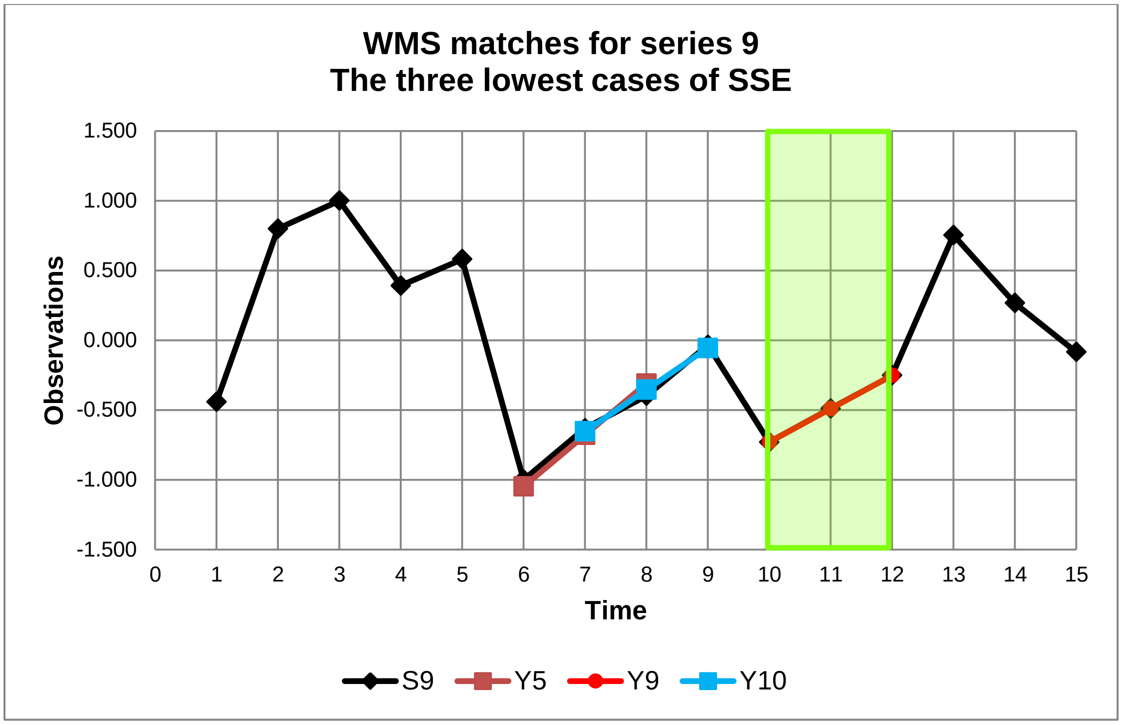


Figure 32. The lowest cases of SSE, WMS matches for scaled Time Series 9.

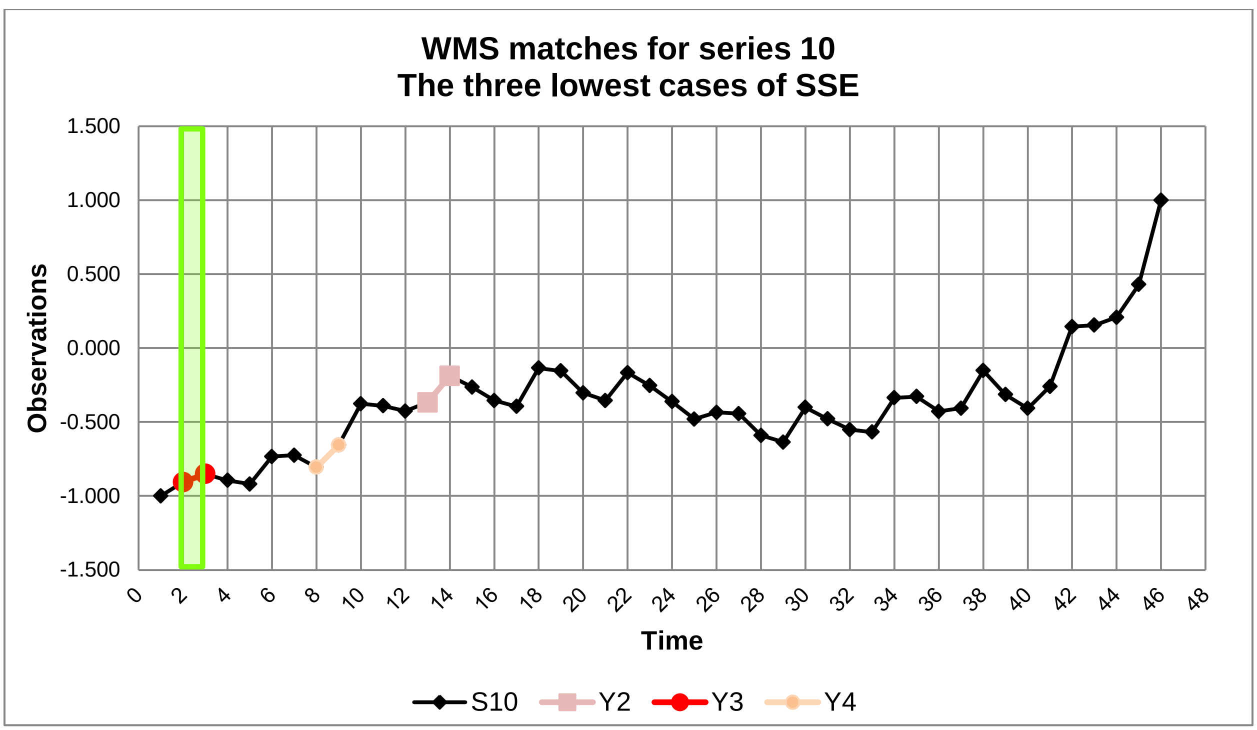


Figure 33. The lowest cases of SSE, WMS matches for scaled Time Series 10.

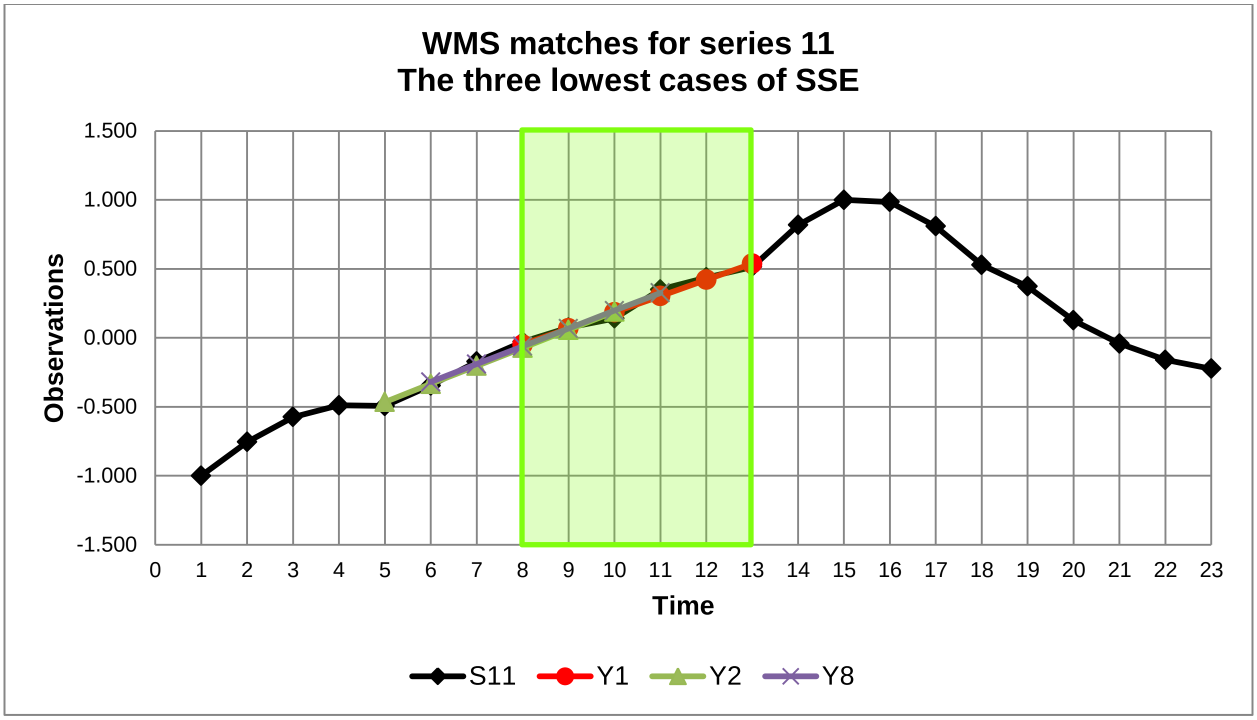


Figure 34. The lowest cases of SSE, WMS matches for scaled Time Series 11.

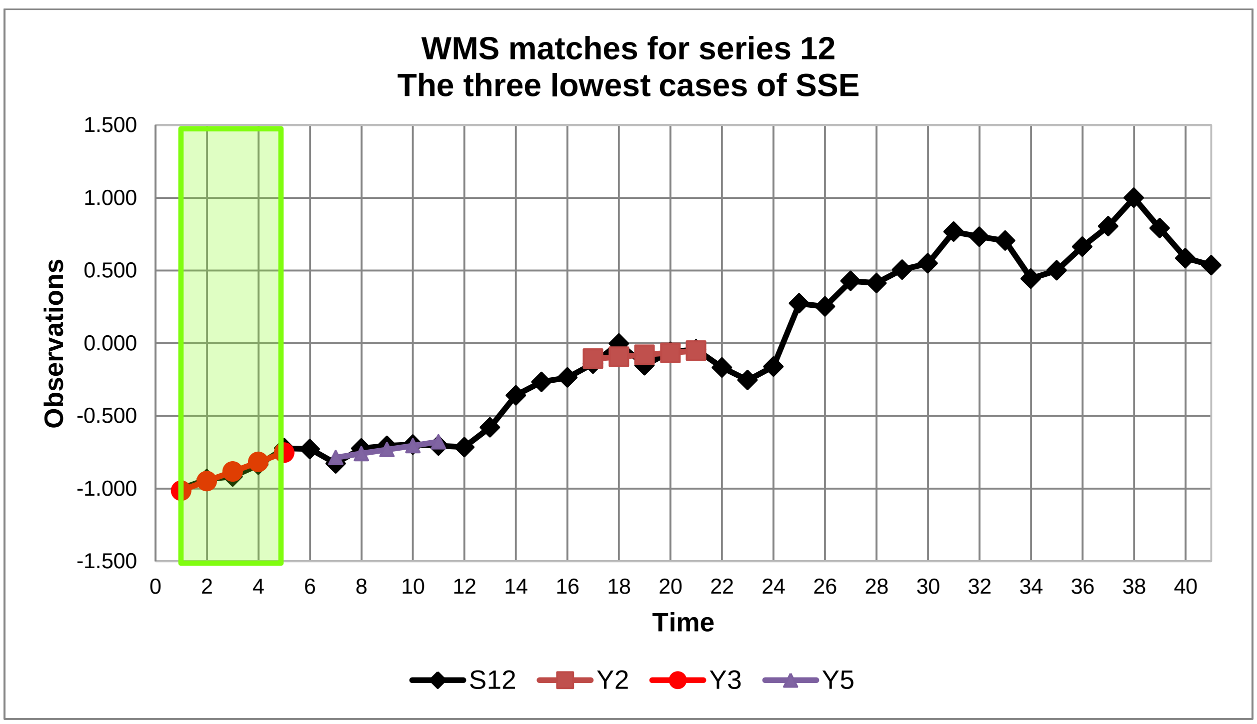


Figure 35. The lowest cases of SSE, WMS matches for scaled Time Series 12.

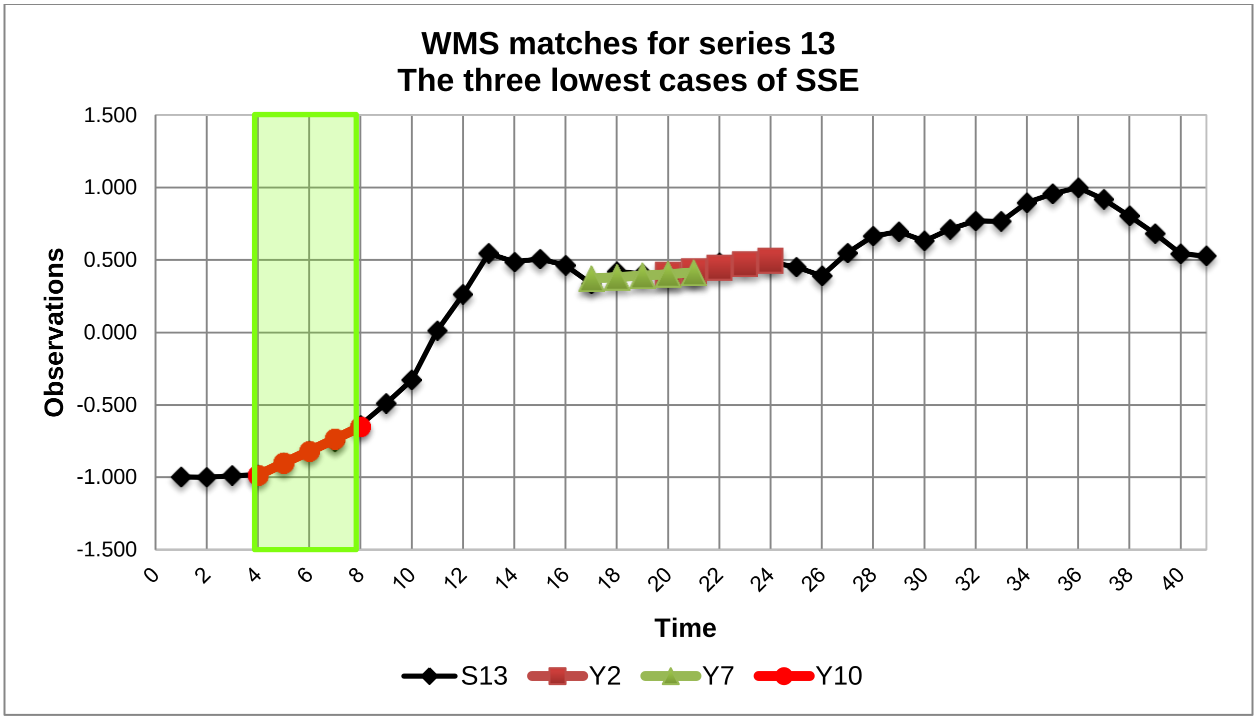


Figure 36. The lowest cases of SSE, WMS matches for scaled Time Series 13.

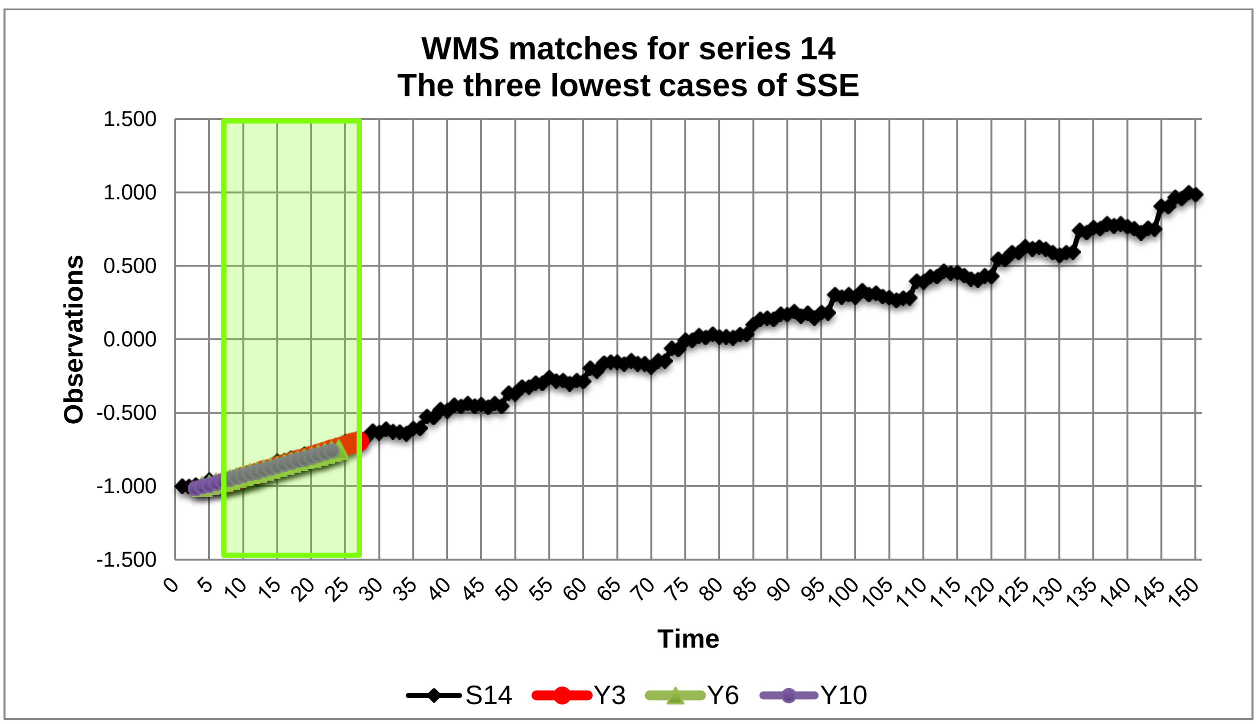


Figure 37. The lowest cases of SSE, WMS matches for scaled Time Series 14.

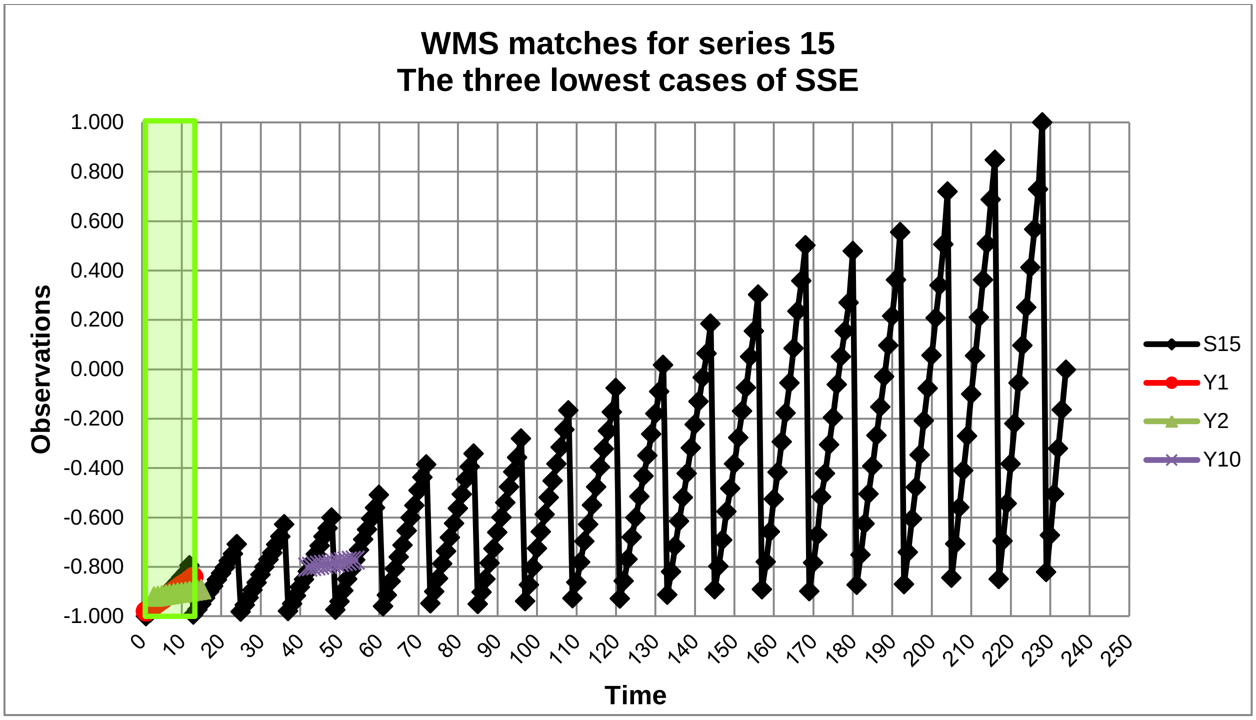


Figure 38. The lowest cases of SSE, WMS matches for scaled Time Series 15.

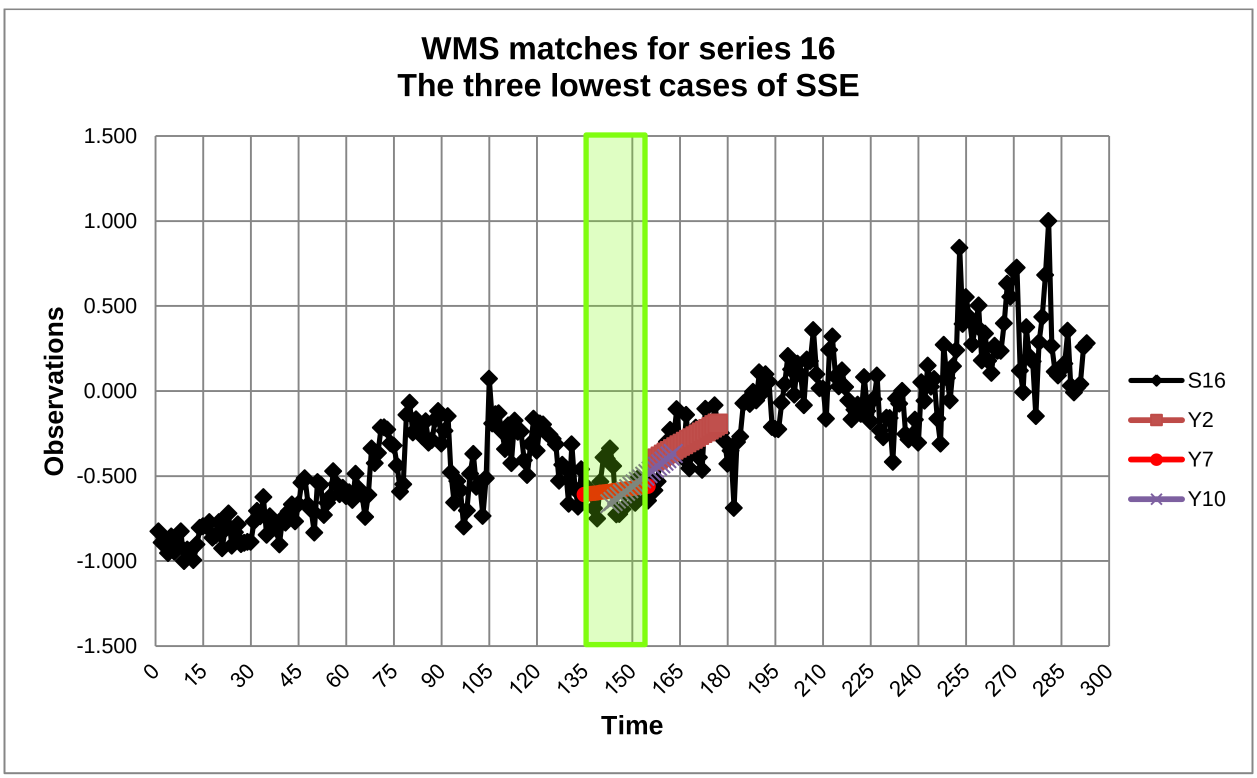


Figure 39. The lowest cases of SSE, WMS matches for scaled Time Series 16.

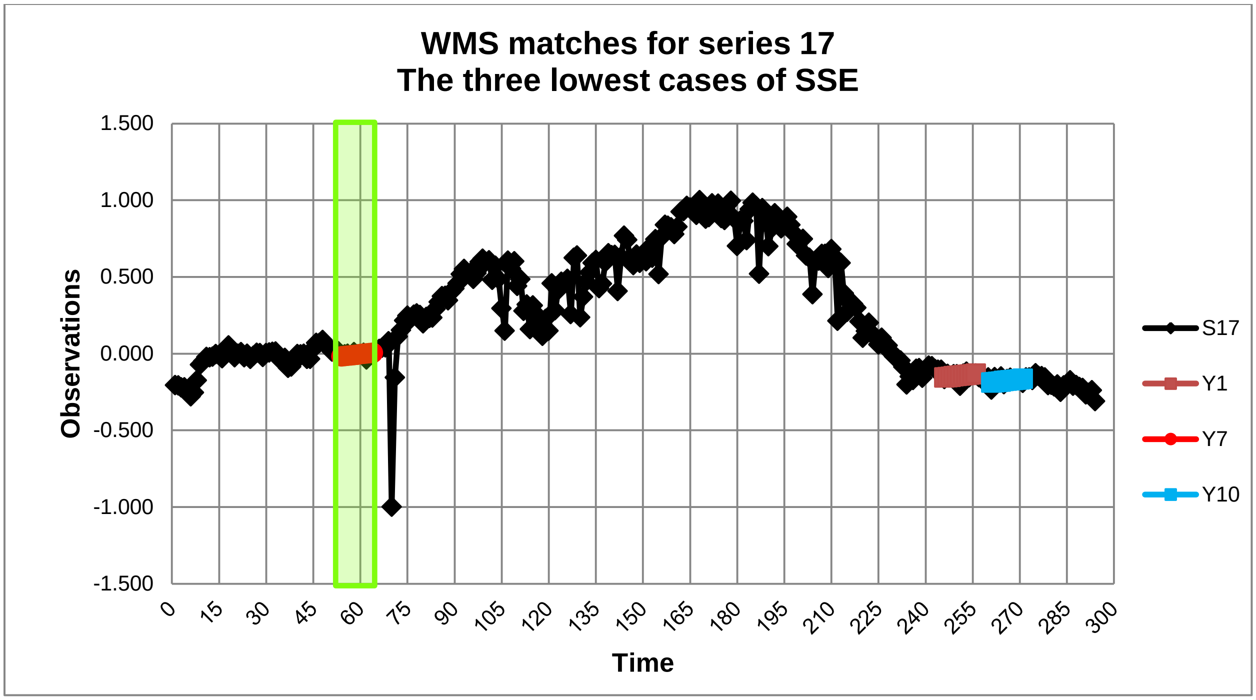


Figure 40. The lowest cases of SSE, WMS matches for scaled Time Series 17.

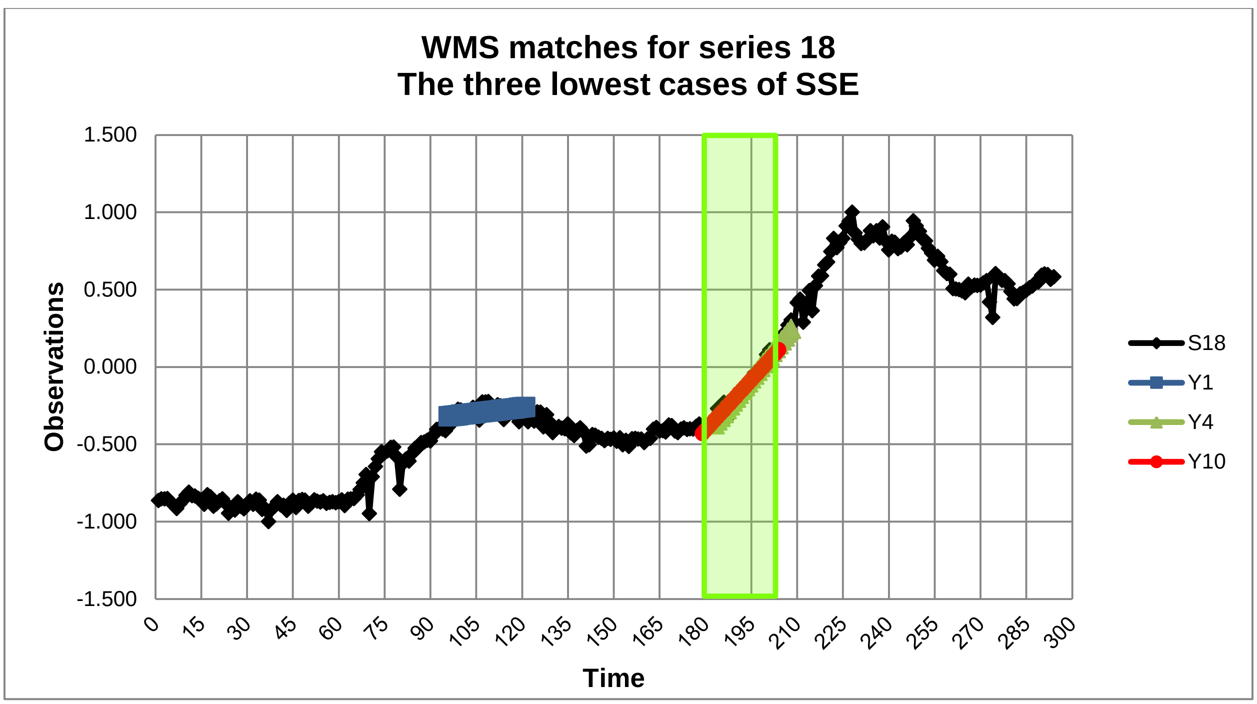


Figure 41. The lowest cases of SSE, WMS matches for scaled Time Series 18.

From figures previously presented it was observed that for the Series 1 to 4, Series 6 to 15, and Series 18, the WMS included a region of data where apparently the linear model with positive slope resulted to be a good descriptor. The WMS in these 15 series indicates good results given that an increasing behavior can be observed in the scaled time series data (and the original data in fact).

In Chapters 3 and 4 it was observed that the results were better using two variables to generate the window. Given evidence of good functionality, it was decided to project the time series to a three dimensional space.

The results in this section were considered reasonable. The next section presents the 3D projection of each time series.

5.1.7 3D projection of the time series

Given that the previous evaluations of this chapter from time series generated results that do not show enough discernment (Section 5.1.1 to 5.1.5) and given that the results were better using two variables (Chapter 3 and 4), a 3D projection to the time series was adapted. Only Series 1 to 13 were considered for the projection. The projection of time series to a three dimension space consisted in adding an auxiliary axis to the already two existent axes. Of this manner, the three axes correspond to the following: x-axis represents the time variable of the series, the y-axis represents the auxiliary axis (or false axis) to project the time series, and z-axis represents the observation values in each scaled time series.

Originally, time series are presented in 2D (data vs. time). In the 3D projection, time series are represented in three axes which are: the data (z-axis), time (x-axis), and a false dimension (the auxiliary axis, y-axis).

The mathematical formulation of the optimization problem is described in (24). In all cases, since the y-axis is a false axis, the epsilon value of the difference for ty was kept to include all points in the time series (maximum size on ty), i.e., a time series with 12 observations will be restricted to take the maximum size on ty: $ty^U - ty^L = t_{max} - 1 = 12 - 1 = 11$ (maximum size of 11). For each scaled series, a WMS size (or epsilon value) was predefined. Table 8 presents the epsilon values for each time series.

Time Series	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13
Epsilon value	3	1	1	1	1	2	1	1	3	3	14	4	10

Table 8. Epsilon values for each Series.

Find tx^U, tx^L, ty^U, ty^L, β_0, β_1, β_2 *for*

Minimize SSE (24)

St.
$1 \le tx^L \le t_{max}$
$1 \le tx^U \le t_{max}$
$1 \le ty^L \le t_{max}$
$1 \le ty^U \le t_{max}$
$tx^U - tx^L \ge$ Epsilon value
$ty^U - ty^L \ge$ maximum size on ty
$0.0024 \le \beta_1 \le 0.9024$
β_0, β_2= unconstrained

The parameters and conditions were considered as following:

- Initializations using multiple starting points with a population size of 100.
- A level of precision of $1\text{x}10^{-9}$.
- A convergence of 1×10^{-4}.

Results

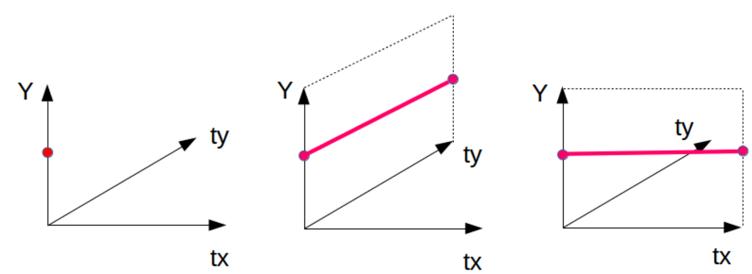
The results are presented in 2D plots: the data in Y and the real time in tx. The false dimension (the auxiliary axis, y-axis) was removed. Figure 42 presents the perspectives of the 3D projection of the time series in the three following cases:

a) When tx = 0 and ty = 0, Y = β_0

b) When tx = 0, Y = $\beta_0 + \beta_2$*ty

c) When ty = 0, Y = $\beta_0 + \beta_1$*tx

The perspective presented in Figure 42 option c) is of interest. Under this perspective in two dimensions, the results for each time series are presented.



a) b) c)

Figure 42. Perspectives of time series in the 3D projection.

Figures 43 to 55 present the result for the lowest cases of SSE. In these figures, the zone of the WMS where the linear model is a good descriptor of data with the best solution match is highlighted. In the case where there were ties of matches (for example in series 3 and 5), these coincidences within the three lower cases of SSE are also presented in the figure. Tables B97 to B109 with detailed results and Figures B1 to B13 with all matches are presented in Appendix B.

It was observed in the figures that the best match included data where there is an increasing behavior. This behavior was observed in 10 out of 13 of the time series evaluated. These ten series are series 1 to 5, 7 and 8, and 11 to 13. For these evaluations it can be concluded that the results are reasonable and that the WMS for these series included a region of data where apparently the linear model with positive slope is a good descriptor.

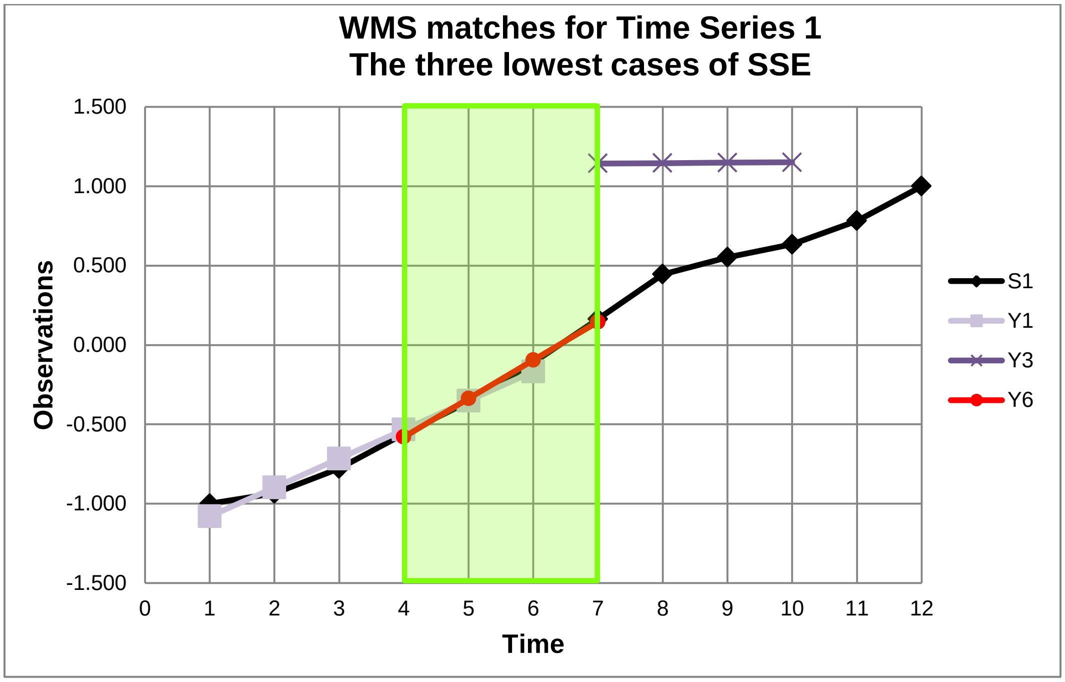


Figure 43. The lowest cases of SSE for Time Series 1, WMS matches from 3D projection.

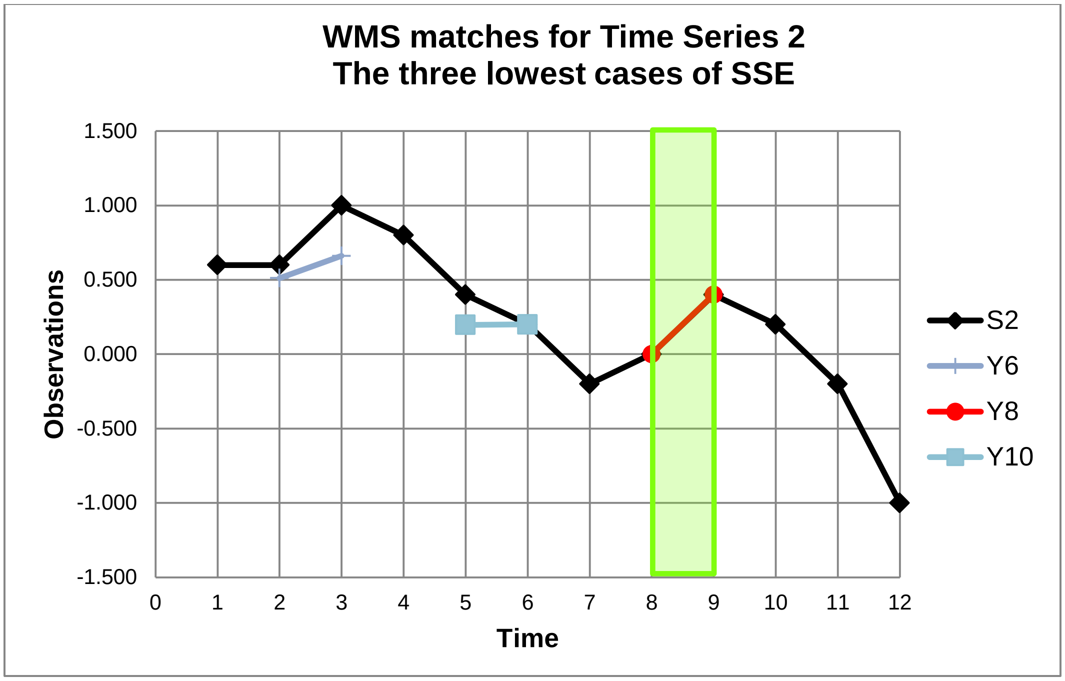


Figure 44. The lowest cases of SSE for Time Series 2, WMS matches from 3D projection.

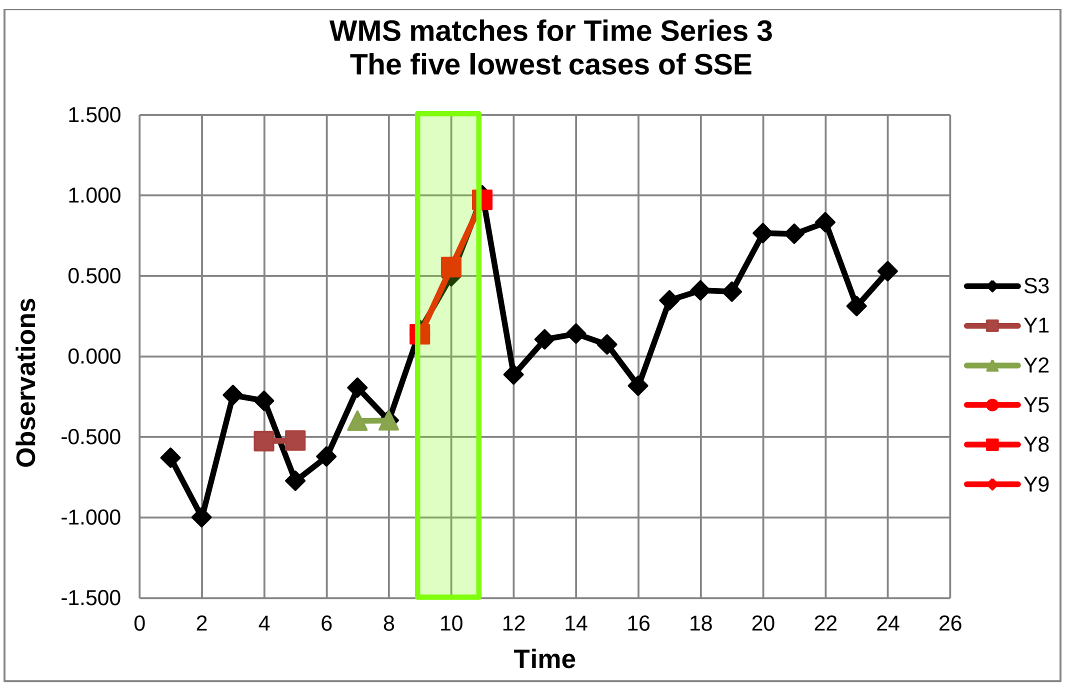


Figure 45. The lowest cases of SSE for Time Series 3, WMS matches from 3D projection.

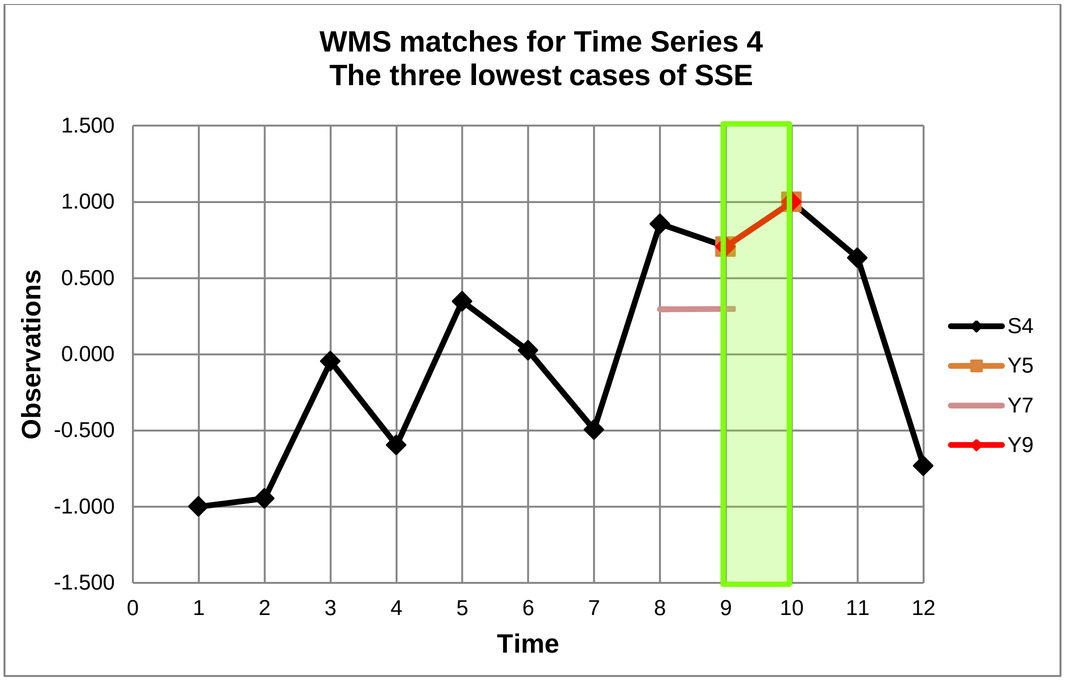


Figure 46. The lowest cases of SSE for Time Series 4, WMS matches from 3D projection.

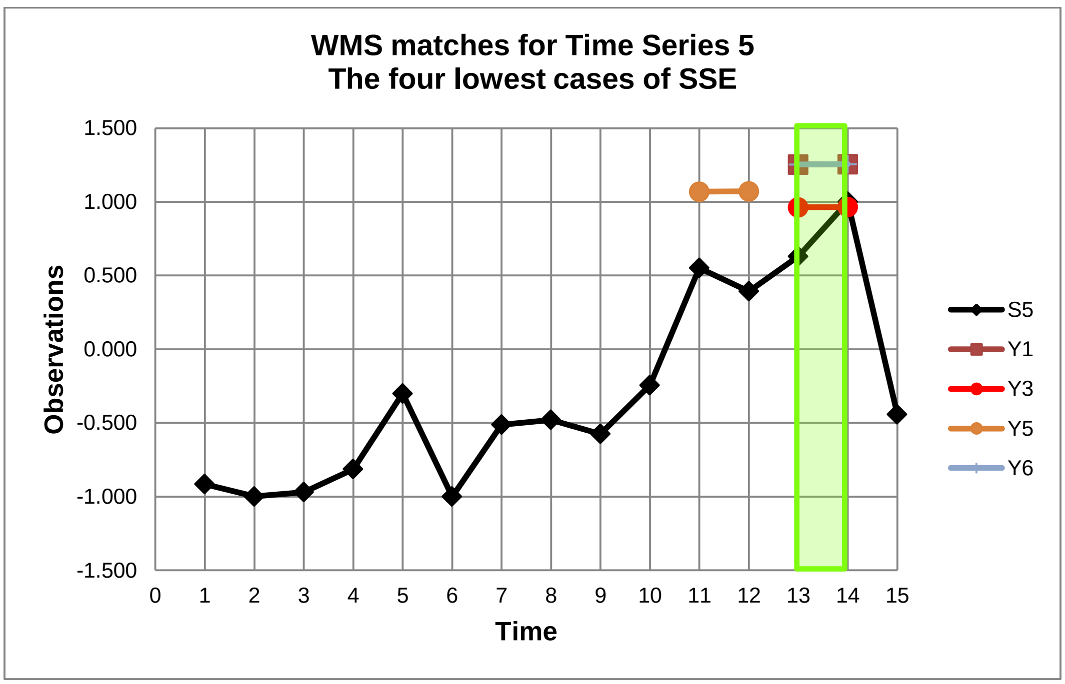


Figure 47. The lowest cases of SSE for Time Series 5, WMS matches from 3D projection.

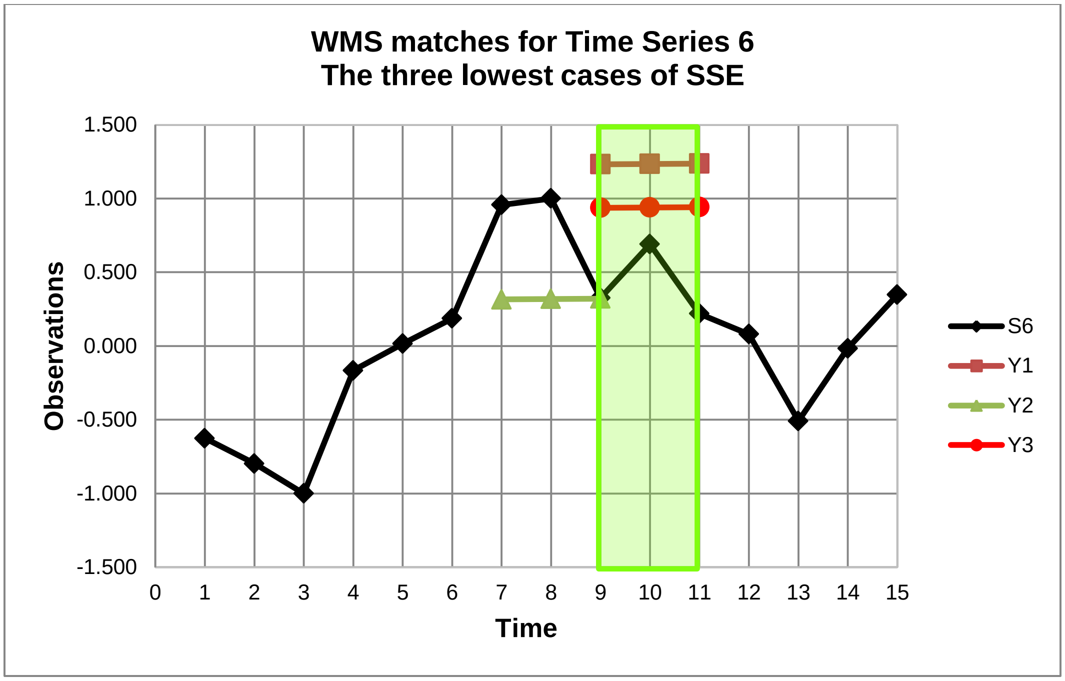


Figure 48. The lowest cases of SSE for Time Series 6, WMS matches from 3D projection.

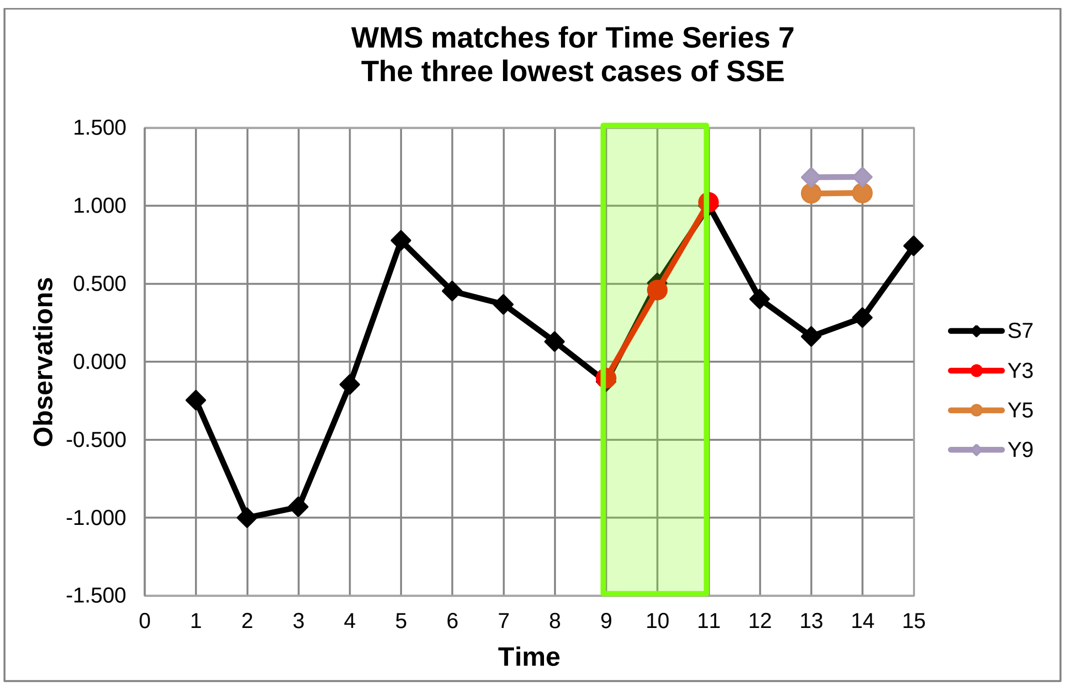


Figure 49. The lowest cases of SSE for Time Series 7, WMS matches from 3D projection.

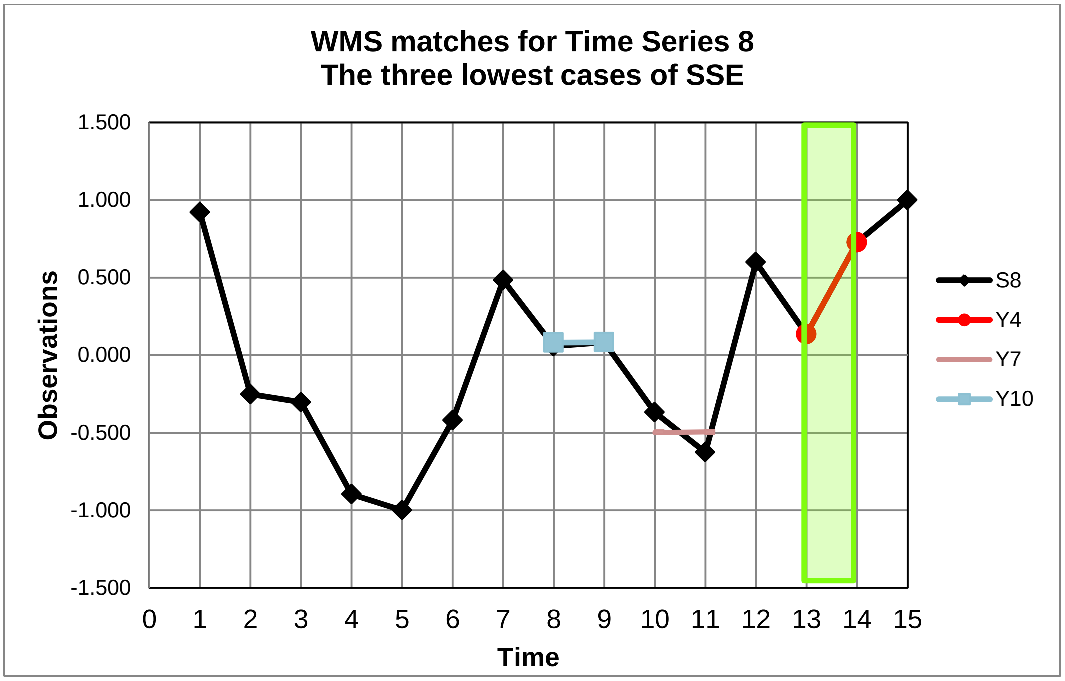


Figure 50. The lowest cases of SSE for Time Series 8, WMS matches from 3D projection.

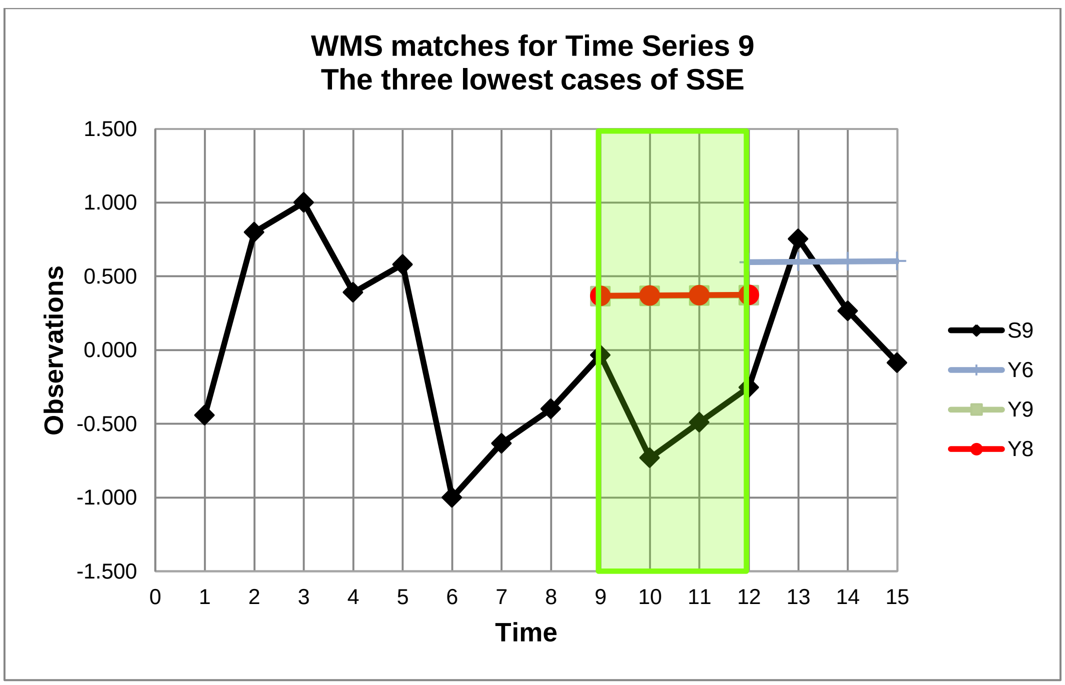


Figure 51. The lowest cases of SSE for Time Series 9, WMS matches from 3D projection.

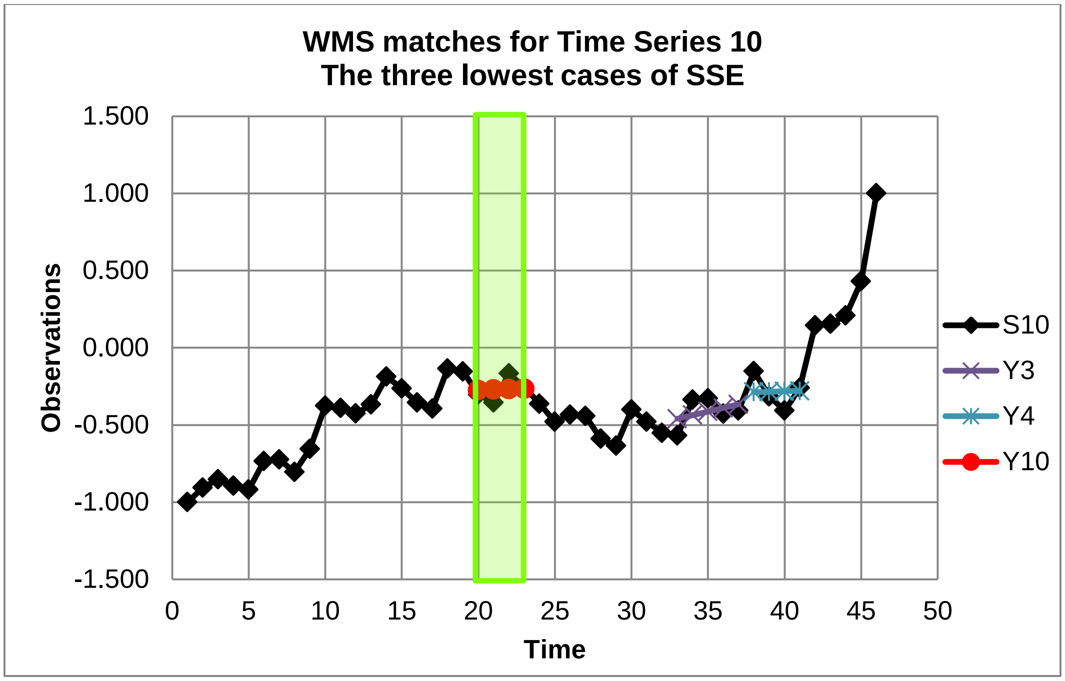


Figure 52. The lowest cases of SSE for Time Series 10, WMS matches from 3D projection.

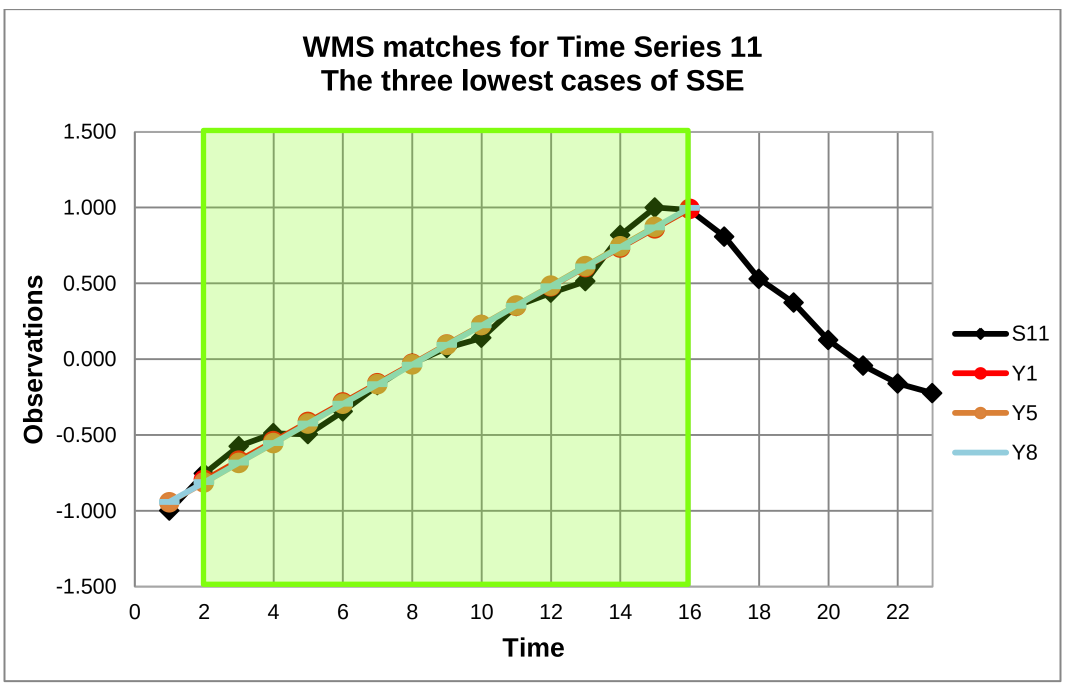


Figure 53. The lowest cases of SSE for Time Series 11, WMS matches from 3D projection.

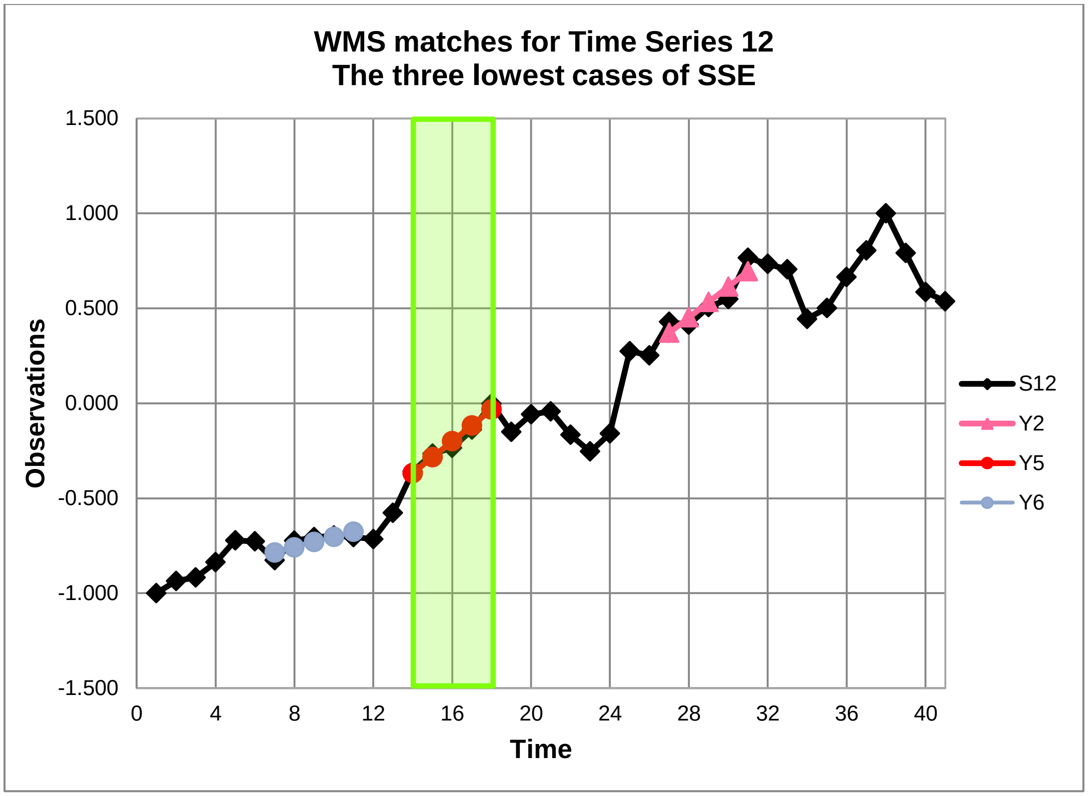


Figure 54. The lowest cases of SSE for Time Series 12, WMS matches from 3D projection.

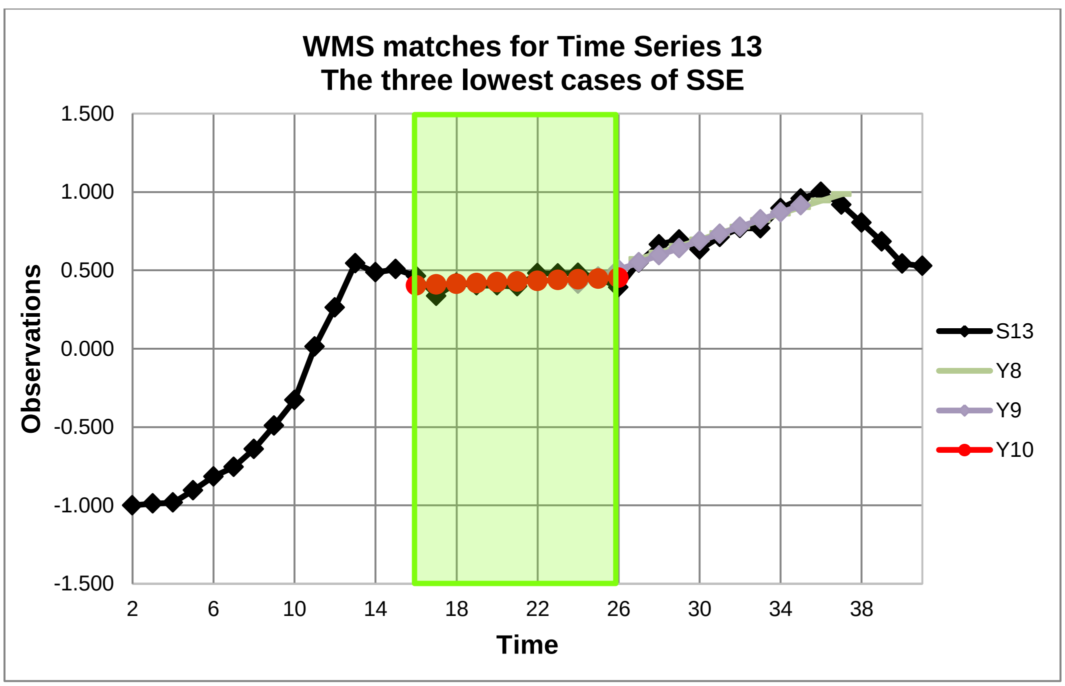


Figure 55. The lowest cases of SSE for Time Series 13, WMS matches from 3D projection.

Linear model for the best WMS match solution from the 3D projection

In order to validate if the data region included in the best solution of WMS, which were found in the evaluations of the 3D projection, exhibit linearity, a linear model regression was applied. The linear model was applied to the original data from the time series which were obtained within the best solution of WMS. Time series 1, 9, 10, 11, 12, and 13 (where the epsilon value was of at least 3) were considered.

If there is sufficient evidence that the data included within of the WMS match presents a linear component, plus the noise component, it is possible to conclude that the method detected the increasing behavior of interest, and therefore, the method will result useful as an explorative tool of regions of interest where a local solution could exist.

Table 9 shows a brief summary of the results obtained for each of the series considered. In the Table 9, the six time series, the respective WMS obtained by the method in the evaluations of the 3D projection for such series, the P-value obtained in the test, and the conclusion if the linearity is or not statistically significant, are presented. From output of linear model, a value less than 0.05 for P-value expresses statistical significance. If the P-value is less than the 0.05, the linear model is statistically significant. The detailed results for the fitted linear model are shown in the Appendix C.

Based on P-values, the linearity in the data (within the WMS match) for the Time Series 1, 11 and 12 was significant. For the remaining three time series (9, 10, and 13), the linearity was non-significant. Only the coefficient (slope) for the variable “Time” in the linear model for the Time Series 9 was negative. However, the P-value of the linear model revealed that the linearity for this data was non-significant. The coefficient (slope) for the variable “Time” in the linear model of each remaining time series was positive.

Time Series	WMS match	P-value	Linearity is:
Series 1	$tx^L = 4$ $tx^U = 7$	0.0024	Significant
Series 9	$tx^L = 9$ $tx^U = 12$	0.822	Non-significant
Series 10	$tx^L = 20$ $tx^U = 23$	0.463	Non-significant
Series 11	$tx^L = 2$ $tx^U = 16$	2.51e-13	Significant
Series 12	$tx^L = 14$ $tx^U = 18$	0.00393	Significant
Series 13	$tx^L = 16$ $tx^U = 26$	0.319	Non-significant

Table 9. Results for the linear model regression to determine if the region of data of the WMS presents linearity.

From this analysis it can be concluded that the results are not perfect but they are good. Recall that in the results evaluations from the 3D projection it was graphically observed that 10 out of 13 cases show an increasing behavior in the data within the WMS. Therefore, it can be concluded that the results are reasonable.

In addition, if the best solution of WMS includes sufficiently data (more than 40 observations) it is needed to explore the Autocorrelation Function (ACF) and the Partial Autocorrelation Function (PACF) plots of the residuals from the model. This exploration will help to verify the time dependence (linear or non-linear time association) that could impact the results.

The data exploration in Time Series 12

To translate the use of windows of maximum similarity into time series data, the example of time series 12 was selected. This series represent:

- Coal (unit of measure: Petajoules), primary energy production, energy sector. Considered period: 1970 - 2010. *Quantity of coal produced, in Petajoules, in the energy sector.* ("Thematic route: Energy > Production of primary energy > Coal", available from http://www.inegi.org.mx/sistemas/bie/).

For the users it could be useful to identify regions where data registered increments in the volume of coal production. The energy sector is a key element of the economy development of many countries. From this point of view, an explorative tool of data would be useful in the detection of period where there were increments which were associated to relevant events in the economy of the countries such as the population consumption, the industry consumption, exportations, the sustainability of resources, in issues related to the environmental protection, etc. (To see detailed information related to the energy sector, is suggested to visit the national energy balance available from http://www.sener.gob.mx)

The best match of WMS found by the method for the time series 12, when the minimum WMS size was predefined to 4 (at least 5 observations included), indicated that between the years 1983 and 1987 ($tx^L = 14$, $tx^U = 18$), an increment in the coal production was registered. Although in the whole time series 12 it can be observed a global trend, according to the method, this series exhibit locally increments in different period of time (1976 - 1980, 1983 - 1987, and 1996 - 2000).

Chapter 6

6.1 Method evaluation using global optimization test functions

In this chapter, an application of the proposed method using unconstrained global optimization test functions is presented. The objective is to find a zone of data with maximum similarity between each test function and a quadratic function.

The optimization test functions included the functions Sphere, Rosenbrock, Rastrigin, Griewank, Goldstein-Price, Easom, and Schwefel. The functions were considered in a two dimensional space. More information about these functions can be found in Pohlheim (2006) and (Surjanovic and Bingham, 2013).

The quadratic function $f(x_1, x_2) = ß_0 + ß_1 x_1^2 + ß_2 x_2^2$, represents the function to superimpose. The ranges for the variables of the quadratic function were the same in which each test function varies. For generating the grid of experimental points, the ranges of the variables of each test function were divided according to a specific value of delta x (Δx) as will be indicated in Table 10.

Model (25) was applied for this problem. In this model, *Min_value* and *Max_value* represent the minimum value and maximum value of the range in which each function varies. The model (25) was developed to automatically generate the WMS size. The WMS size is found through the minimization of a *composite objective function*. This composite objective function includes an objective for the SSE and two objectives for the WMS size (the differences of the bound variables) of two dimensions. In addition, each objective of the composite objective function was weighted to take values of 1/3 for the sum to be 1 (similar to a structure of preferences). Given that, the problem is of minimization, with the minus sign in the differences of the bounds, these differences will tend to take a higher value for the WMS size.

In order to keep in the same order of magnitude the SSE and the differences of the bounds, the logarithm base 10 was incorporated in each term of the composite objective function. The logarithm function is not defined for zero or negative values. Errors of execution of the solver occurred during the preliminary development of the optimization problem, for the initial cases of interest, were relief adding +1 to the SSE value, as well as adding the absolute value and +1 to the differences within logarithm function. The optimization problem was executed in two times using the initialization points presented in Appendix D.

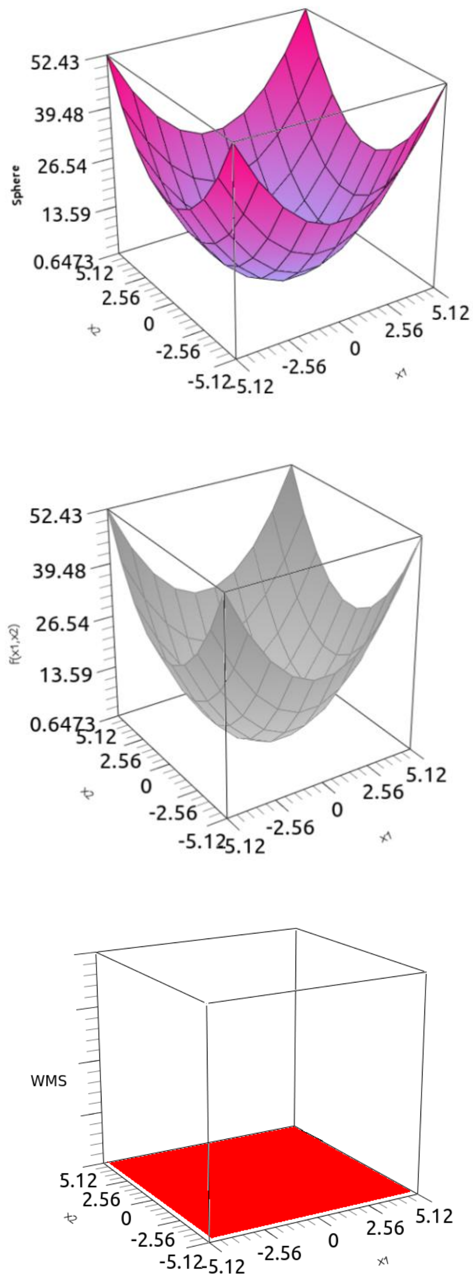
The parameters for the Solver for these evaluations included:

- The use of multiple starting points using a population size of 100.
- A convergence of $1x10^{-4}$.
- Bounds are required on variables.
- A level of precision of $1x10^{-3}$ for the functions Sphere, Rosenbrock, Griewank, Goldstein-Price, Easom, and Schwefel; a level of precision of $1x10^{-9}$ for the function Rastrigin.

$$
\begin{aligned}
&\textit{Find} \quad x_1^L, x_1^U, x_2^L, x_2^U, ß_0, ß_1, ß_2 \quad \textit{for} \\
&\textit{Minimize} \\
&\qquad 1/3[\log(SSE + 1)] - 1/3[\log(|x_1^U - x_1^L| + 1)] - 1/3[\log(|x_2^U - x_2^L| + 1)] \\
&\textit{St.} \\
&\qquad Min\_value \le x_1^L \le Max\_value \\
&\qquad Min\_value \le x_1^U \le Max\_value \\
&\qquad Min\_value \le x_2^L \le Max\_value \\
&\qquad Min\_value \le x_2^U \le Max\_value \\
&\qquad x_1^U - x_1^L \ge 1x10^{-6} \\
&\qquad x_2^U - x_2^L \ge 1x10^{-6} \\
&\qquad -1000 \le ß_0 \le 1000 \\
&\qquad -1000 \le ß_1 \le 1000 \\
&\qquad -1000 \le ß_2 \le 1000
\end{aligned}
\tag{25}
$$

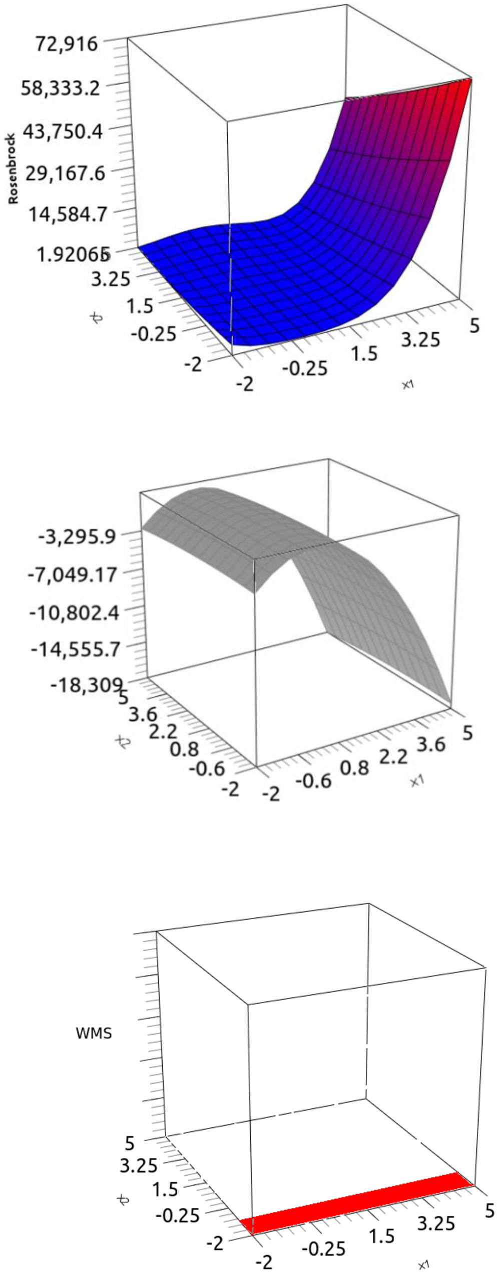
Results

The maximum similarity region between the adjusted quadratic function and each test function is shown in Figures 56 to 62. In addition, the mathematical equation based on two or s quantity of variables followed by the range of each variable is indicated for each figure. The figures were generated in QtiPlot software (version 0.9.8.9) (http://www.qtiplot.com/). Table 10 presents the detailed results for each test function in the two runs executed. This table includes the best solution (bound variables and beta values) followed by the best objective value found composed of SSE and the best objective function value, and the WMS size. The best objective value (minimum value) found in each case is in bold. The figures present the solution highlighted in Table 10 for each test function.



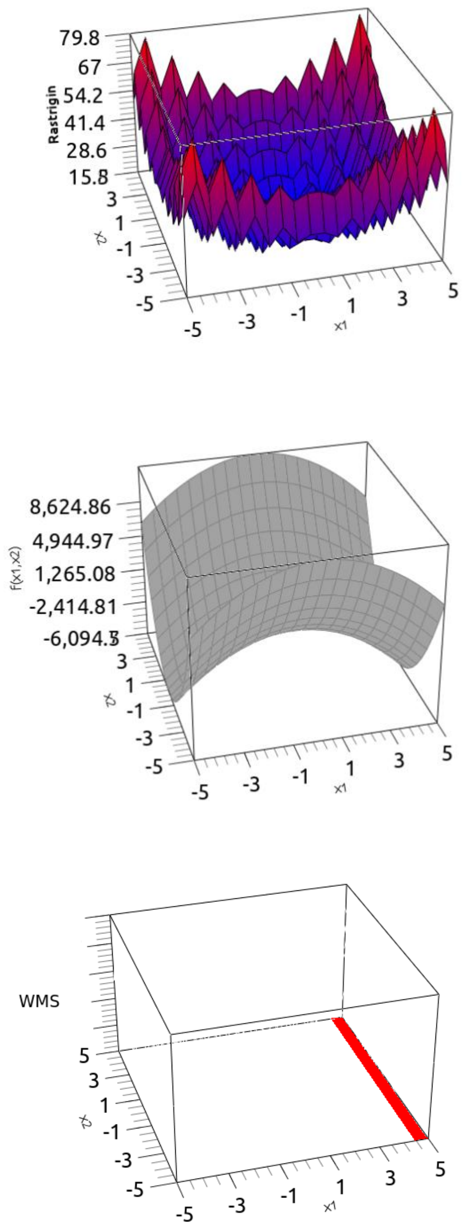
Sphere function (s variables): $(X) = \sum_{r=1}^{s} x_r^2$, -5.12 $\leq x_r \leq$ 5.12

Figure 56. Maximum similarity between Sphere function and quadratic function.



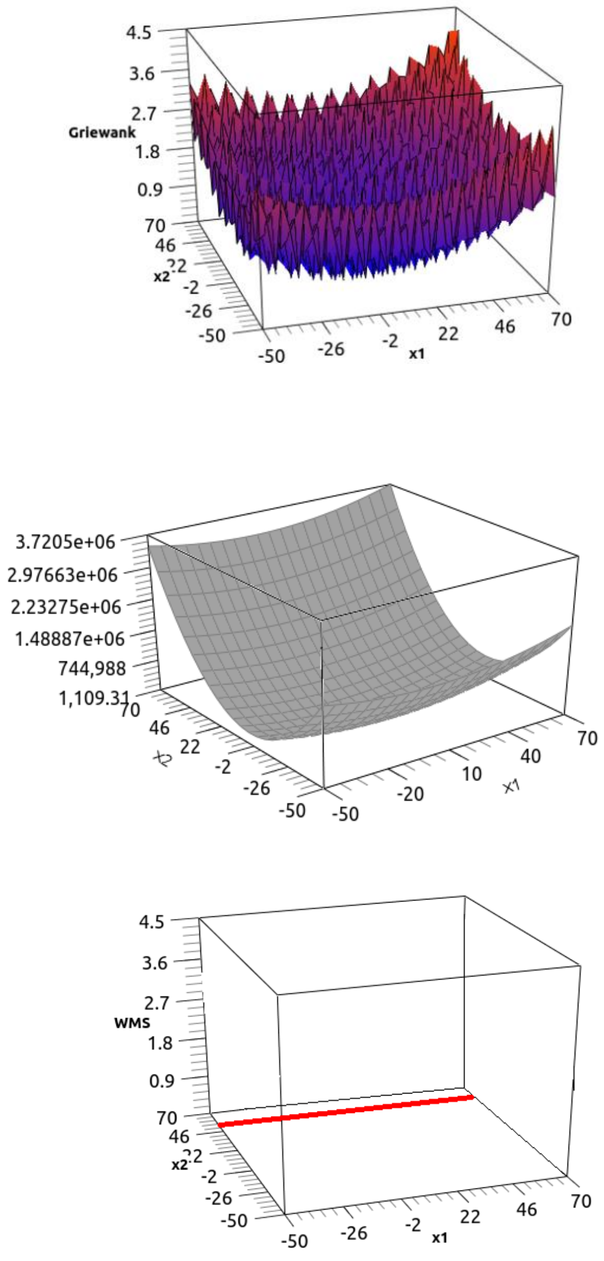
Rosenbrock function (*s* variables): $f(X) = \sum_{r=1}^{s-1}[100 * (x_r^2 - x_{r+1})^2 + (x_r - 1)^2]$, $-2 \le x_r \le 5$

Figure 57. Maximum similarity between Rosenbrock function and quadratic function.



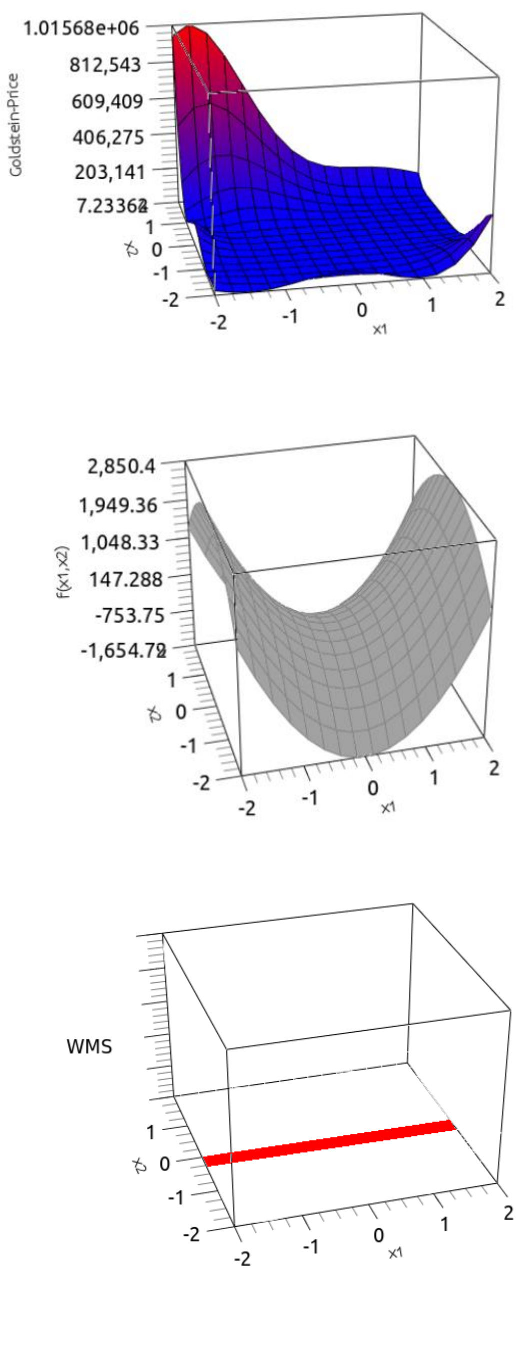
Rastrigin function (*s* variables): $f(X) = 10s + \sum_{r=1}^{s}[x_r^2 - 10cos(2\pi x_r)]$, -5 ≤ x_r ≤ 5

Figure 58. Maximum similarity between Rastrigin function and quadratic function.



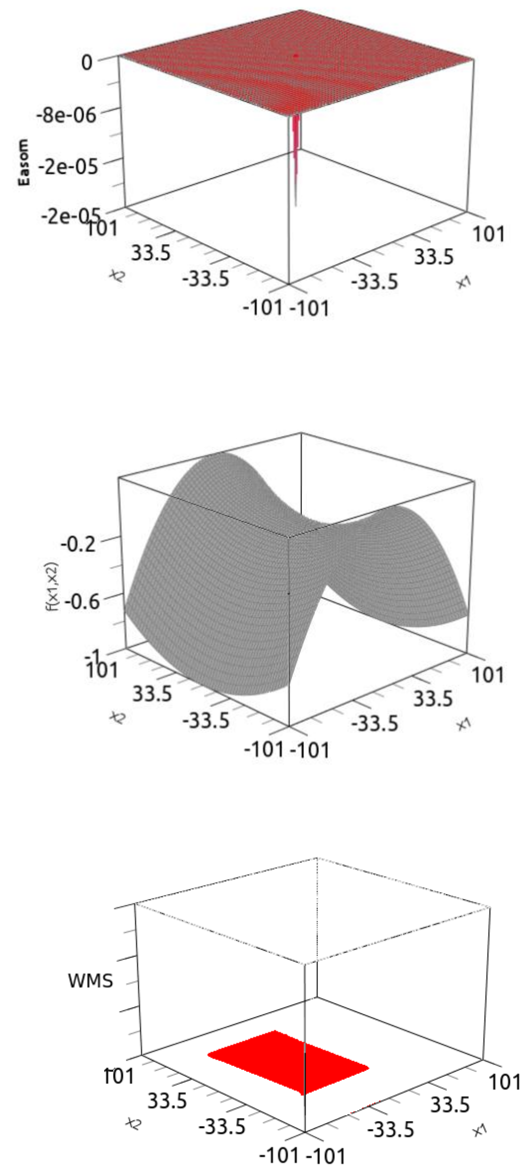
Griewank function (s variables): $f(X) = \sum_{r=1}^{s} \frac{x_r^2}{4000} - \prod_{r=1}^{s} cos\left(\frac{x_r}{\sqrt{r}}\right) + 1$, $-50 \leq x_r \leq 70$

Figure 59. Maximum similarity between Griewank function and quadratic function.



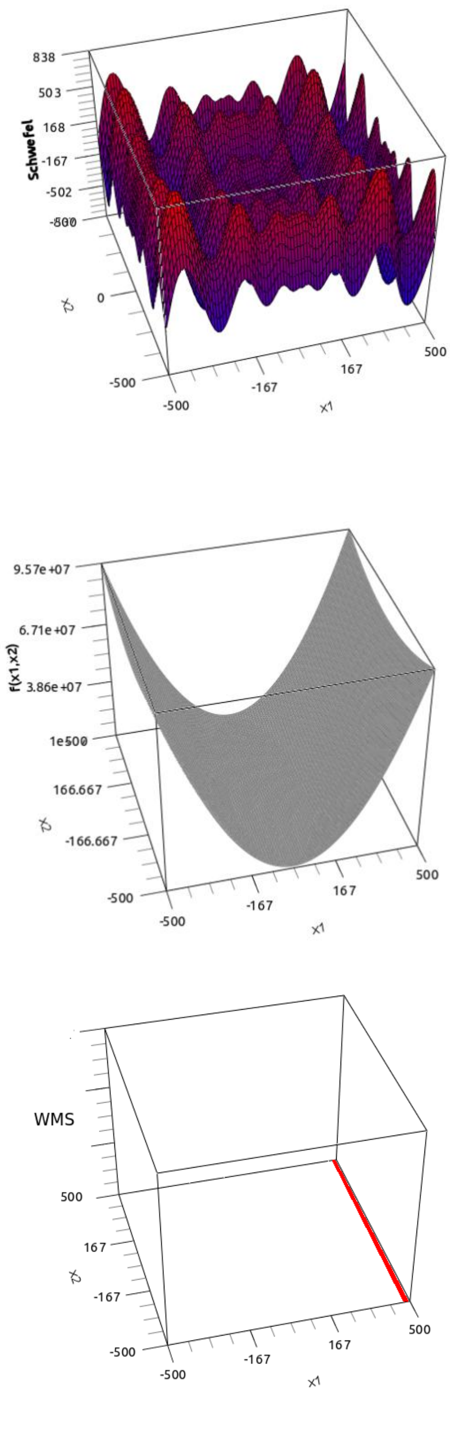
Goldstein-Price function (2 variables): $f(X) = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)] * [30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$, $x_1, x_2 \in [-2, 2]$

Figure 60. Maximum similarity between Goldstein-Price function and quadratic function.



Easom function (2 variables): $f(X) = -cos(x_1)\ cos(x_2)e^{\left(-(x_1-\pi)^2-(x_2-\pi)^2\right)}$, $x_1, x_2 \in$ [-100.5310, 100.5310]

Figure 61. Maximum similarity between Easom function and quadratic function.



Schwefel function (*s* variables): $f(X) = \sum_{r=1}^{s} -x_r sen\left(\sqrt{|x_r|}\right)$, $-500 \leq x_r \leq 500$

Figure 62. Maximum similarity between Schwefel function and quadratic function.

Best solution							Best objective value found		Best objective value found: WMS size	
x_1^L	x_1^U	x_2^L	x_2^U	β_0	β_1	β_2	**SSE**	**Best objective**	x_1^U - x_1^L	x_2^U - x_2^L
				Sphere, Δx= 1.024						
-5.1198	5.1200	-5.1176	5.1200	3.75E-07	1.0000	1.0000	1.7E-10	-0.70048	10.2397	10.2376
-5.1198	5.1200	-5.1198	5.1200	-1.55E-08	1	1	5.0E-11	**-0.70050**	10.2397	10.2397
				Rosenbrock, Δx= 0.5						
-2	5	3.0000	3.5000	-234.2269	-479.1913	-546.9456	0	**-0.35973**	7	0.49999
-2	5	-2.0000	-1.5000	475.1608	-726.2426	-25.1233	0	**-0.35973**	7	0.49998
				Rastrigin, Δx= 0.5						
4.5001	4.9997	-5	5	-180.2282	-237.9641	500.0584	0	**-0.40579**	0.4996	10
-1.0000	-0.5192	-3.9776	2.2641	-595.2606	17.0858	-110.1151	0	-0.34344	0.4808	6.24167
				Griewank, Δx= 2						
-50	70	32.0000	33.9999	207.9203	-643.9052	-947.2928	0	**-0.85330**	120	1.9999
-50	70	50.0000	51.9999	-440.4349	106.7079	652.6685	0	**-0.85330**	120	1.9999
				Goldstein-Price, Δx= 0.25						
-2	2	-0.2500	-0.0016	-107.8186	741.2889	-390.037	0	**-0.26510**	4	0.24837
0.0586	0.2500	-2	2	-412.6185	243.3304	523.4478	0	-0.25834	0.1914	4
				Easom, Δx= π						
-35.1525	31.5537	-35.7937	66.6345	-0.0357	-0.0001	3.81E-05	5.7098	**-1.00619**	66.7063	102.4282
97.3897	100.5302	-100.5310	100.5310	-432.4280	183.2941	662.6869	0	-0.97418	3.1406	201.0619
				Schwefel, Δx= 5						
-391.5594	498.3991	-399.9998	-395.0002	-825.2502	353.8500	-320.358	0	-1.24266	889.9586	4.9997
495.0000	499.9149	-500	500	274.7996	338.0750	44.7754	0	**-1.25746**	4.9149	1000

Table 10. Best solutions for the test functions and the quadratic function.

For functions Rosenbrock and Griewank, their two executions coincided in the best objective value found. Although, in general, the remaining solutions in each function were near to coincide. For functions Sphere, Griewank, and Schwefel, the adjustment of their respective quadratic function followed the test function shape. The WMS found for the Sphere function and their quadratic function was the most evident case where the resulting zone of the WMS was a good descriptor. The WMS generated for function Griewank and Schewefel potentially detected a zone of maximum similarity. Unlike the results of the remaining test functions shown quadratic functions of varied shapes and consequently their WMS were adjusted in varied zones, the obtained results for Sphere function, the simplest shape to adjust a quadratic function, indicate that potentially the WMS can be useful as part of a tool for data exploration of interest.

Chapter 7

In this chapter, the limitations of the method, conclusions, future work, contribution of the thesis, and potential applications in other areas are presented.

7.1 Limitations of the proposed method

In the beginning of the method construction, the obtained results provided evidence to determine the windows of maximum similarity with a better discernment when two variables are analyzed. For the case of the generation of one-dimensional windows, the method shows itself to be less capable of discernment of the matches.

The evaluations of the method took long computational execution times. The evaluations with a longer time of execution, which in some cases took up to 24 hours for initialization, were those presented in the integer case using time series data and for the case of the global optimization test functions.

In many cases, the WMS obtained by the method were limited to take the minimum size or epsilon value assigned.

These limitations open the opportunity to improve the method with the inclusion of other kinds of techniques such as the experimental design and simulation.

7.2 Conclusions

In this work, the use of WMS for future optimization by similarity is proposed. The method intends to find the experimental region where a model with desirable characteristics is a good descriptor of the data at hand. Three progressive trials, under one objective and three-objective approaches, were presented. The evaluations included optimization problems cases that use continuous and integer variables.

An evaluation case using function *AOG 1* was presented. According to these results, the method demonstrates the potential to find regions of similarity between two responses where optimality can be a pattern of interest.

The application of the proposed method, for future optimization by similarity, to 18 real Time Series did help to demonstrate the use of windows of maximum similarity to define a region of data where a model (linear with positive slope) is a good descriptor of data.

Given that in several evaluations using the time series data, the WMS size determined automatically by the method had weak discernment in the WMS matches, a 3D projection of the series adding an auxiliary axis was generated. From 3D projection it was concluded that the results obtained by the method were reasonable.

The evaluations of the method in seven unconstrained global optimization test functions served to show the use of window of maximum similarity in examples of functions with different shapes. Also, it was observed in the evaluations that the WMS method potentially detected zones of maximum similarity between the different test functions and a quadratic function.

The results served to show the use of windows of maximum similarity for future Optimization by Similarity in detecting regions of interest.

In general, the WMS method, although subject to improvement, can be a useful explorative tool of data.

It is important to underlay that the use of this explorative tool can be applied to different kinds of data. The purpose of this explorative tool was not the specialization in determining straight lines, fitted linear models, or regressions in time series data. The evaluations using time series data was an example of the method applicability.

Given that the work presented here corresponds to the initial development of the proposed method, we believe that results obtained point to a useful tool for data exploration of interest with applicability to detect zones in different kinds of data.

7.3 Future work

As future work, this work proposes to improve the WMS method by the application to more examples of functions or other type of data and, additionally, to evaluate the WMS method using a set of initial enumerations generated by an experimental design. The use of windows of maximum similarity will serve as a basis element for future development of the *Optimization by Similarity* method.

In general, to improve the optimization by similarity method, the following points are required:

a) Evaluate the method using additional global optimization test functions.
b) Include more variables to the test functions to evaluate.
c) Apply the strategy to a sufficiently large set of variables.
d) Test to solve the optimization problems by the use of another different algorithm to the Generalized Reduced Gradient (GRG).
e) Code the method in a programming language, for example Octave, Scilab or R, to extend their computational capabilities.

General description of the Optimization by Similarity Method

The Optimization by Similarity method proposes that a characterization of how an optimal solution should look like can be used to explore experimentally or pseudo-experimentally generated data to find at least a local optimum.

The global optimization test functions have optimality properties and commonly are used to test optimization algorithms. The global optimal solution of each test function is known. In Chapter 6, seven examples of these functions were illustrated. These instances will prepare the field for the future testing of Optimization by Similarity.

Considering the global optimization test functions (functions to approximate) and a fixed quadratic function (the function to superimpose) it can be explained Optimization by Similarity. If the method is capable to explore the experimental region such as the shape of the fixed quadratic function is kept, and moving it through the experimental region, the WMS finds a region where the known optimal solution in each test function is located, Optimization by Similarity will be potentially achieved. Given that the method makes a heuristic search of the region with maximal similarity, local solutions can be found by the method. Potentially, these solutions should represent candidate windows of maximum similarity or local optimal solutions.

7.4 Contributions

The methodology developed and reported in this work of thesis contributes to the metamodeling and optimization techniques. The WMS is a new method for future Optimization by Similarity that provides a region of the experimental data. This region of similarity keeps desirable characteristics. The method, along the evaluations, has shown evidence to determine a window of maximum similarity which will serve to detect regions of interest. The method allows generating automatically the WMS size or the size can be user-defined by the optimization model formulation. The method was coded keeping low computational resources.

A large set of evaluations in time series data were used to show the use of windows of maximum similarity as part of the method applicability in determining regions of interest.

The method was constructed under the main objective of finding the experimental data region delimited by a window of similarity where the optimality can exist.

The method has the capability of being applied to diverse kinds of data.

7.5 Potential applications in other areas

The proposed method focuses in the exploration of data from diverse processes of interest and the potential application entails, for example:

- Control Charts

Detect an unusual behavior of the observations recollected to monitor if a process is under statistical control. Data can show signals of a pattern which is not random.

- Human genome

Found regions of interest in the human genome. The proposed method could be applied in the exploration of biological data, for example in sequence alignment of proteins.

- Detecting regions with optimality properties

Detect maximums or minimums in regions of interest for experimental data that need to be investigated in the future.

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Appendix A

In this appendix, tables with the detailed results found through the WMS in Chapter 4 are shown. The proposed method was applied to *AOG 1* function (see Figure 4). Sixteen combinations resulting from a full factorial *DOE* 2^4 were used as initial points for the initializations. For these evaluations, continuous variables truncated to four decimal places were considered.

	Initial points				Best solution				WMS size		Objective value	*SSE*
Run	x_1^L	x_1^U	x_2^L	x_2^U	x_1^L	x_1^U	x_2^L	x_2^U	$x_1^U - x_1^L$	$x_2^U - x_2^L$		
1	-1	-1	-1	-1	Solver could not find a feasible solution.							
2	1	-1	-1	-1	Solver could not find a feasible solution.							
3	**-1**	**1**	**-1**	**-1**	**-2.0614**	**0.7718**	**-2.974**	**0.5314**	**2.8332**	**3.5054**	**-0.4124**	**0**
4	1	1	-1	-1	-4.7186	-4.0548	-4.8698	2.4143	0.6638	7.2841	-0.3798	0
5	-1	-1	1	-1	-1.9399	-1.1408	-4.2262	3.2334	0.7991	7.4596	-0.3941	0
6	1	-1	1	-1	-2.3479	0.9126	-3.5469	-0.613	3.2605	2.9339	-0.4081	0
7	-1	1	1	-1	4	4.75	-3.4419	4.7875	0.75	8.2294	1.6149	1,129,200
8	1	1	1	-1	Solver could not find a feasible solution.							
9	-1	-1	-1	1	Solver could not find a feasible solution.							
10	1	-1	-1	1	-2.3183	-0.9539	-4.8292	1.3484	1.3644	6.1776	1.5582	802,200
11	-1	1	-1	1	-0.9928	1	-3.0577	1	1.9928	4.0577	1.5034	490,100
12	1	1	-1	1	-2.0187	0.991	-3.0732	0.1452	3.0097	3.2184	-0.4094	0
13	-1	-1	1	1	-4.1821	4.6873	-3.9835	-3.8412	8.8694	0.1423	-0.3507	0
14	1	-1	1	1	-4.7886	-4.7262	-3.07	1.2784	0.0624	4.3484	-0.2515	0
15	-1	1	1	1	Solver could not find a feasible solution.							
16	1	1	1	1	3.0994	3.5148	-0.0237	4.4067	0.4154	4.4304	-0.2952	0
Constraint precision of $1x10^{-3}$. Automatic multistarting point initializations.												

Table A1. Best solutions found by the WMS to function *AOG 1*. Variables ranged to [-1, 1] using a level of precision of $1x10^{-3}$ and automatic multistarting point initialization.

	Initial points				Best solution				WMS size		Objective value	***SSE***
Run	x_1^L	x_1^U	x_2^L	x_2^U	x_1^L	x_1^U	x_2^L	x_2^U	$x_1^U - x_1^L$	$x_2^U - x_2^L$		
1	-1	-1	-1	-1	Solver could not find a feasible solution.							
2	1	-1	-1	-1	Solver could not find a feasible solution.							
3	-1	1	-1	-1	Solver could not find a feasible solution.							
4	1	1	-1	-1	Solver could not find a feasible solution.							
5	-1	-1	1	-1	Solver could not find a feasible solution.							
6	1	-1	1	-1	1	2.2	1	2.2	1.2	1.2	1.7569	903,000
7	-1	1	1	-1	-0.9882	1	1	2.2	1.9882	1.2	1.7216	960,600
8	1	1	1	-1	Solver could not find a feasible solution.							
9	-1	-1	-1	1	Solver could not find a feasible solution.							
10	1	-1	-1	1	1	2.2	-0.9882	1	1.2	1.9882	1.6720	681,900
11	**-1**	**1**	**-1**	**1**	**-0.9928**	**1**	**-3.0577**	**1**	**1.9928**	**4.0577**	**1.5034**	**490,100**
12	1	1	-1	1	Solver could not find a feasible solution.							
13	-1	-1	1	1	Solver could not find a feasible solution.							
14	1	-1	1	1	Solver could not find a feasible solution.							
15	-1	1	1	1	Solver could not find a feasible solution.							
16	1	1	1	1	Solver could not find a feasible solution.							
Constraint precision of $1x10^{-3}$. Manual point initializations.												

Table A2. Best solutions found by the WMS to function *AOG* 1. Variables ranged to [-1, 1] using a level of precision of $1x10^{-3}$ and manual point initializations.

	Initial points				Best solution				WMS size		Objective value	***SSE***
Run	x_1^L	x_1^U	x_2^L	x_2^U	x_1^L	x_1^U	x_2^L	x_2^U	$x_1^U - x_1^L$	$x_2^U - x_2^L$		
1	-1	-1	-1	-1	-2.5720	0.65	-2.5720	0.65	3.222	3.222	-0.4170	0
2	1	-1	-1	-1	-1.1174	-0.0338	2.6222	4.9653	1.084	2.343	1.5915	414,500
3	-1	1	-1	-1	-1.5720	1	-2.5720	0.65	2.572	3.222	-0.3928	0
4	1	1	-1	-1	-4.0538	-0.4771	-4.9024	-4.0195	3.577	0.883	-0.3118	0
5	-1	-1	1	-1	4.0327	4.4370	-3.2083	0.1589	0.404	3.367	-0.2626	0
6	1	-1	1	-1	1	2.2	1	2.2	1.2	1.2	1.7569	903,000
7	**-1**	**1**	**1**	**-1**	**-2.3970**	**1.3501**	**-3.7207**	**-0.1446**	**3.747**	**3.576**	**-0.4456**	**0**
8	1	1	1	-1	-3.8069	-3.1811	-2.7157	4.4828	0.626	7.199	-0.3749	0
9	-1	-1	-1	1	1.0979	1.9899	-4.2136	4.5089	0.892	8.723	-0.4216	0
10	1	-1	-1	1	1	2.2	-0.9835	1	1.2	1.984	1.6722	681,900
11	-1	1	-1	1	-2.8994	3.4449	4.3239	4.7236	6.344	0.400	-0.3373	0
12	1	1	-1	1	-4.8988	-4.0737	-2.0570	3.5208	0.825	5.578	-0.3598	0
13	-1	-1	1	1	0.7535	4.9049	-4.9058	3.0591	4.151	7.965	1.6737	4,848,200
14	1	-1	1	1	Solver could not find a feasible solution.							
15	-1	1	1	1	-5	2.4454	-4.5923	-4.4010	7.4454	0.1913	-0.3342	0
16	1	1	1	1	-1.5825	0.9082	-3.2806	0.8030	2.4907	4.0836	-0.4164	0
Constraint precision of $1x10^{-9}$. Automatic multistarting point initializations.												

Table A3. Best solutions found by the WMS to function *AOG 1*. Variables ranged to [-1, 1] using a level of precision of $1x10^{-9}$ and automatic multistarting point initialization.

Run	Initial points x_1^L	x_1^U	x_2^L	x_2^U	Best solution x_1^L	x_1^U	x_2^L	x_2^U	WMS size $x_1^U - x_1^L$	$x_2^U - x_2^L$	Objective value	*SSE*
1	**-1**	**-1**	**-1**	**-1**	**-2.5720**	**0.65**	**-2.5720**	**0.65**	**3.2220**	**3.2220**	**-0.4170**	**0**
2	1	-1	-1	-1	1	2.2	-4.6521	2.2	1.2	6.8521	1.6869	1,986,200
3	-1	1	-1	-1	-1.5720	1	-2.5720	0.65	2.5720	3.2220	-0.3928	0
4	1	1	-1	-1	Solver could not find a feasible solution.							
5	-1	-1	1	-1	-4.6665	2.2	1	2.2	6.8665	1.2	1.7540	3,164,700
6	1	-1	1	-1	1	2.2	1	2.2	1.2	1.2	1.7569	903,000
7	-1	1	1	-1	-0.9835	1	1	2.2	1.9835	1.2	1.7218	960,600
8	1	1	1	-1	Solver could not find a feasible solution.							
9	-1	-1	-1	1	-1.7920	-0.0499	-1.2788	1	1.7421	2.2788	1.4814	250,000
10	1	-1	-1	1	1	2.2	-0.9835	1	1.2	1.9835	1.6722	681,900
11	-1	1	-1	1	The objective cell values do not converge.							
12	1	1	-1	1	Solver could not find a feasible solution.							
13	-1	-1	1	1	Solver could not find a feasible solution.							
14	1	-1	1	1	Solver could not find a feasible solution.							
15	-1	1	1	1	Solver could not find a feasible solution.							
16	1	1	1	1	Solver could not find a feasible solution.							
Constraint precision of $1x10^{-9}$. Manual point initializations.												

Table A4. Best solutions found by the WMS to function *AOG* 1. Variables ranged to [-1, 1] using a level of precision of $1x10^{-9}$ and manual point initializations.

Run	Initial points x_1^L	x_1^U	x_2^L	x_2^U	Best solution x_1^L	x_1^U	x_2^L	x_2^U	WMS size $x_1^U - x_1^L$	$x_2^U - x_2^L$	Objective value	*SSE*
1	-5	-5	-5	-5	-4.9999	-4.5563	-4.4014	1.0604	0.4436	5.4618	-0.3233	0
2	5	-5	-5	-5	Solver could not find a feasible solution.							
3	-5	5	-5	-5	-4.9988	4.9992	-4.75	-4.7499	9.9980	0.0001	-0.3471	0
4	5	5	-5	-5	-2.5574	4.6126	4.7161	4.75	7.1700	0.0339	-0.3089	0
5	-5	-5	5	-5	4.659	4.9937	-5	4.7992	0.3347	9.7992	-0.3863	0
6	5	-5	5	-5	-3.9306	-3.4374	-4.4327	-1.8393	0.4932	2.5934	-0.2432	0
7	-5	5	5	-5	-4.8952	4.9214	-2.9917	2.8937	9.8166	5.8854	1.6503	6,649,700
8	5	5	5	-5	Solver could not find a feasible solution.							
9	-5	-5	-5	5	-1.9708	0.6751	-3.4421	0.9917	2.6459	4.4338	-0.4323	0
10	5	-5	-5	5	-4.8011	2.7932	4.1991	4.5878	7.5943	0.3887	-0.3589	0
11	**-5**	**5**	**-5**	**5**	**-2.8092**	**1.4790**	**-2.7323**	**0.5140**	**4.2882**	**3.2463**	**-0.4504**	**0**
12	5	5	-5	5	Solver could not find a feasible solution.							
13	-5	-5	5	5	-3.8448	0.6416	-4.8125	-4.2579	4.4864	0.5546	-0.3103	0
14	5	-5	5	5	Solver could not find a feasible solution.							
15	-5	5	5	5	Solver could not find a feasible solution.							
16	5	5	5	5	3.2027	3.2873	0.8956	4.2104	0.0846	3.3148	-0.2234	0
Constraint precision of $1x10^{-3}$. Automatic multistarting point initializations.												

Table A5. Best solutions found by the WMS to function *AOG 1*. Variables ranged to [-5, 5] using a level of precision of $1x10^{-3}$ and automatic multistarting point initialization.

	Initial points				Best solution				WMS size		Objective value	*SSE*
Run	x_1^L	x_1^U	x_2^L	x_2^U	x_1^L	x_1^U	x_2^L	x_2^U	$x_1^U - x_1^L$	$x_2^U - x_2^L$		
1	-5	-5	-5	-5	-4.6250	-4.6249	-4.6250	-4.6249	0.0001	0.0001	0.0000	0
2	5	-5	-5	-5	Solver could not find a feasible solution.							
3	-5	5	-5	-5	-4.9988	4.9992	-4.750	-4.7499	9.9980	0.0001	-0.3471	0
4	5	5	-5	-5	Solver could not find a feasible solution.							
5	-5	-5	5	-5	Solver could not find a feasible solution.							
6	5	-5	5	-5	-2.85	3	-2.9530	3	5.8500	5.9530	1.6775	5,131,600
7	-5	5	5	-5	-4.8952	4.9214	-2.9917	2.8937	9.8166	5.8854	1.6503	6,649,700
8	5	5	5	-5	Solver could not find a feasible solution.							
9	**-5**	**-5**	**-5**	**5**	**-4.75**	**-4.7499**	**-4.9991**	**5**	**0.0001**	**9.9991**	**-0.3471**	**0**
10	5	-5	-5	5	-2.9966	3	-4.9612	4.948	5.9966	9.9092	1.6620	7,390,300
11	-5	5	-5	5	-4.9997	4.8173	-4.967	4.75	9.8170	9.7170	1.6696	11,832,400
12	5	5	-5	5	Solver could not find a feasible solution.							
13	-5	-5	5	5	Solver could not find a feasible solution.							
14	5	-5	5	5	Solver could not find a feasible solution.							
15	-5	5	5	5	Solver could not find a feasible solution.							
16	5	5	5	5	Solver could not find a feasible solution.							
Constraint precision of $1x10^{-3}$. Manual point initializations.												

Table A6. Best solutions found by the WMS to function *AOG* 1. Variables ranged to [-5, 5] using a level of precision of $1x10^{-3}$ and manual point initializations.

	Initial points				Best solution				WMS size		Objective value	***SSE***
Run	x_1^L	x_1^U	x_2^L	x_2^U	x_1^L	x_1^U	x_2^L	x_2^U	$x_1^U - x_1^L$	$x_2^U - x_2^L$		
1	-5	-5	-5	-5	-5	-4.7499	-4.8749	-4.6249	0.2501	0.2500	-0.0646	0
2	5	-5	-5	-5	-4.927	-4.5877	-2.5908	4.7759	0.3393	7.3667	-0.3498	0
3	-5	5	-5	-5	-4.9988	4.9985	-4.9986	-4.7482	9.9973	0.2504	-0.3794	0
4	5	5	-5	-5	4.75	4.875	-4.875	-4.7499	0.1250	0.1251	-0.0341	0
5	-5	-5	5	-5	-4.8661	2.9999	-2.9999	3	7.8660	5.9999	1.6708	6,383,800
6	5	-5	5	-5	-4.9853	-4.7928	-4.3269	4.4929	0.1925	8.8198	-0.3562	0
7	-5	5	5	-5	4.0202	4.8616	-4.2694	2.6623	0.8414	6.9317	-0.3882	0
8	**5**	**5**	**5**	**-5**	**-0.9772**	**-0.1163**	**-2.1908**	**4.6958**	**0.8609**	**6.8866**	**-0.3889**	**0**
9	-5	-5	-5	5	-4.9919	-4.7498	-4.9992	4.9988	0.2421	9.9980	-0.3785	0
10	5	-5	-5	5	-2.9842	3	-4.9612	4.948	5.9842	9.9092	1.6623	7,390,300
11	-5	5	-5	5	-3.591	2.8267	-4.9868	-4.7611	6.4177	0.2257	-0.3196	0
12	5	5	-5	5	4.75	4.875	-4.9996	4.9994	0.1250	9.9990	-0.3642	0
13	-5	-5	5	5	-5	-4.7499	4.0612	4.875	0.2501	0.8138	-0.1185	0
14	5	-5	5	5	-2.8943	2.6091	-4.3728	0.1435	5.5034	4.5163	1.5457	1,555,300
15	-5	5	5	5	-4.9993	4.9995	4.75	4.875	9.9988	0.1250	-0.3642	0
16	5	5	5	5	-2.5841	2.4749	4.7942	4.875	5.0590	0.0808	-0.2720	0
Constraint precision of $1x10^{-9}$. Automatic multistarting point initializations.												

Table A7. Best solutions found by the WMS to function *AOG 1*. Variables ranged to [-5, 5] using a level of precision of $1x10^{-9}$ and automatic multistarting point initialization.

	Initial points				Best solution				WMS size		Objective value	*SSE*
Run	x_1^L	x_1^U	x_2^L	x_2^U	x_1^L	x_1^U	x_2^L	x_2^U	$x_1^U - x_1^L$	$x_2^U - x_2^L$		
1	-5	-5	-5	-5	-5	-4.7499	-4.8749	-4.6249	0.2501	0.25	-0.0646	0
2	5	-5	-5	-5	-2.9842	3	-4.9532	3	5.9842	7.9532	1.6682	6,320,800
3	**-5**	**5**	**-5**	**-5**	**-4.9988**	**4.9985**	**-4.9986**	**-4.7482**	**9.9973**	**0.2504**	**-0.3794**	**0**
4	5	5	-5	-5	4.75	4.875	-4.875	-4.7499	0.1250	0.1251	-0.0341	0
5	-5	-5	5	-5	-4.8661	2.9999	-2.9999	3	7.8660	5.9999	1.6708	6,383,800
6	5	-5	5	-5	-2.85	3	-2.953	3	5.8500	5.9530	1.6775	5,131,600
7	-5	5	5	-5	-4.8952	4.9214	-2.9822	2.9009	9.8166	5.8831	1.6503	6,649,700
8	5	5	5	-5	-2.8668	4.9773	-2.9817	2.9999	7.8441	5.9816	1.62611	4,666,000
9	-5	-5	-5	5	-4.9919	-4.7498	-4.9992	4.9988	0.2421	9.9980	-0.3785	0
10	5	-5	-5	5	-2.9842	3	-4.9612	4.948	5.9842	9.9092	1.6623	7,390,300
11	-5	5	-5	5	-4.8842	4.8173	-4.9499	4.75	9.7015	9.6999	1.6714	11,832,400
12	5	5	-5	5	4.75	4.875	-4.9996	4.9994	0.1250	9.9990	-0.3642	0
13	-5	-5	5	5	-5	-4.7499	4.0612	4.875	0.2501	0.8138	-0.1185	0
14	5	-5	5	5	-2.85	3	-2.9612	4.9369	5.85	7.8981	1.6692	6,201,100
15	-5	5	5	5	-4.9993	4.9995	4.75	4.875	9.9988	0.1250	-0.3642	0
16	5	5	5	5	4.75	4.875	4.75	4.875	0.1250	0.1250	-0.0341	0
Constraint precision of $1x10^{-9}$. Manual point initializations.												

Table A8. Best solutions found by the WMS to function *AOG* 1. Variables ranged to [-5, 5] using a level of precision of $1x10^{-9}$ and manual point initializations.

Appendix B

In this appendix the detailed results found through the use of windows of maximum similarity in Chapter 5 are presented. Left to right, the initialization points for the four decision variables in each run are displayed. Next, the best solutions including the best objective function value, SSE value, and the solutions of *t* lower and upper, the intercept and the slope (Final points) provide the WMS size. These results were obtained by setting automatic multistarting points. Additionally, the best objective value of all initializations is highlighted (including if there are repetitions), and for some cases the results are rounded.

From Tables B1 to B82, the initialization points for β_0 and β_1 in the run number 1 was selected according to the beta values from the straight line equation presented in Figure 8. For the remaining initializations, random numbers were used.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-2.709270E-01	0	23.00000	24.00000	-6.97E+29	3.80E+29	1.0000000
2	3	5	0	-5	-2.709269E-01	0	1.00000	2.00000	2.70E+29	8.11E+28	0.9999995
3	-48	81	6	15	-2.709269E-01	0	2.00000	3.00000	9.13E+29	2.48E+29	0.9999993
4	10	-40	-1	-11	-2.709269E-01	0	1.00000	2.00000	-4.58E+29	3.12E+28	0.9999993
5	187	-958	-535	-620	-2.709269E-01	0	1.00000	2.00000	5.64E+29	6.61E+28	0.9999993
6	141	-382	631	494	-2.709269E-01	0	1.00000	2.00000	8.42E+29	7.67E+29	0.9999997
7	-777	-508	-437	-459	-2.709269E-01	0	1.00000	2.00000	-1.44E+29	4.85E+29	0.9999995
8	240	-1356	-964	-1729	-2.709269E-01	0	1.00000	2.00000	7.84E+29	3.15E+28	0.9999997
9	1415	-1505	1164	1167	-2.709270E-01	0	23.00000	24.00000	-7.35E+29	4.09E+29	1.0000000
10	-1018	-1770	-1382	-371	-2.709263E-01	0	1	2.00000	-5.25E+29	1.55E+27	0.9999964
Constraint precision of 1×10^{-9}											

Table B1. Best solutions found by the WMS method in each initialization. Original data from time Series 3, continuous variables, and a level of precision of 1×10^{-9}. Positive slope.

Initialization of the beginning point					Best solution						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-2.709230E-01	0	1.00002	2.00000	-7.40E+29	1.88E+29	0.999979
2	3	5	0	-5	-2.709257E-01	0	4.00000	5.00000	6.90E+29	6.61E+29	0.999994
3	-48	81	6	15	-2.709239E-01	0	8.00000	8.99999	-3.36E+29	7.09E+29	0.999984
4	10	-40	-1	-11	-2.709235E-01	0	3.00001	3.99999	-1.76E+29	5.74E+29	0.999982
5	187	-958	-535	-620	-2.709257E-01	0	14.00000	14.99999	4.09E+29	3.70E+29	0.999993
6	141	-382	631	494	-2.709259E-01	0	1.00000	2.00000	-8.83E+28	5.71E+29	0.999994
7	-777	-508	-437	-459	-2.709239E-01	0	1.00001	2.00000	2.35E+29	8.91E+29	0.999984
8	240	-1356	-964	-1729	-2.709251E-01	0	6.00000	6.99999	-6.70E+29	7.51E+29	0.999990
9	1415	-1505	1164	1167	-2.709216E-01	0	21.00000	21.99998	-2.42E+29	4.70E+29	0.999973
10	-1018	-1770	-1382	-371	-2.709263E-01	0	9.00000	10.00000	3.97E+29	4.44E+29	0.999996
Constraint precision of $1\text{x}10^{-3}$											

Table B2. Best solutions found by the WMS method in each initialization. Original data from time Series 3, continuous variables, and a level of precision of $1\text{x}10^{-3}$. Positive slope.

Initialization: Initial points					Best solution: final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-0.2586	0	11.0587	11.9967	-5.81E+28	8.80E+29	0.9380029
2	3	5	0	-5	-0.2520	0	4.0451	4.9507	1.89E+29	3.22E+29	0.9056064
3	-48	81	6	15	-0.2321	0	10.1641	10.9749	-6.29E+29	9.74E+29	0.8108147
4	10	-40	-1	-11	-0.2631	0	20.0062	20.9666	4.61E+29	5.82E+29	0.9603795
5	187	-958	-535	-620	-0.2608	0	6.0479	6.9969	-7.14E+29	5.57E+29	0.9489798
6	141	-382	631	494	-0.2610	0	12.0151	12.9650	6.86E+29	4.03E+29	0.9499230
7	-777	-508	-437	-459	-0.2528	0	22.0602	22.9695	3.42E+29	4.33E+29	0.9092946
8	240	-1356	-964	-1729	-0.2306	0	12.1542	12.9581	9.83E+29	1.15E+29	0.8039414
9	1415	-1505	1164	1167	-0.2637	0	8.0316	8.9948	7.42E+29	8.54E+29	0.9631928
10	-1018	-1770	-1382	-371	-0.2518	0	17.0726	17.9770	-4.42E+29	5.98E+29	0.9044178
Constraint precision of $1\text{x}10^{-9}$											

Table B3. Best solutions found by the WMS method in each initialization. Original data from time Series 3, continuous variables truncated, and a level of precision of $1\text{x}10^{-9}$. Positive slope.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-0.2652	0	14.0212	14.9922	-9.03E+29	5.76E+29	0.9710650
2	3	5	0	-5	-0.2491	0	22.0755	22.9668	7.36E+29	4.40E+29	0.8913701
3	-48	81	6	15	-0.2124	0	2.2003	2.9224	4.85E+28	4.16E+28	0.7220483
4	10	-40	-1	-11	-0.2209	0	17.0740	17.8337	9.14E+29	9.64E+29	0.7597168
5	187	-958	-535	-620	-0.2378	0	23.0747	23.9121	7.91E+28	5.51E+29	0.8374230
6	141	-382	631	494	-0.2526	0	8.0152	8.9234	-3.13E+29	5.28E+29	0.9082025
7	-777	-508	-437	-459	-0.2575	0	12.0185	12.9511	-8.02E+29	4.21E+29	0.9326277
8	240	-1356	-964	-1729	-0.2681	0	23.0019	23.9878	-1.22E+29	5.55E+29	0.9858467
9	1415	-1505	1164	1167	-0.2445	0	15.1236	15.9930	-5.02E+29	6.76E+29	0.8693109
10	-1018	-1770	-1382	-371	-0.2566	0	14.0484	14.9764	-4.63E+29	7.96E+29	0.9280829
Constraint precision of $1\text{x}10^{-3}$											

Table B4. Best solutions found by the WMS method in each initialization. Original data from time Series 3, continuous variables truncated, and a level of precision of $1\text{x}10^{-3}$. Positive slope.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	5.7502	1.63E+60	6	7	-5.787E+28	1.47257E+29	1
2	3	5	0	-5	5.8864	3.74E+61	23	24	9.39736E+29	1.44083E+29	1
3	-48	81	6	15	5.7553	1.83E+60	16	17	-5.1815E+28	6.10741E+28	1
4	10	-40	-1	-11	5.9011	5.25E+61	6	7	9.48833E+29	6.40427E+29	1
5	187	-958	-535	-620	0.5269	9.52E+07	23	24	88928.4719	2994.8925	1
6	141	-382	631	494	5.8812	3.32E+61	9	10	-4.48445E+29	4.753E+29	1
7	-777	-508	-437	-459	5.7078	6.12E+59	6	7	3.10332E+29	3.73389E+28	1
8	240	-1356	-964	-1729	5.8931	4.36E+61	12	13	3.78256E+29	3.43133E+29	1
9	1415	-1505	1164	1167	5.7647	2.27E+60	3	4	-5.70831E+29	4.60523E+29	1
10	-1018	-1770	-1382	-371	5.8261	9.34E+60	7	8	1.89741E+29	2.6231E+29	1
Constraint precision of $1x10^{-9}$											

Table B5. Best solutions found by the WMS method in each initialization. Original data from time Series 3, integer variables, and a level of precision of $1x10^{-9}$. Positive slope.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	5.8447	1.43E+61	3	4	-8.10631E+28	7.79641E+29	1
2	3	5	0	-5	5.9866	3.76E+62	16	17	1.16569E+29	8.23894E+29	1
3	-48	81	6	15	5.8712	2.64E+61	7	8	3.92511E+29	4.30887E+29	1
4	10	-40	-1	-11	5.7149	7.22E+59	2	3	-5.25327E+29	4.34242E+29	1
5	187	-958	-535	-620	5.7792	3.17E+60	7	8	-1.72095E+29	1.9039E+29	1
6	141	-382	631	494	5.8825	3.42E+61	16	17	1.11242E+29	2.43852E+29	1
7	-777	-508	-437	-459	5.7495	1.60E+60	4	5	6.08767E+29	6.34057E+28	1
8	240	-1356	-964	-1729	5.8024	5.41E+60	3	4	-7.93558E+29	6.86319E+29	1
9	1415	-1505	1164	1167	5.8987	4.97E+61	8	9	8.071E+29	4.91032E+29	1
10	-1018	-1770	-1382	-371	5.8125	6.83E+60	3	4	-5.29111E+29	6.70426E+29	1
Constraint precision of $1x10^{-3}$											

Table B6. Best solutions found by the WMS method in each initialization. Original data from time Series 3, integer variables, and a level of precision of $1x10^{-3}$. Positive slope.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	SSE	t^L	t^U	β_0	β_1	WMS size
1	1	1	-94000	26500	-0.2709269	0	1.0000	2.0000	-6.67E+28	26,472.82	1.000000
2	3	5	0	-5	-0.2709264	0	11.0000	12.0000	4.77E+29	20,110.10	0.999997
3	-48	81	6	15	-0.2709267	0	1.0000	2.0000	2.51E+29	20,099.67	0.999999
4	10	-40	-1	-11	-0.2709266	0	1.0000	2.0000	1.47E+29	22,370.39	0.999998
5	187	-958	-535	-620	-0.2709264	0	1.0000	2.0000	-1.95E+29	23,402.28	0.999997
6	141	-382	631	494	-0.2709269	0	5.0000	6.0000	-5.96E+29	21,287.19	1.000000
7	-777	-508	-437	-459	-0.2709270	0	23.0000	24.0000	-1.09E+29	25,177.45	1.000000
8	240	-1356	-964	-1729	-0.2709270	0	1.0000	2.0000	-6.04E+29	27,045.36	1.000000
9	1415	-1505	1164	1167	-0.2709270	0	1.0000	2.0000	9.69E+29	20,919.15	1.000000
10	-1018	-1770	-1382	-371	-0.2709270	0	23.0000	24.0000	-8.69E+29	26,541.84	1.000000
Constraint precision of $1x10^{-9}$											

Table B7. Best solutions for Series 3 (original data) found by the WMS method in each initialization, for the cases of continuous variables. Slope ranged to [20,000, 30,000] and use of a level of precision of $1x10^{-9}$.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	WMS size
1	1	1	-94000	26500	-0.2709251	0	3.0000	4.0000	4.523E+29	28,489.95	0.999990
2	3	5	0	-5	-0.2709209	0	18.0000	19.0000	-6.062E+29	20,623.57	0.999969
3	-48	81	6	15	-0.2709233	0	16.0000	17.0000	2.752E+29	21,079.64	0.999981
4	10	-40	-1	-11	-0.2709266	0	2.0000	3.0000	3.448E+28	21,876.64	0.999998
5	187	-958	-535	-620	-0.2709239	0	9.0000	10.0000	5.609E+29	26,240.97	0.999984
6	141	-382	631	494	-0.2709233	0	13.0000	14.0000	2.628E+29	26,423.99	0.999981
7	-777	-508	-437	-459	-0.2709247	0	4.0000	5.0000	-8.953E+29	21,714.14	0.999988
8	240	-1356	-964	-1729	-0.2709226	0	18.0000	19.0000	2.635E+29	21,179.70	0.999977
9	1415	-1505	1164	1167	-0.2709232	0	10.0000	11.0000	-3.372E+29	29,556.96	0.999981
10	-1018	-1770	-1382	-371	-0.2709196	0	16.0000	17.0000	-2.140E+28	27,554.15	0.999962
Constraint precision of $1\text{x}10^{-3}$											

Table B8. Best solutions for Series 3 (original data) found by the WMS method in each initialization, for the cases of continuous variables. Slope ranged to [20,000, 30,000] and use of a level of precision of $1\text{x}10^{-3}$.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	SSE	t^L	t^U	β_0	β_1	WMS size
1	1	1	-94000	26500	-0.2615	0	13.0362	13.9888	-1.54E+29	28,388.14	0.9525
2	3	5	0	-5	-0.2633	0	20.0231	20.9847	4.19E+29	26,888.36	0.9616
3	-48	81	6	15	-0.2602	0	15.0542	15.9999	4.06E+29	26,337.47	0.9457
4	10	-40	-1	-11	-0.2629	0	19.0153	19.9747	6.92E+28	28,688.59	0.9594
5	187	-958	-535	-620	-0.2663	0	13.0140	13.9907	8.99E+29	23,195.17	0.9767
6	141	-382	631	494	-0.2522	0	4.0720	4.9785	4.73E+29	24,174.87	0.9065
7	-777	-508	-437	-459	-0.2543	0	5.0805	5.9970	-3.51E+29	20,692.69	0.9166
8	240	-1356	-964	-1729	-0.2521	0	13.0273	13.9334	6.15E+28	22,243.69	0.9061
9	1415	-1505	1164	1167	-0.2461	0	2.0464	2.9234	3.44E+28	22,006.59	0.8770
10	-1018	-1770	-1382	-371	-0.2580	0	1.0117	1.9467	-2.43E+28	23,930.79	0.9350
Constraint precision of $1\text{x}10^{-9}$											

Table B9. Best solutions for Series 3 (original data) found by the WMS method in each initialization, for the cases of continuous variables truncated. Slope ranged to [20,000, 30,000] and use of a level of precision of $1\text{x}10^{-9}$.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	SSE	t^L	t^U	β_0	β_1	WMS size
1	1	1	-94000	26500	-0.2672	0	23.0033	23.9842	8.79E+29	24,908.07	0.9809
2	3	5	0	-5	-0.2429	0	22.0346	22.8961	6.74E+29	28,745.37	0.8615
3	-48	81	6	15	-0.2346	0	3.0498	3.8723	-1.50E+29	22,242.71	0.8224
4	10	-40	-1	-11	-0.2678	0	20.0077	20.9917	-8.53E+29	21,964.70	0.9840
5	187	-958	-535	-620	-0.2601	0	6.0521	6.9977	-9.58E+29	29,960.59	0.9456
6	141	-382	631	494	-0.2625	0	19.0035	19.9610	9.12E+29	20,475.06	0.9575
7	-777	-508	-437	-459	-0.2526	0	22.0605	22.9690	7.35E+29	29,517.22	0.9084
8	240	-1356	-964	-1729	-0.2465	0	8.0305	8.9095	3.50E+28	22,104.23	0.8790
9	1415	-1505	1164	1167	-0.2370	0	18.1499	18.9837	8.21E+29	27,316.56	0.8338
10	-1018	-1770	-1382	-371	-0.2379	0	14.0609	14.8990	1.54E+29	27,701.19	0.8381
Constraint precision of $1\text{x}10^{-3}$											

Table B10. Best solutions for Series 3 (original data) found by the WMS method in each initialization, for the cases of continuous variables truncated. Slope ranged to [20,000, 30,000] and use of a level of precision of $1\text{x}10^{-3}$.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	SSE	t^L	t^U	β_0	β_1	WMS size
1	1	1	-94000	26500	0.6071	6.02E+08	2	3	65,411.00	20,000.00	1
2	3	5	0	-5	-0.0649	5.93E+11	1	24	5.11E-06	20,000.00	23
3	-48	81	6	15	5.6222	8.53E+58	7	8	-2.07E+29	28,939.64	1
4	10	-40	-1	-11	0.5622	2.14E+08	2	3	35,690.21	29,906.93	1
5	187	-958	-535	-620	5.7554	1.83E+60	3	4	-9.57E+29	28,805.57	1
6	141	-382	631	494	0.1358	2.28E+10	15	24	-223,689.12	20,000.00	9
7	-777	-508	-437	-459	5.5935	4.40E+58	23	24	-1.48E+29	23,864.78	1
8	240	-1356	-964	-1729	0.5131	2.66E+09	22	24	-312061.21	20,429.23	2
9	1415	-1505	1164	1167	5.6782	3.10E+59	6	7	3.94E+29	27,203.81	1
10	-1018	-1770	-1382	-371	5.5296	1.01E+58	6	7	7.11E+28	24,645.11	1
Constraint precision of $1x10^{-9}$											

Table B11. Best solutions for Series 3 (original data) found by the WMS method in each initialization, for the cases of integer variables. Slope ranged to [20,000, 30,000] and use of a level of precision of $1x10^{-9}$.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	SSE	t^L	t^U	β_0	β_1	WMS size
1	1	1	-94000	26500	5.7264	9.41E+59	5	6	-6.86E+29	28,557.49	1
2	3	5	0	-5	5.6935	4.41E+59	13	14	-4.70E+29	28,286.15	1
3	-48	81	6	15	5.6757	2.92E+59	1	2	3.82E+29	25,743.80	1
4	10	-40	-1	-11	5.6081	6.17E+58	4	5	-1.76E+29	21,126.28	1
5	187	-958	-535	-620	5.6917	4.23E+59	1	2	-4.60E+29	22,653.57	1
6	141	-382	631	494	5.6636	2.21E+59	2	3	-3.33E+29	28,560.94	1
7	-777	-508	-437	-459	5.7350	1.15E+60	19	20	7.57E+29	27,600.22	1
8	240	-1356	-964	-1729	5.5678	2.44E+58	8	9	1.10E+29	26,522.29	1
9	1415	-1505	1164	1167	5.6180	7.75E+58	2	3	-1.97E+29	26,445.13	1
10	-1018	-1770	-1382	-371	5.7080	6.15E+59	1	2	5.55E+29	20,496.21	1
Constraint precision of $1x10^{-3}$											

Table B12. Best solutions for Series 3 (original data) found by the WMS method in each initialization, for the cases of integer variables. Slope ranged to [20,000, 30,000] and use of a level of precision of $1x10^{-3}$.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	SSE	t^L	t^U	β_0	β_1	WMS size
1	1	1	-94000	26500	-0.27093	0	1.0000	2.0000	-3.33E+29	24,821.86	1.00000
2	3	5	0	-5	-0.27093	0	1.0000	2.0000	-6.30E+29	27,849.64	1.00000
3	-48	81	6	15	-0.27093	0	1.0000	2.0000	-9.65E+29	27,448.00	1.00000
4	10	-40	-1	-11	-0.27093	0	1.0000	2.0000	4.14E+29	27,068.56	1.00000
5	187	-958	-535	-620	-0.27092	0	5.0000	6.0000	-6.31E+28	28,461.46	0.99999
6	141	-382	631	494	-0.27092	0	18.0000	19.0000	7.96E+29	26,307.20	0.99998
7	-777	-508	-437	-459	-0.27093	0	5.0000	6.0000	-8.31E+29	27,525.33	1.00000
8	240	-1356	-964	-1729	-0.27093	0	5.0000	6.0000	4.75E+29	28,117.87	1.00000
9	1415	-1505	1164	1167	-0.27093	0	1.0000	2.0000	-3.83E+29	24,786.76	1.00000
10	-1018	-1770	-1382	-371	-0.27093	0	1.0000	2.0000	5.16E+29	24,567.97	1.00000
Constraint precision of $1x10^{-9}$											

Table B13. Best solutions for Series 3 (original data) found by the WMS method in each initialization, for the cases of continuous variables. Slope ranged to [24,545.1, 28,545.1] and use of a level of precision of $1x10^{-9}$.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	SSE	t^L	t^U	β_0	β_1	WMS size
1	1	1	-94000	26500	-0.27092	0	6.0000	7.0000	5.19E+27	24,558.00	0.99999
2	3	5	0	-5	-0.27092	0	1.0000	2.0000	7.04E+29	26,029.80	0.99998
3	-48	81	6	15	-0.27092	0	19.0000	20.0000	-4.24E+29	26,570.23	0.99998
4	10	-40	-1	-11	-0.27092	0	15.0000	16.0000	5.53E+29	26,928.52	0.99997
5	187	-958	-535	-620	-0.27092	0	11.0000	12.0000	6.62E+28	25,081.46	0.99998
6	141	-382	631	494	-0.27092	0	6.0000	7.0000	-4.27E+29	27,568.70	0.99999
7	-777	-508	-437	-459	-0.27092	0	9.0000	10.0000	4.80E+29	26,541.49	0.99998
8	240	-1356	-964	-1729	-0.27082	0	1.0006	2.0000	8.14E+29	24,613.23	0.99943
9	1415	-1505	1164	1167	-0.27092	0	18.0000	19.0000	-1.02E+29	25,204.26	0.99997
10	-1018	-1770	-1382	-371	-0.27091	0	14.0000	15.0000	-1.05E+29	25,798.88	0.99994
Constraint precision of $1\text{x}10^{-3}$											

Table B14. Best solutions for Series 3 (original data) found by the WMS method in each initialization, for the cases of continuous variables. Slope ranged to [24,545.1, 28,545.1] and use of a level of precision of $1\text{x}10^{-9}$.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	SSE	t^L	t^U	β_0	β_1	WMS size
1	1	1	-94000	26500	-0.2476	0	21.0134	21.8976	-8.41E+29	2.66E+04	0.88412
2	3	5	0	-5	-0.2691	0	18.0073	18.9982	2.97E+29	2.80E+04	0.99093
3	-48	81	6	15	-0.2611	0	19.0471	19.9975	-1.56E+29	2.47E+04	0.95037
4	10	-40	-1	-11	-0.2607	0	16.0515	16.9996	-4.06E+29	2.85E+04	0.94810
5	187	-958	-535	-620	-0.2650	0	2.0289	2.9989	-3.58E+29	2.67E+04	0.97002
6	141	-382	631	494	-0.2642	0	11.0334	11.9991	1.14E+29	2.66E+04	0.96575
7	-777	-508	-437	-459	-0.2087	0	17.1959	17.9017	2.82E+29	2.65E+04	0.70574
8	240	-1356	-964	-1729	-0.2691	0	23.0094	24.0000	-2.39E+28	2.85E+04	0.99056
9	1415	-1505	1164	1167	-0.2298	0	6.1863	6.9866	-1.53E+29	2.76E+04	0.80032
10	-1018	-1770	-1382	-371	-0.2589	0	9.0079	9.9472	1.31E+29	2.56E+04	0.93928
Constraint precision of $1\text{x}10^{-9}$											

Table B15. Best solutions for Series 3 (original data) found by the WMS method in each initialization, for the cases of continuous variables truncated. Slope ranged to [24,545.1, 28,545.1] and use of a level of precision of $1\text{x}10^{-9}$.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	SSE	t^L	t^U	β_0	β_1	WMS size
1	1	1	-94000	26500	-0.2655	0	9.025	9.997	4.27E+29	25,558.41	0.97249
2	3	5	0	-5	-0.2461	0	11.060	11.937	-9.37E+28	24,676.57	0.87703
3	-48	81	6	15	-0.2491	0	18.061	18.952	9.30E+29	26,017.72	0.89119
4	10	-40	-1	-11	-0.2485	0	15.078	15.967	-4.10E+29	27,919.45	0.88867
5	187	-958	-535	-620	-0.2573	0	18.025	18.956	3.50E+29	25,093.93	0.93161
6	141	-382	631	494	-0.2665	0	22.021	22.999	8.86E+29	26,579.22	0.97763
7	-777	-508	-437	-459	-0.2560	0	14.057	14.983	4.63E+29	27,973.25	0.92533
8	240	-1356	-964	-1729	-0.2575	0	19.015	19.948	-4.70E+29	28,331.31	0.93268
9	1415	-1505	1164	1167	-0.2663	0	19.009	19.986	-6.75E+29	27,001.61	0.97656
10	-1018	-1770	-1382	-371	-0.2267	0	21.086	21.873	5.50E+29	24,549.95	0.78619
Constraint precision of $1\text{x}10^{-3}$											

Table B16. Best solutions for Series 3 (original data) found by the WMS method in each initialization, for the cases of continuous variables truncated. Slope ranged to [24,545.1, 28,545.1] and use of a level of precision of $1\text{x}10^{-3}$.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	SSE	t^L	t^U	β_0	β_1	WMS size
1	1	1	-94000	26500	-0.0705	5.21E+11	1	24	-161,894.30	24,545.10	23
2	3	5	0	-5	0.5745	2.84E+08	2	3	42,364.24	27,682.25	1
3	-48	81	6	15	-0.0359	1.16E+12	1	24	6.00	24,545.10	23
4	10	-40	-1	-11	5.7369	1.20E+60	18	19	-7.73357E+29	26,181.26	1
5	187	-958	-535	-620	0.6621	2.14E+09	1	2	31,776.80	24,545.10	1
6	141	-382	631	494	5.6195	8.03E+58	22	23	2.00E+29	24,990.97	1
7	-777	-508	-437	-459	5.7238	8.85E+59	3	4	6.65E+29	26,732.80	1
8	240	-1356	-964	-1729	0.6621	2.14E+09	1	2	31,776.80	24,545.10	1
9	1415	-1505	1164	1167	0.6621	2.14E+09	1	2	31,776.80	24,545.10	1
10	-1018	-1770	-1382	-371	0.2399	128,202.18	6	7	-45,167.68	24,705.95	1
Constraint precision of $1x10^{-9}$											

Table B17. Best solutions for Series 3 (original data) found by the WMS method in each initialization, for the cases of integer variables. Slope ranged to [24,545.1, 28,545.1] and use of a level of precision of $1x10^{-9}$.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	SSE	t^L	t^U	β_0	β_1	WMS size
1	1	1	-94000	26500	5.6474	1.52E+59	1	2	-2.76E+29	24,744.49	1
2	3	5	0	-5	5.7245	9.00E+59	8	9	6.71E+29	24,801.65	1
3	-48	81	6	15	5.6490	1.58E+59	5	6	2.81E+29	28,250.56	1
4	10	-40	-1	-11	5.7581	1.95E+60	20	21	-9.87E+29	27,515.65	1
5	187	-958	-535	-620	5.6775	3.05E+59	6	7	3.90E+29	24,754.73	1
6	141	-382	631	494	5.7041	5.63E+59	6	7	5.30E+29	28,445.32	1
7	-777	-508	-437	-459	5.5353	1.15E+58	13	14	-7.60E+28	25,717.79	1
8	240	-1356	-964	-1729	5.6964	4.71E+59	1	2	4.85E+29	25,983.07	1
9	1415	-1505	1164	1167	5.5977	4.86E+58	22	23	1.56E+29	27,371.11	1
10	-1018	-1770	-1382	-371	5.7285	9.86E+59	10	11	-7.02E+29	25,064.75	1
Constraint precision of $1x10^{-3}$											

Table B18. Best solutions for Series 3 (original data) found by the WMS method in each initialization, for the cases of integer variables. Slope ranged to [24,545.1, 28,545.1] and use of a level of precision of $1x10^{-3}$.

Initialization: Initial points				Best solution: Final points					
Run	t^L	t^U	β_1	Objective	SSE	t^L	t^U	β_1	WMS size
1	1	1	26500	0.6306	1.04E+09	7	8	27,293.16	1
2	3	5	-5	0.5353	1.15E+08	9	10	25,915.00	1
3	-48	81	15	0.5728	2.73E+08	8	9	28,027.78	1
4	10	-40	-11	0.5917	4.23E+08	9	10	28,100.81	1
5	187	-958	-620	0.6241	8.93E+08	10	11	28,444.13	1
6	141	-382	494	0.4571	1.91E+07	10	11	26,694.67	1
7	-777	-508	-459	0.5490	1.58E+08	9	10	25,763.36	1
8	240	-1356	-1729	0.6593	2.01E+09	7	8	25,422.38	1
9	1415	-1505	1167	0.6528	1.73E+09	7	8	25,830.16	1
10	-1018	-1770	-371	0.5265	9.42E+07	8	9	27,420.49	1
Constraint precision of $1x10^{-9}$									

Table B19. Best solutions for Series 3 (original data) found by the WMS method in each initialization, for the cases of integer variables and considering the intercept fixed. Slope ranged to [24,545.1, 28,545.1] and use of a level of precision of $1x10^{-9}$.

Initialization: Initial points				Best solution: Final points					
Run	t^L	t^U	β_1	Objective	SSE	t^L	t^U	β_1	WMS size
1	1	1	26500	0.48835	1.51E+09	7	9	26,286.35	2
2	3	5	-5	0.36774	9.36E+07	8	10	27,037.74	2
3	-48	81	15	0.62096	8.30E+08	7	8	28,492.20	1
4	10	-40	-11	0.63919	1.26E+09	7	8	26,689.35	1
5	187	-958	-620	0.74797	1.55E+10	4	5	26,552.85	1
6	141	-382	494	0.56331	8.46E+09	10	12	26,571.75	2
7	-777	-508	-459	0.46103	2.09E+07	8	9	26,775.00	1
8	240	-1356	-1729	0.63807	1.23E+09	7	8	26,763.19	1
9	1415	-1505	1167	0.66386	2.23E+09	7	8	25,131.89	1
10	-1018	-1770	-371	0.36773	9.36E+07	8	10	27,037.68	2
Constraint precision of $1x10^{-3}$									

Table B20. Best solutions for Series 3 (original data) found by the WMS method in each initialization, for the cases of integer variables and considering the intercept fixed. Slope ranged to [24,545.1, 28,545.1] and use of a level of precision of $1x10^{-3}$.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-1.153220	6.7572	1	24	0	0	23
2	3	5	0	-5	-1.153220	6.7572	1	24	0	0	23
3	-48	81	6	15	-1.186098	2.6385	1	24	-0.6475	0.0583	23
4	10	-40	-1	-11	-1.185967	2.6495	1	24	-0.6764	0.0614	23
5	187	-958	-535	-620	-1.153220	6.7572	1	24	0	0	23
6	141	-382	631	494	-1.153220	6.7572	1	24	0	0	23
7	-777	-508	-437	-459	-1.186101	2.6382	1	24	-0.6543	0.0587	23
8	240	-1356	-964	-1729	-1.153220	6.7572	1	24	0	0	23
9	1415	-1505	1164	1167	-1.186097	2.6386	1	24	-0.6458	0.0582	23
10	-1018	-1770	-1382	-371	-1.185975	2.6488	1	24	-0.6751	0.0613	23
Weights: 0.1 and 0.9. Constraint precision of $1x10^{-9}$											

Table B21. Best solutions for Series 3 (scaled data) found by the WMS method in each initialization. Positive slope and use of a level of precision of $1x10^{-9}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-1.1532199	6.7572	1	24	0	0	23
2	3	5	0	-5	-1.1860983	2.6385	1	24	-0.6475	0.0583	23
3	-48	81	6	15	-1.1860982	2.6385	1	24	-0.6473	0.0583	23
4	10	-40	-1	-11	-1.1859665	2.6495	1	24	-0.6765	0.0614	23
5	187	-958	-535	-620	-1.1532199	6.7572	1	24	0	0	23
6	141	-382	631	494	-1.1532199	6.7572	1	24	0	0	23
7	-777	-508	-437	-459	-1.1532199	6.7572	1	24	0	0	23
8	240	-1356	-964	-1729	-1.1532199	6.7572	1	24	0	0	23
9	1415	-1505	1164	1167	-1.1532199	6.7572	1	24	0	0	23
10	-1018	-1770	-1382	-371	-1.1532199	6.7572	1	24	0	0	23
Weights: 0.1 and 0.9. Constraint precision of $1x10^{-3}$											

Table B22. Best solutions for Series 3 (scaled data) found by the WMS method in each initialization. Positive slope and use of a level of precision of $1x10^{-3}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-123.6581250	2.6382	1	24	-0.6542	0.0587	23
2	3	5	0	-5	-123.3293094	6.7572	1	24	0	0	23
3	-48	81	6	15	-123.6580932	2.6385	1	24	-0.6474	0.0583	23
4	10	-40	-1	-11	-123.6580924	2.6385	1	24	-0.6473	0.0583	23
5	187	-958	-535	-620	-123.3293094	6.7572	1	24	0	0	23
6	141	-382	631	494	-123.6581250	2.6382	1	24	-0.6542	0.0587	23
7	-777	-508	-437	-459	-123.6581055	2.6384	1	24	-0.6489	0.0584	23
8	240	-1356	-964	-1729	-123.3293094	6.7572	1	24	0	0	23
9	1415	-1505	1164	1167	-123.3293094	6.7572	1	24	0	0	23
10	-1018	-1770	-1382	-371	-123.6580686	2.6387	1	24	-0.6451	0.0582	23
Weights: 1 and 90. Constraint precision of $1x10^{-9}$											

Table B23. Best solutions for Series 3 (scaled data) found by the WMS method in each initialization. Positive slope and use of a level of precision of $1x10^{-9}$. Weights level: 1 and 90.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-123.3293094	6.7572	1	24	0	0	23
2	3	5	0	-5	-123.6499525	2.7073	1	24	-0.5437	0.0522	23
3	-48	81	6	15	-123.3293094	6.7572	1	24	0	0	23
4	10	-40	-1	-11	-123.6580914	2.6385	1	24	-0.6472	0.0583	23
5	187	-958	-535	-620	-123.3293094	6.7572	1	24	0	0	23
6	141	-382	631	494	-123.6580936	2.6385	1	24	-0.6474	0.0583	23
7	-777	-508	-437	-459	-123.3293094	6.7572	1	24	0	0	23
8	240	-1356	-964	-1729	-123.3293094	6.7572	1	24	0	0	23
9	1415	-1505	1164	1167	-123.6580936	2.6385	1	24	-0.6474	0.0583	23
10	-1018	-1770	-1382	-371	-123.6580936	2.6385	1	24	-0.6474	0.0583	23
Weights: 1 and 90. Constraint precision of $1x10^{-3}$											

Table B24. Best solutions for Series 3 (scaled data) found by the WMS method in each initialization. Positive slope and use of a level of precision of $1x10^{-3}$. Weights level: 1 and 90.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-1,241.3089699	6.6058	1	24	0.0880	0	23
2	3	5	0	-5	-1,241.6291996	2.6385	1	24	-0.6474	0.0583	23
3	-48	81	6	15	-1,241.6291995	2.6385	1	24	-0.6474	0.0583	23
4	10	-40	-1	-11	-1,241.3004152	6.7572	1	24	0	0	23
5	187	-958	-535	-620	-1,241.6291995	2.6385	1	24	-0.6474	0.0583	23
6	141	-382	631	494	-1,241.3004152	6.7572	1	24	0	0	23
7	-777	-508	-437	-459	-1,241.6281478	2.6473	1	24	-0.6667	0.0608	23
8	240	-1356	-964	-1729	-1,241.6291995	2.6385	1	24	-0.6474	0.0583	23
9	1415	-1505	1164	1167	-1,241.6289079	2.6409	1	24	-0.6702	0.0602	23
10	-1018	-1770	-1382	-371	-1,241.3004152	6.7572	1	24	0	0	23
Weights: 1 and 900. Constraint precision of $1x10^{-9}$											

Table B25. Best solutions for Series 3 (scaled data) found by the WMS method in each initialization. Positive slope and use of a level of precision of $1x10^{-9}$. Weights level: 1 and 900.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-1,241.3089	6.61	1	24	0.0917	0	23
2	3	5	0	-5	-1,241.4088	5.04	1	24	-0.0009	0.0193	23
3	-48	81	6	15	-1,236.1277	1,154,603.35	1	24	6	15	23
4	10	-40	-1	-11	-1,241.5552	3.31	1	24	-1.0007	0.0800	23
5	187	-958	-535	-620	-1,232.6561	3,419,861,472.12	1	24	-535.0000	867.9999	23
6	141	-382	631	494	-1,235.2856	8,026,000.18	1	24	578.3673	0	23
7	-777	-508	-437	-459	-1,233.7073	303,965,773.03	1	24	-437.0000	275.3999	23
8	240	-1356	-964	-1729	-1,231.7534	27,332,377,604.58	1	24	-964.0000	2,420.5999	23
9	1415	-1505	1164	1167	-1,232.3139	7,520,587,588.29	1	24	1,164.0000	1,166.9999	23
10	-1018	-1770	-1382	-371	-1,234.1730	104,021,456.7042	1	24	-1,382.0000	222.5999	23
Weights: 1 and 900. Constraint precision of $1x10^{-3}$											

Table B26. Best solutions for Series 3 (scaled data) found by the WMS method in each initialization. Positive slope and use of a level of precision of $1x10^{-3}$. Weights level: 1 and 900.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-12,421.011473	6.7572	1	24	0	0	23
2	3	5	0	-5	-12,421.011473	6.7572	1	24	0	0	23
3	-48	81	6	15	-12,421.011473	6.7572	1	24	0	0	23
4	10	-40	-1	-11	-12,421.263800	3.3389	1	24	-1.0006	0.0822	23
5	187	-958	-535	-620	-12,421.340289	2.6382	1	24	-0.6543	0.0587	23
6	141	-382	631	494	-12,421.340058	2.6401	1	24	-0.6386	0.0574	23
7	-777	-508	-437	-459	-12,421.340205	2.6389	1	24	-0.6487	0.0581	23
8	240	-1356	-964	-1729	-12,421.340002	2.6406	1	24	-0.6372	0.0581	23
9	1415	-1505	1164	1167	-12,421.340067	2.6401	1	24	-0.6361	0.0576	23
10	-1018	-1770	-1382	-371	-12,421.340288	2.6382	1	24	-0.6554	0.0588	23
Weights: 1 and 9000. Constraint precision of $1x10^{-9}$											

Table B27. Best solutions for Series 3 (scaled data) found by the WMS method in each initialization. Positive slope and use of a level of precision of $1x10^{-9}$. Weights level: 1 and 9000.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-12,409.5670	2,158,408,057,458.3	1	24	-93,999.4600	26,499.5937	23
2	3	5	0	-5	-12,421.1235	4.9929	1	24	-0.0077	0.0192	23
3	-48	81	6	15	-12,415.8387	1,154,603.3490	1	24	6.0000	15.0000	23
4	10	-40	-1	-11	-12,421.2086	3.9269	1	24	-1.0006	0.0688	23
5	187	-958	-535	-620	-12,412.3672	3,419,862,115.2722	1	24	-535.0000	868.0000	23
6	141	-382	631	494	-12,420.6681	16.1032	1	24	-2.0568	0.1667	23
7	-777	-508	-437	-459	-12,415.8404	1,150,255.9199	1	24	-449.9024	25.9070	23
8	240	-1356	-964	-1729	-12,411.4645	27,332,379,725.896	1	24	-964.0000	2,420.6000	23
9	1415	-1505	1164	1167	-12,412.0249	7,520,588,350.5295	1	24	1,164.0000	1,167.0000	23
10	-1018	-1770	-1382	-371	-12,413.8841	104,021,534.1223	1	24	-1,382.0000	222.6000	23
Weights: 1 and 9000. Constraint precision of $1x10^{-3}$											

Table B28. Best solutions for Series 3 (scaled data) found by the WMS method in each initialization. Positive slope and use of a level of precision of $1x10^{-3}$. Weights level: 1 and 9000.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-1.186098	2.638	1	24	-0.6475	0.0583	23
2	3	5	0	-5	-1.186101	2.638	1	24	-0.6543	0.0587	23
3	-48	81	6	15	-1.185967	2.650	1	24	-0.6766	0.0614	23
4	10	-40	-1	-11	-1.170198	2.577	1	23	-0.6766	0.0614	22
5	187	-958	-535	-620	-1.169038	2.674	2	24	-0.5688	0.0548	22
6	141	-382	631	494	5.758687	1.98E+60	3	4	-9.94E+29	0.7692	1
7	-777	-508	-437	-459	-1.186002	2.647	1	24	-0.6755	0.0611	23
8	240	-1356	-964	-1729	-1.037257	111.03	1	24	0.0000	0.1657	23
9	1415	-1505	1164	1167	-1.186089	2.64	1	24	-0.6528	0.0582	23
10	-1018	-1770	-1382	-371	-1.185980	2.648	1	24	-0.6732	0.0612	23
Weights: 0.1 and 0.9. Constraint precision of $1x10^{-9}$											

Table B29. Best solutions for Series 3 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	5.6623749	2.15E+59	2	3	-3.28E+29	0.4542	1
2	3	5	0	-5	-1.1859669	2.649	1	24	-0.6766	0.0614	23
3	-48	81	6	15	-1.1861014	2.638	1	24	-0.6543	0.0587	23
4	10	-40	-1	-11	-1.1861014	2.638	1	24	-0.6543	0.0587	23
5	187	-958	-535	-620	-1.1640113	5.050	1	24	0	0.0182	23
6	141	-382	631	494	5.7457364	1.47E+60	5	6	8.57E+29	0.3135	1
7	-777	-508	-437	-459	5.5269627	9.53E+57	1	2	-6.90E+28	0.6577	1
8	240	-1356	-964	-1729	5.6236362	8.82E+58	1	2	-2.10E+29	0.2092	1
9	1415	-1505	1164	1167	5.6706704	2.61E+59	11	12	-3.61E+29	0.0336	1
10	-1018	-1770	-1382	-371	5.4612918	2.10E+57	2	3	-3.24E+28	0.8360	1
Weights: 0.1 and 0.9. Constraint precision of $1x10^{-3}$											

Table B30. Best solutions for Series 3 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-86.762870	2.859E+37	1	24	-1.0914E+18	0.5633	23
2	3	5	0	-5	-120.289923	2.3738	3	24	-0.5439	0.0522	21
3	-48	81	6	15	-123.658090	2.6385	1	24	-0.6470	0.0583	23
4	10	-40	-1	-11	-105.395572	1.8356	10	24	-0.5163	0.0486	14
5	187	-958	-535	-620	-121.996991	2.6184	1	23	-0.7190	0.0668	22
6	141	-382	631	494	-123.657123	2.6466	1	24	-0.6755	0.0611	23
7	-777	-508	-437	-459	-123.656816	2.6492	1	24	-0.6755	0.0613	23
8	240	-1356	-964	-1729	-123.658125	2.6382	1	24	-0.6543	0.0587	23
9	1415	-1505	1164	1167	-123.657388	2.6444	1	24	-0.6755	0.0609	23
10	-1018	-1770	-1382	-371	-123.656845	2.6489	1	24	-0.6747	0.0613	23
Weights: 1 and 90. Constraint precision of $1x10^{-9}$											

Table B31. Best solutions for Series 3 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 1 and 90.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-112.490860	11,605,009,721	2	24	-22474.1584	0.9024	22
2	3	5	0	-5	-123.652265	2.688	1	24	-0.5689	0.0548	23
3	-48	81	6	15	-123.658081	2.639	1	24	-0.6464	0.0583	23
4	10	-40	-1	-11	-105.385526	1.902	10	24	-0.5168	0.0468	14
5	187	-958	-535	-620	-117.382018	6,870,590.12	1	24	-534.9998	0.0027	23
6	141	-382	631	494	-117.223425	9,898,891	1	24	630.9988	0.9024	23
7	-777	-508	-437	-459	-117.557986	4,581,690.72	1	24	-436.9996	0.0123	23
8	240	-1356	-964	-1729	-116.870609	22,305,045.52	1	24	-963.9998	0.0030	23
9	1415	-1505	1164	1167	-116.698567	33,147,071.27	1	24	1163.9993	0.9024	23
10	-1018	-1770	-1382	-371	-116.557753	45,841,487.14	1	24	-1382.0000	0.0024	23
Weights: 1 and 90. Constraint precision of $1x10^{-3}$											

Table B32. Best solutions for Series 3 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 1 and 90.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-1241.628300	2.646	1	24	-0.6471	0.0595	23
2	3	5	0	-5	-1241.629199	2.638	1	24	-0.6473	0.0583	23
3	-48	81	6	15	-1241.627999	2.649	1	24	-0.6731	0.0612	23
4	10	-40	-1	-11	-1241.629231	2.638	1	24	-0.6542	0.0587	23
5	187	-958	-535	-620	-1241.628366	2.645	1	24	-0.6648	0.0605	23
6	141	-382	631	494	-1241.628356	2.646	1	24	-0.6716	0.0609	23
7	-777	-508	-437	-459	-1241.629227	2.638	1	24	-0.6519	0.0586	23
8	240	-1356	-964	-1729	-1234.851906	21,787,698.54	1	24	-963.9785	0.9024	23
9	1415	-1505	1164	1167	-1241.629099	2.639	1	24	-0.6527	0.0582	23
10	-1018	-1770	-1382	-371	-1241.629194	2.639	1	24	-0.6469	0.0583	23
Weights: 1 and 900. Constraint precision of $1x10^{-9}$											

Table B33. Best solutions for Series 3 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 1 and 900.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-1230.97846	162,801,966,431.96	1	24	-82372.6914	0.9024	23
2	3	5	0	-5	-1241.56477	3.22	1	24	-0.3328	0.0392	23
3	-48	81	6	15	-1238.29126	7,921.50	1	24	6.0000	0.9024	23
4	10	-40	-1	-11	-1241.55594	3.31	1	24	-0.9986	0.0802	23
5	187	-958	-535	-620	-1235.35312	6,870,581.17	1	24	-534.9967	0.0025	23
6	141	-382	631	494	-1235.19454	9,898,753.51	1	24	630.9944	0.9024	23
7	-777	-508	-437	-459	-1235.52887	4,584,012.31	1	24	-437.0000	0.0035	23
8	240	-1356	-964	-1729	-1234.84171	22,305,312.19	1	24	-963.9999	0.0026	23
9	1415	-1505	1164	1167	-1234.66967	33,147,108.23	1	24	1163.9999	0.9024	23
10	-1018	-1770	-1382	-371	-1234.52886	45,841,311.92	1	24	-1381.9999	0.0026	23
Weights: 1 and 900. Constraint precision of $1x10^{-3}$											

Table B34. Best solutions for Series 3 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 1 and 900.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-12384.6750	1.683E+37	1	24	-8.375E+17	0.7605	23
2	3	5	0	-5	-12421.1177	5.074	1	24	0.0033	0.0182	23
3	-48	81	6	15	-12383.0182	7.637E+38	1	24	-5.641E+18	0.4546	23
4	10	-40	-1	-11	-12421.2645	3.332	1	24	-0.9998	0.0820	23
5	187	-958	-535	-620	-12421.3403	2.638	1	24	-0.6543	0.0587	23
6	141	-382	631	494	-12421.3401	2.640	1	24	-0.6533	0.0581	23
7	-777	-508	-437	-459	-12421.3403	2.638	1	24	-0.6549	0.0586	23
8	240	-1356	-964	-1729	-12421.3388	2.651	1	24	-0.6217	0.0556	23
9	1415	-1505	1164	1167	-12421.3401	2.640	1	24	-0.6522	0.0580	23
10	-1018	-1770	-1382	-371	-12421.3403	2.638	1	24	-0.6550	0.0586	23
Weights: 1 and 9000. Constraint precision of $1x10^{-9}$											

Table B35. Best solutions for Series 3 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 1 and 9000.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-12410.57481	212,015,912,293	1	24	-94000.5417	0.9024	23
2	3	5	0	-5	-12421.10145	5.306	1	24	0.0006	0.0258	23
3	-48	81	6	15	-12418.00231	7921.500894	1	24	6.0000	0.9024	23
4	10	-40	-1	-11	-12421.24931	3.486	1	24	-0.9998	0.0859	23
5	187	-958	-535	-620	-12415.06418	6,870,671.64	1	24	-535.0000	0.0024	23
6	141	-382	631	494	-12414.92114	9,550,804.84	1	24	630.8826	0.0024	23
7	-777	-508	-437	-459	-12415.23990	4,584,307	1	24	-437.0000	0.0024	23
8	240	-1356	-964	-1729	-12414.55277	22,305,354.14	1	24	-964.0000	0.0025	23
9	1415	-1505	1164	1167	-12414.38073	33,147,111.92	1	24	1164.0000	0.9024	23
10	-1018	-1770	-1382	-371	-12414.23992	45,841,485.38	1	24	-1382.0000	0.0024	23
Weights: 1 and 9000. Constraint precision of $1x10^{-3}$											

Table B36. Best solutions for Series 3 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 1 and 9000.

Initialization: Initial points				Best solution: Final points					
Run	t^L	t^U	β_1	Objective	*SSE*	t^L	t^U	β_1	*WMS size*
1	1	1	26500	-1.059963808	65.4145	1	24	0.2717	23
2	3	5	-5	-1.059963808	65.4145	1	24	0.2717	23
3	-48	81	15	-1.059963813	65.4145	1	24	0.2717	23
4	10	-40	-11	-1.059963808	65.4145	1	24	0.2717	23
5	187	-958	-620	-1.059963808	65.4145	1	24	0.2717	23
6	141	-382	494	-1.059963808	65.4145	1	24	0.2717	23
7	-777	-508	-459	-1.059963810	65.4145	1	24	0.2717	23
8	240	-1356	-1729	-1.059963811	65.4145	1	24	0.2717	23
9	1415	-1505	1167	-1.059963809	65.4145	1	24	0.2717	23
10	-1018	-1770	-371	-1.059963808	65.4145	1	24	0.2717	23
Weights: 0.1 and 0.9. Constraint precision of $1x10^{-9}$									

Table B37. Best solutions for Series 3 (scaled data) fixing the intercept found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 0.1 and 0.9.

Initialization: Initial points				Best solution: Final points					
Run	t^L	t^U	β_1	Objective	*SSE*	t^L	t^U	β_1	*WMS size*
1	1	1	26500	-1.059964194	65.41394	1	24	0.2716	23
2	3	5	-5	-1.059998649	65.36127	1	24	0.2710	23
3	-48	81	15	-1.059963808	65.41453	1	24	0.2717	23
4	10	-40	-11	-1.059963808	65.41453	1	24	0.2717	23
5	187	-958	-620	-1.059963808	65.41453	1	24	0.2717	23
6	141	-382	494	-1.059963801	65.41454	1	24	0.2717	23
7	-777	-508	-459	-1.059964166	65.41398	1	24	0.2716	23
8	240	-1356	-1729	-1.059963753	65.41461	1	24	0.2717	23
9	1415	-1505	1167	-1.059963808	65.41453	1	24	0.2717	23
10	-1018	-1770	-371	-1.059987273	65.37865	1	24	0.2712	23
Weights: 0.1 and 0.9. Constraint precision of $1x10^{-3}$									

Table B38. Best solutions for Series 3 (scaled data) fixing the intercept found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 0.1 and 0.9.

Initialization: Initial points				Best solution: Final points					
Run	t^L	t^U	β_1	Objective	*SSE*	t^L	t^U	β_1	*WMS size*
1	1	1	26500	-122.39674874	65.4145	1	24	0.2717	23
2	3	5	-5	-122.39674866	65.4145	1	24	0.2717	23
3	-48	81	15	-122.39674866	65.4145	1	24	0.2717	23
4	10	-40	-11	-122.39674866	65.4145	1	24	0.2717	23
5	187	-958	-620	-122.39674866	65.4145	1	24	0.2717	23
6	141	-382	494	-122.39674866	65.4145	1	24	0.2717	23
7	-777	-508	-459	-122.39674866	65.4145	1	24	0.2717	23
8	240	-1356	-1729	-122.39674866	65.4145	1	24	0.2717	23
9	1415	-1505	1167	-122.39674866	65.4145	1	24	0.2717	23
10	-1018	-1770	-371	-122.39674866	65.4145	1	24	0.2717	23
Weights: 1 and 90. Constraint precision of $1x10^{-9}$									

Table B39. Best solutions for Series 3 (scaled data) fixing the intercept found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 1 and 90.

Initialization: Initial points				Best solution: Final points					
Run	t^L	t^U	β_1	Objective	*SSE*	t^L	t^U	β_1	*WMS size*
1	1	1	26500	-122.39674866	65.41453	1	24	0.2717	23
2	3	5	-5	-122.39674866	65.41453	1	24	0.2717	23
3	-48	81	15	-122.39674867	65.41453	1	24	0.2717	23
4	10	-40	-11	-122.39674866	65.41453	1	24	0.2717	23
5	187	-958	-620	-122.39674866	65.41453	1	24	0.2717	23
6	141	-382	494	-122.39674867	65.41453	1	24	0.2717	23
7	-777	-508	-459	-122.39674868	65.41453	1	24	0.2717	23
8	240	-1356	-1729	-122.39674866	65.41453	1	24	0.2717	23
9	1415	-1505	1167	-122.39674866	65.41453	1	24	0.2717	23
10	-1018	-1770	-371	-122.39674867	65.41453	1	24	0.2717	23
Weights: 1 and 90. Constraint precision of $1x10^{-3}$									

Table B40. Best solutions for Series 3 (scaled data) fixing the intercept found by the MWS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 1 and 90.

Initialization: Initial points				Best solution: Final points					
Run	t^L	t^U	β1	Objective	*SSE*	t^L	t^U	β1	*WMS size*
1	1	1	26500	-1240.36785447	65.4145	1	24	0.2717	23
2	3	5	-5	-1240.36785445	65.4145	1	24	0.2717	23
3	-48	81	15	-1240.36785445	65.4145	1	24	0.2717	23
4	10	-40	-11	-1240.36785445	65.4145	1	24	0.2717	23
5	187	-958	-620	-1240.36785446	65.4145	1	24	0.2717	23
6	141	-382	494	-1240.36785445	65.4145	1	24	0.2717	23
7	-777	-508	-459	-1240.36785446	65.4145	1	24	0.2717	23
8	240	-1356	-1729	-1240.36785445	65.4145	1	24	0.2717	23
9	1415	-1505	1167	-1240.36785445	65.4145	1	24	0.2717	23
10	-1018	-1770	-371	-1240.36785447	65.4145	1	24	0.2717	23
Weights: 1 and 900. Constraint precision of $1x10^{-9}$									

Table B41. Best solutions for Series 3 (scaled data) fixing the intercept found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 1 and 900.

Initialization: Initial points				Best solution: Final points					
Run	t^L	t^U	β1	Objective	*SSE*	t^L	t^U	β1	*WMS size*
1	1	1	26500	-1240.37035410	65.03337	1	24	0.2598	23
2	3	5	-5	-1240.37056880	65.00073	1	24	0.2618	23
3	-48	81	15	-1240.37057028	65.00050	1	24	0.2618	23
4	10	-40	-11	-1240.36990170	65.10219	1	24	0.2670	23
5	187	-958	-620	-1240.36394110	66.01568	1	24	0.2768	23
6	141	-382	494	-1240.36988963	65.10403	1	24	0.2578	23
7	-777	-508	-459	-1240.36366188	66.05878	1	24	0.2772	23
8	240	-1356	-1729	-1240.37044677	65.01928	1	24	0.2645	23
9	1415	-1505	1167	-1240.37056763	65.00091	1	24	0.2618	23
10	-1018	-1770	-371	-1240.36785445	65.41453	1	24	0.2717	23
Weights: 1 and 900. Constraint precision of $1x10^{-3}$									

Table B42. Best solutions for Series 3 (scaled data) fixing the intercept found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 1 and 900.

Initialization: Initial points				Best solution: Final points					
Run	t^L	t^U	β1	Objective	*SSE*	t^L	t^U	β1	*WMS size*
1	1	1	26500	-12420.07891231	65.4145	1	24	0.2717	23
2	3	5	-5	-12420.07891230	65.4145	1	24	0.2717	23
3	-48	81	15	-12420.07891238	65.4145	1	24	0.2717	23
4	10	-40	-11	-12420.07891231	65.4145	1	24	0.2717	23
5	187	-958	-620	-12420.07891235	65.4145	1	24	0.2717	23
6	141	-382	494	-12420.07891235	65.4145	1	24	0.2717	23
7	-777	-508	-459	-12420.07891235	65.4145	1	24	0.2717	23
8	240	-1356	-1729	-12420.07891233	65.4145	1	24	0.2717	23
9	1415	-1505	1167	-12420.07891233	65.4145	1	24	0.2717	23
10	-1018	-1770	-371	-12420.07891231	65.4145	1	24	0.2717	23
Weights: 1 and 9000. Constraint precision of $1x10^{-9}$									

Table B43. Best solutions for Series 3 (scaled data) fixing the intercept found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 1 and 9000.

Initialization: Initial points				Best solution: Final points					
Run	t^L	t^U	β_1	Objective	*SSE*	t^L	t^U	β_1	*WMS size*
1	1	1	26500	-12420.08051	65.17020	1	24	0.2684	23
2	3	5	-5	-12420.08046	65.17867	1	24	0.2685	23
3	-48	81	15	-12420.08159	65.00586	1	24	0.2637	23
4	10	-40	-11	-12420.08082	65.12412	1	24	0.2675	23
5	187	-958	-620	-12420.08046	65.17849	1	24	0.2685	23
6	141	-382	494	-12420.07446	66.09818	1	24	0.2774	23
7	-777	-508	-459	-12420.08053	65.16831	1	24	0.2683	23
8	240	-1356	-1729	-12420.08046	65.17852	1	24	0.2685	23
9	1415	-1505	1167	-12420.08087	65.11628	1	24	0.2673	23
10	-1018	-1770	-371	-12420.08157	65.00922	1	24	0.2639	23
Weights: 1 and 9000. Constraint precision of $1x10^{-3}$									

Table B44. Best solutions for Series 3 (scaled data) fixing the intercept found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 1 and 9000.

Tables B45 to B78 show the results obtained for the time series presented in section 5.1.5 of Chapter 5.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-0.9680	0.078	1	12	-1.3167	0.1989	11
2	3	5	0	-5	-0.8563	0.061	4	12	-1.2137	0.1893	8
3	-48	81	6	15	-0.9677	0.086	1	12	-1.3399	0.2036	11
4	10	-40	-1	-11	-0.9681	0.076	1	12	-1.2905	0.1974	11
5	187	-958	-535	-620	-0.9677	0.086	1	12	-1.3401	0.2036	11
6	141	-382	631	494	5.6779	3.07E+59	11	12	-3.92E+29	0.4579	1
7	-777	-508	-437	-459	-0.9681	0.075	1	12	-1.2891	0.1970	11
8	240	-1356	-964	-1729	-0.9680	0.078	1	12	-1.3167	0.1989	11
9	1415	-1505	1164	1167	-0.9682	0.074	1	12	-1.2681	0.1933	11
10	-1018	-1770	-1382	-371	-0.9681	0.076	1	12	-1.2911	0.1974	11
Constraint precision of $1x10^{-9}$											

Table B45. Best solutions for Series 1 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	5.6692	2.52E+59	9	10	-3.55E+29	0.2929	1
2	3	5	0	-5	-0.9682	0.074	1	12	-1.2774	0.1944	11
3	-48	81	6	15	-0.9682	0.074	1	12	-1.2773	0.1944	11
4	10	-40	-1	-11	-0.9681	0.076	1	12	-1.2906	0.1974	11
5	187	-958	-535	-620	5.6415	1.33E+59	6	7	2.58E+29	0.1166	1
6	141	-382	631	494	5.6055	5.81E+58	3	4	-1.70E+29	0.6886	1
7	-777	-508	-437	-459	5.6545	1.80E+59	6	7	3.00E+29	0.5516	1
8	240	-1356	-964	-1729	5.6390	1.26E+59	2	3	2.51E+29	0.2148	1
9	1415	-1505	1164	1167	5.7421	1.35E+60	3	4	8.21E+29	0.7925	1
10	-1018	-1770	-1382	-371	5.6201	8.14E+58	1	2	2.02E+29	0.3302	1
Constraint precision of $1x10^{-3}$											

Table B46. Best solutions for Series 1 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-0.8961	1.582	1	11	0.3044	0.0024	10
2	3	5	0	-5	-0.9073	3.359	1	12	0.3044	0.0024	11
3	-48	81	6	15	-0.9082	3.269	1	12	0.2177	0.0024	11
4	10	-40	-1	-11	-0.9073	3.359	1	12	0.3044	0.0024	11
5	187	-958	-535	-620	-0.9073	3.359	1	12	0.3044	0.0024	11
6	141	-382	631	494	-0.8651	1.232	1	10	0.3868	0.0024	9
7	-777	-508	-437	-459	-0.9082	3.269	1	12	0.2177	0.0024	11
8	240	-1356	-964	-1729	-0.9073	3.359	1	12	0.3044	0.0024	11
9	1415	-1505	1164	1167	-0.9082	3.269	1	12	0.2177	0.0024	11
10	-1018	-1770	-1382	-371	-0.9082	3.269	1	12	0.2177	0.0024	11

Table B47. Best solutions for Series 2 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	5.6048	5.72E+58	2	3	1.69E+29	0.8842	1
2	3	5	0	-5	5.6805	3.27E+59	6	7	-4.04E+29	0.7521	1
3	-48	81	6	15	-0.9082	3.269	1	12	0.2178	0.0024	11
4	10	-40	-1	-11	-0.8943	1.687	1	11	0.2687	0.0074	10
5	187	-958	-535	-620	5.7564	1.88E+60	3	4	-9.69E+29	0.2168	1
6	141	-382	631	494	5.6945	4.51E+59	4	5	4.75E+29	0.7135	1
7	-777	-508	-437	-459	5.6884	3.92E+59	9	10	-4.43E+29	0.7516	1
8	240	-1356	-964	-1729	5.6733	2.77E+59	7	8	3.72E+29	0.8648	1
9	1415	-1505	1164	1167	5.6768	3.00E+59	3	4	3.87E+29	0.8281	1
10	-1018	-1770	-1382	-371	5.7435	1.39E+60	4	5	8.35E+29	0.3489	1
Constraint precision of $1x10^{-3}$											

Table B48. Best solutions for Series 2 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-0.900738	4.073	1	12	-0.5761	0.0920	11
2	3	5	0	-5	-0.901415	3.994	1	12	-0.7481	0.1119	11
3	-48	81	6	15	-0.893975	4.928	1	12	-0.9965	0.1761	11
4	10	-40	-1	-11	-0.901415	3.994	1	12	-0.7481	0.1119	11
5	187	-958	-535	-620	-0.901414	3.994	1	12	-0.7469	0.1114	11
6	141	-382	631	494	-0.894019	4.922	1	12	-0.9929	0.1756	11
7	-777	-508	-437	-459	-0.901414	3.994	1	12	-0.7438	0.1111	11
8	240	-1356	-964	-1729	-0.900738	4.073	1	12	-0.5761	0.0920	11
9	1415	-1505	1164	1167	-0.901413	3.995	1	12	-0.7400	0.1109	11
10	-1018	-1770	-1382	-371	-0.901415	3.994	1	12	-0.7481	0.1119	11
Constraint precision of $1x10^{-9}$											

Table B49. Best solutions for Series 4 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	5.6320	1.07E+59	2	3	-2.31E+29	0.6124	1
2	3	5	0	-5	-0.8940	4.931	1	12	-0.9985	0.1764	11
3	-48	81	6	15	-0.9014	3.994	1	12	-0.7481	0.1119	11
4	10	-40	-1	-11	-0.9014	3.994	1	12	-0.7481	0.1119	11
5	187	-958	-535	-620	5.6185	7.83E+58	4	5	-1.98E+29	0.2687	1
6	141	-382	631	494	5.5063	5.92E+57	4	5	5.44E+28	0.4897	1
7	-777	-508	-437	-459	5.6876	3.85E+59	2	3	-4.39E+29	0.4669	1
8	240	-1356	-964	-1729	5.6544	1.79E+59	7	8	2.99E+29	0.8405	1
9	1415	-1505	1164	1167	5.6636	2.21E+59	6	7	-3.33E+29	0.7200	1
10	-1018	-1770	-1382	-371	5.6934	4.39E+59	2	3	-4.69E+29	0.8884	1
Constraint precision of $1x10^{-3}$											

Table B50. Best solutions for Series 4 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-0.9200	2.258	4	15	-1.3236	0.1235	11
2	3	5	0	-5	-0.9200	2.258	4	15	-1.3236	0.1235	11
3	-48	81	6	15	-1.0064	2.315	1	15	-1.2296	0.1148	14
4	10	-40	-1	-11	-1.0064	2.315	1	15	-1.2296	0.1148	14
5	187	-958	-535	-620	-1.0064	2.315	1	15	-1.2296	0.1148	14
6	141	-382	631	494	-0.6403	2.986	10	15	-2.6125	0.2566	5
7	-777	-508	-437	-459	-0.9995	2.889	1	15	-1.5193	0.1581	14
8	240	-1356	-964	-1729	-1.0064	2.315	1	15	-1.2296	0.1148	14
9	1415	-1505	1164	1167	-1.0064	2.315	1	15	-1.2296	0.1148	14
10	-1018	-1770	-1382	-371	-0.9995	2.892	1	15	-1.5226	0.1583	14
Constraint precision of $1x10^{-9}$											

Table B51. Best solutions for Series 5 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	5.4503	1.63E+57	12	13	2.85E+28	0.0678	1
2	3	5	0	-5	-1.0059	2.355	1	15	-1.3380	0.1249	14
3	-48	81	6	15	-1.0064	2.315	1	15	-1.2295	0.1148	14
4	10	-40	-1	-11	-1.0064	2.315	1	15	-1.2302	0.1148	14
5	187	-958	-535	-620	5.6688	2.50E+59	10	11	-3.53E+29	0.1673	1
6	141	-382	631	494	5.6779	3.08E+59	5	6	3.92E+29	0.7490	1
7	-777	-508	-437	-459	5.5620	2.13E+58	5	6	1.03E+29	0.1471	1
8	240	-1356	-964	-1729	5.7535	1.76E+60	1	2	9.37E+29	0.8808	1
9	1415	-1505	1164	1167	5.6639	2.23E+59	6	7	-3.34E+29	0.0073	1
10	-1018	-1770	-1382	-371	5.6771	3.02E+59	6	7	-3.89E+29	0.5380	1
Constraint precision of $1x10^{-3}$											

Table B52. Best solutions for Series 5 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-0.9879	4.085	1	15	-0.3941	0.0569	14
2	3	5	0	-5	-0.9206	2.209	4	15	0.2760	0.0024	11
3	-48	81	6	15	-0.9878	4.086	1	15	-0.4246	0.0601	14
4	10	-40	-1	-11	-0.9879	4.082	1	15	-0.4076	0.0569	14
5	187	-958	-535	-620	-0.9876	4.114	1	15	-0.3109	0.0478	14
6	141	-382	631	494	-0.9175	2.445	1	12	-0.7066	0.1206	11
7	-777	-508	-437	-459	-0.9877	4.107	1	15	-0.3244	0.0498	14
8	240	-1356	-964	-1729	-0.9879	4.082	1	15	-0.4078	0.0569	14
9	1415	-1505	1164	1167	-0.9879	4.082	1	15	-0.3964	0.0558	14
10	-1018	-1770	-1382	-371	-0.9876	4.110	1	15	-0.3196	0.0495	14
Constraint precision of $1x10^{-9}$											

Table B53. Best solutions for Series 6 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	5.7311	1.05E+60	5	6	7.2E+29	0.0597	1
2	3	5	0	-5	-0.9369	3.537	3	15	-0.0346	0.0252	12
3	-48	81	6	15	-0.9879	4.082	1	15	-0.4075	0.0569	14
4	10	-40	-1	-11	-0.9879	4.082	1	15	-0.4076	0.0569	14
5	187	-958	-535	-620	5.7521	1.70E+60	2	3	-9.2E+29	0.0720	1
6	141	-382	631	494	5.4029	5.47E+56	12	13	1.7E+28	0.2108	1
7	-777	-508	-437	-459	5.7515	1.68E+60	6	7	9.2E+29	0.7159	1
8	240	-1356	-964	-1729	5.4010	5.24E+56	4	5	1.6E+28	0.2865	1
9	1415	-1505	1164	1167	5.5303	1.03E+58	1	2	-7.2E+28	0.2729	1
10	-1018	-1770	-1382	-371	5.6866	3.76E+59	7	8	-4.3E+29	0.4509	1
Constraint precision of $1x10^{-3}$											

Table B54. Best solutions for Series 6 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-0.5078	1.191	1	4	-0.9720	0.0024	3
2	3	5	0	-5	-1.0003	2.822	1	15	-0.5426	0.0858	14
3	-48	81	6	15	-1.0004	2.811	1	15	-0.4869	0.0806	14
4	10	-40	-1	-11	-1.0004	2.811	1	15	-0.4830	0.0799	14
5	187	-958	-535	-620	-1.0003	2.822	1	15	-0.5428	0.0858	14
6	141	-382	631	494	-1.0003	2.822	1	15	-0.5428	0.0858	14
7	-777	-508	-437	-459	-1.0003	2.821	1	15	-0.5415	0.0857	14
8	240	-1356	-964	-1729	-1.0003	2.822	1	15	-0.5428	0.0858	14
9	1415	-1505	1164	1167	-1.0004	2.811	1	15	-0.4852	0.0804	14
10	-1018	-1770	-1382	-371	-1.0003	2.821	1	15	-0.5404	0.0856	14
Constraint precision of $1x10^{-9}$											

Table B55. Best solutions for Series 7 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	5.7521	1.70E+60	3	4	-9.22E+29	0.8561	1
2	3	5	0	-5	-0.9477	2.539	3	15	0.0972	0.0308	12
3	-48	81	6	15	-1.0004	2.811	1	15	-0.4869	0.0806	14
4	10	-40	-1	-11	-0.9730	6.151	1	15	0.4632	0.0037	14
5	187	-958	-535	-620	5.6755	2.91E+59	9	10	3.81E+29	0.3008	1
6	141	-382	631	494	5.7587	1.98E+60	6	7	-9.95E+29	0.2289	1
7	-777	-508	-437	-459	5.5945	4.51E+58	6	7	1.50E+29	0.5415	1
8	240	-1356	-964	-1729	-0.9830	4.686	1	15	0	0.0629	14
9	1415	-1505	1164	1167	5.7449	1.44E+60	7	8	-8.48E+29	0.0564	1
10	-1018	-1770	-1382	-371	5.6904	4.10E+59	2	3	-4.53E+29	0.0759	1
Constraint precision of $1x10^{-3}$											

Table B56. Best solutions for Series 7 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-0.97966	5.141	1	15	-0.6065	0.0587	14
2	3	5	0	-5	-0.97830	5.337	1	15	-0.7910	0.0818	14
3	-48	81	6	15	-0.98202	4.816	1	15	-0.4308	0.0551	14
4	10	-40	-1	-11	-0.97512	2.664	2	15	-0.9672	0.0989	13
5	187	-958	-535	-620	-0.97512	2.664	2	15	-0.9675	0.0989	13
6	141	-382	631	494	5.72029	8.17E+59	5	6	6.39E+29	0.8117	1
7	-777	-508	-437	-459	-0.98202	4.816	1	15	-0.4290	0.0548	14
8	240	-1356	-964	-1729	-0.98202	4.816	1	15	-0.4325	0.0551	14
9	1415	-1505	1164	1167	-0.98201	4.817	1	15	-0.4141	0.0529	14
10	-1018	-1770	-1382	-371	-0.97834	5.330	1	15	-0.7879	0.0815	14
Constraint precision of $1x10^{-9}$											

Table B57. Best solutions for Series 8 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	5.5880	3.88E+58	8	9	1.39E+29	0.7645	1
2	3	5	0	-5	-0.9489	2.443	3	15	-1.3130	0.1362	12
3	-48	81	6	15	-0.9820	4.816	1	15	-0.4310	0.0551	14
4	10	-40	-1	-11	-0.9783	5.336	1	15	-0.7909	0.0818	14
5	187	-958	-535	-620	5.6901	4.07E+59	2	3	4.51E+29	0.6453	1
6	141	-382	631	494	5.7498	1.61E+60	4	5	-8.97E+29	0.3274	1
7	-777	-508	-437	-459	5.6334	1.10E+59	3	4	2.35E+29	0.5726	1
8	240	-1356	-964	-1729	-0.9776	5.445	1	15	0	0.0133	14
9	1415	-1505	1164	1167	5.7233	8.76E+59	11	12	6.62E+29	0.8403	1
10	-1018	-1770	-1382	-371	5.6841	3.55E+59	4	5	-4.21E+29	0.1116	1
Constraint precision of $1x10^{-3}$											

Table B58. Best solutions for Series 8 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-0.9779	5.395	1	15	-0.0316	0.0024	14
2	3	5	0	-5	-0.9779	5.395	1	15	-0.0316	0.0024	14
3	-48	81	6	15	-0.9779	5.395	1	15	-0.0316	0.0024	14
4	10	-40	-1	-11	-0.9779	5.394	1	15	-0.0376	0.0024	14
5	187	-958	-535	-620	-0.9779	5.394	1	15	-0.0376	0.0024	14
6	141	-382	631	494	-0.9778	5.416	1	15	0.0002	0.0024	14
7	-777	-508	-437	-459	-0.9778	5.416	1	15	0.0002	0.0024	14
8	240	-1356	-964	-1729	-0.9225	5.316	2	14	-0.0931	0.0024	12
9	1415	-1505	1164	1167	-0.9779	5.394	1	15	-0.0376	0.0024	14
10	-1018	-1770	-1382	-371	-0.9778	5.416	1	15	0.0002	0.0024	14
Constraint precision of $1x10^{-9}$											

Table B59. Best solutions for Series 9 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	5.744	1.41E+60	6	7	-8.38E+29	0.3445	1
2	3	5	0	-5	-0.928	4.569	3	15	-0.0101	0.0024	12
3	-48	81	6	15	-0.978	5.394	1	15	-0.0376	0.0024	14
4	10	-40	-1	-11	-0.955	9.886	1	15	-0.9994	0.0960	14
5	187	-958	-535	-620	5.717	7.51E+59	6	7	-6.13E+29	0.7956	1
6	141	-382	631	494	5.752	1.69E+60	7	8	9.19E+29	0.4740	1
7	-777	-508	-437	-459	5.440	1.27E+57	2	3	-2.52E+28	0.3196	1
8	240	-1356	-964	-1729	5.622	8.52E+58	7	8	-2.06E+29	0.6197	1
9	1415	-1505	1164	1167	5.655	1.83E+59	1	2	-3.03E+29	0.6828	1
10	-1018	-1770	-1382	-371	5.677	2.99E+59	1	2	-3.86E+29	0.8816	1
Constraint precision of $1x10^{-3}$											

Table B60. Best solutions for Series 9 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-1.1582	2415.1496	1	46	0	0.2603	45
2	3	5	0	-5	-1.4085	3.1353	4	46	-0.6792	0.0129	42
3	-48	81	6	15	-1.4389	2.0900	1	45	-0.7696	0.0158	44
4	10	-40	-1	-11	-1.4341	3.2080	1	46	-0.7451	0.0150	45
5	187	-958	-535	-620	-1.4344	3.1752	1	46	-0.7696	0.0158	45
6	141	-382	631	494	-1.4389	2.0900	1	45	-0.7696	0.0158	44
7	-777	-508	-437	-459	-1.4389	2.0900	1	45	-0.7696	0.0158	44
8	240	-1356	-964	-1729	-1.4341	3.2066	1	46	-0.7456	0.0150	45
9	1415	-1505	1164	1167	-1.4350	3.1194	1	46	-0.7641	0.0166	45
10	-1018	-1770	-1382	-371	-1.4343	3.1882	1	46	-0.7588	0.0155	45
Constraint precision of $1x10^{-9}$											

Table B61. Best solutions for Series 10 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	5.6981	4.90E+59	12	13	4.95E+29	0.7945	1
2	3	5	0	-5	-1.4177	3.1096	3	46	-0.7315	0.0146	43
3	-48	81	6	15	-1.3746	15.5341	1	46	0	0.0047	45
4	10	-40	-1	-11	-1.3639	2.7894	9	46	-0.6284	0.0130	37
5	187	-958	-535	-620	5.7433	1.39E+60	31	32	8.33E+29	0.8767	1
6	141	-382	631	494	-1.4353	3.0909	1	46	-0.7931	0.0178	45
7	-777	-508	-437	-459	5.7036	5.56E+59	2	3	5.27E+29	0.2775	1
8	240	-1356	-964	-1729	5.7209	8.29E+59	32	33	-6.44E+29	0.4376	1
9	1415	-1505	1164	1167	-1.4354	3.0840	1	46	-0.8187	0.0189	45
10	-1018	-1770	-1382	-371	5.7320	1.07E+60	29	30	-7.31E+29	0.4765	1
Constraint precision of $1x10^{-3}$											

Table B62. Best solutions for Series 10 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	5.69807	4.90E+59	12	13	4.95E+29	0.7157	1
2	3	5	0	-5	-1.13670	4.186	2	23	-0.3858	0.0462	21
3	-48	81	6	15	-1.15199	4.440	1	23	-0.4970	0.0482	22
4	10	-40	-1	-11	-1.15107	4.557	1	23	-0.4794	0.0522	22
5	187	-958	-535	-620	-1.15199	4.440	1	23	-0.4931	0.0479	22
6	141	-382	631	494	-1.15144	4.510	1	23	-0.3830	0.0409	22
7	-777	-508	-437	-459	-1.15076	4.596	1	23	-0.5780	0.0586	22
8	240	-1356	-964	-1729	-1.15197	4.443	1	23	-0.4735	0.0467	22
9	1415	-1505	1164	1167	-1.15198	4.441	1	23	-0.4820	0.0472	22
10	-1018	-1770	-1382	-371	-1.15199	4.440	1	23	-0.4904	0.0477	22
Constraint precision of $1x10^{-9}$											

Table B63. Best solutions for Series 11 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	5.7118	6.72E+59	11	12	-5.80E+29	0.4497	1
2	3	5	0	-5	-1.1514	4.5108	1	23	-0.3825	0.0410	22
3	-48	81	6	15	-1.1520	4.4401	1	23	-0.4969	0.0482	22
4	10	-40	-1	-11	-1.1520	4.4401	1	23	-0.4970	0.0482	22
5	187	-958	-535	-620	5.6963	4.70E+59	5	6	4.85E+29	0.7124	1
6	141	-382	631	494	5.7129	6.90E+59	1	2	-5.87E+29	0.1127	1
7	-777	-508	-437	-459	5.3534	1.75E+56	4	5	9.36E+27	0.6225	1
8	240	-1356	-964	-1729	5.7396	1.28E+60	2	3	7.99E+29	0.8422	1
9	1415	-1505	1164	1167	5.7471	1.51E+60	5	6	-8.70E+29	0.5263	1
10	-1018	-1770	-1382	-371	5.7550	1.82E+60	5	6	-9.54E+29	0.8345	1
Constraint precision of $1x10^{-3}$											

Table B64. Best solutions for Series 11 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-1.03796	0.6042	2	16	-0.6664	0.0024	14
2	3	5	0	-5	-1.40067	1.0555	3	41	-1.1150	0.0511	38
3	-48	81	6	15	-1.42060	1.0371	1	41	-1.0754	0.0489	40
4	10	-40	-1	-11	-1.42023	1.0549	1	41	-1.0971	0.0505	40
5	187	-958	-535	-620	-1.42057	1.0388	1	41	-1.0628	0.0485	40
6	141	-382	631	494	-1.42055	1.0397	1	41	-1.0598	0.0484	40
7	-777	-508	-437	-459	-1.42034	1.0496	1	41	-1.0870	0.0500	40
8	240	-1356	-964	-1729	-1.42017	1.0574	1	41	-1.1023	0.0507	40
9	1415	-1505	1164	1167	-1.42060	1.0371	1	41	-1.0759	0.0489	40
10	-1018	-1770	-1382	-371	-1.42032	1.0505	1	41	-1.0970	0.0503	40
Constraint precision of $1x10^{-9}$											

Table B65. Best solutions for Series 12 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	5.6757	2.92E+59	1	2	3.82E+29	0.4601	1
2	3	5	0	-5	-1.4206	1.0372	1	41	-1.0788	0.0491	40
3	-48	81	6	15	-1.3386	12.4735	1	41	0	0.0103	40
4	10	-40	-1	-11	-1.4206	1.0371	1	41	-1.0758	0.0489	40
5	187	-958	-535	-620	5.7410	1.32E+60	1	2	-8.11E+29	0.3284	1
6	141	-382	631	494	-1.4206	1.0371	1	41	-1.0758	0.0489	40
7	-777	-508	-437	-459	5.7340	1.12E+60	12	13	7.48E+29	0.5617	1
8	240	-1356	-964	-1729	5.7465	1.49E+60	23	24	8.64E+29	0.8152	1
9	1415	-1505	1164	1167	-1.4206	1.0371	1	41	-1.0758	0.0489	40
10	-1018	-1770	-1382	-371	5.5647	2.27E+58	1	2	-1.07E+29	0.7492	1
Constraint precision of $1x10^{-3}$											

Table B66. Best solutions for Series 12 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-1.3469	10.125	1	41	0.0684	0.0164	40
2	3	5	0	-5	-1.3812	4.044	1	41	-0.7518	0.0479	40
3	-48	81	6	15	-1.3816	4.000	1	41	-0.7180	0.0455	40
4	10	-40	-1	-11	-1.3812	4.043	1	41	-0.7514	0.0479	40
5	187	-958	-535	-620	-1.3490	9.583	1	41	0	0.0241	40
6	141	-382	631	494	-1.3816	4.000	1	41	-0.7196	0.0454	40
7	-777	-508	-437	-459	-1.3813	4.033	1	41	-0.7512	0.0477	40
8	240	-1356	-964	-1729	-1.3816	4.000	1	41	-0.7186	0.0455	40
9	1415	-1505	1164	1167	-1.3816	4.000	1	41	-0.7184	0.0455	40
10	-1018	-1770	-1382	-371	-1.3814	4.030	1	41	-0.7415	0.0474	40
Constraint precision of $1x10^{-9}$											

Table B67. Best solutions for Series 13 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	5.7444	1.42E+60	5	6	-8.44E+29	0.6035	1
2	3	5	0	-5	-1.3645	3.7289	3	41	-0.5795	0.0407	38
3	-48	81	6	15	-1.3511	9.1020	1	41	0	0.0200	40
4	10	-40	-1	-11	-1.3812	4.0431	1	41	-0.7512	0.0479	40
5	187	-958	-535	-620	5.7481	1.55E+60	1	2	8.80E+29	0.5375	1
6	141	-382	631	494	-1.1671	697.7573	1	41	0	0.1896	40
7	-777	-508	-437	-459	5.6932	4.38E+59	1	2	4.68E+29	0.4730	1
8	240	-1356	-964	-1729	5.7299	1.02E+60	33	34	7.14E+29	0.6908	1
9	1415	-1505	1164	1167	-1.0318	15750.3753	1	41	0	0.8325	40
10	-1018	-1770	-1382	-371	5.7272	9.57E+59	7	8	-6.92E+29	0.5507	1
Constraint precision of $1x10^{-3}$											

Table B68. Best solutions for Series 13 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-1.8471	2.582	21	150	-1.3589	0.0162	129
2	3	5	0	-5	-1.9404	0.263	4	150	-1.0373	0.0131	146
3	-48	81	6	15	-1.9483	0.264	1	150	-1.0358	0.0130	149
4	10	-40	-1	-11	-1.9483	0.264	1	150	-1.0358	0.0130	149
5	187	-958	-535	-620	-1.9433	0.336	1	149	-1.0399	0.0128	148
6	141	-382	631	494	-1.9481	0.271	1	150	-1.0416	0.0130	149
7	-777	-508	-437	-459	-1.9482	0.268	1	150	-1.0264	0.0129	149
8	240	-1356	-964	-1729	-1.9483	0.264	1	150	-1.0364	0.0130	149
9	1415	-1505	1164	1167	-1.9459	0.259	1	149	-1.0355	0.0130	148
10	-1018	-1770	-1382	-371	-1.9478	0.279	1	150	-1.0180	0.0128	149
Constraint precision of $1x10^{-9}$											

Table B69. Best solutions for Series 14 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	5.6462	1.48E+59	16	17	-2.72E+29	0.2488	1
2	3	5	0	-5	-1.9483	0.264	1	150	-1.0362	0.0130	149
3	-48	81	6	15	-1.7966	40.553	1	150	0	0.0033	149
4	10	-40	-1	-11	-1.9361	0.234	1	145	-1.0313	0.0129	144
5	187	-958	-535	-620	-1.7965	4.06E+01	1	150	0	0.0034	149
6	141	-382	631	494	-1.9483	0.264	1	150	-1.0364	0.0130	149
7	-777	-508	-437	-459	5.7272	9.57E+59	21	22	6.92E+29	0.7435	1
8	240	-1356	-964	-1729	5.7486	1.57E+60	66	67	-8.86E+29	0.5814	1
9	1415	-1505	1164	1167	-1.7957	41.457	1	150	0	0.0038	149
10	-1018	-1770	-1382	-371	5.7316	1.06E+60	98	99	7.28E+29	0.3479	1
Constraint precision of $1x10^{-3}$											

Table B70. Best solutions for Series 14 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-1.7426	7,879.678	1	234	0	0.0406	233
2	3	5	0	-5	-1.9922	24.177	1	234	-0.9739	0.0042	233
3	-48	81	6	15	-1.9922	24.177	1	234	-0.9743	0.0042	233
4	10	-40	-1	-11	-1.9922	24.182	1	234	-0.9783	0.0042	233
5	187	-958	-535	-620	-1.9884	24.408	1	232	-0.9116	0.0038	231
6	141	-382	631	494	-1.9888	25.194	2	234	-0.8475	0.0033	232
7	-777	-508	-437	-459	-1.3909	25,909,520.521	1	234	-433.6836	0.9024	233
8	240	-1356	-964	-1729	-1.9887	26.310	1	234	-0.8859	0.0030	233
9	1415	-1505	1164	1167	-1.9864	27.771	1	234	-0.7596	0.0024	233
10	-1018	-1770	-1382	-371	-1.9921	24.215	1	234	-0.9789	0.0042	233
Constraint precision of $1x10^{-9}$											

Table B71. Best solutions for Series 15 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	5.753	1.72E+60	130	131	-9.28E+29	0.7962	1
2	3	5	0	-5	-1.988	26.672	1	234	-0.7722	0.0031	233
3	-48	81	6	15	-1.894	239.527	1	234	0	0.0041	233
4	10	-40	-1	-11	-1.992	24.177	1	234	-0.9741	0.0042	233
5	187	-958	-535	-620	-1.906	182.841	1	234	0	0.0029	233
6	141	-382	631	494	-1.587	281,040.860	1	234	0	0.2537	233
7	-777	-508	-437	-459	-1.899	211.962	1	234	0	0.0036	233
8	240	-1356	-964	-1729	5.676	2.96E+59	73	74	3.85E+29	0.5873	1
9	1415	-1505	1164	1167	-1.899	215.041	1	234	0	0.0036	233
10	-1018	-1770	-1382	-371	5.694	4.48E+59	215	216	-4.73E+29	0.1020	1
Constraint precision of $1x10^{-3}$											

Table B72. Best solutions for Series 15 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-0.9792	2.57E+12	1	293	-93,804.1209	0.9024	292
2	3	5	0	-5	-2.1033	13.311	2	293	-0.8595	0.0039	291
3	-48	81	6	15	-2.1046	13.312	1	293	-0.8601	0.0039	292
4	10	-40	-1	-11	-2.1046	13.313	1	293	-0.8567	0.0039	292
5	187	-958	-535	-620	-2.0967	16.167	1	293	-0.6623	0.0028	292
6	141	-382	631	494	-2.1022	14.124	1	293	-0.7591	0.0033	292
7	-777	-508	-437	-459	-2.1045	13.33	1	293	-0.8423	0.0038	292
8	240	-1356	-964	-1729	-2.1013	14.435	1	293	-0.7578	0.0032	292
9	1415	-1505	1164	1167	-2.0924	17.942	1	293	-0.6414	0.0024	292
10	-1018	-1770	-1382	-371	-2.1039	13.543	1	293	-0.8035	0.0036	292
Constraint precision of $1x10^{-9}$											

Table B73. Best solutions for Series 16 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	5.6582	1.96E+59	2	3	-3.13E+29	0.4576	1
2	3	5	0	-5	-2.1046	13.312	1	293	-0.8611	0.0039	292
3	-48	81	6	15	-1.9720	302.519	1	293	0	0.0048	292
4	10	-40	-1	-11	-2.0150	16.344	1	238	-1.0000	0.0057	237
5	187	-958	-535	-620	-1.9712	307.824	1	293	0	0.0048	292
6	141	-382	631	494	-1.8488	114.201	102	293	0	0.0031	191
7	-777	-508	-437	-459	0.2871	380,316.03	156	157	-437.0000	0.0024	1
8	240	-1356	-964	-1729	-1.9058	1,391.51	1	293	0	0.0120	292
9	1415	-1505	1164	1167	-1.5973	1,692,333.50	1	293	-132.4739	0.9024	292
10	-1018	-1770	-1382	-371	5.6308	1.04E+59	1	2	-2.28E+29	0.6951	1
Constraint precision of $1x10^{-3}$											

Table B74. Best solutions for Series 16 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-2.0456	56.458	1	294	-0.1035	0.0024	293
2	3	5	0	-5	-2.0393	54.141	4	291	-0.0964	0.0024	287
3	-48	81	6	15	-2.0456	56.454	1	294	-0.1070	0.0024	293
4	10	-40	-1	-11	-2.0456	56.457	1	294	-0.1038	0.0024	293
5	187	-958	-535	-620	-2.0443	56.447	2	294	-0.1031	0.0024	292
6	141	-382	631	494	-2.0456	56.454	1	294	-0.1070	0.0024	293
7	-777	-508	-437	-459	-2.0456	56.454	1	294	-0.1070	0.0024	293
8	240	-1356	-964	-1729	-2.0456	56.454	1	294	-0.1071	0.0024	293
9	1415	-1505	1164	1167	-2.0456	56.454	1	294	-0.1070	0.0024	293
10	-1018	-1770	-1382	-371	-2.0456	56.454	1	294	-0.1070	0.0024	293
Constraint precision of $1x10^{-9}$											

Table B75. Best solutions for Series 17 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	5.5679	2.45E+58	79	80	-1.11E+29	0.3824	1
2	3	5	0	-5	-2.0399	58.698	3	293	0	0.0024	290
3	-48	81	6	15	-2.0456	56.454	1	294	-0.1069	0.0024	293
4	10	-40	-1	-11	-1.9904	153.648	10	294	-1.0000	0.0064	284
5	187	-958	-535	-620	-2.0082	134.840	1	294	0	0.0044	293
6	141	-382	631	494	-1.8845	52.058	102	294	-0.1404	0.0024	192
7	-777	-508	-437	-459	5.7485	1.56E+60	88	89	-8.84E+29	0.6341	1
8	240	-1356	-964	-1729	-1.2900	62.321	222	264	0	0.0045	42
9	1415	-1505	1164	1167	-2.0456	56.454	1	294	-0.1061	0.0024	293
10	-1018	-1770	-1382	-371	5.7546	1.80E+60	291	292	9.49E+29	0.1467	1
Constraint precision of $1x10^{-3}$											

Table B76. Best solutions for Series 17 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	-2.0499	50.964	1	294	-0.5371	0.0024	293
2	3	5	0	-5	-2.0953	17.305	1	294	-1.1240	0.0064	293
3	-48	81	6	15	-2.0953	17.305	1	294	-1.1242	0.0064	293
4	10	-40	-1	-11	-2.0856	16.855	9	294	-1.1480	0.0065	285
5	187	-958	-535	-620	-2.0935	18.058	1	294	-1.0234	0.0059	293
6	141	-382	631	494	-2.0941	17.238	2	294	-1.1285	0.0064	292
7	-777	-508	-437	-459	-2.0949	17.47	1	294	-1.1147	0.0062	293
8	240	-1356	-964	-1729	-2.0622	38.207	1	294	-0.6666	0.0032	293
9	1415	-1505	1164	1167	-2.0928	17.208	2	293	-1.1297	0.0064	291
10	-1018	-1770	-1382	-371	-2.0948	17.506	1	294	-1.1071	0.0062	293
Constraint precision of $1x10^{-9}$											

Table B77. Best solutions for Series 18 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-9}$. Weights level: 0.1 and 0.9.

Initialization: Initial points					Best solution: Final points						
Run	t^L	t^U	β_0	β_1	Objective	*SSE*	t^L	t^U	β_0	β_1	*WMS size*
1	1	1	-94000	26500	5.6273	9.59E+58	164	165	2.19E+29	0.1486	1
2	3	5	0	-5	-2.0940	17.279	1	293	-1.1336	0.0064	292
3	-48	81	6	15	-2.0045	147.005	1	294	0	0.0027	293
4	10	-40	-1	-11	-1.8660	2.722	10	146	-1.0000	0.0058	136
5	187	-958	-535	-620	-1.7956	11.572	164	294	-1.6622	0.0088	130
6	141	-382	631	494	-1.9374	14.709	102	294	-1.3979	0.0077	192
7	-777	-508	-437	-459	0.2871	380,468.551	156	157	-437.0000	0.0024	1
8	240	-1356	-964	-1729	-1.9414	231.021	1	263	0	0.0047	262
9	1415	-1505	1164	1167	-2.0953	17.304	1	294	-1.1249	0.0064	293
10	-1018	-1770	-1382	-371	-2.0050	145.291	1	294	0	0.0027	293
Constraint precision of $1x10^{-3}$											

Table B78. Best solutions for Series 18 (scaled data) found by the WMS method in each initialization. Slope restricted to [0.0024, 0.9024] and use of a level of precision of $1x10^{-3}$. Weights level: 0.1 and 0.9.

Tables B79 to B96 correspond to the results obtained for the evaluations presented in section 5.1.6, about the exploration of time series data under one-objective approach. Random numbers were used to generate the initial points. Also, the initial points can arbitrarily vary in order to test other start points. For example, in the initial points of initialization in run 8 of Table B81, the intercept value was intentionally selected according to the function $f(t_e) = 0.4524t - 3.9816$, as presented in Figure 9. The initialization of the run number 1 (up to Table B82) is another selected example where the initial points of the beta values are near to the value of slope and intercept from data presented in Figure 8.

	Initialization: Initial points				Best solution: Final points					
Run	t^L	t^U	β_0	β_1	*SSE*	t^L	t^U	β_0	β_1	WMS size
1	1	1	-94000	26500	0.0120	7	11	-0.7666	0.1425	4
2	3	5	0	-5	0.0008	2	5	-1.3408	0.1951	3
3	-48	81	6	15	0.0120	7	11	-0.7666	0.1425	4
4	10	-40	-1	-11	0.0009	8	11	-0.4329	0.1091	3
5	187	-958	-535	-620	0.0005	3	6	-1.4222	0.2156	3
6	141	-382	631	494	0.0008	5	8	-1.6685	0.2610	3
7	-777	-508	-437	-459	0.0009	8	11	-0.4329	0.1091	3
8	240	-1356	-964	-1729	0.0011	2	5	-1.3128	0.1873	3
9	1415	-1505	1164	1167	0.0570	2	11	-1.3401	0.2036	9
10	-1018	-1770	-1382	-371	0.0009	8	11	-0.4329	0.1091	3

Table B79. Best solutions for Series 1 (scaled data) found by the WMS method in each initialization, using optimization model (23).

	Initialization: Initial points				Best solution: Final points					
Run	t^L	t^U	β_0	β_1	*SSE*	t^L	t^U	β_0	β_1	WMS size
1	1	1	-94000	26500	0.027076	1	3	0.3140	0.2125	2
2	3	5	0	-5	0.006667	7	9	-2.3315	0.2998	2
3	-48	81	6	15	0.026667	1	3	0.3334	0.2000	2
4	10	-40	-1	-11	0.026667	1	3	0.3334	0.2000	2
5	187	-958	-535	-620	0.274096	3	5	0.8916	0.0024	2
6	141	-382	631	494	0.026668	1	3	0.3347	0.1994	2
7	-777	-508	-437	-459	0.026668	1	3	0.3351	0.1992	2
8	240	-1356	-964	-1729	0.152047	1	3	0.5874	0.0076	2
9	1415	-1505	1164	1167	0.026675	1	3	0.3378	0.1980	2
10	-1018	-1770	-1382	-371	0.026669	1	3	0.3310	0.2008	2

Table B80. Best solutions for Series 2 (scaled data) found by the WMS method in each initialization, using optimization model (23).

	Initialization: Initial points				Best solution: Final points					
Run	t^L	t^U	β_0	β_1	*SSE*	t^L	t^U	β_0	β_1	WMS size
1	1	1	-94000	26500	0.0326	17	20	-1.8223	0.1245	3
2	3	5	0	-5	0.1325	19	22	0.7147	0.0024	3
3	-48	81	6	15	0.1022	6	9	-1.8787	0.2156	3
4	10	-40	-1	-11	0.0210	12	15	-0.7535	0.0597	3
5	187	-958	-535	-620	0.0210	12	15	-0.7535	0.0597	3
6	141	-382	631	494	0.0374	12	15	0.0194	0.0024	3
7	-777	-508	-437	-459	0.1374	14	17	-0.4725	0.0366	3
8	8	11	-3.98	0.045	0.0675	13	16	-0.0003	0.0024	3
9	1415	-1505	1164	1167	0.2090	3	6	-0.4886	0.0024	3
10	-1018	-1770	-1382	-371	0.0210	12	15	-0.7535	0.0597	3

Table B81. Best solutions for Series 3 (scaled data) found by the WMS method in each initialization, using optimization model (23).

Initialization: Initial points					Best solution: Final points					
Run	t^L	t^U	β_0	β_1	*SSE*	t^L	t^U	β_0	β_1	WMS size
1	9	6	-94000	26500	2.47E-11	2	3	-2.7456	0.8996	1
2	11	4	0	-5	3.89E-06	2	3	-2.7525	0.9024	1
3	10	8	6	15	3.14E-12	9	10	-1.9369	0.2937	1
4	11	8	-1	-11	0.0009	4	5	-4.1855	0.9024	1
5	8	10	-535	-620	1.8172	7	8	0.8362	0.0024	1
6	1	3	631	494	0.3928	8	9	-3.5919	0.4776	1
7	11	2	-437	-459	2.48E-11	2	3	-2.7456	0.8996	1
8	7	9	-964	-1729	0.0272	9	10	0.2033	0.0723	1
9	5	9	1164	1167	0.0009	4	5	-4.1855	0.9024	1
10	10	12	-1382	-371	7.88E-11	9	10	-1.9368	0.2937	1

Table B82. Best solutions for Series 4 (scaled data) found by the WMS method in each initialization, using optimization model (23).

Initialization: Initial points					Best solution: Final points					
Run	t^L	t^U	β_0	β_1	*SSE*	t^L	t^U	β_0	β_1	WMS size
1	2	14	7.86	1.34	4.20E-19	2	3	-1.0584	0.0297	1
2	7	13	9.23	10.36	4.87E-18	2	3	-1.0584	0.0297	1
3	2	8	1.74	4.73	4.87E-18	2	3	-1.0584	0.0297	1
4	5	10	5.13	2.35	4.46E-11	2	3	-1.0584	0.0297	1
5	13	13	6.99	9.67	4.86E-18	2	3	-1.0584	0.0297	1
6	10	14	4.14	3.7	4.20E-19	2	3	-1.0584	0.0297	1
7	6	15	9.31	2.55	2.74E-19	2	3	-1.0584	0.0297	1
8	1	4	2.75	7.48	4.64E-11	2	3	-1.0584	0.0297	1
9	8	10	5.43	8.02	4.46E-11	2	3	-1.0584	0.0297	1
10	6	14	6.82	7.47	5.56E-18	2	3	-1.0584	0.0297	1

Table B83. Best solutions for Series 5 (scaled data) found by the WMS method in each initialization, using optimization model (23).

Initialization: Initial points					Best solution: Final points					
Run	t^L	t^U	β_0	β_1	*SSE*	t^L	t^U	β_0	β_1	WMS size
1	1	13	3.04	9.6	4.17E-11	3	4	-3.5027	0.8342	1
2	1	3	1.85	1.36	5.49E-11	3	4	-3.5027	0.8342	1
3	12	14	3.72	7.1	4.17E-11	3	4	-3.5027	0.8342	1
4	1	8	8.19	1.99	6.05E-12	4	5	-0.8931	0.1818	1
5	4	11	3.07	3.11	2.01E-11	3	4	-3.5027	0.8342	1
6	2	5	5.62	6.92	1.06E-10	3	4	-3.5027	0.8342	1
7	3	9	0.76	9.13	5.10E-12	4	5	-0.8931	0.1818	1
8	2	12	8.11	3.52	4.17E-11	3	4	-3.5027	0.8342	1
9	1	9	8.59	4.74	2.69E-11	3	4	-3.5027	0.8342	1
10	13	15	6.07	7.4	4.17E-11	3	4	-3.5027	0.8342	1

Table B84. Best solutions for Series 6 (scaled data) found by the WMS method in each initialization, using optimization model (23).

Initialization: Initial points					Best solution: Final points					
Run	t^L	t^U	β_0	β_1	*SSE*	t^L	t^U	β_0	β_1	WMS size
1	10	1	5.17	0.75	0.6267	4	6	0.6022	0.0024	2
2	9	2	9.63	5.28	0.0195	13	15	-3.6722	0.2906	2
3	10	3	6.84	3.86	0.0195	13	15	-3.6724	0.2906	2
4	14	4	3.94	7.01	0.0031	3	5	-3.5185	0.8547	2
5	5	5	1.13	8.11	0.0292	12	14	0.2509	0.0024	2
6	4	6	7.3	6.52	0.0195	13	15	-3.6723	0.2906	2
7	6	7	5.06	2.01	0.7901	8	10	-5.8205	0.6325	2
8	4	8	9.33	8.82	0.0031	9	11	-5.1823	0.5641	2
9	5	9	7.6	9.2	0.0859	2	4	-1.9744	0.4274	2
10	8	10	9.72	4.44	0.0031	9	11	-5.1797	0.5638	2

Table B85. Best solutions for Series 7 (scaled data) found by the WMS method in each initialization, using optimization model (23).

Initialization: Initial points					Best solution: Final points					
Run	t^L	t^U	β_0	β_1	*SSE*	t^L	t^U	β_0	β_1	WMS size
1	12	2	4.35	9.16	3.47E-11	5	6	-3.9032	0.5806	1
2	6	1	5.54	1.32	3.45E-11	5	6	-3.9032	0.5806	1
3	9	8	6.31	8.96	5.13E-10	8	9	-0.1481	0.0258	1
4	9	7	3.95	8.67	4.18E-11	5	6	-3.9032	0.5806	1
5	4	8	5.78	6.66	3.41E-11	5	6	-3.9032	0.5806	1
6	6	8	0.5	9.63	4.08E-11	5	6	-3.9032	0.5806	1
7	14	13	4.71	2.42	4.08E-11	5	6	-3.9032	0.5806	1
8	7	12	4.82	5.56	1.43E-16	14	15	-3.0645	0.2710	1
9	14	7	6.89	3.16	3.41E-07	6	7	-5.8333	0.9024	1
10	15	15	8.49	9.82	5.67E-06	13	14	-7.5352	0.5902	1

Table B86. Best solutions for Series 8 (scaled data) found by the WMS method in each initialization, using optimization model (23).

Initialization: Initial points					Best solution: Final points					
Run	t^L	t^U	β_0	β_1	*SSE*	t^L	t^U	β_0	β_1	WMS size
1	12	9	7.79	0.49	2.89E-03	7	9	-2.7455	0.2988	2
2	6	15	4.36	1.78	2.89E-03	7	9	-2.7455	0.2988	2
3	7	13	3.37	1.36	9.85E-02	11	13	-7.4515	0.6213	2
4	1	6	9.92	4.61	1.96E-01	3	5	0.6475	0.0024	2
5	9	5	7.34	7.1	1.24E-02	6	8	-3.2619	0.3689	2
6	13	10	9.22	2.52	1.96E-01	3	5	0.6475	0.0024	2
7	7	15	9.47	7.13	1.96E-01	3	5	0.6475	0.0024	2
8	7	13	8.69	1.84	9.85E-02	11	13	-7.4514	0.6213	2
9	9	12	7.94	7.31	3.14E-06	10	12	-3.1113	0.2382	2
10	7	8	4.89	4.23	2.89E-03	7	9	-2.7455	0.2988	2

Table B87. Best solutions for Series 9 (scaled data) found by the WMS method in each initialization, using optimization model (23).

Initialization: Initial points					Best solution: Final points					
Run	t^L	t^U	β_0	β_1	SSE	t^L	t^U	β_0	β_1	WMS size
1	4	9	6.97	2.6	2.22E-10	9	10	-3.1805	0.2805	1
2	4	13	2.08	1.01	1.39E-10	13	14	-2.7043	0.1797	1
3	1	46	7.78	5.82	3.78E-11	2	3	-1.0179	0.0555	1
4	6	8	7.49	1.16	8.01E-11	8	9	-2.0022	0.1496	1
5	4	12	8.94	7.26	1.64E-10	12	13	-1.1292	0.0586	1
6	1	16	6.1	6.01	2.47E-04	18	19	-0.1886	0.0024	1
7	2	9	3.48	5.71	2.22E-10	9	10	-3.1805	0.2805	1
8	7	12	5.75	0.27	1.64E-10	12	13	-1.1292	0.0586	1
9	16	15	0.23	1.38	3.02E-03	14	15	-0.2604	0.0024	1
10	4	13	1.21	1.02	1.39E-10	13	14	-2.7043	0.1797	1

Table B88. Best solutions for Series 10 (scaled data) found by the WMS method in each initialization, using optimization model (23).

Initialization: Initial points					Best solution: Final points					
Run	t^L	t^U	β0	β1	SSE	t^L	t^U	β0	β1	WMS size
1	2	3	9.35	9.18	0.0056	8	13	-0.9885	0.1174	5
2	3	23	0.87	3.78	0.0065	5	10	-1.1184	0.1307	5
3	1	20	8.88	0.04	0.2859	12	18	0.1740	0.0369	6
4	6	20	4.87	8.65	0.0210	7	14	-1.1015	0.1303	7
5	15	19	4.55	4.42	0.0075	5	12	-1.1225	0.1315	7
6	10	13	4.76	8.17	0.0312	9	14	-0.9913	0.1177	5
7	21	20	6.47	7.73	0.0190	10	15	-1.5590	0.1679	5
8	3	20	0.53	8.82	0.0059	6	11	-1.0947	0.1291	5
9	3	14	1.18	9.19	0.0176	9	14	-1.2268	0.1405	5
10	3	18	4.03	5.41	0.2260	13	18	0.7384	0.0024	5

Table B89. Best solutions for Series 11 (scaled data) found by the WMS method in each initialization, using optimization model (23).

Initialization: Initial points					Best solution: Final points					
Run	t^L	t^U	β0	β1	SSE	t^L	t^U	β0	β1	WMS size
1	30	40	6.52	2.88	0.1004	36	40	0.6780	0.0024	4
2	17	21	8.67	2.34	0.0146	17	21	-0.3380	0.0136	4
3	2	5	5.79	8.59	0.0028	1	5	-1.0787	0.0655	4
4	17	41	4.75	0.56	0.0234	17	22	-0.1406	0.0024	5
5	39	11	9.68	2.07	0.0044	7	11	-0.9753	0.0271	4
6	31	36	1.87	3.47	0.0531	34	39	-2.8134	0.0963	5
7	15	3	4.73	9.56	0.0028	1	5	-1.0788	0.0655	4
8	27	31	6.08	7.86	0.0681	32	36	0.5280	0.0024	4
9	37	26	4.11	6.35	0.0695	22	26	-3.2665	0.1357	4
10	21	41	4.61	0.06	0.0203	25	32	-1.6259	0.0743	7

Table B90. Best solutions for Series 12 (scaled data) found by the WMS method in each initialization, using optimization model (23).

Initialization: Initial points					Best solution: Final points					
Run	t^L	t^U	β0	β1	SSE	t^L	t^U	β0	β1	WMS size
1	20	40	2.34	6.57	0.0089	16	21	0.3595	0.0024	5
2	15	22	5.85	1.29	0.0021	20	24	-0.0841	0.0242	4
3	37	36	0.43	8.45	0.0393	33	38	0.4299	0.0130	5
4	4	10	8.73	0.22	0.0043	6	10	-1.5983	0.1240	4
5	13	18	5.24	4.77	0.0134	16	20	0.1935	0.0104	4
6	13	29	7.64	8.9	0.0086	27	32	-0.3250	0.0338	5
7	1	20	5.37	6.37	0.0031	17	21	0.1935	0.0104	4
8	31	30	9.38	3.87	0.0052	29	33	-0.1558	0.0281	4
9	1	36	9.14	1.32	0.0033	32	36	-1.3230	0.0647	4
10	9	8	1.06	1.53	0.0006	4	8	-1.3186	0.0832	4

Table B91. Best solutions for Series 13 (scaled data) found by the WMS method in each initialization, using optimization model (23).

Initialization: Initial points					Best solution: Final points					
Run	t^L	t^U	β0	β1	*SSE*	t^L	t^U	β0	β1	WMS size
1	12	73	4.31	6.04	0.0269	56	77	-1.0502	0.0134	21
2	32	138	7.35	4.07	0.0446	123	144	-0.6986	0.0104	21
3	5	4	7.68	6.98	0.0133	7	27	-1.0810	0.0144	20
4	57	74	5.14	7.79	0.0268	57	78	-1.0837	0.0139	21
5	12	145	6.62	1.21	0.0145	1	21	-1.0515	0.0129	20
6	16	131	1.85	3.11	0.0136	4	24	-1.0544	0.0127	20
7	50	92	2.77	9.41	0.0730	15	92	-1.0459	0.0134	77
8	16	147	0.84	4.78	0.0142	2	22	-1.0531	0.0129	20
9	144	128	0.62	0.4	0.0378	108	128	-1.1669	0.0140	20
10	46	119	0.43	8.68	0.0135	3	23	-1.0525	0.0128	20

Table B92. Best solutions for Series 14 (scaled data) found by the WMS method in each initialization, using optimization model (23).

Initialization: Initial points					Best solution: Final points					
Run	t^L	t^U	β0	β1	*SSE*	t^L	t^U	β0	β1	WMS size
1	173	23	7.15	8.68	0.0317	1	13	-0.9917	0.0115	12
2	16	126	4.46	6.75	0.0471	3	15	-0.9279	0.0024	12
3	43	209	2.9	7.89	2.7088	197	209	-0.6098	0.0024	12
4	164	77	6.57	8.83	0.9348	32	77	-0.8942	0.0027	45
5	174	215	3.04	3.44	3.4381	209	221	-0.5464	0.0024	12
6	1	90	0.23	4.21	1.5551	2	90	-0.9297	0.0033	88
7	114	174	0.96	1.09	2.1483	171	183	-1.0399	0.0044	12
8	141	208	3.77	9.64	2.7944	196	208	-0.6233	0.0024	12
9	53	72	0.96	6.69	0.3838	67	79	-0.8406	0.0024	12
10	36	54	3.81	8.9	0.1594	42	54	-0.9025	0.0024	12

Table B93. Best solutions for Series 15 (scaled data) found by the WMS method in each initialization, using optimization model (23).

Initialization: Initial points					Best solution: Final points					
Run	t^L	t^U	β0	β1	*SSE*	t^L	t^U	β0	β1	WMS size
1	4	91	1.22	1.67	0.3242	71	91	-0.4605	0.0024	20
2	160	167	1.64	1.3	0.2937	157	177	-2.1059	0.0108	20
3	52	88	3.63	6.11	0.3410	69	89	-0.4674	0.0024	20
4	108	220	0.8	2.02	0.5222	207	227	-0.5002	0.0024	20
5	141	186	6.61	6.57	0.4729	166	186	-0.7072	0.0024	20
6	45	175	7.97	6.76	0.4153	155	175	-0.7370	0.0024	20
7	126	153	8.56	2.24	0.2764	135	155	-0.9323	0.0024	20
8	180	205	6.29	7.18	0.3675	197	218	-0.4110	0.0024	21
9	155	164	6.59	5.72	0.3211	152	172	-2.7567	0.0146	20
10	81	160	3.55	0.05	0.3181	143	163	-2.7799	0.0148	20

Table B94. Best solutions for Series 16 (scaled data) found by the WMS method in each initialization, using optimization model (23).

Initialization: Initial points					Best solution: Final points					
Run	t^L	t^U	β0	β1	*SSE*	t^L	t^U	β0	β1	WMS size
1	32	241	2.21	8.13	0.0062	246	256	-0.7466	0.0024	10
2	191	181	9.73	4.6	0.0752	171	181	0.4804	0.0024	10
3	58	147	0.11	7.93	0.0860	138	148	0.1100	0.0036	10
4	226	10	3.51	5.83	0.0224	4	14	-0.4446	0.0346	10
5	87	79	7.85	7.05	0.8774	69	79	-4.9519	0.0677	10
6	53	125	2.53	5.33	0.0537	1	11	-0.2419	0.0083	10
7	265	51	8.63	2.22	0.0028	54	64	-0.1491	0.0024	10
8	237	63	9.38	4.29	0.0079	269	280	-0.8262	0.0024	11
9	110	286	3.03	6.18	0.0068	278	288	-0.8825	0.0024	10
10	101	104	0.52	6.66	0.0057	261	271	-0.8161	0.0024	10

Table B95. Best solutions for Series 17 (scaled data) found by the WMS method in each initialization, using optimization model (23).

Initialization: Initial points					Best solution: Final points					
Run	t^L	t^U	β0	β1	*SSE*	t^L	t^U	β0	β1	WMS size
1	8	110	1.59	8.51	0.0453	96	121	-0.5507	0.0024	25
2	202	257	2.1	7.95	0.1465	232	257	0.2237	0.0024	25
3	293	280	7.58	5.8	0.0979	269	294	-0.1519	0.0024	25
4	174	207	0.94	2.72	0.0464	183	208	-4.8898	0.0247	25
5	58	284	3.88	3.95	0.0483	1	26	-0.9037	0.0024	25
6	184	219	8.84	0.05	0.0482	185	210	-5.3045	0.0268	25
7	4	185	3.59	5.27	3.3203	7	185	-0.9400	0.0038	178
8	249	259	0.82	9.98	0.7408	245	272	0.0372	0.0024	27
9	19	79	4.32	3.91	0.1267	54	79	-1.8378	0.0163	25
10	121	203	8.7	8.79	0.0365	179	204	-4.3075	0.0217	25

Table B96. Best solutions for Series 18 (scaled data) found by the WMS method in each initialization, using optimization model (23).

Tables B97 to B109 and Figures B1 to B13 show the results for the Time Series from 3D projection (Section 5.1.7). The tables include the initial point of each run, the SSE value obtained, the best solution obtained for such run, the beta values of each match and the WMS associated to such run.

Initialization: Initial points								Best solution: Final points								
Run	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	*SSE*	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	WMS size
1	9	6	6	5	2	0.97	1.45	0.1885	1	6	1	12	0.1825	0.0000	-1.2629	5
2	11	4	3	4	0.5	0.14	1.62	1033.7344	1	4	1	12	0.5000	0.1400	1.6200	3
3	10	8	11	7	0	0.98	1.04	6.6553	7	10	1	12	0.0024	-0.0798	1.1266	3
4	11	8	10	11	0.75	0.3	1.59	4374.1821	7	10	1	12	0.7500	0.3000	1.5900	3
5	8	10	7	8	1	0.54	1.64	15.9762	6	10	1	12	0.0024	-0.1130	1.2526	4
6	1	3	7	11	0.2	0.23	1.72	0.0169	4	7	1	12	0.2417	-5.48E-08	-1.5461	3
7	11	2	10	11	0.8	0.12	1.87	1820.9732	2	5	1	12	0.8	0.12	1.87	3
8	7	9	12	4	2	0.23	1.58	310.4247	8	12	1	12	0.0024	0.0030	2.9062	4
9	5	9	11	12	1	0.57	1.97	6.9911	7	10	1	12	0.0024	-0.0978	1.0312	3
10	10	12	1	8	2	0.74	1.53	11252.8043	9	12	1	12	0.9024	0.74	1.53	3

Table B97. Best solutions for the Time Series 1 from 3D projection.

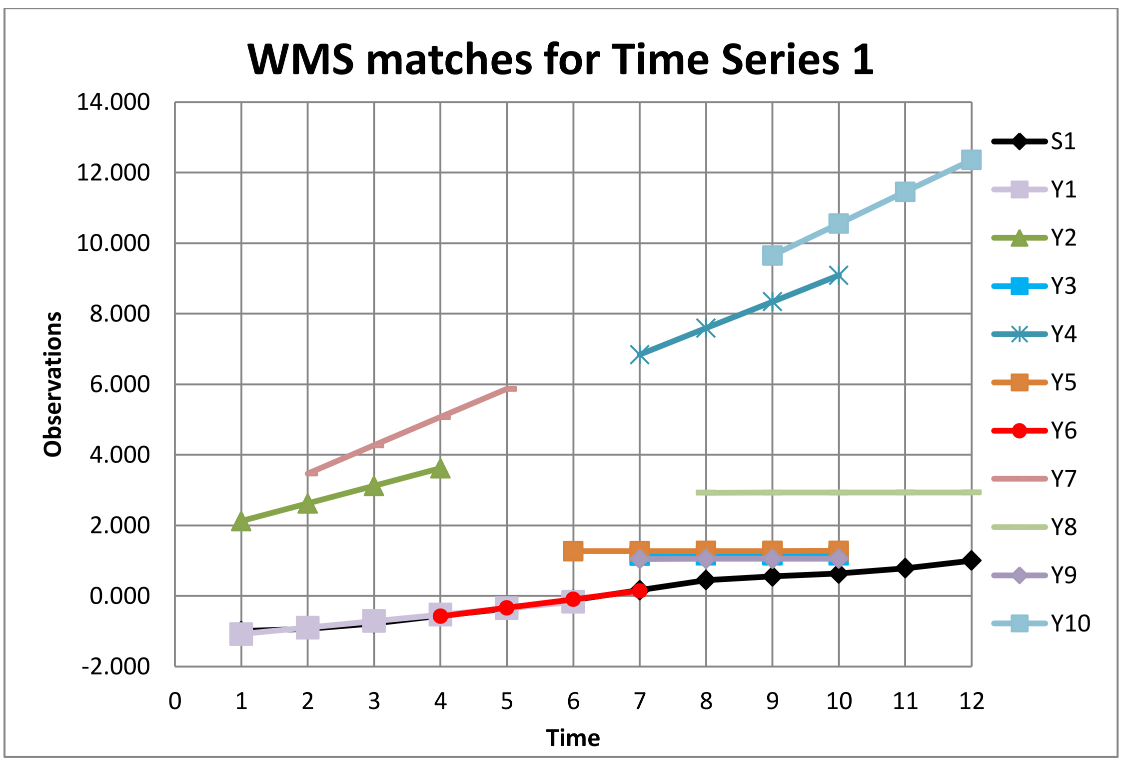

Figure B1. Results of all matches for Time Series 1, WMS matches from 3D projection.

Initialization: Initial points								Best solution: Final points								
Run	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	*SSE*	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	WMS size
1	1	3	5	6	2	0.97	1.45	1.9431	6	7	1	12	0.0024	0.0000	-0.2168	1
2	11	4	3	4	0.5	0.14	1.62	388.5528	2	4	1	12	0.5000	0.1400	1.6200	2
3	10	8	11	7	0	0.98	1.04	4.2050	8	9	1	12	0.0024	-0.0654	0.8956	1
4	11	8	10	11	0.75	0.3	1.59	2087.7204	7	8	1	12	0.7500	0.3000	1.5900	1
5	8	10	7	12	0	0.54	1.64	2.3718	9	10	1	12	0.0024	-0.0786	0.9104	1
6	1	3	7	11	0.2	0.23	1.72	0.6370	2	3	1	12	0.1494	0.0294	0.2124	1
7	11	2	10	11	0.8	0.12	1.87	1.4063	1	2	1	12	0.2206	0.0621	-0.1572	1
8	7	9	4	12	2	0.23	1.58	0.0000	8	9	1	12	0.3999	0.0000	-3.1991	1
9	7	9	11	12	1	0.57	2	24.3317	1	9	1	12	0.0024	-0.0808	1.0622	8
10	2	3	11	12	2	0.74	1.53	0.4916	5	6	1	12	0.0024	0.0000	0.1856	1

Table B98. Best solutions for the Time Series 2 from 3D projection.

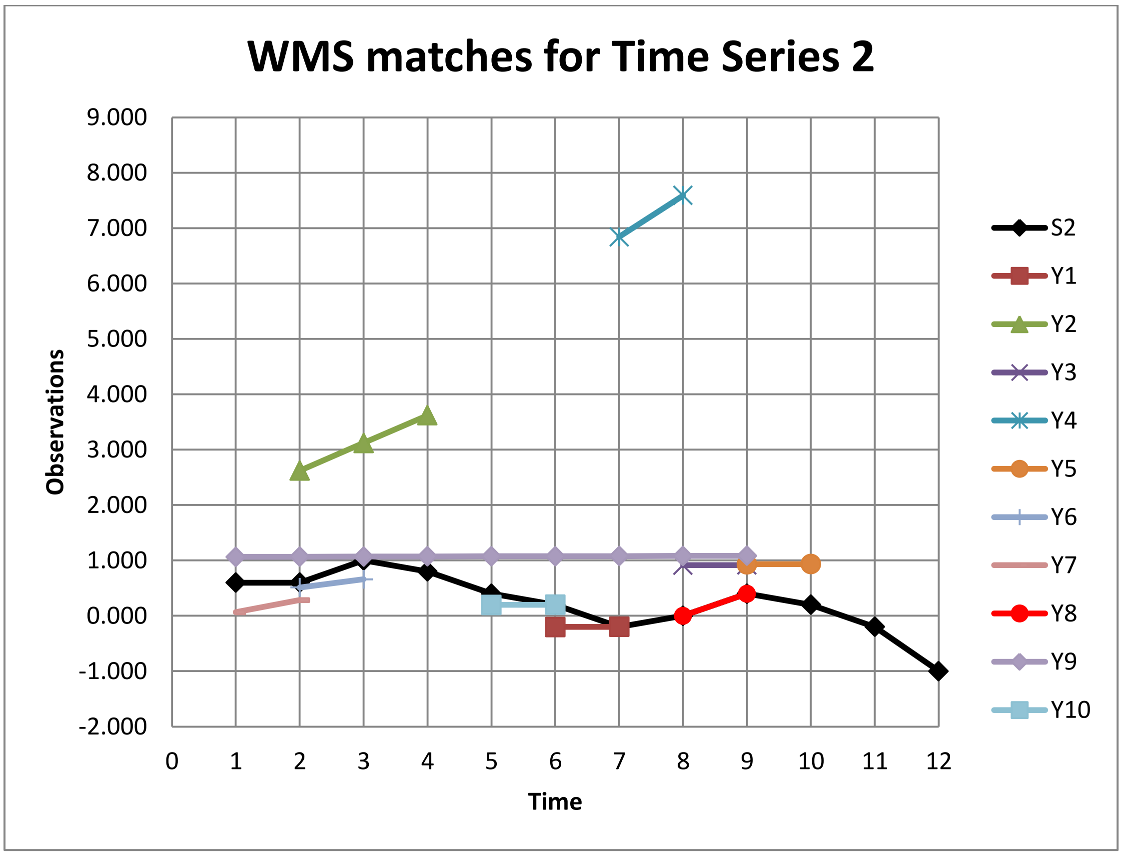

Figure B2. Results of all matches for Time Series 2, WMS matches from 3D projection.

Initialization: Initial points								Best solution: Final points								
Run	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	*SSE*	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	WMS size
1	1	3	5	6	2	0.97	1.45	3.0006	4	5	1	24	0.0024	-2.66E-08	-0.5352	1
2	11	4	3	4	0.5	0.14	1.62	1.0136	7	8	1	24	0.0024	7.23E-08	-0.4168	1
3	10	8	11	7	0	0.98	1.04	4.1560	18	19	1	24	0.0024	-0.0372	0.9690	1
4	1	24	10	11	0.75	0.3	1.59	139,108.8289	1	24	1	24	0.75	0.3	1.59	23
5	8	10	7	12	0	0.54	1.64	0.1172	9	11	1	24	0.4171	-1.86E-07	-3.6171	2
6	21	23	1	20	0.2	0.23	1.72	5,224.3762	21	23	1	24	0.2	0.23	1.72	2
7	23	24	11	12	0.8	0.12	1.87	22,744.0828	23	24	1	24	0.8	0.12	1.87	1
8	7	9	4	12	2	0.23	1.58	0.1172	9	11	1	24	0.4171	1.014E-08	-3.6171	2
9	7	9	11	12	1	0.57	2	0.1172	9	11	1	24	0.4171	5.587E-07	-3.6171	2
10	7	9	11	12	2	0.74	1.53	7.5498	8	9	1	24	0.0024	-1.43E-07	0.1441	1

Table B99. Best solutions for the Time Series 3 from 3D projection.

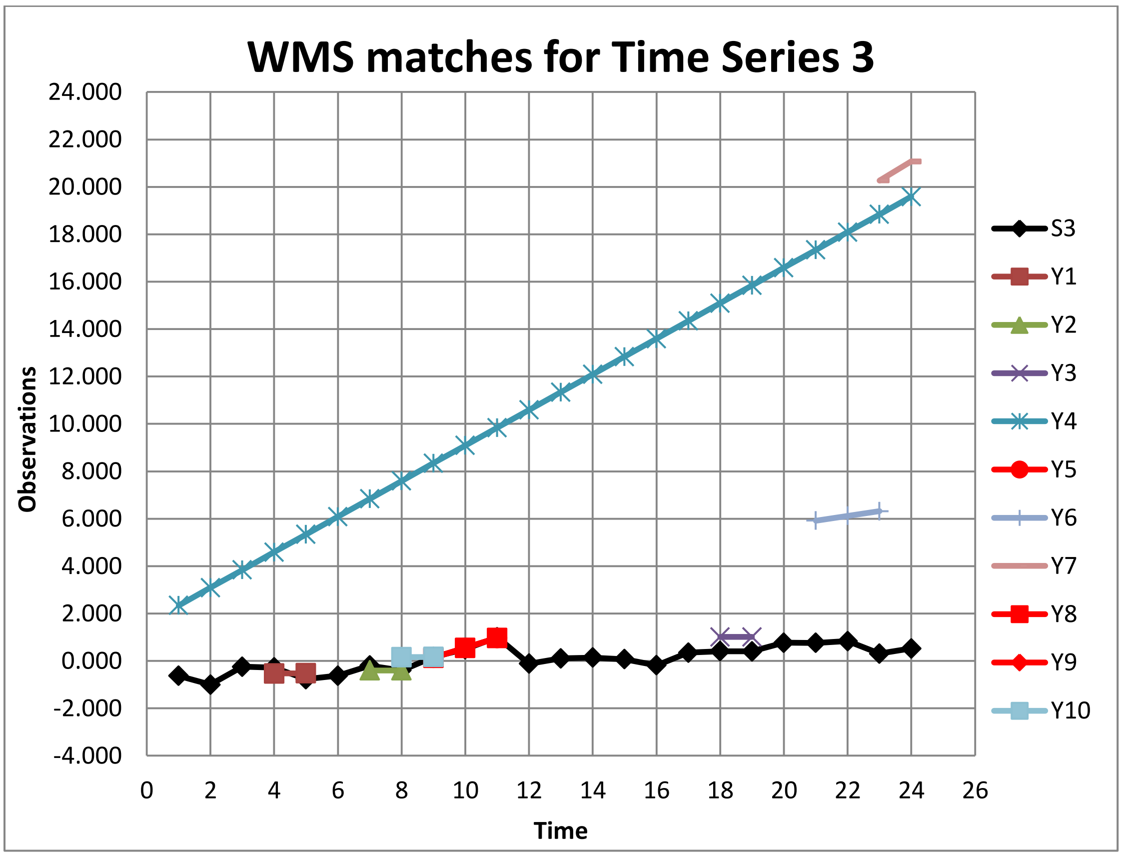

Figure B3. Results of all matches for Time Series 3, WMS matches from 3D projection.

Initialization: Initial points								Best solution: Final points								
Run	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	*SSE*	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	WMS size
1	1	3	5	6	2	0.97	1.45	14.1631	5	6	1	12	0.0024	-0.1948	1.1055	1
2	11	4	3	4	0.5	0.14	1.62	763.6737	2	4	1	12	0.5000	0.1400	1.6200	2
3	10	8	11	7	0	0.98	1.04	520.4165	3	4	1	12	0.5000	0.1400	1.6200	1
4	11	8	10	11	0.75	0.3	1.59	11.0825	7	8	1	12	0.0024	-0.0237	0.3439	1
5	8	10	7	12	0	0.54	1.64	1.63E-07	9	10	1	12	0.2935	-2.0E-06	-1.9354	1
6	1	3	7	11	0.2	0.23	1.72	1.6511	8	9	1	12	0.1575	-0.0446	-0.4154	1
7	3	12	11	12	0.8	0.12	1.87	1.6155	8	9	1	12	0.0024	0.0717	0.2756	1
8	4	6	1	12	0	0.23	1.58	3.4877	8	9	1	12	0.0024	-0.1039	1.5405	1
9	7	9	11	12	0	0.57	2	1.50E-07	9	10	1	12	0.2935	-1.9E-06	-1.9354	1
10	8	10	11	12	0	0.74	1.53	20.5711	7	10	1	12	0.0024	-0.0732	1.0960	3

Table B100. Best solutions for the Time Series 4 from 3D projection.

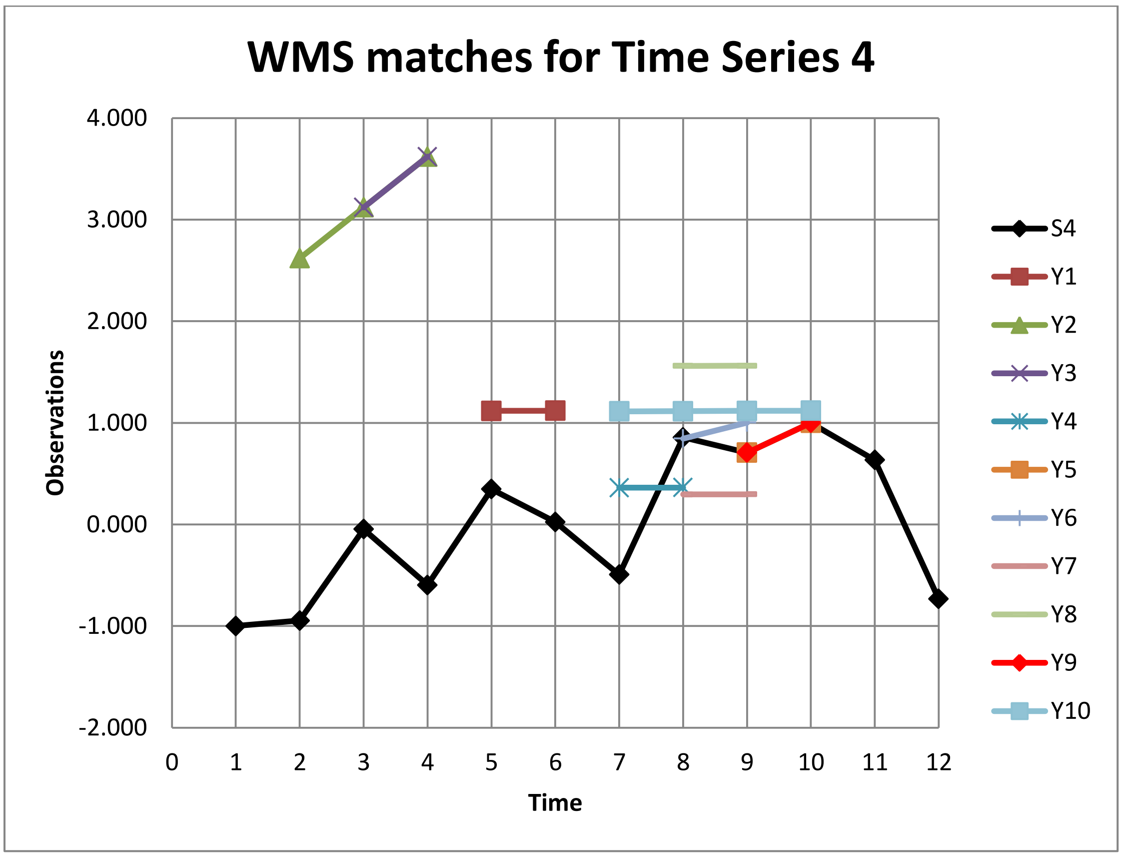

Figure B4. Results of all matches for Time Series 4, WMS matches from 3D projection.

Initialization: Initial points								Best solution: Final points								
Run	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	*SSE*	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	WMS size
1	1	14	5	6	2	0.97	1.45	2.321	13	14	1	15	0.0024	-0.0428	1.2223	1
2	11	4	3	4	0.5	0.14	1.62	934.045	4	5	1	15	0.5000	0.1400	1.6200	1
3	10	8	11	7	0	0.98	1.04	1.161	13	14	1	15	0.0024	-0.0144	0.9308	1
4	11	8	10	11	0.75	0.3	1.59	3120.656	7	8	1	15	0.7500	0.3000	1.5900	1
5	8	10	7	12	0	0.54	1.64	5.270	11	12	1	15	0.0024	-0.0906	1.0422	1
6	1	3	7	11	0.2	0.23	1.72	2.321	13	14	1	15	0.0024	-0.0428	1.2223	1
7	3	12	11	12	0.8	0.12	1.87	118.004	12	13	1	15	0.0024	0.1087	1.5353	1
8	4	6	1	12	0	0.23	1.58	289.01	12	13	1	15	0.0024	0.2300	1.5800	1
9	7	9	11	12	0	0.57	2	1290.67	12	13	1	15	0.0024	0.5700	2	1
10	8	10	11	12	0	0.74	1.53	17.70	7	8	1	15	0.0024	-0.1676	1.0833	1

Table B101. Best solutions for the Time Series 5 from 3D projection.

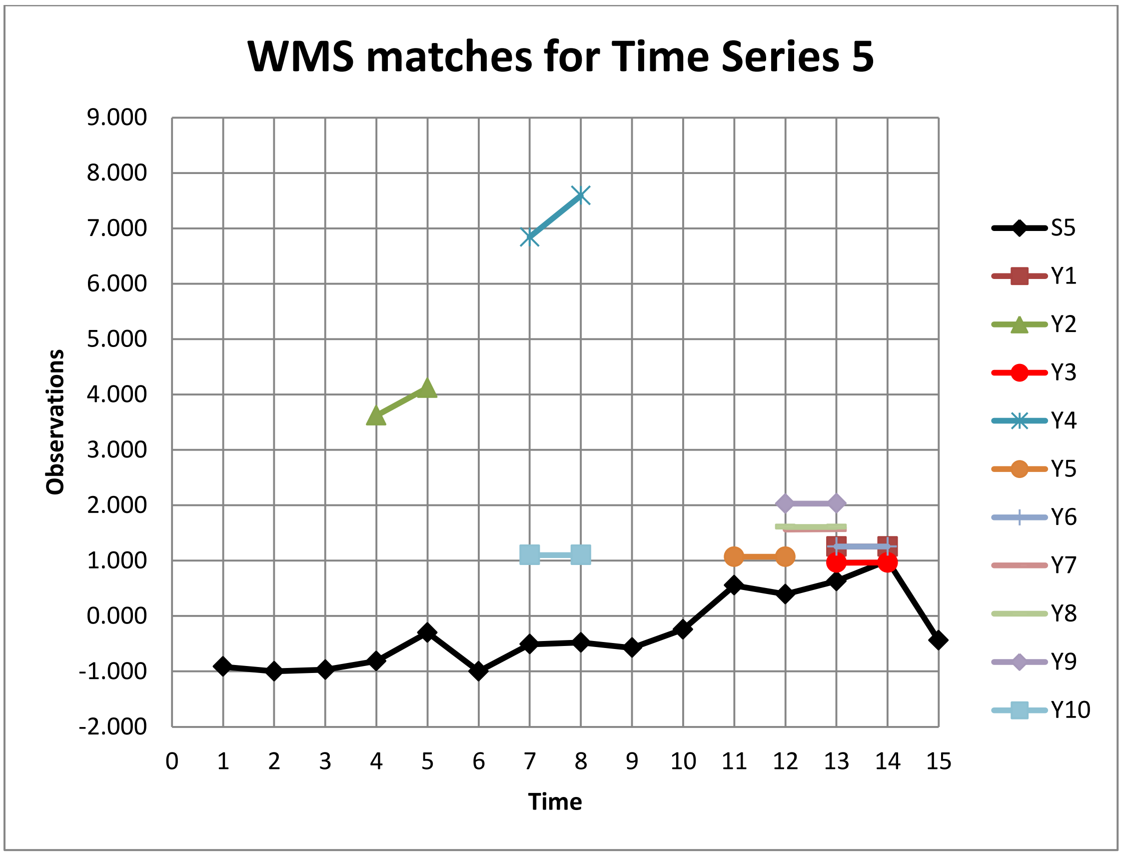

Figure B5. Results of all matches for Time Series 5, WMS matches from 3D projection.

Initialization: Initial points								Best solution: Final points								
Run	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	*SSE*	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	WMS size
1	1	11	1	11	2	0.97	1.45	8.7180	9	11	1	15	0.0024	-0.0801	1.2108	2
2	7	9	9	11	0.5	0.14	1.62	6.4913	7	9	1	15	0.0024	0.0499	0.2997	2
3	9	11	8	14	1	0.98	1.04	4.6648	9	11	1	15	0.0024	-0.0512	0.9156	2
4	8	10	1	14	0.75	0.3	1.59	4664.2609	8	10	1	15	0.7500	0.3000	1.5900	2
5	4	14	1	7	0	0.54	1.64	17.4064	12	14	1	15	0.0024	-0.1180	1.0106	2
6	3	10	4	6	0.2	0.23	1.72	1039.8478	8	10	1	15	0.2000	0.2300	1.7200	2
7	6	6	4	13	0.8	0.12	1.87	3115.3451	2	6	1	15	0.8000	0.1200	1.8700	4
8	1	2	2	14	0	0.23	1.58	459.8755	9	11	1	15	0.0024	0.2300	1.5798	2
9	9	13	3	11	0	0.57	2	19.5007	11	13	1	15	0.0024	-0.1211	1.1136	2
10	10	3	10	11	0	0.74	1.53	19.1705	5	7	1	15	0.0024	-0.0889	1.4190	2

Table B102. Best solutions for the Time Series 6 from 3D projection.

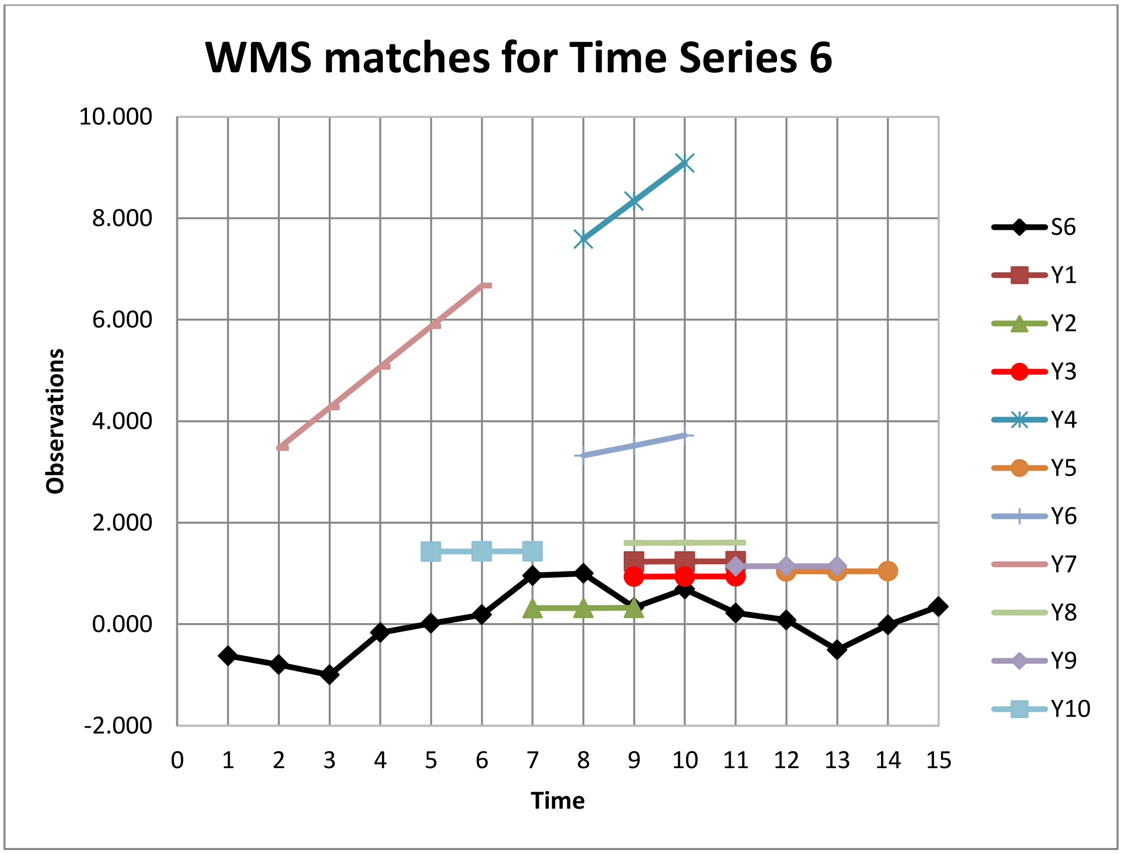

Figure B6. Results of all matches for Time Series 6, WMS matches from 3D projection.

Initialization: Initial points								Best solution: Final points								
Run	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	*SSE*	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	WMS size
1	1	11	1	11	2	0.97	1.45	11235.5107	11	12	1	15	0.9021	0.9695	1.4499	1
2	7	9	9	11	0.5	0.14	1.62	104.4109	10	12	1	15	0.0024	0.1105	1.1472	2
3	7	10	6	14	0	0.98	1.04	0.0468	9	11	1	15	0.5641	0.0000	-5.1824	2
4	8	10	1	14	0.75	0.3	1.59	3632.4550	9	10	1	15	0.7500	0.3000	1.5900	1
5	4	14	1	7	0	0.54	1.64	5.1074	13	14	1	15	0.0024	-0.0851	1.0484	1
6	3	10	4	6	0.2	0.23	1.72	864.8355	9	10	1	15	0.2000	0.2300	1.7200	1
7	6	6	4	13	0.8	0.12	1.87	2926.0151	2	6	1	15	0.8000	0.1200	1.8700	4
8	1	2	2	14	0	0.23	1.58	290.5811	14	15	1	15	0.0024	0.2299	1.5794	1
9	9	13	3	11	0	0.57	2	6.3964	13	14	1	15	0.0024	-0.0961	1.1520	1
10	10	3	10	11	0	0.74	1.53	7.8193	6	7	1	15	0.0024	-0.0842	1.4236	1

Table B103. Best solutions for the Time Series 7 from 3D projection.

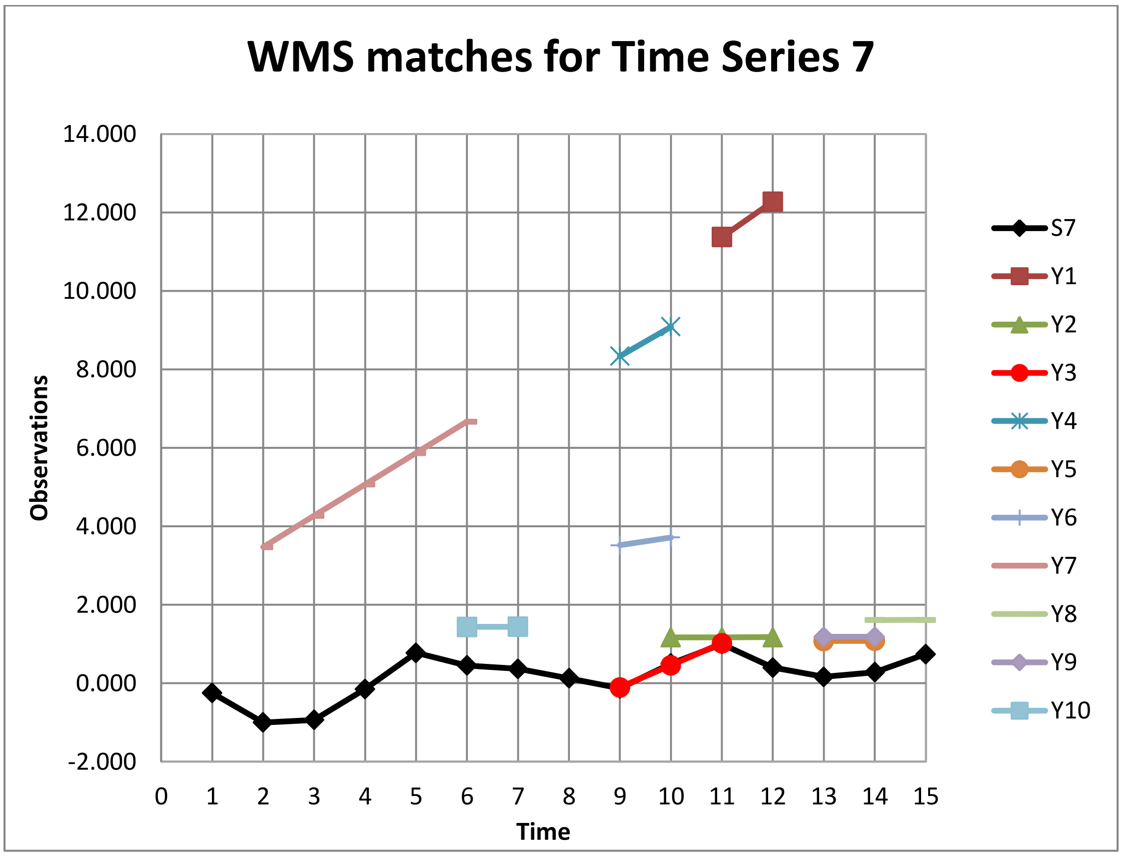

Figure B7. Results of all matches for Time Series 7, WMS matches from 3D projection.

Initialization: Initial points								Best solution: Final points								
Run	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	*SSE*	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	WMS size
1	1	12	2	11	0.78	0.08	0.06	4.559	14	15	1	15	0.0024	0.0741	0.0596	1
2	3	6	4	6	0.21	0.74	0.97	33.046	8	9	1	15	0.1649	-0.2361	0.7949	1
3	4	10	1	15	0.74	0.68	0.44	5379.957	9	10	1	15	0.7400	0.6800	0.4400	1
4	6	14	7	11	0.55	0.51	0.43	4.26E-07	13	14	1	15	0.5933	0.0000	-7.5776	1
5	2	3	11	12	0.67	0.51	0.25	15.365	2	3	1	15	0.3702	-0.1437	0.2335	1
6	8	9	1	13	1	0.06	0.73	1.487	14	15	1	15	0.0024	0.0402	0.4709	1
7	6	11	7	9	0.71	0.91	0.68	0.509	10	11	1	15	0.0024	0.0000	-0.5220	1
8	5	7	1	8	0.28	0.97	0.91	34.15	7	8	1	15	0.2144	-0.2119	0.8031	1
9	2	3	1	15	0.93	0.2	0.26	608.29	2	3	1	15	0.9024	0.2000	0.2600	1
10	3	4	10	15	0	0.43	0.36	0.008	8	9	1	15	0.0024	0.0000	0.0623	1

Table B104. Best solutions for the Time Series 8 from 3D projection.

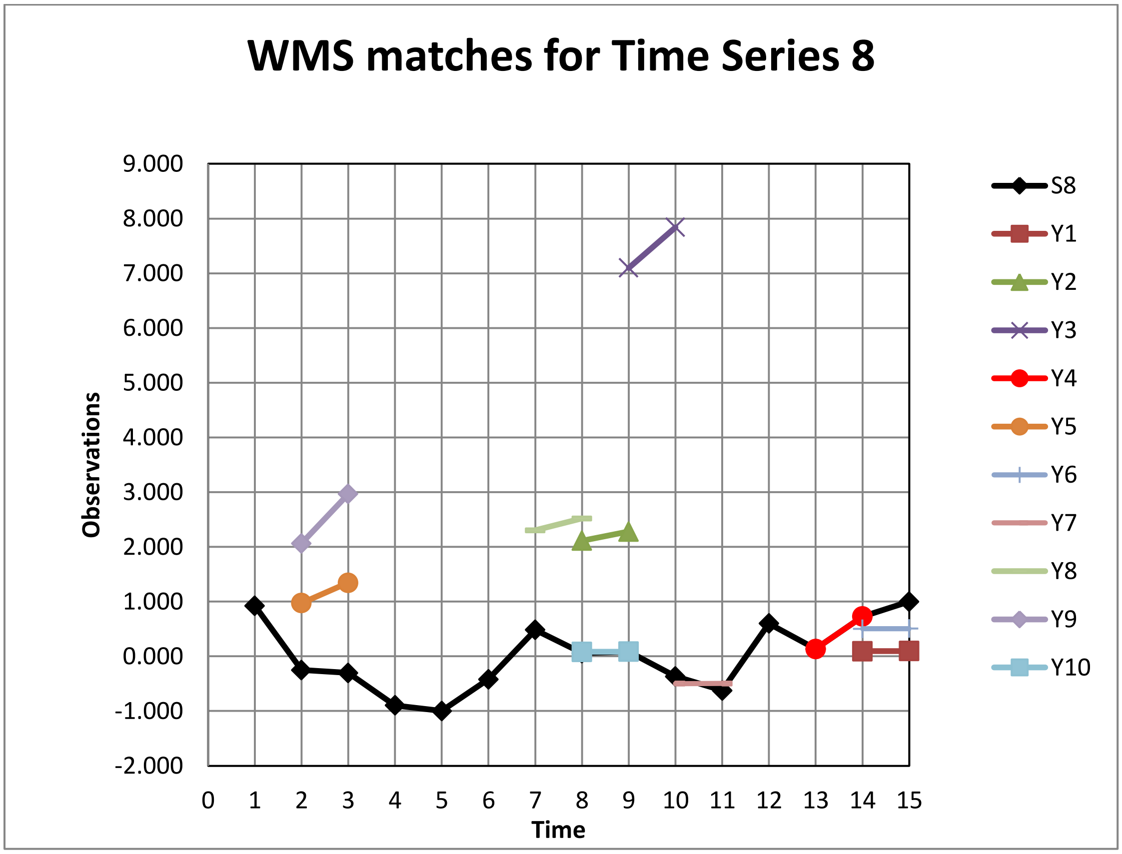

Figure B8. Results of all matches for Time Series 8, WMS matches from 3D projection.

Initialization: Initial points								Best solution: Final points								
Run	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	*SSE*	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	WMS size
1	3	13	1	12	0.68	0.98	0.8	17691.44	10	13	1	15	0.6799	0.9799	0.8000	3
2	1	9	1	14	0.33	0.27	0.73	2156.65	6	9	1	15	0.3300	0.2700	0.7300	3
3	2	11	8	12	0.8	0.43	0.49	8825.32	8	11	1	15	0.7998	0.4299	0.4900	3
4	13	13	1	14	0.81	0.09	0.35	8470.48	12	15	1	15	0.8100	0.0900	0.3500	3
5	4	5	1	10	0.83	0.06	0.8	188.98	6	9	1	15	0.0024	0.0569	0.7329	3
6	12	13	7	11	0	0.7	0.63	16.99	12	15	1	15	0.0024	-0.0752	0.5674	3
7	4	7	14	14	0.82	0.13	0.09	18.31	6	9	1	15	0.0024	-0.0252	0.0821	3
8	3	12	9	12	0	0.64	0.37	11.66	9	12	1	15	0.0024	-0.0725	0.3463	3
9	4	6	1	5	0	0.19	0.57	13.67	9	12	1	15	0.0024	-0.0515	0.3427	3
10	3	14	4	15	0.96	0.49	0.8	15525.69	11	14	1	15	0.9024	0.4899	0.8000	3

Table B105. Best solutions for the Time Series 9 from 3D projection.

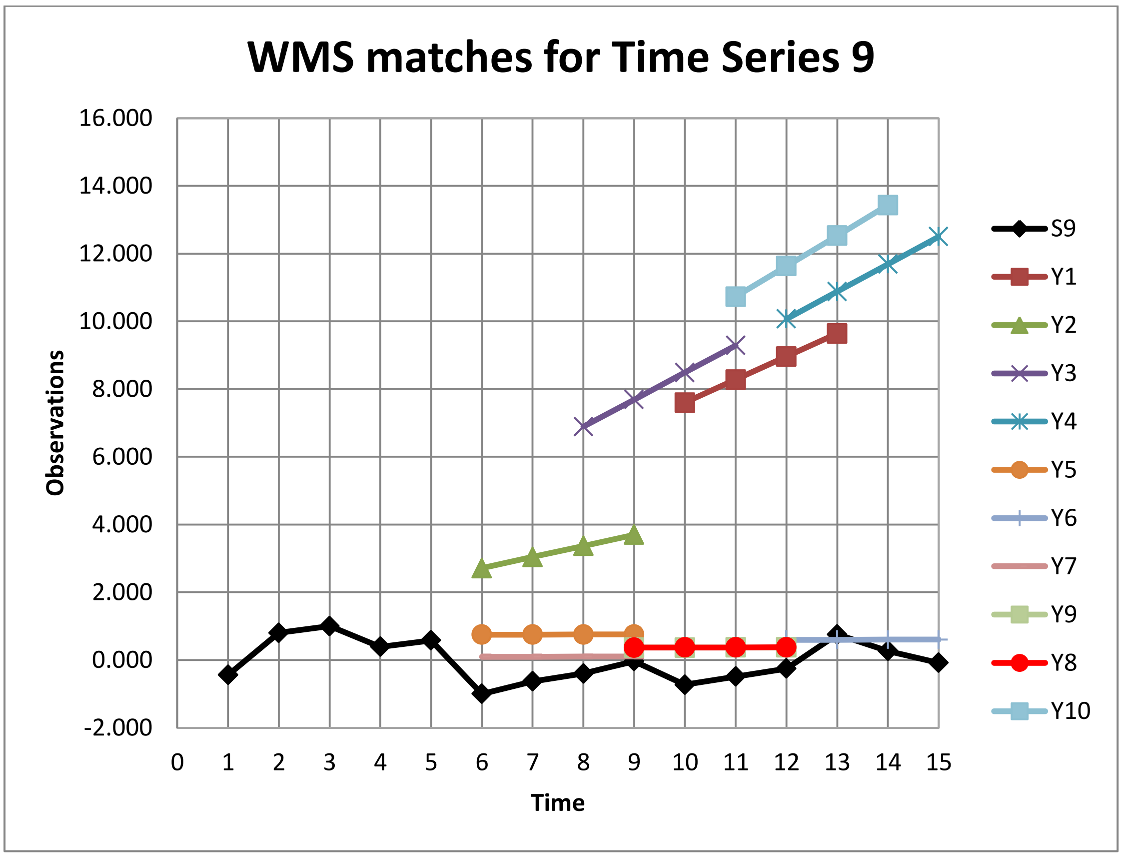

Figure B9. Results of all matches for Time Series 9, WMS matches from 3D projection.

Initialization: Initial points								Best solution: Final points								
Run	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	*SSE*	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	WMS size
1	15	34	20	40	0	0.75	0.8	2.8452	39	42	1	46	0.1526	0.0000	-6.3889	3
2	5	18	1	46	0	0.79	0.34	20.0379	20	23	1	46	0.0024	-0.0208	0.3351	3
3	29	35	1	46	0.51	0.19	0.52	1.4733	33	37	1	46	0.0232	0.0000	-1.2256	4
4	8	39	16	30	0.13	0.45	0.15	1.6162	38	41	1	46	0.0024	0.0000	-0.3781	3
5	13	41	17	22	0.43	0.07	0.9	140,374.17	40	46	1	46	0.4300	0.0700	0.9000	6
6	15	46	5	40	0.79	0.45	0.78	396,760.29	43	46	1	46	0.7897	0.4499	0.7800	3
7	14	37	10	43	0	0.65	0.83	37.24	41	44	1	46	0.0024	-0.0269	0.7939	3
8	32	46	13	37	0.52	0.43	0.68	78.44	35	40	1	46	0.0024	-0.0345	0.6378	5
9	3	37	8	34	0.14	0	0.15	2.0055	34	38	1	46	0.0105	-0.0009	-0.7000	4
10	16	22	20	37	0.72	0.75	0.06	0.8494	20	23	1	46	0.0024	0.0000	-0.3211	3

Table B106. Best solutions for the Time Series 10 from 3D projection.

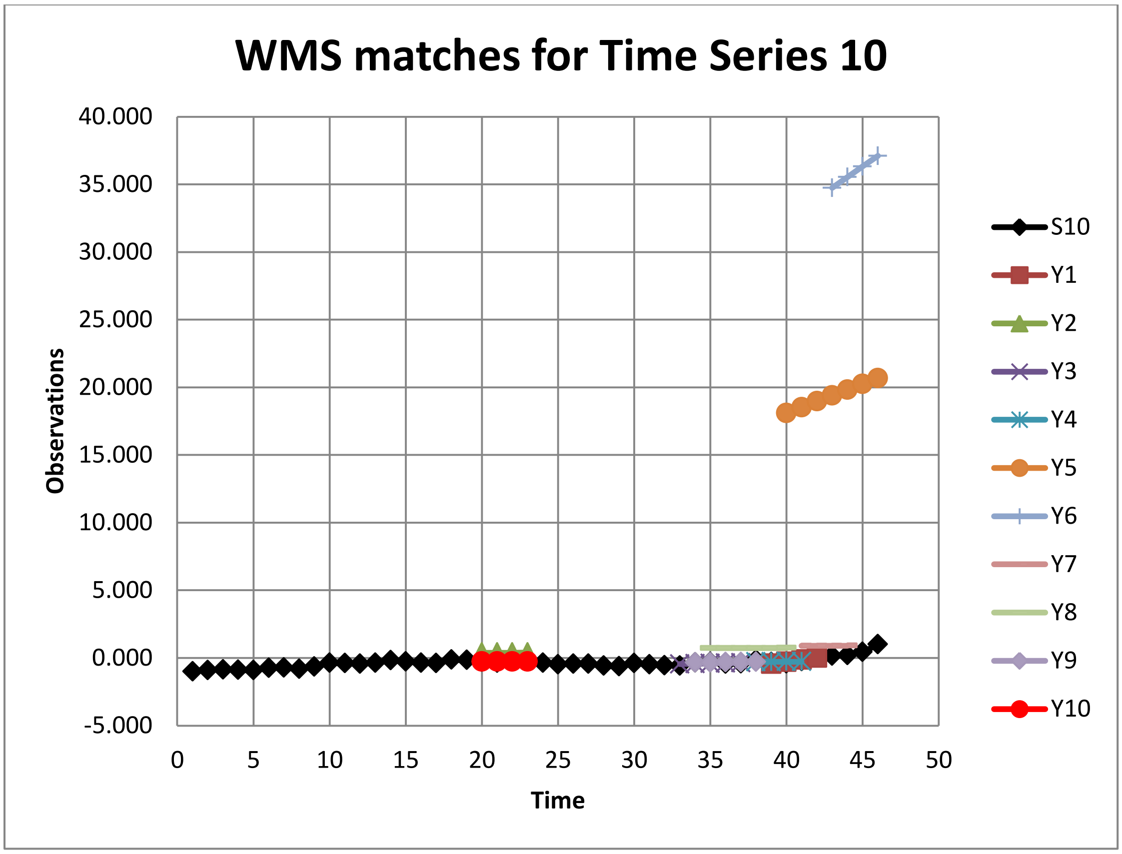

Figure B10. Results of all matches for Time Series 10, WMS matches from 3D projection.

Initialization: Initial points								Best solution: Final points								
Run	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	*SSE*	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	WMS size
1	3	13	7	7	0.7	0.34	0.74	1.55	2	16	1	23	0.1277	-6.45E-07	-1.0520	14
2	3	15	6	11	0.77	0.97	0.02	3.26	3	17	1	23	0.1211	2.52E-07	-1.0098	14
3	9	19	1	14	0.48	0.88	0.82	133,834.62	9	23	1	23	0.4800	0.8800	0.8200	14
4	15	20	9	23	0.55	0.81	0.89	33.32	6	20	1	23	0.0489	0.0000	-0.2346	14
5	11	15	9	17	0.88	1	0.18	1.65	1	15	1	23	0.1298	0.0000	-1.0732	14
6	2	18	1	19	0.14	0.21	0.71	7,657.01	4	18	1	23	0.1400	0.2100	0.7100	14
7	4	18	3	21	0.61	0.89	0.86	125,132.22	4	18	1	23	0.6100	0.8900	0.8600	14
8	3	9	8	9	0.84	0.92	0.33	1.65	1	16	1	23	0.1294	0.0000	-1.0709	15
9	11	18	14	22	0.67	0.75	0.08	3.26	3	17	1	23	0.1211	0.0000	-1.0098	14
10	4	21	1	22	0.29	0.27	0.3	19,502.71	7	21	1	23	0.2900	0.2700	0.3000	14

Table B107. Best solutions for the Time Series 11 from 3D projection.

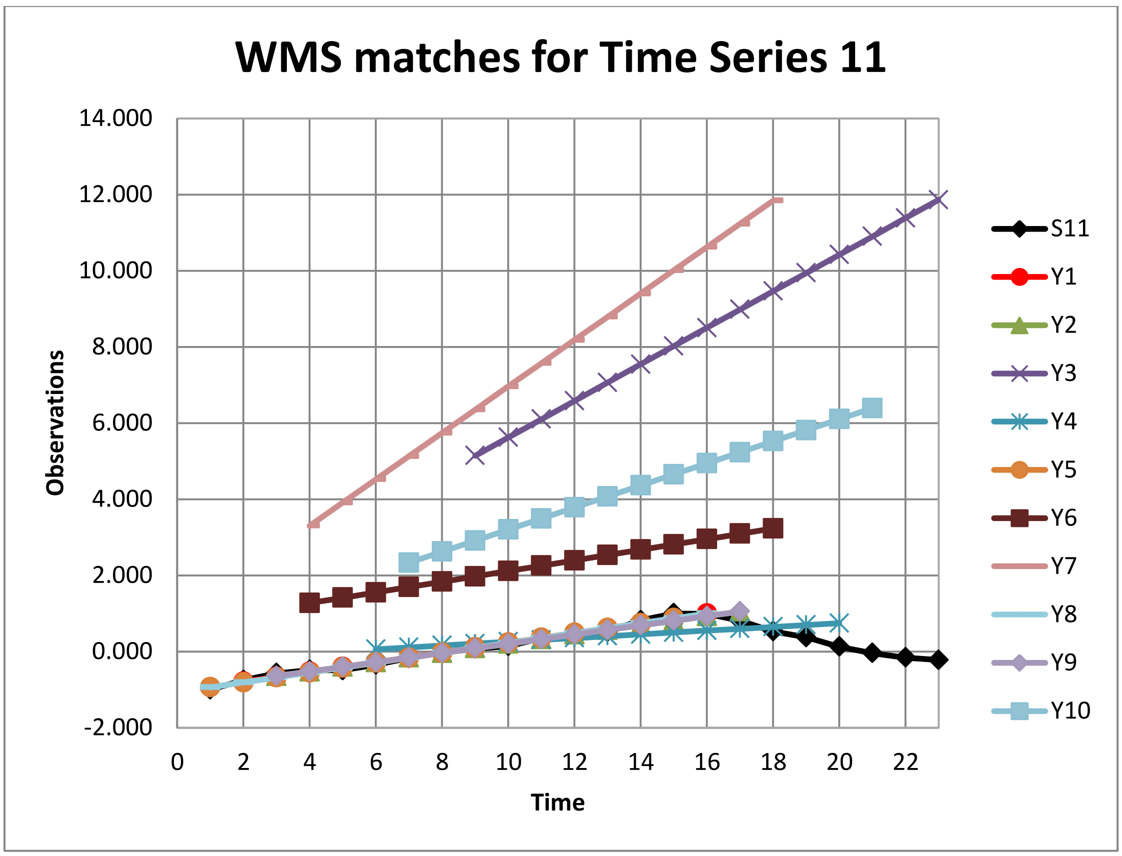

Figure B11. Results of all matches for Time Series 11, WMS matches from 3D projection.

Initialization: Initial points								Best solution: Final points								
Run	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	*SSE*	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	WMS size
1	1	29	25	31	0.8	0.51	0.33	1.8064	25	29	1	41	0.0027	-0.0007	0.3215	4
2	23	41	24	30	0.51	0.75	0.16	0.6052	27	31	1	41	0.0811	0.0000	-1.8186	4
3	7	22	14	22	0.22	0.54	0.22	1.2822	19	23	1	41	0.0024	0.0000	-0.1850	4
4	12	20	6	23	0.62	0.65	0.72	0.6844	17	21	1	41	0.0024	-0.0007	-0.1174	4
5	14	18	5	33	0.03	0.06	0.59	0.1313	14	18	1	41	0.0837	0.0000	-1.5388	4
6	6	41	23	30	0.39	0.63	0.46	0.1819	7	11	1	41	0.0271	0.0000	-0.9755	4
7	7	24	4	19	0.43	0.35	0.27	6.8551	21	25	1	41	0.0030	-0.0008	-0.1164	4
8	14	33	1	40	0.05	0.56	0.5	2.7923	32	36	1	41	0.0024	0.0000	0.5280	4
9	22	38	7	19	0.08	0.43	0.19	779.1715	10	38	1	41	0.0646	-0.0589	0.1864	28
10	29	30	26	26	0.44	0.84	0.67	1.2937	19	23	1	41	0.0024	0.0003	-0.1983	4

Table B108. Best solutions for the Time Series 12 from 3D projection.

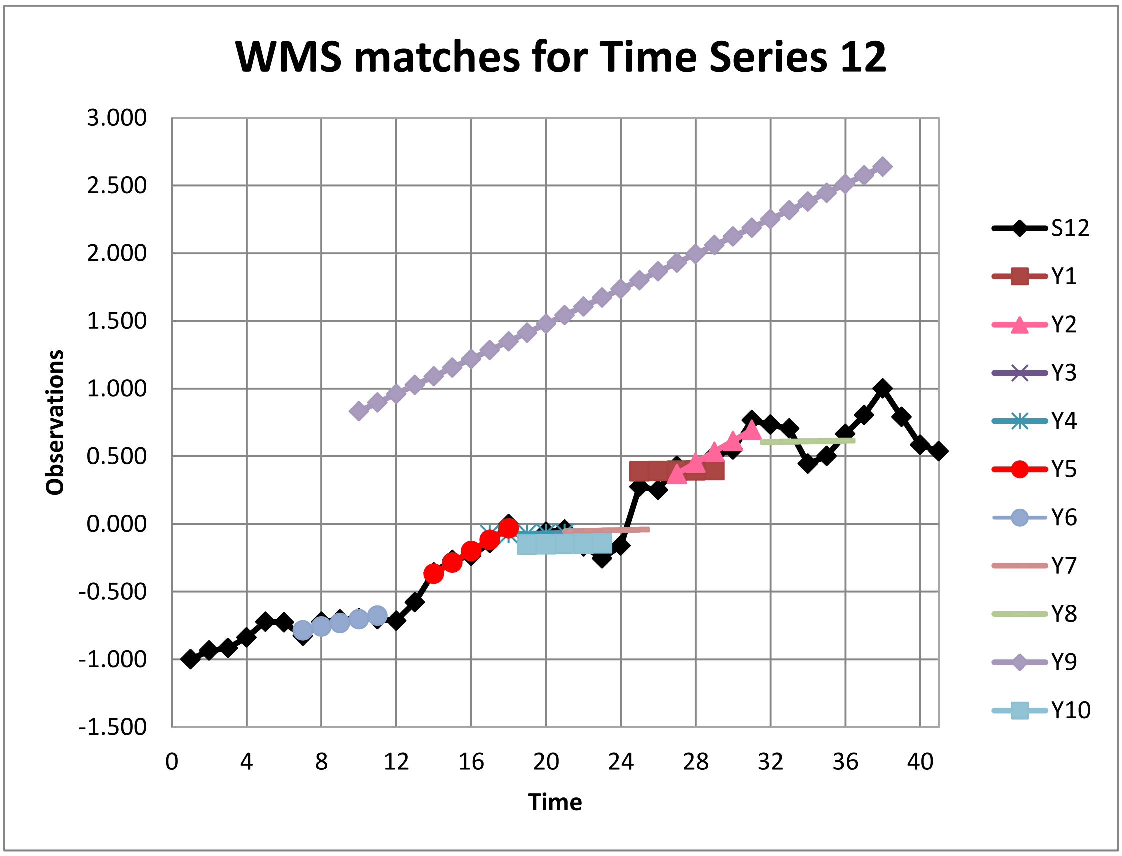

Figure B12. Results of all matches for Time Series 12, WMS matches from 3D projection.

Initialization: Initial points								Best solution: Final points								
Run	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	*SSE*	tx^L	tx^U	ty^L	ty^U	β_1	β_2	β_0	WMS size
1	18	21	11	29	0.72	0.19	0.82	45.9184	3	21	1	41	0.0967	-6E-07	-1.2486	18
2	18	25	11	38	0.09	0.22	0.91	1.4619	18	28	1	41	0.0171	-4E-07	0.0716	10
3	25	32	11	31	0.44	0.31	0.17	1.4379	23	33	1	41	0.0363	-2E-06	-0.4168	10
4	1	10	23	27	0.45	0.81	0.49	192.5505	4	14	1	41	0.0024	-0.028	0.4240	10
5	19	37	5	34	0.45	0.74	0.49	555231.0210	29	41	1	41	0.45	0.74	0.49	12
6	27	37	4	22	0.73	0.64	0.98	853978.5499	29	41	1	41	0.73	0.64	0.98	12
7	12	32	7	40	0.17	0.91	0.2	3.2762	22	32	1	41	0.0145	-9E-04	0.2090	10
8	9	32	18	40	0.17	0.27	0.53	0.9980	27	37	1	41	0.0414	-9E-07	-0.5465	10
9	16	34	2	41	0.98	0.91	0.25	1.4331	24	35	1	41	0.0455	8E-08	-0.6783	11
10	10	25	16	19	0.2	0.24	0.21	0.8178	16	26	1	41	0.0047	-2E-08	0.3287	10

Table B109. Best solutions for the Time Series 13 from 3D projection.

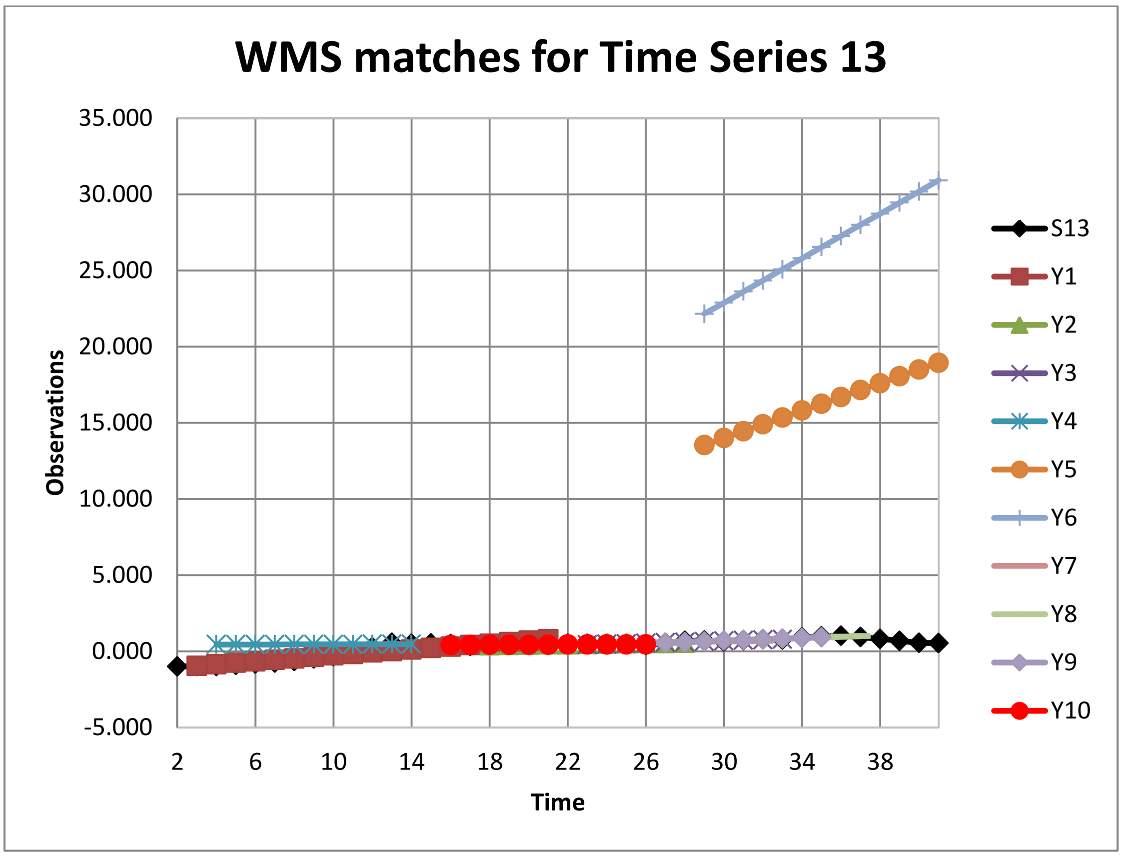

Figure B13. Results of all matches for Time Series 13, WMS matches from 3D projection.

Appendix C

This appendix shows the fitted linear model for data of the WMS obtained in the 3D projection of the time series (Chapter 5, Section 5.1.7). This test statistics was generated by R statistical software (R Core Team, 2013).

Linear model for data within WMS obtained in the 3D projection of Time Series 1

```
> summary(lm.window)

Call:
lm(formula = Obvs_wms_S1 ~ Time)

Residuals:
       1        2        3        4 
 6841671 -7300924 -5923165  6382418 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)   
(Intercept) -44785380   23541091  -1.902   0.1975   
Time         85534559    4194414  20.392   0.0024 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 9379000 on 2 degrees of freedom
Multiple R-squared:  0.9952,	Adjusted R-squared:  0.9928 
F-statistic: 415.9 on 1 and 2 DF,  p-value: 0.002396
```

Linear model for data within WMS obtained in the 3D projection of Time Series 9

```
> summary(lm.window)

Call:
lm(formula = Obvs_wms_S9 ~ Time)

Residuals:
      1       2       3       4 
  84897 -113388  -27914   56406 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   352892     517573   0.682    0.566
Time          -12558      49016  -0.256    0.822

Residual standard error: 109600 on 2 degrees of freedom
Multiple R-squared:  0.03178,	Adjusted R-squared:  -0.4523 
F-statistic: 0.06565 on 1 and 2 DF,  p-value: 0.8217
```

Linear model for data within WMS obtained in the 3D projection of Time Series 10

```
> summary(lm.window)

Call:
lm(formula = Obvs_wms_S10 ~ Time)

Residuals:
     1      2      3      4 
 1.812 -7.288  9.140 -3.664 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   50.978     84.315   0.605    0.607
Time           3.523      3.916   0.900    0.463

Residual standard error: 8.757 on 2 degrees of freedom
Multiple R-squared:  0.288,	Adjusted R-squared:  -0.06796 
F-statistic: 0.8091 on 1 and 2 DF,  p-value: 0.4633
```

Linear model for data within WMS obtained in the 3D projection of Time Series 11

```
> summary(lm.window)

Call:
lm(formula = Obvs_wms_S11 ~ Time)

Residuals:
    Min      1Q  Median      3Q     Max 
-744.61 -403.83  -51.61  370.21 1080.63 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  6812.49     339.91   20.04 3.70e-11 ***
Time         1009.83      34.05   29.66 2.51e-13 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1 

Residual standard error: 569.7 on 13 degrees of freedom
Multiple R-squared:  0.9854,	Adjusted R-squared:  0.9843 
F-statistic: 879.7 on 1 and 13 DF,  p-value: 2.515e-13
```

Linear model for data within WMS obtained in the 3D projection of Time Series 12

```
> summary(lm.window)

Call:
lm(formula = Obvs_wms_S12 ~ Time)

Residuals:
     1      2      3      4      5 
 1.019  1.958 -3.750 -2.449  3.222 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)   
(Intercept)  -15.393     17.427  -0.883  0.44214   
Time           8.786      1.085   8.098  0.00393 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1 

Residual standard error: 3.431 on 3 degrees of freedom
Multiple R-squared:  0.9563,	Adjusted R-squared:  0.9417 
F-statistic: 65.58 on 1 and 3 DF,  p-value: 0.003935
```

Linear model for data within WMS obtained in the 3D projection of Time Series 13

```
> summary(lm.window)

Call:
lm(formula = Obvs_wms_S13 ~ Time)

Residuals:
    Min      1Q  Median      3Q     Max 
-242.22  -85.03   15.41  136.46  195.18 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  5346.99     316.21  16.910 3.97e-08 ***
Time           15.72      14.89   1.055    0.319    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 156.2 on 9 degrees of freedom
Multiple R-squared:  0.1101,	Adjusted R-squared:  0.01126 
F-statistic: 1.114 on 1 and 9 DF,  p-value: 0.3187
```

Appendix D

In this appendix the initial points in each initialization for the test functions used in Chapter 6, are presented (Table D1). For more details about these functions, refer to Pohlheim (2006) and (Surjanovic and Bingham, 2013).

	Initial points
Test function	$(x_1^L, x_1^U, x_2^L, x_1^U, \beta_0, \beta_1, \beta_2)$
Sphere	(-5, 5, -5, 5, 0, 1, 1)
	(-2.5, 2.5, -2.5, 2.5, 0, 1, 1)
Rosenbrock	(-2, 5, -2, 5, random-between(-1000,1000), random-between(-1000,1000), random-between (-1000,1000))
	(-1, 2.5, -1, 2.5, random-between(-1000,1000), random-between(-1000,1000), random-between(-1000,1000))
Rastrigin	(-5, 5, -5, 5, random-between(-1000,1000), random-between(-1000,1000), random-between(-1000,1000))
	(-2.5, 2.5, -2.5, 2.5, random-between(-1000,1000), random-between(-1000,1000), random-between(-1000,1000))
Griewank	(-25, 35, -25, 35, random-between(-1000,1000), random-between(-1000,1000), random-between(-1000,1000))
	(-50, 70, -50, 70, random-between(-1000,1000), random-between(-1000,1000), random-between(-1000,1000))
Goldstein-Price	(-2, 2, -2, 2, random-between(-1000,1000), random-between(-1000,1000), random-between(-1000,1000))
	(-1, 1, -1, 1, random-between(-1000,1000), random-between(-1000,1000), random-between(-1000,1000))
Easom	(-100.53, 100.53,-100.53, 100.53, random(), random(), random())
	(-50, 50, -50, 50, random-between(-1000,1000), random-between(-1000,1000), random-between(-1000,1000))
Schwefel	(-500, 500, -500, 500, random-between(-1000,1000), random-between(-1000,1000), random-between(-1000,1000))
	(-250, 250, -250, 250, random-between(-1000,1000), random-between(-1000,1000), random-between(-1000,1000))

Table D1. Initial points used in the initializations of the WMS method for each test function.

Appendix E

Comparison of the Inverse Metamodeling with Response Surface Methodology (RSM)

The WMS method works with a focus of inverse metamodeling that can be compared with the Response Surface Methodology since point of view of a system. RSM receives as input a set of known variables (x_1 and x_2), which undergo a process and generates a function ($y = f(x_1, x_2)$) as output. Inverse metamodeling is a premise where known variables are the input of a process or system which produces a region of the experimental data under study as output. The proposed WMS method determines this experimental region through least squares. The experimental region is defined by a window of maximum similarity. Figure E1 presents the system diagrams of this comparison.

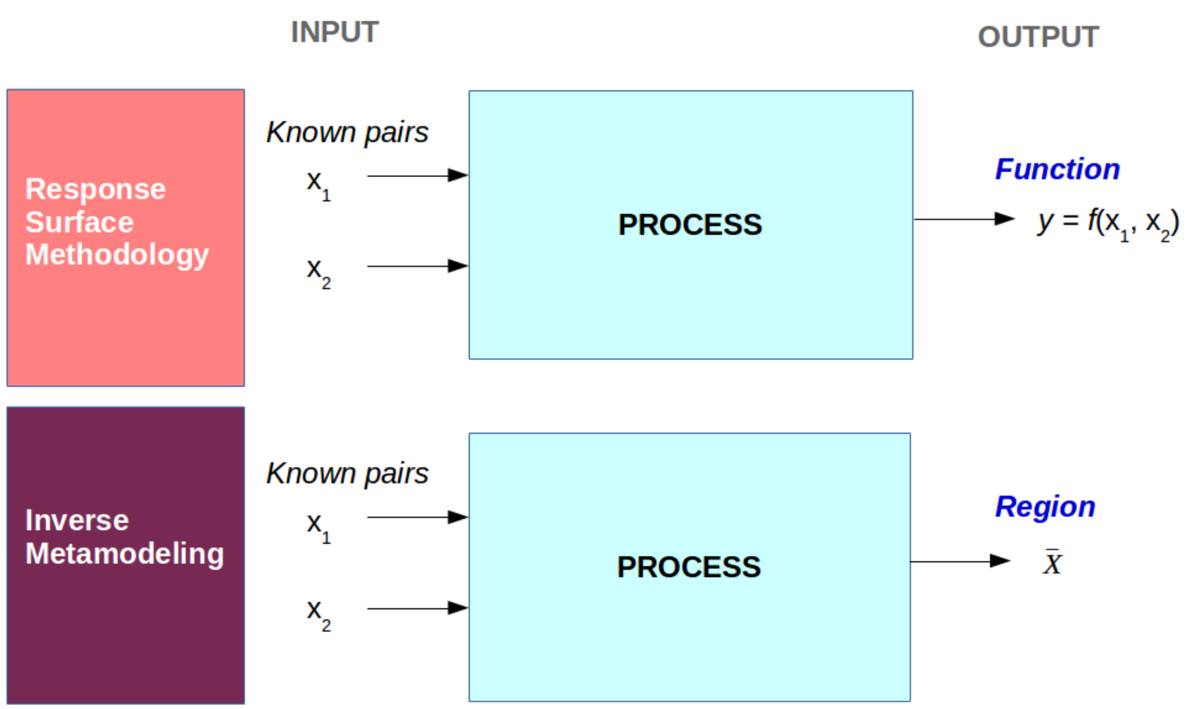

Figure E1. System diagrams comparing the Response Surface Methodology and the Inverse Metamodeling technique.

In general, the WMS method for future optimization by similarity method intends to provide an exploratory tool of experimentally or pseudo-experimentally generated data to find a region where the maximum similarity is located.