# Finding the Best Cost-Efficient Food Assortment for a Non-for-Profit Firm

by

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# Abstract

Consider a non-profit firm that provides food to consumers (e.g. schools cafeterias) where operational costs are highly affected by the assortment to offer. The food items offered in the assortment belong to one or more categories and must comply or exceed with daily nutritional standards established by the United States Department of Agriculture (USDA); which are measured by the quantity per serving per food category. Furthermore, the demand of such assortment is uncertain and influenced by the presence of other items within the assortment. In order to find the most cost-efficient assortment for these firms, we developed a mathematical formulation to minimize operational costs while the assortment composition is satisfied. Also, the model allows for some sources of revenue. To verify the model and describe the structure of an optimal assortment, we used a numerical study inspired in the Puerto Rico School Meal Program (PRSMP) who offers services to public and private schools in Puerto Rico.

# Resumen

Considera una empresa sin fines de lucro que provee alimento a los consumidores (ej. los comedores escolares), donde los costos operacionales son altamente afectados por el surtido a ofrecer. Los alimentos ofrecidos en el surtido pertenecen a una o más categorías y deben cumplir o exceder los estándares diarios de nutrición que son establecidos por el Departamento de Agricultura de Estados Unidos (USDA, por sus siglas en inglés); los cuales son medidos en cantidades por servicio por categoría de alimento. Además, se considera que la demanda de este surtido es incierta y está influenciada por la presencia de otros productos en el surtido. Con el fin de encontrar el surtido más costo-eficiente para estas empresas, desarrollamos una formulación matemática para minimizar los costos operacionales mientras la composición del surtido es satisfecha. En adición, el modelo permite algunas fuentes de ingresos. Para verificar el modelo y describir la estructura de un surtido óptimo, utilizamos un estudio numérico inspirado en el Programa de Comedores Escolares de Puerto Rico, el cual ofrece servicios a escuelas públicas y privadas en Puerto Rico.

*To my parents,*
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## 1. Introduction

Consider a non-profit firm that provides food to its consumers, e.g. school cafeterias operated by governmental agencies. United States' schools cafeterias aims to provide each student with a food tray composed of a food assortment (or menu) that complies with or exceeds daily nutritional requirements set by the USDA (United States Department of Agriculture)[1]. In this example, as is the case of Puerto Rico's Department of Education, the menu or food assortment decision is made considering two factors: students' preference for each food item and the nutritional composition of the daily menu. None of these factors consider direct or indirect costs of providing such food items. Hence, one can argue that the menu offered considering only these two factors will likely require high operational costs compared to other acceptable menus, i.e. menus that also comply with nutritional requirements while keeping a suitable level of consumer demand. This thesis presents a cost minimization model that aims to find a cost efficient menu that meets nutritional requirements and maintains an acceptable demand level.

We model the 'menu' (or assortment) as a combination of food items, e.g. lasagna and bread, which is offered by the firm on any particular day. Each food item in this menu will have its own unit purchasing and cooking costs. Furthermore, the combination of items influences demand. The firms observe daily uncertain demand which is naturally influenced by the menu (or assortment) offered that day. In this scenario, there is a particular demand characteristic that poses a challenge and it is that consumer's preferences on a particular item are influenced by the presence of other items offered. For example, spaghetti does not have the same demand influence when paired with chicken than when paired with beef.

The supply chain scenario considered consists of distributions centers that carry inventory and distribute the items among the firms following a distribution cycle (see Figure 1). In this distribution cycle each firm is visited once. In every visit the firms receive the items for them to store. This inventory is stored at the firm level until offered to the consumers.

[1] http://www.fns.usda.gov/cnd/lunch/

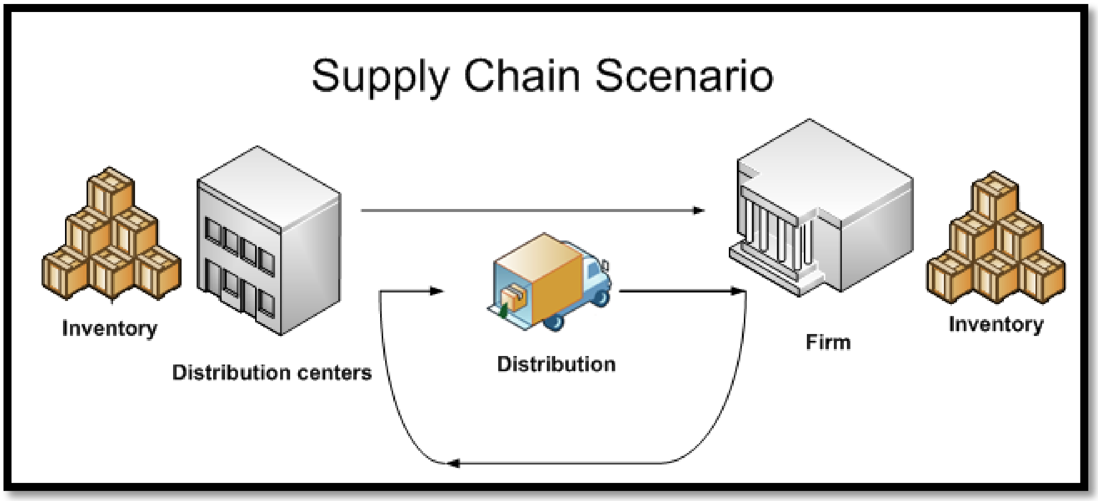


**Figure 1. Supply chain scenario and distribution cycle**

Our model considers a peculiar situation where inventory and demand may differ for two types of items, prepared and unprepared items. Once a food item is prepared (cooked), there exists the possibility of demand-inventory mismatch because of stochastic demand. Continuing with the motivating example, this demand-inventory mismatch will occur when the cafeteria chooses to prepare the food for a certain demand and the demand realization did not match. If the demand is smaller than the quantities offered, we assume the items will be sold at a certain salvage value. In contrast, unprepared items will incur holding costs.

Although this work presents a cost minimization model, we include in the model two types of revenues, which are inspired by the motivational example. These non-profit firms receive external funds as refund for each consumer served[2]. Hence, we included a refund per consumer served by the firm. Also, as another source of revenue, the leftover cooked items can be sold at a low price to a third party. For example, the excess cooked food items are sold to farmers to feed the animals (e.g. pigs). Hence, we included revenue in the form of a salvage value.

In the literature there are fields of studies that are related with the work presented here. This mathematical formulation has relevance in the food management and assortment and

[2] http://www.camaraderepresentantes.org/files/pdf/%7B94AEC159-ECC8-459D-8A79-FD38EF9EF33F%7D.doc

inventory decision areas. For the most part, food management related works have focused on menu selection that maximizes consumer preferences or maximizes nutritional value, e.g. the works of Kashima, Matsumoto and Ishii (2008) and (2009). Nonetheless, there are some works that has focused on cost minimization for the customer, e.g. Ford (2006). Our work contributes to this stream by studying a cost minimization model for the firm (not the consumer) taking into account nutritional constraints and consumer preferences. On the other hand, defining menu as a combination of items relates us with the assortment decision's literature. There is vast literature interested in the question on how to plan the assortment. As mentioned in Kök and Vaidyanathan (2008), most of these literature models present an assortment for a single product category. Nonetheless, in the operations literature there is recent interest in modeling multiple categories products while considering operational costs. Moreover, the demand model considered in Rodríguez, B. and Aydin, G. (2011) presents a multiple categories problem where the item utility does not depend on other items in the assortment. Our work contributes to this stream of research by explicitly considering interplay between items on the assortment preferences.

We highlight the relevance of our model using a numerical study inspired in the Puerto Rico School Meal Program (PRSMP), which provides foods to public and private schools in the island. We used the data of a single school within the PRSMP with a reduced number of items. To verify the model, we used a subset of nine food items of five different food categories from one hundred eighty-six different food items that has been used by the firm during 2010 and we obtained an optimal assortment. With this simplified scenario, using real and estimated parameters, we have found an optimal assortment compound by an item of each family. Using the same scenario, we structured a sample data for ten food items using it as a base model where we had two items per food category with the same parameters values. Changing the parameter values we were able to describe the structure of the optimal assortment in different instances and illustrate how the firm should take the assortment planning decisions taking into consideration the food items characteristics.

This thesis points to some important directions for future research about the modeling work presented in this thesis. An immediate extension of our model is to consider more than one period. If the plan horizon is extended, an important characteristic of these food items must be considered, which is that they are perishable. Therefore, incorporating risk of inventory-loss to the model will have an important contribution to our model, which will affect the inventory holding cost calculation. On the other hand, the supply chain studied can be extended incorporating more echelons, for example: distribution centers and purchasing firm offices, incorporating how operational costs are affected and including new features that in the lower echelon does not exists.

In the next section we review the related literature. In Section 3, we explain in details the problem, the model formulation, and the assumptions made. The motivational example is described in Section 4. Results of our model are presented in Section 5, using a numerical study. Finally, we conclude and present the future work in Section 6. The proofs and model in Lingo are included in the Appendix.

## 2. Literature Review

This work has relevance in different research areas. This section will be organized in two general subsections, which we labeled Food Management and Assortment and Inventory Decision. Food Management includes work related to the research of the optimum menu planning given different restrictions such as nutritional constraints. Assortment and Inventory Decisions focuses on finding the optimum assortment and inventory level that maximizes profits. Next we highlight the contributions of our work to both of the above mentioned areas.

### 2.1 Food Management

There are several studies made in the area of food management, which as Balintfy (1975) defined, "is concerned with the decision making problems of feeding a given population by converting raw food into menu items". These researches have studied this area in different aspects but all with the same goal, including the same decision variable as ours, the menu planning. In 1975, Balintfy (1975) worked with mathematical programming to find the combination of menu items for a series of days, which satisfied nutritional requirements at the lowest cost. He also considered the structural, compatibility and variety of constraints for a nonselective menu[3]. Balintfy (1975) applied his work in different feeding institutions, like hospitals, in order to demonstrate its validity, reducing costs from 10 to 15 percent in such institutions. Other studies, like this one, show researchers that have the common goal of minimizing costs. Sklan and Dariel (1993) used a mixed integer-programming algorithm to plan nutritional diets for humans at minimum costs. The algorithm calculates three different menus per day for one week, including breakfast, lunch and dinner, while minimizing the costs of the diet, achieving more than five percent in economic benefits. The diet presented only takes into consideration nutritional requirements based in low cholesterol and low energy program, which differs from ours because we consider all daily nutrients needed measured by quantity per serving per food category and we also incorporate consumer preferences for the menu.

[3] Nonselective menu refers to when the consumers do not have the opportunity to choose from the menu the items that they prefer.

Similar to previous works, Ford (2006) demonstrated in his thesis the use of linear programming to find the cheapest combinations of foods that meet with the minimal daily nutrients required by an individual, using a university student's weekly budget as an example. The objective function was a simple sum of purchasing costs per servings of each item multiplied by the number of servings that specific item would be eaten over the course of a week. Ford (2006) first studied the scenario considering only the nutritional constraints, but the amount that he had to eat of each item didn't meet his taste requirements.

Goal programming has also been used to ensure optimal health while reducing costs. Ferguson, et al. (2006) used goal programming to design the optimal food-based complementary feeding recommendations (CFRs) through four phases. This programming considered goals instead of constraints, where they consider diet costs as one of them.

There have been authors who have researched previous work and applied new and updated data. One example is Garner-Garille and Gass (2001), who reworked the Stigler's Diet Problem of 1945. Stigler (1945) implemented linear programming to determine the minimum diet cost while satisfying nutritional requirements. Garner-Garille and Gass (2001) used this work to incorporate updated nutritional and costs data to show how this past research impact in cost-effective diets. There were three different scenarios presented using the 1945 work as a basis. In the first scenario, they used the nutrients considered by Stigler with the updated food prices; in the second, they use the updated food and nutritional data; and for the final scenario, the Stigler food prices of 1939 and the updated nutritional data. With this study they concluded that the previous diet problem had some nutritional inadequacy that motivated others researches to evaluate new questions about integer and goal programming.

The aforementioned literature deals with menu planning but only considering the unit purchasing cost of the food item on the market. We contribute to this literature by taking into consideration additional costs, such as: unit cooking and inventory holding. Moreover, our work models consumer demand whereas previous work only offered the cost-effective menu planning for one consumer at a time.

The food management literature has not been limited only on minimizing costs; Kashima, Matsumoto and Ishii worked in two different models based on maximizing the individual taste satisfaction. In 2008, they worked with fuzzy mathematical programming to address a well-balanced menu planning while satisfying individual tastes. In 2009, they formed a genetic algorithm to understand health problems using multidimensional 0/1 knapsack problem. The model was an integer programming of 0/1, acceptance or reject of each dish to find out only one menu that maximizes the individual satisfaction while the negatives and positives nutritional requirements are satisfied. In our model, we implicitly capture consumer satisfaction by modeling consumer demand, which is influenced by the items offered.

Others studies in this area have focused on scheduling serving frequencies for a finite time horizon. Balintfy, Ross, Sinha and Zoltners (1978) worked with a two-phase model to maximize frequencies or minimize costs. In the first phase, the least cost servings are obtained and then used in the second phase to obtain a meal-by-meal day plus day menu, maintaining a nutritional and cost control. In other studies, there have been authors, like Darmon, Furgeson and Briend (2002), which used cost constraint linear programming models to show how the budget directly influences food selection and therefore diet quality. Contrary to our proposed model, that will focus on minimizing cost while satisfying nutritional requirements. Also, as in previous works, these works are focused in consumers, contrary to our work where we are focused in a firm.

## 2.2 Assortment and Inventory Decisions

As mentioned in Kök, Fisher and Vaidymathan (2008), most of the literature in assortment planning focuses on single category or subcategory of product at a given time. In our model, we are focusing the cost minimization model for a single period considering more than one category. These previous works that are focused in single category differ among themselves and with our work in many of its assumptions. van Ryzin and Mahajan (1999) were the first to study single period assortment planning and inventory decisions under Multinomial Logit Model (MNL). Their objective was to find the optimal assortment and the newsboy inventory level that maximizes the expected profit, assuming equal retail prices and unit costs. Like this previous study, Bish and Maddah (2004) studied the price, the inventory and the assortment size for a single category with stochastic demand and equal costs that maximizes expected profit using MNL model. They studied two situations: what are the assortment size and inventory level if the price was fixed and, the opposite, what is the inventory level and price if the assortment size was fixed. Assuming stochastic demand, equal costs and process, Gaur and Honhon (2005), considered a single period retail assortment planning and newsboy inventory management to find the optimal assortment, inventory decisions and product location that maximize the expected profit. Differing from the previous mentioned researches, they used the Locational Choice Model to represent the stochastic demand that arrived according the Poisson distribution. The problem was first studied using static substitution and then was considered dynamic substitution, which can always occur between two products. In our case, the question of considering substitution or not is not relevant since in the scenario considered it is very likely that at most one item per category will be offered. On the other hand, Kök and Fisher (2007) proposed a single period model assuming stock-out based substitution within category and equal costs and prices. They used a heuristic to find the best assortment and inventory level of a single subcategory that maximizes profit subject to shelf space allocation. Then, they explained how to expand it to multiple subcategories.

The proposed model differs from these previous works in that we consider multiple product categories and unequal operational costs, studying its effect in the optimal assortment, instead of the price effect. Like them, we are working with stochastic demand and single period plan horizon, but we are adding to our model a demand model that acknowledges the demand influence between items and restrictions in the assortment composition.

There have been some works that considered different operational costs similar to our model. Li (2007) studied a single period assortment optimization using unequal costs parameters in a process using MNL model. He assumed that the store traffic is a continuous random variable and that the customer does not substitute if his favorite product is out of stock. Using a heuristic, he concluded that the optimal assortment should contain products that have the highest profit rate. Kök and Xu (2010) also made his study with a static substitution and stochastic demand but with a different objective than previous work. They wanted to compare how the optimal assortment that maximizes the profit of a category with different brands behaved according to different assortment management: centralized and decentralized category management. They concluded that the optimal assortment depends on the hierarchical structure of the consumer choice process.

These next researchers assumed a stochastic demand and unequal operational costs and prices as we do. Smith and Agrawal (2000) used an exogenous demand to model a probabilistic demand in which the objective was to find the inventory level of an assortment that maximizes the profit. In addition, they formulated the problem using non-linear integer programming. This mathematical formulation was subject to various restrictions such as floor space, assortment size and open-to-buy budget, which are not considered in our study. Like ours, they considered a fixed type-1 service level, but they used this to ensure that their inventory decision variable achieved this level. In our case this inventory level is a given parameter that is defined as the firm complies with the item demand.

Honhon, Gaur and Seshadri (2006) considered dynamic substitution with different costs and prices to maximize profit. Consumer preferences were taken into account but they used it to defined customer types, which buy the most ranked product. Their focus was to compare how the assortment and inventory level behave if all consumers are from the same type and how behave if they are from different ones.

All previously discussed literature was focused on single product category, contrary to ours. There are few works that consider multiple categories as we do. Cachon and Kök (2007) took into consideration multiple products categories with unequal costs like ours, but with different focus because we do not consider the competition between categories and they do. Their purpose was to demonstrate that category management (CM) never finds the optimal solution and how it affects prices, which lead to poor decisions contrary to centralized management, which is focused on how the optimal assortment takes all the categories into consideration at the same time. Another work that considers multiple categories is Rodríguez and Aydin (2011) who study the assortment selection and pricing for configurable products under demand uncertainty. They find that the optimal prices are the ones where all variants of a component share the same effective margin. As we do, they consider the demand influence by combinations of items, but contrary as our work their utility contribution of an item is independent of which item is included in the assortment (with which item it is matched). In our work, the items' demand contribution for the assortment is influenced by which is the combination of items. Also, Rodríguez and Aydin (2011) do not have restrictions on assortment composition, as we have.

In the assortment and inventory decision areas there are few works that take into consideration multiple periods planning. Hariga, Al-Ahmari and Mohamend (2007) and Flapper et al. (2010) considered multiple periods making it a more general model, but they simplify the model by assuming deterministic demand, contrary to ours. Hariga, Al-Ahmari and Mohamend (2007) considered no substitution and multiple categories. As decision variables, they defined the variety of products, display locations, ordering quantities and shelf space allocation. We only consider the product variety. The constraints considered here were the shelf space capacity for the display area and backroom. They do not allow shortages, contrary from our model, which permits them. On the other hand, Flapper et al. (2010) didn't allowed substitution and considered multiple categories, but their focus was to make a comparison between product and customer based strategies, where their objective was to find which of these strategies provided more profit.

Our model has some similarities and differences with all previous assortment planning and inventory decision literature. Among the similarities with some of these works are that most of them considered stochastic demand and single period plan horizon like us. In terms of costs there are some works that considered unequal costs as we do, but on the contrary with other works this is a difference because they assumed equal costs. Most of the works in this area studied the price effect in the optimal assortment, but in our work we are not considering price. We are studying the optimal assortment that minimizes the operational costs, and then we are studying the effect of the costs instead of price. On the other hand, there are works that have as given parameter the assortment to find other assortment characteristics like inventory level or to compare categories management strategies, contrary as our where the assortment is the decision variable. Two main differences of our work with all previous works are that we consider that the demand is influenced by the presence of the combination of items within the assortment and also in that we have restrictions in the assortment composition.

In summary, to the best of our knowledge, there is no literature that focuses on which is the optimal assortment that minimizes costs for a non-profit firm, assuming a given stochastic demand that is influenced by the presence of other items within the assortment. Adding unequal operational costs parameters (like unit holding costs and unit purchasing and cooking costs), funding revenues and products salvage values with a given inventory level, subject to the availability of the entire batch that satisfied a type-1 service level and assortment composition constraints.

## 3. Problem Description

We consider a single period problem where a firm offers a daily assortment composed of a combination of items. Each item, denoted with *i*, can belongs to one or more food categories *k*, e.g. chicken tenders and rice with beans, where the former belongs to the meats category and the latter belongs to the cereals and grains food categories. Let $X_i$ be our binary decision variable, and define as

$$X_i = \begin{cases} 1, & \textit{if item i should be offered in the assortment} \\ 0, & \textit{otherwise.} \end{cases}$$

Consider a two-echelon supply chain scenario that consists of a distribution center and a firm, which have the objective in fulfilling a consumer request. In both echelons (or parties) inventory is stored. Furthermore, demand is observed at the lower echelon and inventory is received periodically, i.e. inventory is received every fix number of periods (e.g. each month). (For the sake of model tractability, in this work we model only the costs at the lower echelon level of the supply chain, e.g. school cafeteria).

At the lower echelon there are several events that take place. Figure 2 has a flowchart representation of those events.

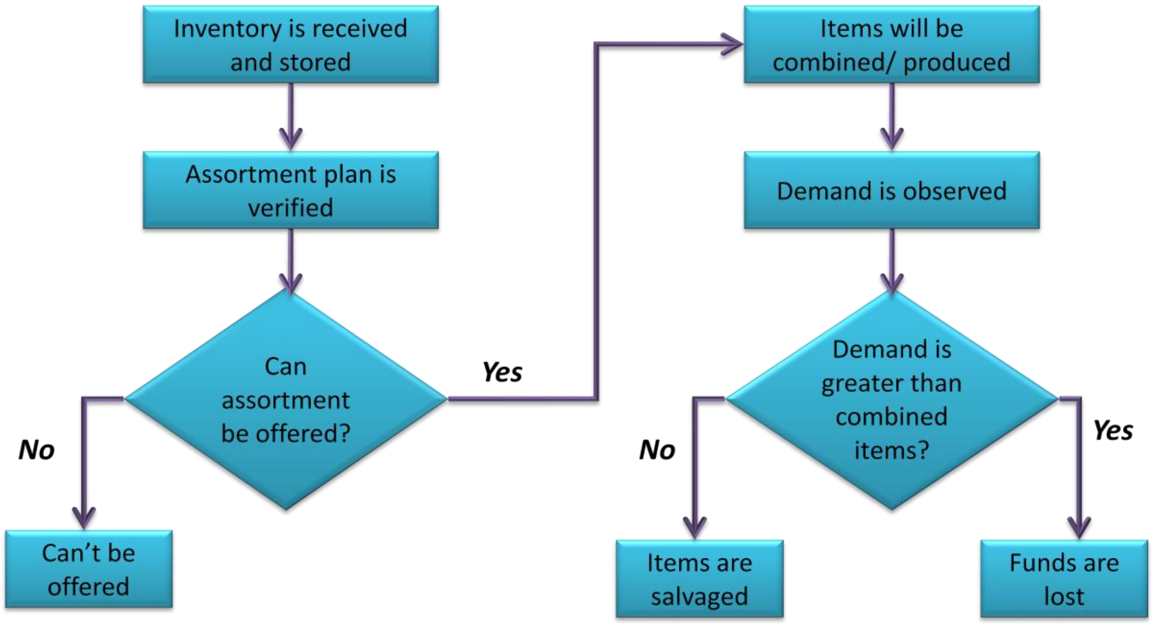


**Figure 2. Chronological events at lower echelon level of the supply chain**

Observe from Figure 2 that after the food items are received at the lower echelon, the firm stores the item-level inventory until the assortment planning indicates that it needs to be offered. Before items are combined (or produced), the firm has to determine if the inventory meets or exceeds the necessary quantity to satisfy a type-1 service level, which is the probability that the firm complies with the demand of the food item $i$ on any given period. If there are enough raw materials, then the items will be combined as the total cost of all individual units purchasing and cooking costs of the items included in the assortment. Then, demand is observed. At the end of the period the firm receives revenues from external funds per consumer who received certain quantity of food items[4]. Alternatively, if there are leftover items they will be salvaged at a predetermined value. We next describe the consumer demand model.

## 3.1 Demand Model

We assume that demand observed by the firm is uncertain and it is influenced by the combination of items offered. This demand has a characteristic that poses a challenge, which is

[4] An established minimum quantity of items must be selected by the customer in order for the firm to get the external funds. This requirement (constraint) is based on the case were federal agencies reimburse schools only if the students get a minimum quantity set by the agency.

that the number of consumers that assist to the firm when a particular item is offered are influenced by the presence of other items offered within the assortment. For example, we can offer rice or spaghetti only, and the number of consumers that will assist to the firm may be different for each one. Then, if we denote β as the number of consumers that assist to the firm when a particular item is offered, or in other words, how many consumers an item provides to the assortment's demand, they will have its own β(rice) and β(spaghetti), respectively. But, when these food items are combined with meatballs, these quantities of consumers that participate in the firm change. Then, we can have a change in the number of consumers' participation when we offered rice with meatballs, β(rice, meatballs), and when we offer spaghetti with meatballs, β(spaghetti, meatballs). Furthermore, we are assuming that these changes may reflect an increase or decrease in the consumers' participation if the influence of the combinations of items is favorable or not for them. Then, we allow the number of consumers that participate in the firm to be different when we offer these items individually. Then, we are assuming that

$$\beta(rice, meatballs) \neq \beta(rice)$$

and

$$\beta(spaghetti, meatballs) \neq \beta(spaghetti).$$

In addition, we are considering that these changes in the number of consumer's participation can be greater for one assortment than for the other one, which is a novel feature in the model that we will be presenting. Then, we are assuming that

$$\beta(spaghetti, meatballs) - \beta(spaghetti) > \beta(rice, meatballs) - \beta(rice).$$

Therefore, meatballs do not have the same demand influence when paired with rice than when paired with spaghetti.

As explained in the previous example, we are assuming that we have a number of consumer's participation for each food item *i* that will be denoted as $\beta_i$. Also, we are considering that we can have an adjustment in the number of consumer's participation when we offer a combination of two food items, which will be denoted as $\beta_{ij}$. These consumers' participation measured the quantity of the demand (consumers) that a food item *i* provides to the total assortment's demand and the quantity of demand that is adjusted when the firm offer the combination of food item *i* with food item *j*, respectively.

Denote a particular assortment with the letter *S*. We allow each consumer to choose, from the assortment offered, *S*, any combination of items $i \in S$. In our model, we consider a unit of demand any consumer that chose to take at least one item from the assortment offered. Hence, there is a demand per assortment, which is influenced by the presence of other food items within that particular assortment *S*. Then, the expected demand per assortment *S*, $Y_s$, as we model, could be calculated using a multi-regression analysis, as

$$Y_s = \beta_0 + \sum_{i=1}^{n} \beta_i X_i + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \beta_{ij} X_i X_j, \quad (1)$$

where $\beta_i$ is the number of consumers' participation for the food item *i* and $\beta_{ij}$ is the adjustment in consumers' participation for the combination (interaction) of food item *i* with the food item *j*.

The demand observed by the firm is uncertain and so far $Y_s$ is the expected value of the assortment demand. To model the uncertain demand we decided to incorporate an error term $\varepsilon_s$ to the demand formulation. In the literature, there are some demand models that are used to model this kind of uncertain demand. There are two demand models relevant to our work that can be applied to our formulation, which are the *multiplicative* and *additive* demand models. The *multiplicative* demand model has been used before by some authors in the literature of pricing when the demand depends on price, for example Rodríguez and Aydin (2011), Song, Ray and Boyaci (2007), and Yao, Chen and Yan (2005). On the other hand, the *additive* demand model has been used in works were demand uncertainty is studied and pricing is involved, e.g. Sošić (2010). Also, there are authors that work with the two demand models and compare each other, like Petruzzi and Dada (1999). We incorporated uncertainty in our demand model using an error term $\varepsilon_s$, and we compared the two-demand model, multiplicative and additive demand models. Using the multiplicative demand model, the demand of the assortment *S*, $D_s$, will be given by

$$D_s = Y_s \varepsilon_s, \tag{2}$$

where $\varepsilon_s$ is an independent and identically distributed (i.i.d) normal random variable with mean one ($\mu_{\varepsilon_s} = 1$) and standard deviation $\sigma_{\varepsilon_s}$. On the other hand, using the additive demand model, the demand of the assortment *S*, will be given by

$$D_s = Y_s + \varepsilon_s, \tag{3}$$

where $\varepsilon_s$ is an i.i.d normal random variable with mean zero ($\mu_{\varepsilon_s} = 0$) and standard deviation $\sigma_{\varepsilon_s}$.

These models differ from each other by their coefficient of variation. For the multiplicative demand the coefficient of variation is given by

$$CV = \frac{\sqrt{Var(Y_s \varepsilon_s)}}{E(Y_s \varepsilon_s)} = \sigma_{\varepsilon_s}, \tag{4}$$

where $\sigma_{\varepsilon_s}$ is the standard deviation of the normal random variable $\varepsilon_s$ for the multiplicative demand. On the other hand, the coefficient of variation for the additive demand is given by,

$$CV = \frac{\sqrt{Var(Y_s + \varepsilon_s)}}{E(Y_s + \varepsilon_s)} = \frac{\sigma_{\varepsilon_s}}{Y_s} \tag{5}$$

where $\sigma_{\varepsilon_s}$ is the standard deviation of the normal random variable $\varepsilon_s$ for the additive demand. Notice that the coefficient of variation for the multiplicative demand is constant and equal to the standard deviation $\sigma_{\varepsilon_s}$, and on the contrary for the additive demand the coefficient of variation varies with respect of $Y_s$. Then, we assume that no matter how large or small the demand is, the coefficient of variation is the same, hence we decided to use the multiplicative demand model as establish in (2). Furthermore, Driver and Valletti (2003) studied the effects of multiplicative demand and the additive demand models with the certain values and the multiplicative demand model was characterized as the model with neutral effects. Therefore, the expected demand (as described before) and standard deviation of the assortment *S* will be given by

$$E(D_s) = E(Y_s \varepsilon_s) = Y_s \tag{6}$$

and

$$\sigma_{D_s} = Y_s \sigma_{\varepsilon_s}, \tag{7}$$

respectively.

Given the demand model, we describe next how it is incorporated in the firm's optimization model. It is assumed that the firm has the knowledge that the consumer's preference per each food item *i* is different and that the consumers can choose the quantity of food items *i* that they desired from the assortment. For example, they can offer rice with sausage and pink beans with an expected assortment demand of two hundreds of consumers ($Y_s = 200$). But, they expected that all or most of them will choose rice with sausage and other expected percent will choose pink beans. Then, they will offer rice with sausage for the same quantity of the expected assortment demand (200 consumers or 100% of $Y_s$) and for pink beans only the expected percentage of this demand that will eat it (e.g. 90% of $Y_s$). Then, the firm cook different quantity per each food item *i* in order to reduce the excess of cooked food items. Hence, there is a demand per food item *i* that is requested at some uncertain rate by the consumers and we are assuming that this rate can be different for the same food item *i* because its demand is influenced by the combination of items offered. Let $\gamma_i$ be the uncertain rate of the expected assortment demand at which the food item $i$ is requested by the consumers with an expected rate of $\mu_{\gamma_i}$ and standard deviation of $\sigma_{\gamma_i}$. Then, the demand of food item $i$, $D_i$, is given by

$$D_i = Y_s \gamma_i. \tag{8}$$

Therefore, the expected demand and the standard deviation of the food item $i$ is given by

$$\mu_i = E(D_i) = Y_s \mu_{\gamma_i} \tag{9}$$

and

$$\sigma_i = Y_s \sigma_{\gamma_i}, \tag{10}$$

respectively.

With the demand model formulated we next explain the inventory related costs.

## 3.2 Related Costs and Revenues

Suppose for a moment that the assortment for the period is fixed. The firm will decide whether it will be able to offer (or not) that assortment based on the availability of raw material needed to produce that combination of items. The alternative to producing this assortment is to produce one of the other available assortments assigned to another period. For example, school cafeterias have the daily assortment or menu schedules for approximately three months. If any given day, the cafeteria determines it does not has enough items to meet the demand of a certain day, then the cafeteria can choose to offer any other menu assigned to the same week. We incorporate this decision in our model by only considering as feasible assortments the ones that the quantity available inventory measures up to the demand's type-1service level.

Provided the items' quantity to offer per consumer serving, we measure the item's level quantity as consumers' servings instead of actual quantity, which simplifies the model's exposition. Given that our demand follows a continuous distribution, food item $i$ quantity (measured in number of consumers) to offer will be given by

$$Q_i = \mathrm{F}^{-1}(\alpha), \tag{11}$$

where $\alpha$ represents the in stock rate which is the type-1 service level that will be exogenously fixed by the firm and $\mathrm{F}^{-1}$ is the inverse of a cumulative distribution function. For the sake of simplicity, we assume that food item $i$ demand follows a normal distribution[5] and the number of consumers to offer will be,

$$Q_i = Z_{Q_i}\sigma_i + \mu_i, \tag{12}$$

where,

$$Z_{Q_i} = \Phi^{-1}(\alpha) \tag{13}$$

and $Z_{Q_i}$ is the standard normal random variable and $\Phi$ is the standard normal cumulative distribution function.

The firm offers the food items $i$ measured by the amount per serving (e.g. ounces) per food category $k$. Let $\lambda_{i,k}$ be the amount per serving per food category $k$ of food item $i$. Each food

[5] To support our assumption, we studied the demand for a three month period of a local school. We observed that some of the items studied had appeared to follow a continuous distribution for the demand. However, due the limited repetitions of items offered in our motivational example, we were not able to statistically prove that they follow a specific distribution. Hence, we assume that the item's demand $(D_i)$ can be modeled as a normal distribution, given that this distribution is frequently used in the assortment and inventory decision and often result in a close estimation of the demand behavior.

item *i* can belong to more than one food category *k*. For example, hotdog is composed by bread from the cereals category and sausage from the meats category, and hence it belongs to both of those categories. After it is cooked, it will be served by the total amount per serving per food item *i* (e.g. hotdog). Therefore, the total amount served, will be the total amount per serving of all food categories that compose the food item *i*, which will be needed to calculate the total associated operational costs per each food item *i*. This last expression can be formulated as

$$U_i = \sum_k \lambda_{i,k} \quad \forall\, i \tag{14}$$

where $U_i$ is the total amount per serving per food item *i* that must be offered. Then, the total quantity of food item *i* to offer to satisfy a predetermined type-1 service level[6] will be equal to $Q_i U_i$.

Any particular assortment will have its own unit purchasing and cooking costs. If we let $C_i$ denote the unit purchasing and cooking cost for food item *i*, which is defined as

$$C_i = c_{p_i} + c_{c_i} \tag{15}$$

where $c_{p_i}$ is the unit purchasing cost and $c_{c_i}$ is the unit cooking cost for food item *i*; then, the total cost incurred by the firm per period (each time they prepared it) will be

$$Purchasing\ and\ cooking\ costs \;=\; \sum_i C_i\, U_i Q_i X_i. \tag{16}$$

where $U_i$ is item *i* quantity per serving that must be offered.

[6] Recall, that the type-1 service level is the probability that the firm complies with the demand of the food item *i* on any given period.

Recall that when the firm knows the total quantity needed to satisfy the desired service level, $Q_iU_i$, they verified if the inventory level in stock satisfy this amount. Then, let $I_i$ denote the inventory level of food item $i$. If the inventory level, $I_i$, is equal or greater than the amount needed to satisfy the service level, $Q_iU_i$, the food item *i* can be a candidate for the assortment. If not, the food item *i* is not considered for the assortment. In order to guarantee this feasibility we introduce the following constraint,

$$X_i(I_i - Q_iU_i) \geq 0 \quad \forall\, i. \tag{17}$$

If food item $i$'s availability, $(I_i - Q_iU_i)$, is positive the decision variable $X_i$ could take a value of one, making it a candidate to be part of the assortment, otherwise $X_i$ will be forced to be zero. If an inventory level $I_i$ for food item *i* is greater than the amount needed to satisfy the desired service level then the excess will be carried to the next period incurring in an inventory holding cost $h_i$. For one period problem, we are considering only the elements of costs that are affected by the number of unprepared items at inventory, assuming that the work load, space and indirect costs remain fixed. Then, we are considering that the inventory holding costs, $h_i$, will be the opportunity cost of having an unit of food item *i* in stock, which is calculated as

$$h_i = i_R * c_{p_i} \tag{18}$$

where $i_R$ is an interest rate per period and $c_{p_i}$ is the unit purchasing cost for item *i*. Then, the total inventory holding costs will be

$$Inventory\ holding\ \ costs = \sum_i h_i(I_i - Q_iU_i)X_i. \tag{19}$$

We assume that the firm will only carry items in the inventory that are scheduled to be offered. In our model, this means that the firm will not store any item that is not in the assortment for the particular period. Hence the firm will incur only in holding costs for the excess quantity of the items *i* that will be offered (i.e. items with $X_i = 1$), and otherwise the items not in the optimal assortment (i.e. items with $X_i = 0$)) will not incur in holding costs.

The scenario changes when the inventory-demand mismatch occurs with the prepared (cooked) food items. Once an item is prepared (cooked), there exists the possibility of demand-inventory mismatch because of stochastic demand. Continuing with the motivating example, this demand-inventory mismatch will occur when the school cafeteria chooses to prepare the food for a certain demand and the demand realization did not match. If demand is smaller than the offered items, we assume the excess of prepared food items will be sold at a certain salvage value, in contrast with unprepared items that incur in holding costs. In our motivating example, PRSMP sells the quantity excess of cooked food items to farms (e.g. pigs farms) to feed the animals. Let $g_i$ denote the revenue per quantity of excess of food item *i*, then the total revenue received is

$$Expected\ salvage\ value = \sum_i g_i U_i E(Q_i - D_i)^+ X_i \tag{20}$$

where $U_i$ is the total amount per serving that must be offered of food item *i*, as was calculated in (14), and $E(Q_i - D_i)^+$ is the expected excess demand for cooked food item *i* [7]. Because the consumer demand is assumed to follow a continuous distribution, the expected excess demand for prepared (cooked) food item *i*, $E(Q_i - D_i)^+$, can be expressed by definition as

$$E(Q_i - D_i)^+ = \int_{-\infty}^{Q_i} (Q_i - D_i) F(D_i)\, dD_i. \tag{21}$$

Assuming that the demand follows a normal distribution, the previous expression can be expressed as

[7] $E(Q_i - D_i)^+$ is the maximum between zero and $Q_i - D_i$. Hence, $E(Q_i - D_i)^+ = \max(0, Q_i - D_i)$.

$$E(Q_i - D_i)^+ = \sigma_i \left[ Z_{Q_i} + \phi\left(Z_{Q_i}\right) - Z_{Q_i}\left(1 - \Phi\left(Z_{Q_i}\right)\right)\right], \tag{22}$$

where $Z_{Q_i}$ is the standard normal random variable and $\phi\left(Z_{Q_i}\right)$ and $\Phi\left(Z_{Q_i}\right)$ are the normal standard probability density and cumulative distribution of demand, respectively (see Appendix A for details on the proof for this result).

Although this work presents a cost minimization model for a non-profit food firm, in addition to the food item salvage value, it is assumed that the firm receives other sources of revenues that are inspired by the motivational example. These firms receive external funds as refund for each consumer served[8]. In our motivational example the federal agency, who gives these funds, consider as "consumer served" the consumers who choose three or more food items. Hence, we included in our model a refund per consumer served who find and choose *m* or more food items. We model the total quantity of consumers that find and choose *m* or more food items as the product of the expected assortment demand, $Y_s$, and the probability that a consumer find and choose *m* or more food items. Let $f$ denote the fund received per consumer satisfied. Then, the expected funding revenue that the firm will receive is given by

$$Expected\ Funding\ revenue = fY_s P(l \geq m) \tag{23}$$

where $l$ is the number of items to find and choose and $P(\cdot)$ stands for probability.

## 3.3 Feasibility Constraints

We are considering that the quantity per food category must comply (or exceeds) with the daily nutritional requirements established by the USDA agency. These daily nutritional standards are measured by the amount per serving per food categories $k$. To guarantee that the nutritional requirements are satisfied the next set of constraints was established

[8] http://www.camaraderepresentantes.org/files/pdf/%7B94AEC159-ECC8-459D-8A79-FD38EF9EF33F%7D.doc

$$\sum_i X_i \lambda_{i,k} \geq R_k \quad \forall\, k \tag{24}$$

where $R_k$ is the standard amount per serving per food category $k$ that must be served in each period.

On the other hand, as mentioned before, the quantity offered of items per each category is limited, that is, we are considering that they offer an established quantity of items per each food category $k$ (e.g. one meat or two vegetables, etc.). In our model, we are assuming that this quantity must be greater or equal than one item per food category. Then, a second set of constraints is formulated to ensure that the desired composition is offered

$$\sum_i X_i F_{i,k} \geq 1 \quad \forall\, k, \tag{25}$$

where $F_{i,k}$ is a binary parameter that has a value of one if $\lambda_{i,k}$ is positive ($\lambda_{i,k} > 0$) or zero otherwise.

### 3.4 Summary

In summary, the objective function of our model is formulated as

$$\min_{X_i} \sum_i C_i\, U_i Q_i X_i + \sum_i h_i (I_i - Q_i U_i) X_i - \, f Y_s P(\geq m) - \sum_i g_i U_i E(Q_i - D_i)^{+} X_i. \tag{26}$$

There are a few constraints that need to be satisfied by the assortment offered. Some of these constraints are due to the requirement set for the firm, e.g. nutritional requirements constraint, and others are for modeling purposes. We next provide the constraints for our model. (Note that the first three sets of constraints are the equations 24, 25, and 17; respectively.)

- ✓ $\sum_i X_i \lambda_{i,k} \geq R_k \quad \forall\, k$

- ✓ $\sum_i X_i F_{i,k} \geq 1 \quad \forall\, k$

- ✓ $X_i (I_i - Q_i U_i) \geq 0 \quad \forall\, i$

- ✓ $X_i \in \{0,1\}$ (27)

The first set of constraints is established to ensure that nutritional requirements per each food category *k*, established by the USDA agency are met. The second set of constraints guarantees that the quantity of food items offered per each food category *k* is greater or equal than one. Therefore, this constraint guarantees that the firm will offer at least one item of each food category *k*. The third set of constraints guarantees that if a food item *i* is offered, the firm has the necessary quantity to satisfy the desired type-1 service level in stock, otherwise this constraint guarantees that the food item *i* will not be offered. Finally, the last constraint is the standard integrality constraint for the decision variable.

## 4. Motivational Example

To highlight the relevance of our model we next present an example of the application of our model using as scenario the Puerto Rico School Meal Program (PRSMP). In this section we describe the scenario studied.

The PRSMP is operated by Puerto Rico's Department of Education. PRSMP provide food service to public and private schools' cafeterias in Puerto Rico. They offer nutritional lunches and breakfasts services to children between kindergarten to high schools (primarily children between 5 to 18 years old) during the fall, spring and in some schools during summer.

The supply chain of the PRSMP consists of six distribution centers where food and equipment is stored (see Figure 3). This food is purchased by the government and then it is transported to each distribution center by the supplier. Deliveries to schools are made by the PRSMP by trucks once a month in the case of frozen and dry food (no fresh food). Fresh food items, such as milk, are delivered by a third party on a more frequent basis. Each distribution center has to make deliveries to an average of 220 schools that are at different locations in the school district area. Once the food items arrive at each school they are stored and served.

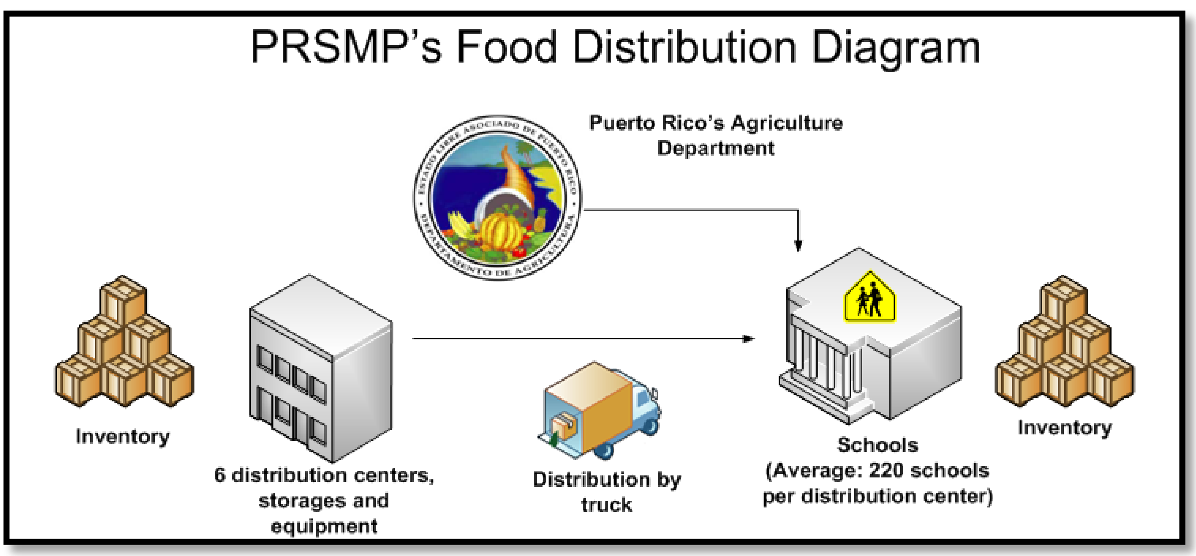


**Figure 3. PRSMP Supply Chain**

After deliveries are made, the schools have to serve the meals as scheduled by the PRSMP. These meals must offer all the nutrients required to ensure the "good health" on every child. These nutrients are divided into six different food categories, which are: meats, cereals, grains, fruit, vegetables and milk (these last one will not be considered in our study because it must be offered every day). The amounts of nutrients that they must satisfy are more specific compared with the nutritional constraints that we presented in the more general model (model in Section 3). However, the constraints presented in the general model were modified in order to model the PRSMP. The general model's nutritional constraints in (24) apply for the food categories $k$ equal to meats (*k=1*) and cereals (*k=2*). As for the remaining categories on the PRSMP case, the amount per serving is measured together to comply with only one standard nutritional quantity. Then, letting $\rho$ as the standard amount per total servings of the remaining food categories $k$, where the food categories $k$ are equal to vegetables (*k=3*), grains (*k=4*) and fruits (*k=5*), the next set of constraints was established

$$\sum_{i}\sum_{k=3}^{5} X_i \lambda_{i,k} \geq \rho \,. \tag{28}$$

Furthermore, the PRSMP has a minimum requirement for the meats, cereals and fruit categories (*k=1, 2 & 5*) in which they must offer one of each. As for the vegetable category (*k=3*) they can offer more than one food items *i*. Contrary to the previous categories, a maximum of one food item *i* of grains category (*k=4*) can be offered. Therefore, the next three constraints substitute the set of constraints in (25),

$$\sum_{i} X_i F_{i,k} = 1 \quad \forall\, k = 1, 2 \,\&\, 5, \tag{29}$$

$$\sum_{i} X_i F_{i,k} \geq 1 \quad \forall\, k = 3, \tag{30}$$

and

$$\sum_i X_i F_{i,k} \leq 1 \quad \forall\, k = 4. \tag{31}$$

The numbers of students that assist daily to these cafeterias is uncertain. Although, the school receives a good estimate by asking each day during the morning how many students wants to participate from the program and with this estimate the school decides the quantities to cook. After the firm offered all the food services, PRSMP receives federal funds for every student that received three or more food items from the assortment offered. Then, for this scenario the amount of students that is expected to receive three or more food items will be given by the product of the expected assortment demand, $Y_s$, and the probability that the student find and choose three or more items. If we define the probability of find three or more items, defined in (23), as the multiplication of the probability of find three or more items by the probability of choose three or more, we have

$$P(l \geq m) = \omega\eta \tag{32}$$

where $\omega$ is the probability of find and $\eta$ is the probability of choose, $m$ or more food items. Since the firm meets a type-1 service level, for this scenario we can model the probability of finding three ($m=3$) or more food items as

$$\omega = \sum_{l=3}^{t} [\alpha^l (1-\alpha)^{t-l}], \tag{33}$$

where, $\alpha$ is the in-stock rate established by the firm, $l$ is the number of items to find and $t$ is the total number of food items offered in the assortment ($t = \sum_i X_i$). The PRSMP has the characteristic that they offer a maximum of six items per tray including milk. Therefore, considering this characteristic the total number of food items offered in the assortment is less or equal than five $(t \leq 5)$ in this example. On the other hand for this scenario, we also can calculate the probability that a consumer choose three or more food items from the assortment, which is given by

$$\eta = 1 - P(l < 3) = 1 - P(l = 1) - P(l = 2). \tag{34}$$

Recall that we defined in Section 3.1 the expected rate of the expected assortment demand at which the food item $i$ is requested by the consumers, denoted by $\mu_{\gamma_i}$. Then, with this rate we can calculate, using formulation in (34), the probability that a consumer choose three or more food items from the assortment as

$$\eta = 1 - \sum_{i} (\mu_{\gamma_i} X_i \prod_{j \neq i} (1 - \mu_{\gamma_j} X_j)) - \sum_{i} \sum_{j \geq i+1} (\mu_{\gamma_i} \mu_{\gamma_j} X_i X_j \prod_{n \neq i \neq j} (1 - \mu_{\gamma_n} X_n). \tag{35}$$

Then substituting (33) and (35) in (32), the expected funding that the PRSMP will received for their service will be given by,

$$Expected\ funding\ revenue = f Y_s \sum_{l=3}^{t} [\alpha^l (1-\alpha)^{t-l}] \left( 1 - \sum_i (\mu_{\gamma_i} X_i \prod_{j \neq i} (1 - \mu_{\gamma_j} X_j)) - \sum_i \sum_{j \geq i+1} (\mu_{\gamma_i} \mu_{\gamma_j} X_i X_j \prod_{n \neq i \neq j} \left(1 - \mu_{\gamma_n} X_n\right)\right). \tag{36}$$

This scenario is an example that applies to our study because has the characteristics of the firm that is taken into account in our model. This non-profit program offers a nutritional food assortment to an uncertain demand of consumers, which have certain food preferences influenced by the combination of food items offered. Also, PRSMP incurred in all the operational costs taken into account in our model, like purchasing, cooking and inventory holding. The objective of this program is to provide each student with a nutritional food assortment, and as a public firm that is operated by the government, it will be useful the minimization of operational costs. On the other hand, as mentioned before, this firm has some sources of revenues that inspired the formulation of it in our model.

# 5. Results

## 5.1 Model Verification

To illustrate the results of the model presented, a numerical study was used inspired in our motivational example. One of the challenges with these types of public firms is the time it takes to get real input data for the model verification. Therefore, it was decided for purpose of verification of this model that we were going to use a combination of real and estimated data of a single school within the PRSMP with a reduced number of items.

The PRSMP has more than one hundred eighty-six different food items that belong to one or more food categories, from the five categories that we are going to consider. To verify our model we used a subset of nine food items, which are presented in the next table with its corresponding food categories (see Table 1).

**Table 1. Subset of food items used in the verification of the model**

| *i* | Food item | Food categories |
|---|---|---|
| **1** | Turkey Stew | Meats |
| **2** | White Rice | Cereals |
| **3** | Pinto Beans | Grains |
| **4** | Carrots | Vegetables |
| **5** | Peaches | Fruits |
| **6** | Rice /w sausage | Cereals & Meats |
| **7** | Pink Beans | Grains |
| **8** | Green bean salad /w carrots | Vegetables |
| **9** | Pears | Fruits |

For these nine food items, we have real and estimated data for the next parameters presented in the Table 2. For this one single period scenario, we are assuming that the initial inventory level is given by the total quantity per serving needed to satisfy the type 1-service level. Therefore, we are assuming for this illustration that

$$I_i = Q_i U_i \quad \forall\, i. \tag{37}$$

**Table 2. Real and estimated input data**

| *Real Data* | *Estimated Data* |
|---|---|
| $f$ | $c_{pi}$ |
| $\lambda_{i,k}$ | $c_{ci}$ |
| $U_i$ | $C_i$ |
| $F_{i,k}$ | $h_i$ |
| $R_k$ | $g$ |
| $\rho$ | $\alpha$ |
| | $\beta_i$ , $\beta_{i,j}$ |
| | $\mu_{\gamma i}$ |
| | $\sigma_{\gamma i}$ |
| | $Y_s$ |
| | $\mu_i$ |
| | $\sigma_i$ |
| | $Q_i$ |
| | $Z_{qi}$ |
| | $I_i$ |
| | $\omega$ |
| | $\eta$ |

The model presented was introduced in optimization software that has the solution technique needed to solve an integer non-linear programming, like our model. The software that we used is the eleventh version of LINGO©, which have the “Global Solver” technique to solve nonlinear models. See Appendix B for the formulation of the model using the optimization software. The “Global Solver” technique employs branch-and-bound methods to break a model down into many convex sub-regions to find a number of locally optimal points and then it reports the global solutions to the non-convex model contrary to other nonlinear solvers that typically will converge to a local or sub-optimal point. After the model was introduced with the input data for the subset of nine food items the software was run and after forty-one seconds we obtained an optimal menu. The model for this subset of food items consists of sixty-two variables (with 40 nonlinear and 9 integers variables), seventy constraints (where 17 constrains are nonlinear) and the software makes 20,418 iterations to find the optimal solution. The optimal assortment for this subset consists of five food items, which are: Turkey Stew, White Rice, Carrots, Pinto Beans and Peaches. Some input and output data and related costs for this menu is presented in the next tables (see Tables 3 and 4, see Appendix C for the complete data).

**Table 3. Input Data**

| *Parameters* | *Notation* | *Value* |
|---|---|---|
| **Salvage value** | $g$ | $ 0.05 |
| **Funds** | $f$ | $ 3.50 |
| **In stock rate** | $\alpha$ | 0.90 |
| **Nutritional standards** | $R_1$ | 2.00 oz |
| | $R_2$ | 1.50 oz |
| | $\rho$ | 6.00 oz |

**Table 4. Output Data**

| *Output* | *Notation* | *Value* |
|---|---|---|
| **Expected menu demand** | $Y_s$ | 233 |
| **Probability of find 3 or more items** | $\omega$ | 0.663 |
| **Probability of choose 3 or more items** | $\eta$ | 0.975 |
| **Total purchasing and cooking costs** | | $166.58 |
| **Total inventory costs** | | $0.00 |
| **Total salvage value** | | $18.92 |
| **Total funds** | | $489.76 |
| **Objective function value** | | **$ -342.11** |

To verify that this is the optimal menu, we ran the model with all feasible menus that we can obtained with those nine food items and we can observed that our optimal menu is which obtained the minimal costs, as can be seen in Figure 4. All results obtained show a negative value, which indicate that in every run we obtained profit, being greater for some menus than others.

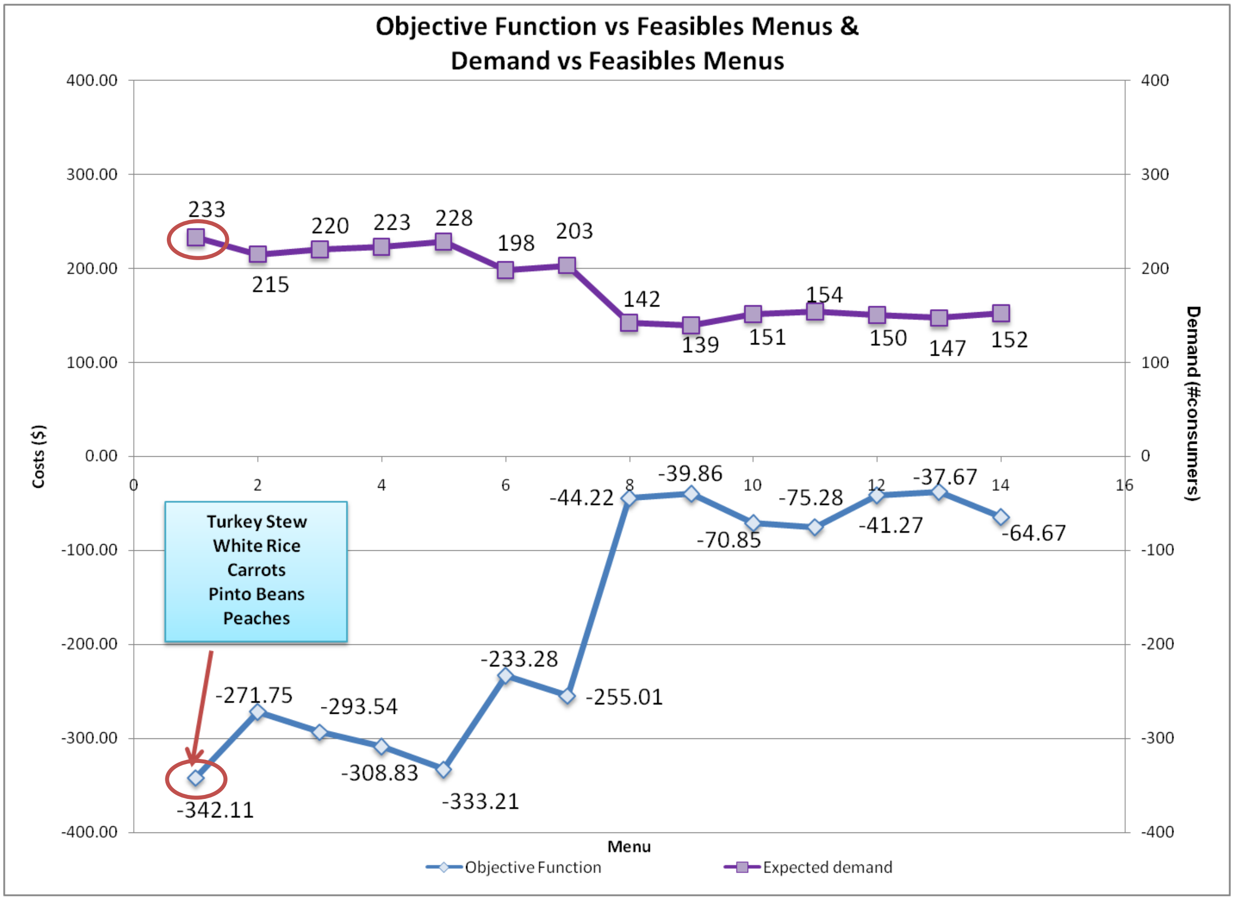


**Figure 4. Objective function vs. feasible assortments and demands**

Also, it is observed from Figure 4 that the optimal menu has the highest expected demand and its cost (profit) is approximately nine times lower than the menu with the higher costs that has lower expected demand, which strengthen the argument that the current menu planning technique could be carrying unnecessary higher costs.

To identify others characteristics of the model, all feasible menus were sorted from the lower cost to higher cost (see Table 5). We can observe that the menus with the lower costs contain five items, one more that the menus with the highest costs. The items that make these menus different are the Turkey Stew and White Rice (for the menus with lower costs) and Rice with sausage (the ones with higher costs). Although the menus with lower costs have more foods items, the two items that are different in these menus have the total purchasing and cooking cost per serving lower than the cost per serving for the Rice with sausage. Also, the menus that offer five food items have higher demands than the ones that offer four items. In addition, we can observe that the greater the number of foods to offer, lower is the probability of find three or more items but greater is the probability of choose three or more items. On the other hand, we can observe that the two menus that offer the lower costs (menus 1 and 5) differ in the food items that belong to the grains and fruits categories. These food items differ in their purchasing and cooking costs per serving, having Pinto Beans the lower cost comparing with Pink Beans, but Peaches being the one with higher cost comparing with Pears. The menu 1 has the lower total purchasing and cooking costs than menu 5, and the total funding being greater than the menu 5.

**Table 5. All feasible menus with their respective objective value (from lower to higher costs)**

| Menu | Objective Function | Expected demand | Assortment | | | | | ω | η |
|---|---|---|---|---|---|---|---|---|---|
| | | | Meats | Cereals | Grains | Vegetables | Fruits | | |
| 1 | -342.11 | 233 | Turkey Stew | White Rice | Pinto Beans | Carrots | Peaches | 0.663 | 0.975 |
| 5 | -333.21 | 228 | Turkey Stew | White Rice | Pink Beans | Carrots | Pears | 0.663 | 0.979 |
| 4 | -308.83 | 223 | Turkey Stew | White Rice | Pink Beans | Carrots | Peaches | 0.663 | 0.985 |
| 3 | -293.54 | 220 | Turkey Stew | White Rice | Pinto Beans | Green bean salad /w carrots | Pears | 0.663 | 0.968 |
| 2 | -271.75 | 215 | Turkey Stew | White Rice | Pinto Beans | Green bean salad /w carrots | Peaches | 0.663 | 0.978 |
| 7 | -255.01 | 203 | Turkey Stew | White Rice | Pink Beans | Green bean salad /w carrots | Pears | 0.663 | 0.981 |
| 6 | -233.28 | 198 | Turkey Stew | White Rice | Pink Beans | Green bean salad /w carrots | Peaches | 0.663 | 0.987 |
| 11 | -75.28 | 154 | Rice /w sausage | | Pink Beans | Carrots | Pears | 0.729 | 0.854 |
| 10 | -70.85 | 151 | Rice /w sausage | | Pink Beans | Carrots | Peaches | 0.729 | 0.884 |
| 14 | -64.67 | 152 | Rice /w sausage | | Pinto Beans | Carrots | Peaches | 0.729 | 0.821 |
| 8 | -44.22 | 142 | Rice /w sausage | | Pink Beans | Green bean salad /w carrots | Pears | 0.729 | 0.868 |
| 12 | -41.27 | 150 | Rice /w sausage | | Pinto Beans | Green bean salad /w carrots | Pears | 0.729 | 0.808 |
| 9 | -39.86 | 139 | Rice /w sausage | | Pink Beans | Green bean salad /w carrots | Peaches | 0.729 | 0.896 |
| 13 | -37.67 | 147 | Rice /w sausage | | Pinto Beans | Green bean salad /w carrots | Peaches | 0.729 | 0.838 |

*Note: Recall that ω is the probability of find m=3 or more food items and η is the probability of choose m=3 or more food items.*

To understand and evaluate in more detail the previous results we performed a numerical study in order to answer questions about the characteristics of the optimal assortment. The following section outlines which are the characteristics that possess an item to be attractive to belong to an assortment.

## 5.2 Numerical Study

To perform the numerical analysis we first ran our model for different number of items in order to select the number of food items to be used in the study to obtain an accurate and promptly results. The next graph (see Figure 5 and Table 6) shows how running time increases as the number of items in the model increases. See Appendix D for the results of each run with different number of items.

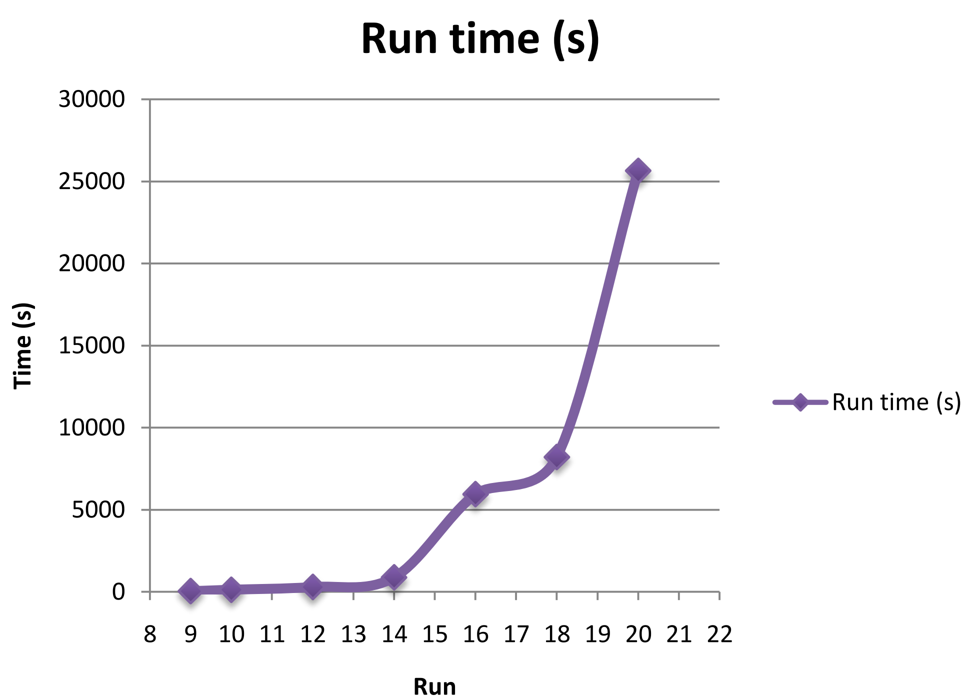


**Figure 5. Run time vs. Number of items**

**Table 6. Run times for different number of items**

| Number of items | Run time (s) | hh:mm:ss |
|---|---|---|
| **9** | 41 | 00:00:41 |
| **10** | 133 | 00:02:13 |
| **12** | 300 | 00:05:00 |
| **14** | 880 | 00:14:40 |
| **16** | 5947 | 01:39:07 |
| **18** | 8212 | 02:16:52 |
| **20** | 25639 | 07:07:19 |

Therefore, to obtain a promptly results and to be able to run as many instances we need to do the analysis, we used ten food items, two items per food category $k$.

For the numerical study, using PRSMP scenario characteristics, we defined all the parameters for the food items that belong to the same food category, $i \in k,$ with an equal value, being this scenario known as the base model scenario (see Table 7 and Appendix E for all values used). In order to describe the structure of the optimal assortment and illustrate how the firm should take the assortment planning decisions taking into consideration the food items characteristics, we made several runs, using Lingo© software, isolating the effects of one or more parameters to identify when a food item becomes attractive to be carried in the assortment compared to the base items[9]. The next section outlines the results and observations that were obtained in order to explain how the difference in value of one or more parameters can determine which items are attractive to belong to an assortment while the value of all others parameters of the food items of the same category remained equal.

[9] The base item is the other item of the same food category that has all parameters values equally set as in the base model.

**Table 7. Base model parameters values**

| Item | Food Category | $c_{p_i}$ | $c_{c_i}$ | $C_i$[10] | $\mu_{\gamma_i}$ | $\sigma_{\gamma_i}$ | $\beta_i$[11] | $U_i$ |
|---|---|---|---|---|---|---|---|---|
| **1** | Meats | 0.38 | 0.13 | 0.50 | 0.95 | 0.04 | 9 | 2.01 |
| **2** | Cereals | 0.11 | 0.04 | 0.15 | 0.93 | 0.05 | 19 | 3.02 |
| **3** | Grains | 0.04 | 0.01 | 0.05 | 0.70 | 0.20 | 13 | 2.16 |
| **4** | Vegetables | 0.08 | 0.03 | 0.10 | 0.85 | 0.10 | 10 | 2.40 |
| **5** | Fruits | 0.15 | 0.05 | 0.20 | 0.90 | 0.12 | 12 | 3.36 |
| **6** | Cereals | 0.11 | 0.04 | 0.15 | 0.93 | 0.05 | 19 | 3.02 |
| **7** | Grains | 0.04 | 0.01 | 0.05 | 0.70 | 0.20 | 13 | 2.16 |
| **8** | Vegetables | 0.08 | 0.03 | 0.10 | 0.85 | 0.10 | 10 | 2.40 |
| **9** | Fruits | 0.15 | 0.05 | 0.20 | 0.90 | 0.12 | 12 | 3.36 |
| **10** | Meats | 0.38 | 0.13 | 0.50 | 0.95 | 0.04 | 9 | 2.01 |

### 5.2.1 Purchasing and cooking costs parameter's characteristic

In preparation for the next results, we considered the PRSMP scenario characteristics with the assumption that that the initial inventory level is given by the total quantity per serving needed to satisfy the type 1-service level, $I_i = Q_i U_i \quad \forall\, i$. For all observations, the parameters values were changed to higher values and lower values using as reference the base model value to identify if this characteristic has significance when a firm has to select an item to be part of the assortment. (For complete output data obtained in each run see Appendices F through L)

As first observation, the model was ran changing the purchasing and cooking costs for one item $i$ of each food category $k$. The results obtained for a food item $i$, that has a base value $C_i = 0.50$, are show in the next Figure 6. Figure 6 shows the costs incurred by the firm if item $i$ is offered, total costs incurred by the firm if base item is offered in the assortment (turning point)

[10] The units' purchasing and cooking costs $(C_i)$ were obtained using 2003-04 CNPP Food Prices Database (http://www.cnpp.usda.gov/usdafoodplanscostoffood.htm). On the other hand, the unit purchasing costs can be calculated dividing the total cost of the food batch between the total quantities (i.e. ounces, units). In like manner, unit cooking costs can be estimated by calculating labor costs to prepare the items and dividing such cost per unit produced.

[11] In a real scenario, we could calculate $\beta_i$ values by performing a regression analysis given that sufficient observations are used. For the purposes of this work (and due to insufficient data) $\beta_i$ were selected so that we could observe similar assortment demand values as those observed at the school selected for the study.

and the optimal region (minimum total costs in each run). The interpretation is defined in Observation 1.

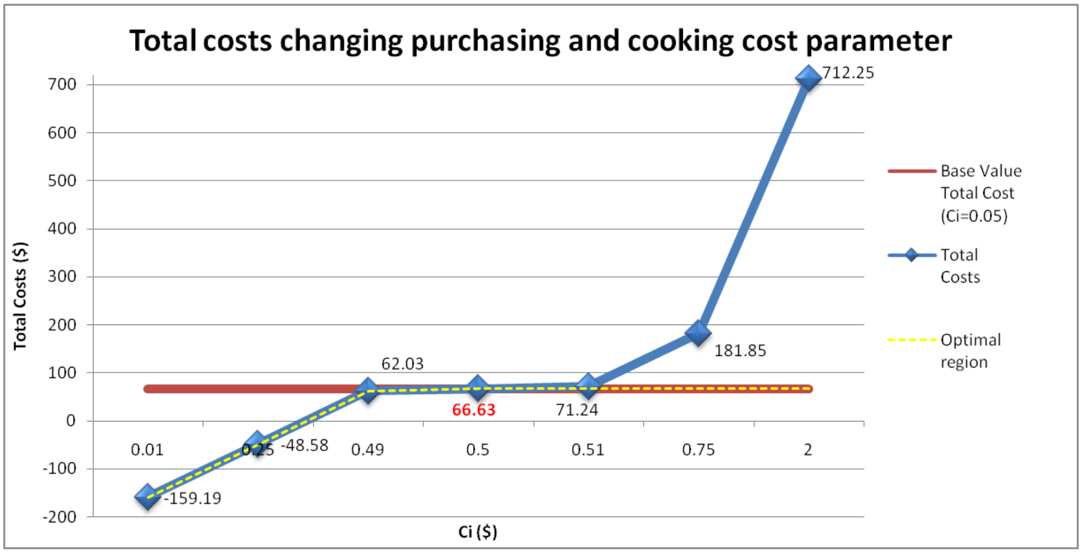


**Figure 6. Total costs changing purchasing and cooking costs parameter**

**Observation 1.** *If two items i of the same category k have the same parameters' values except for the value of the purchasing and cooking cost* $C_i$ *, then the item with the lower purchasing and cooking cost* $C_i$ *is more attractive to belongs to the assortment.*

The previous observation indicates that the purchasing and cooking costs parameter has significance when a firm has to decide between two or more items that are identical except in this value and which can also be equally combined with the other food category items. For this instance, to obtain a minimal total cost, the item with lower purchasing and cooking cost must be selected.

### 5.2.2 Number of consumer's participation parameter's characteristic

Using the same scenario that we used with the purchasing and cooking cost parameter, we next changed the number of consumer's participation parameter for one item of each food category *k*, maintaining the other parameters with its base values. This parameter was studied for two different cases, when the firm receives lower funding revenues and when they receive higher funding revenues. Figure 7 shows when an item *i* is attractive to belong to the assortment (when the firm incur in lower total costs or are inside the optimal region) in the two cases. Two observations were formulated, which are demonstrated in next example whose item *i* presented has a base value of $\beta_i = 9$.

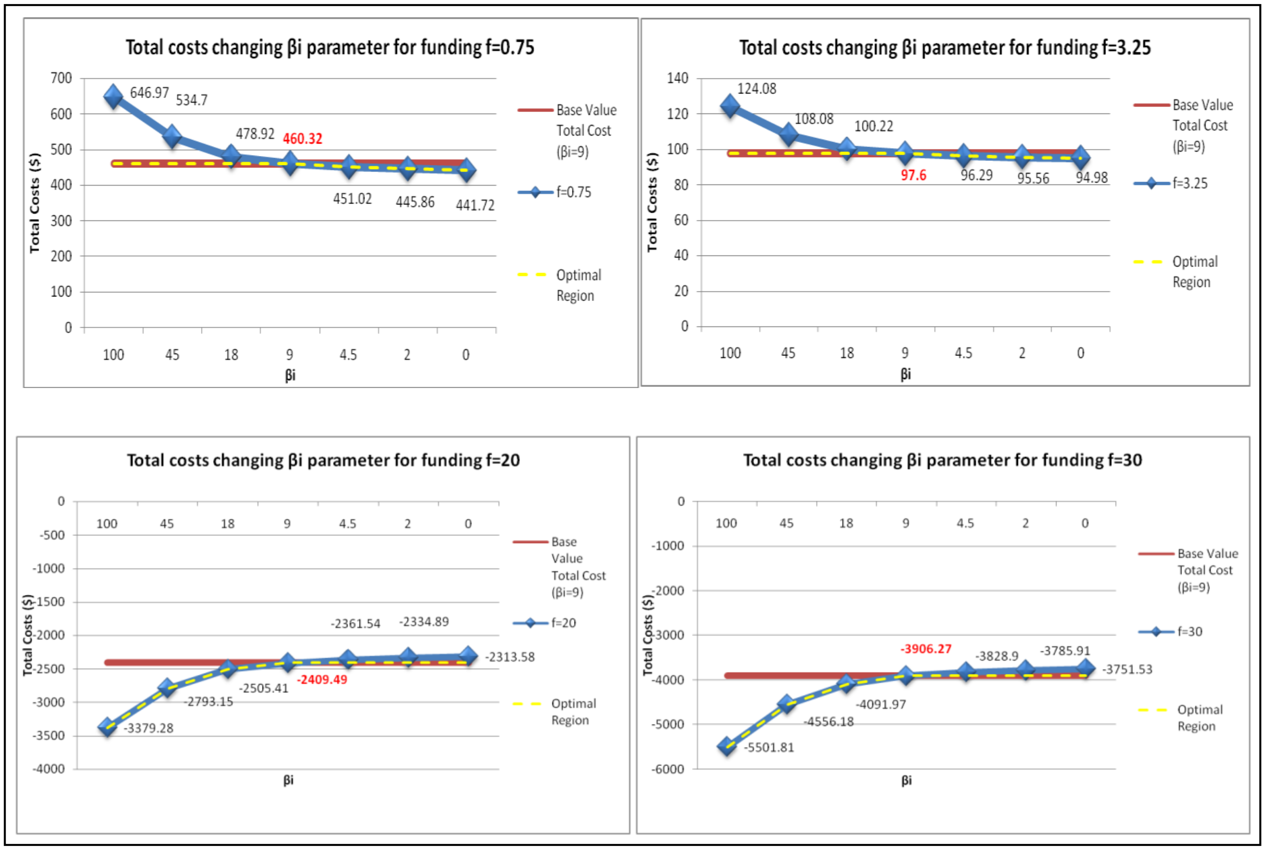


**Figure 7. Total costs changing $\beta_i$ parameter for lower and higher refunds**

**Observation 2a.** *For lower refund values, if two items i of the same category k have the same parameters' values except for the value of the number of consumer's participation* $\beta_i$ *, then the item with the lower number of consumer's participation* $\beta_i$ *is more attractive to belongs to the assortment.*

**Observation 2b.** *For higher refund values, if two items i of the same category k have the same parameters' values except for the value of the number of consumer's participation* $\beta_i$ *, then the* *item with the higher number of consumer's participation $\beta_i$ is more attractive to belongs to the assortment.*

From the results, can be observed that for lower revenues values it is more attractive to offer an item that has lower values in the number of consumer's participation $\beta_i$. This result is based in the fact that the lower the revenue the higher the likelihood for negative profits (losses) and then is more attractive to offer an item that has lower consumer's participation to incur in less total costs. On the other hand, it can be noted that for higher revenues values it is more attractive to offer an item that has higher values in the number of consumer's participation $\beta_i$. Contrary as the case of lower revenues, in this instance the likelihood for obtain profits are higher and then is more attractive to offer an item that has higher consumer's participation to receive more revenues.

### 5.2.3 Variability parameter's characteristic

We next explore how a change of value in the item *i* variability can affect the assortment decision. As was formulated in section 3.1, the variability of food item $i$ is calculated as the squared product of the expected assortment demand $Y_s$ and the standard deviation of the rate that food item $i$ is requested by the consumers, $\sigma_{\gamma_i}$. Then, for this observation, the standard deviation $\sigma_{\gamma_i}$ was changed to a higher and lower value for one item of each food category $k$ to identify if the variability of a food item has significance in the decision variable. In this case, the results obtained per each item didn't have the same behavior; for some items the results obtained were opposite to the ones obtained with other items of other categories $k$ (see the runs with the base purchasing and cooking cost values in Appendix G). The assortment demand standard deviation, $\sigma_i$, affects two terms of the objective function in our model, the total purchasing and cooking costs and the total expected salvage value. Therefore, this parameter affects cost's terms and revenue's terms, affecting the net profit value. Using PRSMP example and assuming that the initial inventory level is given by the total quantity per serving needed to satisfy the type 1-service level, $I_i = Q_i U_i \;\; \forall\, i$, the total inventory cost term is equal to zero and the total funding revenue remain equal for any value of $\sigma_i$. Then, to understand these results, we changed the purchasing and cooking cost parameter, $C_i$, together with the variability parameter, $\sigma_{\gamma_i}$, for different items $i$ (see Appendix G).

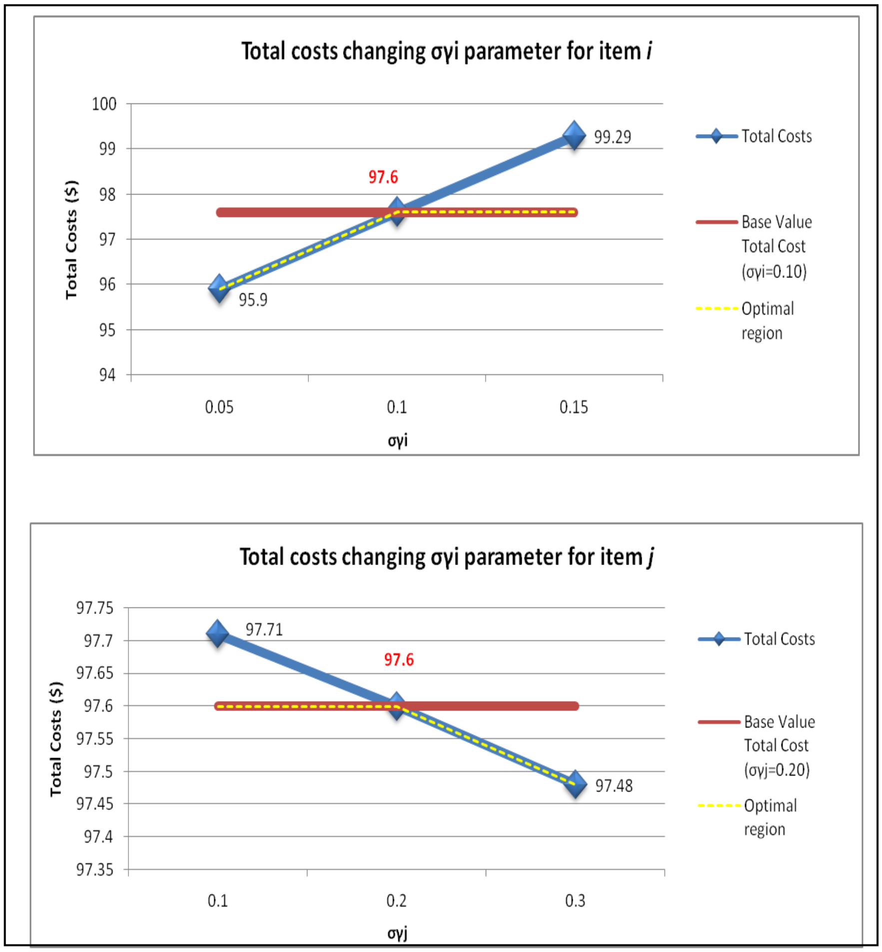


**Figure 8.Total costs changing variability parameter for two items of different categories *k***

In the previous example (see Figure 8) item $i$ has a standard deviation base value of $\sigma_{\gamma_i} = 0.10$ and the item $j$ has a base value of $\sigma_{\gamma_j} = 0.20$. Note from above that when item $i$ increasing the coefficient of variability ($\sigma_{\gamma_i}$) from 0.05 to 0.15 leaves the item out of the assortment (total cost is out the optimal region). In this case, the firm should favor the item with less variability that is fairly intuitive. For item $j$ the opposite occurs item $j$ increasing the coefficient of variability ($\sigma_{\gamma_j}$) from 0.10 to 0.30 the model will include the item in the assortment (total cost is inside the optimal region). The last result suggests that it is possible that in optimality the firm will favor items with higher variability in the assortment. The last two examples trigger the next observation.

**Observation 3.** *Given two items that are the same in all respects except for the value of the standard deviation $\sigma_{\gamma_i}$, the choice on which item to add to carry in the assortment is not trivial.*

The explanation for the results is related to the inventory decision. Recall that the model assumes that the inventory decision is exogenously fixed once a service level ("in-stock rate") is picked. Therefore, the inventory decision will not necessarily be the one that minimizes cost (i.e. $Q_i$ may not be optimal). However, we know that there exists an optimal service level for any assortment, meaning that, for a fixed assortment, the cost minimization function is concave with respect to $Q_i$. Now, observe from equation (12) that $Q_i$ is linear with respect to $\sigma_{\gamma_i}$. Putting together the last two facts (i.e. concavity of cost function with respect to $Q_i$ and $Q_i$'s linearity with respect to $\sigma_i$), explains when having two items that are the same with the exception of the variability parameter, the firm may (or may not) sometimes pick the one with high variability (i.e. will pick the one that brings the item closer to the optimal $Q_i$).

### 5.2.4 Mean parameter's characteristic

Consider that the firm offers its products for the whole assortment expected demand, i.e. $\mu_{\gamma_i} = 1$. Hence, the expected demand for food item *i* will be equal to the assortment expected demand, i.e. $\mu_i = Y_{s.}$ To determine how $\mu_{\gamma_i}$ affects the decision variable with the previous PRSMP assumptions, we ran several instances where one of the food items had the parameter $\mu_{\gamma_i}$ set to equal to one (for results of each food item see Appendix H). For the next example (see Figure 9), the expected rate base value for item *i* is equal to $\mu_{\gamma_i} = 0.95$.

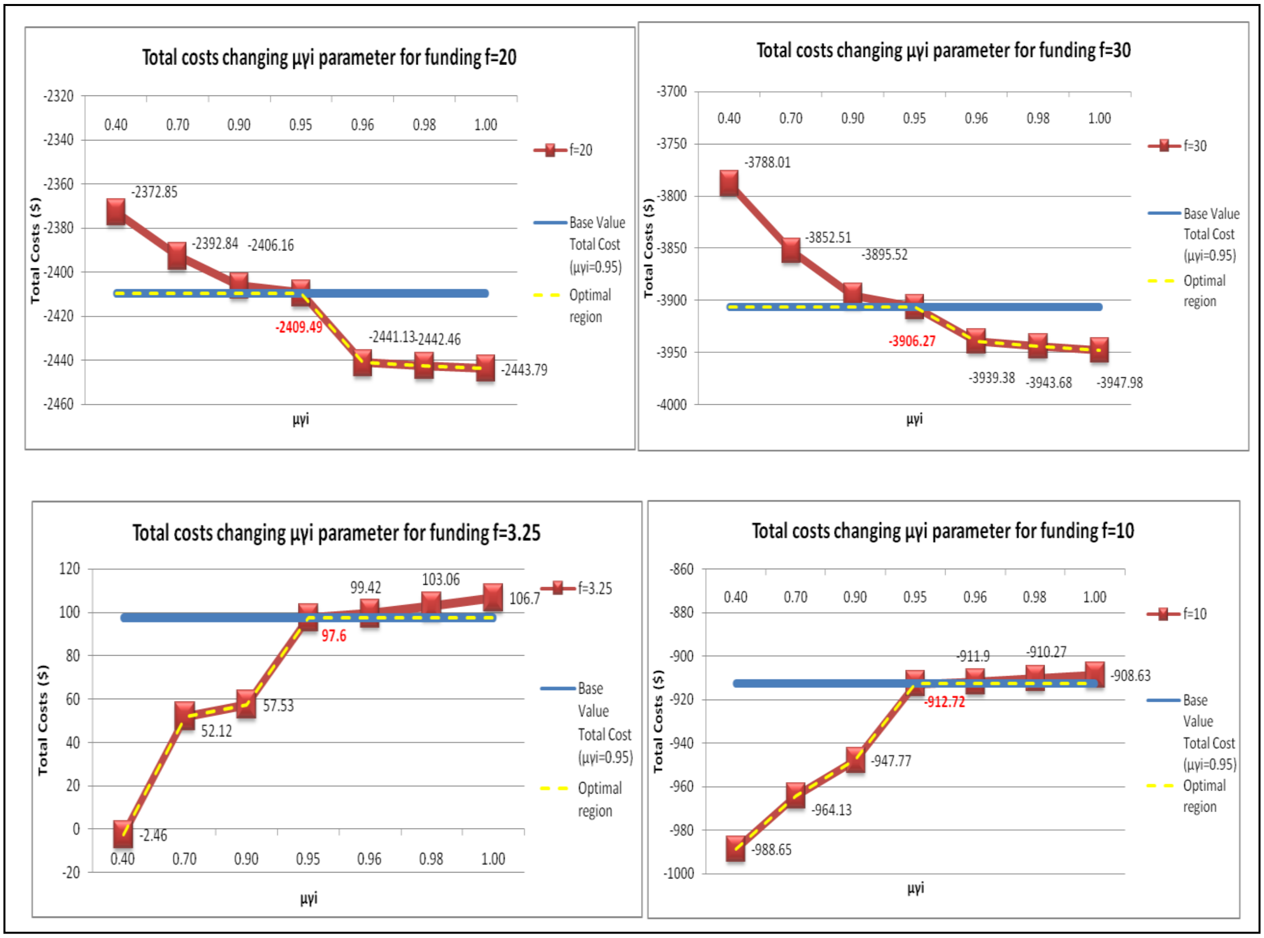


**Figure 9. Total costs changing $\mu_{\gamma_i}$ parameter for lower and higher refunds**

**Observation 4a.** The *higher the refund values, the more attractive are the items with higher* $\mu_{\gamma_i}$.

**Observation 4b.** The *lower the refund values, the more attractive are the items with lower* $\mu_{\gamma_i}$.

Similar to the number of consumer's participation parameter (Observations 2a and 2b), note from above that for higher revenue values it is more attractive to offer an item that has a greater expected demand having a greater net profit. This result is grounded in the fact that the higher the refund the higher the likelihood for profiting on offering the item. But, if the revenue is lower, it is more attractive to offer an item that is less attractive. This last result is grounded in the fact that the lower the revenue the higher the likelihood for negative profits (losses) and hence it is desirable to meet a lower consumer demand. For example, if the firm receives higher revenues per food tray served, it will like to offer an item that is more attractive (i.e. steak) because more consumers will choose it and hence a refund (revenue) will be granted making some profit. But, if the revenues are lower, the firm has to offer an item less attractive (i.e. sausage) because less consumers will choose it and by doing this, the firm will incur in less associates costs (i.e. purchasing and cooking costs).

### 5.2.5 Holding cost parameter's characteristic

In preparation for the next results, we know considered the PRSMP scenario characteristics with the assumption that the initial inventory level is greater than the total quantity per serving needed to satisfy the type 1-service level. To consider how the assortment composition can be affected when inventory costs are greater than zero, we explored this observation changing the value of the holding cost parameter, considering an interest per period of 0.068% (annual interest of 25%[12]). Following the same procedure as the previous observations the next interpretation was done based on the results obtained for item $i$, which are shown in Figure 10. The item *i* presented in next example has a base value of $h_i = 8\ x\ 10^{-5}$.

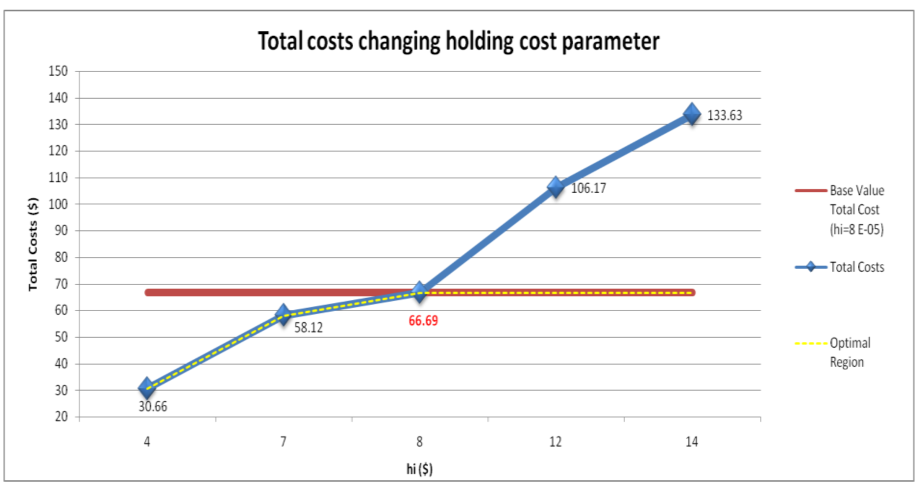


**Figure 10. Total costs changing holding cost parameter**

[12] Using as reference the capital cost used by the United States Department of Transportation Federal Transit Administration (FTA) in November 1999 during the San Juan, Puerto Rico Minillas Extension. http://www.fta.dot.gov/publications/reports/reports_to_congress/planning_environment_2947.html

**Observation 5.** *If two items $i$ of the same category $k$ have the same parameters' values except for the value of the holding cost $h_i$, then the item with the lower holding cost $h_i$ is more attractive to belongs to the assortment.*

The previous observation indicates that the holding cost parameter has significance on the food optimal assortment in the sense that items with lower holding cost are favored. The above result is not surprising once given Observation 1 and the fact that the holding cost is linear with respect to $c_{p_i}$ and $c_{c_i}$ is maintained constant. However, it illustrates that the result holds even when now it is assumed that the holding cost does in fact play a role in the cost function.

### 5.2.5 In-stock rate discussion

In order to verify our model's results veracity, we decided to study other parameters to observe the optimal assortment behavior and composition. One of the parameters selected was the in-stock rate. Recall that the in-stock rate or type-1 service level is the probability that the firm complies with the demand of the food item $i$ on any given period. We study this parameter for different instances; cases where the firm receives revenues and cases where it does not receives revenues (funding revenue equals zero, *f=0*) for meeting consumer demand, assuming $I_i = Q_i U_i \quad \forall\, i$. For the first case, is expected that for higher revenues the firm desires to offer the assortment to more consumers because it will translate to higher profits (or, less total costs). On the contrary, if they receive funding revenues close to zero (or, any revenue), is expected that the firm will preferred to offer their assortment to the minimal required quantity of consumers to incur in the minimal total costs. Then, it is expected that if the firm receives revenues, they will have more profit when they increase the in-stock rate parameter; and vice versa.

To verify that our model is adapted to reality, we ran the model for a specific assortment, using the base model values, for the previous in-stock rate scenario ($f = 0 \; and \; f \geq 0$). As observed in Appendix J, the expected result that were obtained for the two instances show that our model's behavior is in-tuned with common intuition.

### 5.2.7 Assumptions $\boldsymbol{I_i = Q_i U_i \ \ \forall\, i\ vs.\ I_i > Q_i U_i}$

To understand how our inventory assumption, which is that the initial inventory level is given by the total quantity per serving needed to satisfy the type 1-service level, $I_i = Q_i U_i \ \ \forall\, i$, can affect our results, we ran again two of the instances but this time assuming that this inventory level can be greater than the quantity needed to satisfy the type-1 service level, $I_i > Q_i U_i \ \ \forall\, i$. The instances that we studied were changes in the number of consumer participation parameter, $\beta_{i,}$ and changes in variability parameter, $\sigma_i$. The results obtained are shown in next tables (Table 8 and 9).

**Table 8. Number of consumer participation change for food item *I* and decision variables results assuming $\boldsymbol{I_i > Q_i U_i \ \ \forall\, i}$**

| $\boldsymbol{\beta_i}$ | $\boldsymbol{X_i}$ |
|---|---|
| **100** | 0 |
| **45** | 0 |
| **18** | 0 |
| **4.5** | 1 |
| **2** | 1 |
| **0** | 1 |

**Table 9. Decision variable results for purchasing and cooking costs $\boldsymbol{C_i}$ change in parallel with the standard deviation $\boldsymbol{\sigma_{\gamma_i}}$ assuming $\boldsymbol{I_i > Q_i U_i \ \ \forall\, i}$ for item *i* and *j***

| **Item *i*** | | | **Item *j*** | | |
|---|---|---|---|---|---|
| $\boldsymbol{C_i}$ | $\boldsymbol{\sigma_{\gamma_i}}$ | $\boldsymbol{X_i}$ | $\boldsymbol{C_i}$ | $\boldsymbol{\sigma_{\gamma_i}}$ | $\boldsymbol{X_i}$ |
| **0.10** | 0.15 | 0 | **0.05** | 0.30 | 1 |
| **0.10** | 0.05 | 1 | **0.05** | 0.10 | 0 |

Comparing this results and the ones presented in Appendix K and L with the previous results when the inventory assumption ($I_i = Q_i U_i \ \forall\, i$) was used, can be seen that there is no change in the decision variable results. Then, we can conclude that our assumption does not affect the results and it represents accurate observations of the instances studied.

The following section outlines the conclusion of this research and some future work that can extend the knowledge of this topic.

## 6. Conclusions and Future Work

In conclusion, we present a cost minimization model subject to assortment composition constraints where demand influence between items is considered. The demand of the items that we considered is influenced by the presence of other items in the assortment, because the consumer preference of a particular item is influenced by the combination of items. On the other hand, this model minimizes costs for a firm (e.g. non-profit firms, like Publics School Meals Programs) contrary to some works in food management area that has an objective of minimize costs for the consumers. The minimization model has constraints to guarantee nutritional requirements, assortment composition and food items availability, which can be adjusted to different real scenarios, e.g. PRSMP. The work that we presented considers some operational costs, like purchasing and cooking costs and inventory holding costs. Furthermore, this model considers two types of revenues, like funding and items' salvage value. In addition to the case study presented, this work can be applied to other non-profits firms that offer nutritional food assortment incurring in costs and receiving external revenues, like hospitals, Department of Corrections and Rehabilitation[13], Food Banks and other organizations like Meal on Wheels Program[14] and Child & Adult Care Food Program[15].

To verify the model, it was introduced into an optimization software and several instances were solved. It was found that as the number of items increased the software running time also increased but always obtaining an optimal assortment. To describe the structure of the optimal assortment different instances were modeled illustrating how the firm should take the assortment planning decisions based solely on food items' characteristics. We preformed a numerical analysis using the Puerto Rico School Meal Program (PRSMP) as motivational example and several observations were obtained. The most influential parameter on the assortment composition found was the purchasing and cooking cost $C_i$, which between two items from the same category $k$ and all other parameter been equal, the item with the lower purchasing and cooking costs is the most attractive to be part of the assortment. This observation can be obtained

[13] http://www.cdcr.ca.gov/Regulations/Adult_Operations/docs/DOM/NCDOM/2010NCDOM/10-15/DOM%20Chp5%20Art51%20Food%20Service.pdf
[14] http://www.mowaa.org
[15] http://www.fns.usda.gov/cnd/care/

regardless of the inventory level $I_i$ is equal or not to the quantity needed to satisfy a type-1 service level, $Q_i U_i$. As for the items demand variability, it was observed that it is not a trivial decision of what item to carry based solely on this parameter, which highlights the importance of the inventory decision for the problem.

Furthermore, we observed that if the firm receives lower funding revenues the lower the value of the number of consumer participation $\beta_i$ the more attractive is item *i*, and vice versa. Similarly, if the firm has two items from the same category *k* with equal parameters values, except for the expected rate value at which an item is requested by the consumers $\mu_{\gamma_i}$, if the firm receives higher revenues is more attractive to offer the item that is more attractive to the consumer, therefore the item with higher $\mu_{\gamma_i}$; whereas, if the firm receives lower revenues is more attractive to offer the item with lower $\mu_{\gamma_i}$, incurring in less total costs. Finally, the model was verified and the parameters studied, and we can conclude that our model complies with its function and an optimal menu can be found for a firm.

With this model, we are providing to non-profit firms a tool to plan their nutritional food assortment at lower costs. On the other hand, if these firms do not have the resources to obtain all the necessary data or to run a programming like this, we are providing some guides that describe an optimal menu and how they should plan their assortment considering some food items characteristics in order to offer a nutritional assortment at the minimum total costs.

An immediate extension for this work is to consider several periods. The scope of this work was concentrated in one period plan horizon. As next step, this model can be extended to consider more than one period. Considering more than one period, some features can be included, as is that the items that we are considering are perishables, then contrary as we considered in this work, exists the possibility that these items are damaged and can't be offered to the consumers. This event will affect the inventory holding costs calculation and the assortment planning for future periods. Also, the inventory holding cost formulation can be modified adding the cost that the firm will incurred in holding inventory of items that has not be offered in a single period and are expected to be offered in next periods. On the other hand, the consumers' preferences also can be affected when more than one period is considered. A consumer may like some items but not necessarily consecutively repeating the same. As a further extension, can be incorporated in the analysis more echelons of this Supply Chain (for example: distribution centers and the purchasing process at the firm main offices), including how the operational costs are affected in the whole supply chain and considering how this echelons are connected.

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# 8 Appendices

## A. Proof for Expected Excess Demand for cooked food item *i* in Section 3.2

The following mathematical formulation proofs how must be calculated the expected excess demand for cooked food item *i*, assuming that the demand follows a normal distribution. As presented in Section 3.2, the expected excess demand for cooked food item *i*, where the demand is stochastic and continuous is defined as

$$E(Q_i - D_i)^+ = \int_{-\infty}^{Q_i} (Q_i - D_i) F(D_i)\, dD_i$$

where,

$$E(Q_i - D_i)^+ = \int_{-\infty}^{Q_i} (Q_i - D_i) F(D_i)\, dD_i + \int_{Q_i}^{\infty} (Q_i - D_i) F(D_i)\, dD_i - \int_{Q_i}^{\infty} (Q_i - D_i) F(D_i)\, dD_i$$

$$E(Q_i - D_i)^+ = \int_{-\infty}^{\infty} (Q_i - D_i) F(D_i)\, dD_i - \int_{Q_i}^{\infty} (Q_i - D_i) F(D_i)\, dD_i$$

$$E(Q_i - D_i)^+ = E(Q_i - D_i) - \int_{Q_i}^{\infty} (Q_i - D_i) F(D_i)\, dD_i$$

$$E(Q_i - D_i)^+ = E(Q_i) - E(D_i) - \int_{Q_i}^{\infty} (Q_i - D_i) F(D_i)\, dD_i$$

$$E(Q_i - D_i)^+ = Q_i - \mu_i - \int_{Q_i}^{\infty} (Q_i - D_i) F(D_i)\, dD_i$$

$$E(Q_i - D_i)^+ = Q_i - \mu_i + \int_{Q_i}^{\infty} (D_i - Q_i) F(D_i)\, dD_i. \tag{38}$$

Assuming that the demand follows a standard normal distribution, then

$$\begin{aligned} (D_i - Q_i) &= (Z_{D_i}\sigma_i + \mu_i) - (Z_{Q_i}\sigma_i + \mu_i), \\ (D_i - Q_i) &= \sigma_i (Z_{D_i} - Z_{Q_i}), \\ P(D_i \le Q_i) &= P(Z_{D_i} \le Z_{Q_i}). \end{aligned} \tag{39}$$

Substituting (39) in (38) we have,

$$E(Q_i - D_i)^+ = (Z_{Q_i}\sigma_i + \mu_i) - \mu_i + \int_{Z_{Q_i}}^{\infty} \sigma_i (Z_{D_i} - Z_{Q_i}) F\left(Z_{D_i}\right) dZ_{D_i},$$

$$E(Q_i - D_i)^+ = Z_{Q_i}\sigma_i + \sigma_i \int_{Z_{Q_i}}^{\infty} (Z_{D_i} - Z_{Q_i}) F\left(Z_{D_i}\right) dZ_{D_i}. \tag{40}$$

The unit normal loss function is defined by

$$I_N\left(Z_{Q_i}\right) = \int_{Z_{Q_i}}^{\infty} (Z_{D_i} - Z_{Q_i}) F\left(Z_{D_i}\right) dZ_{D_i} = \phi\left(Z_{Q_i}\right) - Z_{Q_i}\left(1 - \Phi\left(Z_{Q_i}\right)\right). \tag{41}$$

Finally, substituting (41) in (40), we have

$$E(Q_i - D_i)^+ = \sigma_i \left[Z_{Q_i} + \phi\left(Z_{Q_i}\right) - Z_{Q_i}\left(1 - \Phi\left(Z_{Q_i}\right)\right)\right]. \tag{42}$$

## B. Model formulation using LINGO© optimization software

```
Title: Food Optimal Assortment;

Sets:
Product/1..10/:C, Q, U, X, h, Inv, Expected_excess, miu, sigma, Zq, Beta,
m_gamma, s_gamma; !i or j;
Menu/1/: Y, BetaZero; !s;
Category/1..5/:R; !k;
Component(Product, Category):lambda;
Composition(Product, Category):F;
Interaction(Product, Product):BetaInter;

End sets

!Objective Function;
min= @sum(Product(i):C(i)*Q(i)*U(i)*X(i)) + @sum(Product(i): h(i)*((Inv(i)-
Q(i)*U(i))*X(i))) - g*@sum(Product(i):Expected_excess(i)*U(i)*X(i))
- funding*@Sum(Menu(s):Y(s))*Probability*Choose;

P_C=@sum(Product(i):C(i)*Q(i)*U(i)*X(i)); !Total purchasing and cooking
costs;
IC=@sum(Product(i): h(i)*((Inv(i)-Q(i)*U(i))*X(i)));!Total inventory costs;
SV=g*@sum(Product(i):Expected_excess(i)*U(i)*X(i));!Salvage value;
FR=funding*@Sum(Menu(s):Y(s))*Probability*Choose;!Total funding;

!Constraints;
!Minimum nutritional requirements per food category k= cereals and k= meats;
@For(Category(k)|(k#EQ#1)#AND#(k#EQ#2):@sum(Product(i):X(i)*lambda(i,k)) >=
R(k));

!Minimum total nutritional requirement for categories k=vegetables, k=grains
and k=fruits;
@Sum(Category(k)|(k#GE#3)#AND#(k#LE#5):@sum(Product(i):X(i)*lambda(i,k))) >=
rho;

!To guarantee product availability;
@For(Product(i): X(i)*(Inv(i)-Q(i)*U(i)) >= 0);

!Daily Food Offer;
@For(Category(k)|(k#EQ#1):@SUM(Product(i):X(i)*F(i,k)) = 1); !Only one meat;
@For(Category(k)|(k#EQ#2):@SUM(Product(i):X(i)*F(i,k)) = 1);!Only one
cereals;
@For(Category(k)|(k#EQ#3):@SUM(Product(i):X(i)*F(i,k)) >= 1);!One or more
vegetables;
@For(Category(k)|(k#EQ#4):@SUM(Product(i):X(i)*F(i,k)) <= 1);!One or no
grains;
@For(Category(k)|(k#EQ#5):@SUM(Product(i):X(i)*F(i,k)) = 1); !Only one fruit;

@sum(Product(i):X(i))<=5;

!Binary variables;
@For(Product(i): @Bin(X(i)));
```

```
!Parameters Definitions:
!Inventory level for product i;
@For(Product(i):Inv(i)=Q(i)*U(i));

!Total quantity per serving per product i;
@For(Product(i):U(i)=@Sum(Category(k):lambda(i,k)));

!Menu expected demand;
@For(Menu(s):
Y(s)=BetaZero(s)+@Sum(Product(i):Beta(i)*X(i))+@Sum(Interaction(i,j)|j#GE#i+1
:BetaInter(i,j)*X(i)*X(j)));

!Expected demand for product i;
@For(Product(i):@For(Menu(s):miu(i)= Y(s)*m_gamma(i)));

!Standard deviation for product i;
@For(Product(i):@For(Menu(s):sigma(i)= Y(s)*s_gamma(i)));

!Inverse of the standard normal cumulative distribution for a probability of
x% of service level (assuming Normal distribution demand);
@For(Product(i):Zq(i)= @Normsinv(in_stock));

!Order-up to point per product for x% service level (assuming Normal
distribution demand);
@For(Product(i): Q(i)= Zq(i)*sigma(i) + miu(i));

!Expected_excess of product i (assuming Normal distribution demand);
@For(Product(i): Expected_excess(i)=sigma(i)*(Zq(i)+@PSL(Zq(i))));

!Total number of product offer;
t=@sum(Product(i):X(i));

!Probability to find m or more items;

Probability=@if(t#EQ#3,(in_stock)^(3),0) +
@if(t#EQ#4,(in_stock^(3))*((1-in_stock)^(1)) + (in_stock^(4)),0) +
@if(t#EQ#5,(in_stock^(3))*((1-in_stock)^(2)) + (in_stock^(4))*((1-
in_stock)^(1)) + (in_stock^(5)),0) +
@if(t#EQ#6,(in_stock^(3))*((1-in_stock)^(3)) + (in_stock^(4))*((1-
in_stock)^(2)) + (in_stock^(5))*((1-in_stock)^(1)) + (in_stock^(6)),0);

!Probability to choose 3 or more items;

Choose=1-(@sum(Product(i):(m_gamma(i)*X(i))*@prod(Product(j)|j#NE#i:(1-
m_gamma(j)*X(j))))+
@sum(Interaction(i,j)|j#GE#i+1:(m_gamma(i)*m_gamma(j)*X(i)*X(j))*@prod(Produc
t(k)|(k#NE#i)#AND#(k#NE#j):(1-m_gamma(k)*X(k)))));

@For(Product(i):@bnd(-10,Zq(i),10));

Data:
g=0.05; !salvage value;
funding=3.25;!funding;
in_stock=.90; !type-1 service level;
rho=6; !quantity per serving standard for vegetables, grains and fruits;

!Excel input data;
```

```
C= @Ole(,'FoodCost'); !Purchasing and cooking cost;
lambda= @Ole(,'Servings'); !Serving to offer per food item;
R= @Ole(,'Standard'); !Standard requirements per food category (meats and
cereals);
F= @Ole(,'Composition'); !Food category composition;
h=@Ole(,'Holding'); !Holding cost;
m_gamma=@Ole(,'Avg_gamma');!expected rate;
s_gamma=@Ole(,'stdev_gamma');!standard deviation for rate;

Beta=@Ole(,'Beta_i');
BetaZero=@Ole(,'Beta_zero');
BetaInter=@Ole(,'Betas_ij');

!Excel output data;
@Ole(,'FundsProb')=Probability;
@Ole(,'Offer')=X;
@Ole(,'Expected_demand')=miu;
@Ole(,'Standard_deviation')=sigma;
@Ole(,'Excess')=Expected_excess;
@Ole(,'Menu_demand')=Y;
@Ole(,'service')=Q;
@Ole(,'instockrate')=in_stock;
@ole(,'PandC')=P_C;
@ole(,'inventorycosts')=IC;
@ole(,'salvagevalue')=SV;
@ole(,'funds')=FR;
@ole(,'salvage')=g;
@Ole(,'total_items')=t;
@Ole(,'Choose3')=Choose;
@Ole(,'inventory')=Inv;
end data

END
```

# C. Input data used and output data obtained in the model verification for nine food items

| E | F | G | H | I |
|---|---|---|---|---|
| Real Data | | | | |
| | | Ci | Cp | Cc |
| Turkey stew | 1 | 0.167 | 0.125 | 0.042 |
| White rice | 2 | 0.010 | 0.008 | 0.003 |
| Pinto beans | 3 | 0.017 | 0.013 | 0.004 |
| Carrots | 4 | 0.057 | 0.043 | 0.014 |
| Peaches | 5 | 0.072 | 0.054 | 0.018 |
| Rice w/ sausage | 6 | 0.276 | 0.207 | 0.069 |
| Pink beans | 7 | 0.055 | 0.041 | 0.014 |
| Green bean salad /w carrots | 8 | 0.150 | 0.113 | 0.038 |
| Pears | 9 | 0.063 | 0.047 | 0.016 |

| N | O | P | Q |
|---|---|---|---|
| Estimated Data | Gamma | | |
| | | Avg | stdev |
| Turkey stew | 1 | 0.95 | 0.04 |
| White rice | 2 | 0.95 | 0.05 |
| Pinto beans | 3 | 0.7 | 0.2 |
| Carrots | 4 | 0.6 | 0.13 |
| Peaches | 5 | 0.92 | 0.12 |
| Rice w/ sausage | 6 | 0.95 | 0.1 |
| Pink beans | 7 | 0.85 | 0.01 |
| Green bean salad /w carrots | 8 | 0.65 | 0.06 |
| Pears | 9 | 0.85 | 0.05 |

| E | F | G |
|---|---|---|
| | | Estimated Data |
| | | Holding |
| Turkey stew | 1 | 8.57913E-05 |
| White rice | 2 | 5.30097E-06 |
| Pinto beans | 3 | 8.69418E-06 |
| Carrots | 4 | 2.9432E-05 |
| Peaches | 5 | 3.69612E-05 |
| Rice w/ sausage | 6 | 0.000141947 |
| Pink beans | 7 | 2.82233E-05 |
| Green bean salad /w carrots | 8 | 7.71845E-05 |
| Pears | 9 | 3.25049E-05 |

| I | J |
|---|---|
| Real Data | |
| Rk | oz |
| 1 | 2 |
| 2 | 1.5 |
| 3 | 0 |
| 4 | 0 |
| 5 | 0 |

| BetaInter | | Turkey stew | White rice | Pinto beans | Carrots | Peaches | Rice w/ sausage | Pink beans | Green bean salad /w | Pears |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Turkey stew | 1 | 0 | 10 | 19 | 17 | 10 | 5 | 11 | 13 | 13 |
| White rice | 2 | 10 | 0 | 13 | 20 | 13 | 0 | 15 | 10 | 15 |
| Pinto beans | 3 | 19 | 13 | 0 | 18 | 20 | 15 | 0 | 19 | 15 |
| Carrots | 4 | 17 | 20 | 18 | 0 | 14 | 15 | 16 | 5 | 18 |
| Peaches | 5 | 10 | 13 | 20 | 14 | 0 | 17 | 15 | 14 | 0 |
| Rice w/ sausage | 6 | 5 | 0 | 15 | 15 | 17 | 0 | 18 | 14 | 20 |
| Pink beans | 7 | 11 | 15 | 0 | 16 | 15 | 18 | 0 | 10 | 10 |
| Green bean salad /w carrots | 8 | 13 | 10 | 19 | 5 | 14 | 14 | 10 | 0 | 18 |
| Pears | 9 | 13 | 15 | 15 | 18 | 0 | 20 | 10 | 18 | 0 |

| Real Data (oz) | λik | Meats | Cereals | Vegetables | Grains | Fruits | Ui |
|---|---|---|---|---|---|---|---|
| Turkey stew | 1 | 2.01 | 0 | 0 | 0 | 0 | 2.01 |
| White rice | 2 | 0 | 3.015 | 0 | 0 | 0 | 3.015 |
| Pinto beans | 3 | 0 | 0 | 0 | 2.16 | 0 | 2.16 |
| Carrots | 4 | 0 | 0 | 1.2 | 0 | 0 | 1.2 |
| Peaches | 5 | 0 | 0 | 0 | 0 | 3.36 | 3.36 |
| Rice w/ sausage | 6 | 2 | 2.25 | 0 | 0 | 0 | 4.25 |
| Pink beans | 7 | 0 | 0 | 0.0 | 2.4 | 0 | 2.4 |
| Green bean salad /w carrots | 8 | 0 | 0 | 2.4 | 0 | 0 | 2.4 |
| Pears | 9 | 0 | 0 | 0 | 0 | 2.6 | 2.6 |

| Estimated Data | | Beta_i |
|---|---|---|
| Turkey stew | 1 | 9 |
| White rice | 2 | 19 |
| Pinto beans | 3 | 13 |
| Carrots | 4 | 10 |
| Peaches | 5 | 18 |
| Rice w/ sausage | 6 | 2 |
| Pink beans | 7 | 16 |
| Green bean salad /w ca | 8 | 5 |
| Pears | 9 | 19 |
| BetaZero | 10 | |

| Real Data | Fik | Meats | Cereals | Vegetables | Grains | Fruits |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| Turkey stew | 1 | 1 | 0 | 0 | 0 | 0 |
| White rice | 2 | 0 | 1 | 0 | 0 | 0 |
| Pinto beans | 3 | 0 | 0 | 0 | 1 | 0 |
| Carrots | 4 | 0 | 0 | 1 | 0 | 0 |
| Peaches | 5 | 0 | 0 | 0 | 0 | 1 |
| Rice w/ sausage | 6 | 1 | 1 | 0 | 0 | 0 |
| Pink beans | 7 | 0 | 0 | 0 | 1 | 0 |
| Green bean salad /w carrots | 8 | 0 | 0 | 1 | 0 | 0 |
| Pears | 9 | 0 | 0 | 0 | 0 | 1 |

**Figure 11. Input data for model verification**

| Food item | i | Daily Offer (Decision) | Expected demand | Standard deviation | Expected excess | Q(i) |
|---|---|---|---|---|---|---|
| Turkey stew | 1 | 1 | 221.35 | 9.32 | 12.3853 | 233.2940605 |
| White rice | 2 | 1 | 221.35 | 11.65 | 15.48162 | 236.2800757 |
| Pinto beans | 3 | 1 | 163.1 | 46.6 | 61.92649 | 222.8203027 |
| Carrots | 4 | 1 | 139.8 | 30.29 | 40.25222 | 178.6181968 |
| Peaches | 5 | 1 | 214.36 | 27.96 | 37.1559 | 250.1921816 |
| Rice w/ sausage | 6 | 0 | 221.35 | 23.3 | 30.96325 | 251.2101514 |
| Pink beans | 7 | 0 | 198.05 | 2.33 | 3.096325 | 201.0360151 |
| Green bean salad /w carrots | 8 | 0 | 151.45 | 13.98 | 18.57795 | 169.3660908 |
| Pears | 9 | 0 | 198.05 | 11.65 | 15.48162 | 212.9800757 |

| Input/Output data | |
|---|---|
| Expected Menu demand | 233 |
| Total items offered | 5 |
| g (salvage value) | 0.05 |
| Probability of find | 0.66339 |
| in-stock rate | 0.9 |
| Probability of choose | 0.97493 |

| Costs/Revenues | |
|---|---|
| Purchasing and cooking | 166.5760391 |
| Inventory | 0 |
| Salvage Value | 18.92396271 |
| Funds | 489.7581109 |
| **Objective value** | **-342.1060345** |

**Figure 12. Output data for model verification**

# D. Results for each run made to study run time (ten to twenty food items)

| Food item | i | Daily Offer (Decision) | Expected demand | Standard deviation | Expected excess | Q(i) |
|---|---|---|---|---|---|---|
| Turkey stew | 1 | 1 | 221.35 | 9.32 | 12.3853 | 233.2940605 |
| White rice | 2 | 1 | 221.35 | 11.65 | 15.48162 | 236.2800757 |
| Pinto beans | 3 | 1 | 163.1 | 46.6 | 61.92649 | 222.8203027 |
| Carrots | 4 | 1 | 139.8 | 30.29 | 40.25222 | 178.6181968 |
| Peaches | 5 | 1 | 214.36 | 27.96 | 37.1559 | 250.1921816 |
| Rice w/ sausage | 6 | 0 | 221.35 | 23.3 | 30.96325 | 251.2101514 |
| Pink beans | 7 | 0 | 198.05 | 2.33 | 3.096325 | 201.0360151 |
| Green bean salad /w carrots | 8 | 0 | 151.45 | 13.98 | 18.57795 | 169.3660908 |
| Pears | 9 | 0 | 198.05 | 11.65 | 15.48162 | 212.9800757 |
| Spaghetti w/ chicken | 10 | 0 | 221.35 | 20.97 | 27.86692 | 248.2241362 |

| Input/Output data | |
|---|---|
| Expected Menu demand | 233 |
| Total items offered | 5 |
| g (salvage value) | 0.05 |
| Probability of find | 0.66339 |
| in-stock rate | 0.9 |
| Probability of choose | 0.97493 |
| **Costs/Revenues** | |
| Purchasing and cooking | 166.5760391 |
| Inventory | 0 |
| Salvage Value | 18.92396271 |
| Funds | 489.7581109 |
| **Objective value** | -342.1060345 |

Figure 13. Results for ten food items

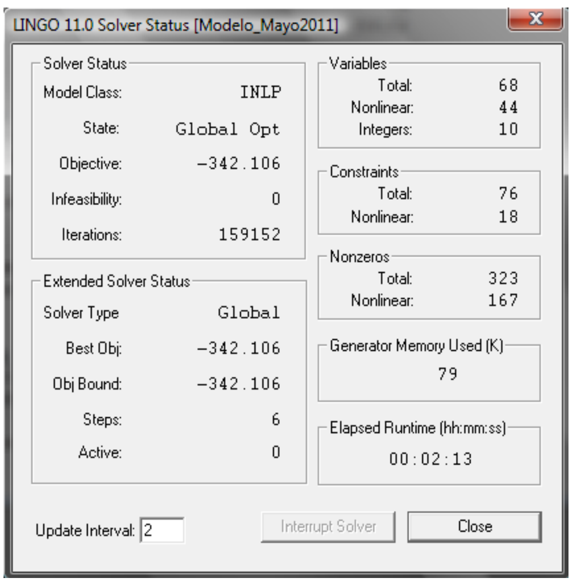


Figure 14. Lingo's solver status for ten food items

| Food item | i | Daily Offer (Decision) | Expected demand | Standard deviation | Expected excess | Q(i) |
|---|---|---|---|---|---|---|
| Turkey stew | 1 | 1 | 221.35 | 9.32 | 12.3853 | 233.2940605 |
| White rice | 2 | 1 | 221.35 | 11.65 | 15.48162 | 236.2800757 |
| Pinto beans | 3 | 1 | 163.1 | 46.6 | 61.92649 | 222.8203027 |
| Carrots | 4 | 1 | 139.8 | 30.29 | 40.25222 | 178.6181968 |
| Peaches | 5 | 1 | 214.36 | 27.96 | 37.1559 | 250.1921816 |
| Rice w/ sausage | 6 | 0 | 221.35 | 23.3 | 30.96325 | 251.2101514 |
| Pink beans | 7 | 0 | 198.05 | 2.33 | 3.096325 | 201.0360151 |
| Green bean salad /w carrots | 8 | 0 | 151.45 | 13.98 | 18.57795 | 169.3660908 |
| Pears | 9 | 0 | 198.05 | 11.65 | 15.48162 | 212.9800757 |
| Spaghetti w/ chicken | 10 | 0 | 221.35 | 20.97 | 27.86692 | 248.2241362 |
| Rice w/ red beans | 11 | 0 | 209.7 | 6.99 | 9.288974 | 218.6580454 |
| Meatballs sandwish | 12 | 0 | 233 | 9.32 | 12.3853 | 244.9440605 |

| Input/Output data | |
|---|---|
| Expected Menu demand | 233 |
| Total items offered | 5 |
| g (salvage value) | 0.05 |
| Probability of find | 0.66339 |
| in-stock rate | 0.9 |
| Probability of choose | 0.97493 |
| **Costs/Revenues** | |
| Purchasing and cooking | 166.5760391 |
| Inventory | 0 |
| Salvage Value | 18.92396271 |
| Funds | 489.7581109 |
| **Objective value** | **-342.1060345** |

**Figure 15. Results for twelve food items**

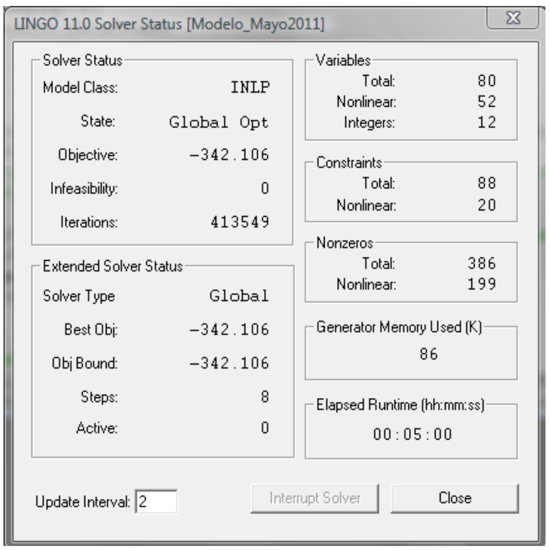


**Figure 16. Lingo's solver status for twelve food items**

| Food item | i | Daily Offer (Decision) | Expected demand | Standard deviation | Expected excess | Q(i) |
|---|---|---|---|---|---|---|
| Turkey stew | 1 | 0 | 238.45 | 10.04 | 13.3421 | 251.3167777 |
| White rice | 2 | 1 | 238.45 | 12.55 | 16.67763 | 254.5334721 |
| Pinto beans | 3 | 1 | 175.7 | 50.2 | 66.71052 | 240.0338884 |
| Carrots | 4 | 1 | 150.6 | 32.63 | 43.36184 | 192.4170274 |
| Peaches | 5 | 1 | 230.92 | 30.12 | 40.02631 | 269.520333 |
| Rice w/ sausage | 6 | 0 | 238.45 | 25.1 | 33.35526 | 270.6169442 |
| Pink beans | 7 | 0 | 213.35 | 2.51 | 3.335526 | 216.5666944 |
| Green bean salad /w carrots | 8 | 0 | 163.15 | 15.06 | 20.01315 | 182.4501665 |
| Pears | 9 | 0 | 213.35 | 12.55 | 16.67763 | 229.4334721 |
| Spaghetti w/ chicken | 10 | 0 | 238.45 | 22.59 | 30.01973 | 267.4002498 |
| Rice w/ red beans | 11 | 0 | 225.9 | 7.53 | 10.00658 | 235.5500833 |
| Meatballs sandwish | 12 | 0 | 251 | 10.04 | 13.3421 | 263.8667777 |
| Chicken nuggets | 13 | 1 | 251 | 22.59 | 30.01973 | 279.9502498 |
| Chicken stew | 14 | 0 | 245.98 | 25.1 | 33.35526 | 278.1469442 |

| Input/Output data | |
|---|---|
| Expected Menu demand | 251 |
| Total items offered | 5 |
| g (salvage value) | 0.05 |
| Probability of find | 0.66339 |
| in-stock rate | 0.9 |
| Probability of choose | 0.98304 |
| **Costs/Revenues** | |
| Purchasing and cooking | 175.9136863 |
| Inventory | 0 |
| Salvage Value | 23.06766248 |
| Funds | 531.9823122 |
| **Objective value** | **-379.1362884** |

**Figure 17. Results for fourteen food items**

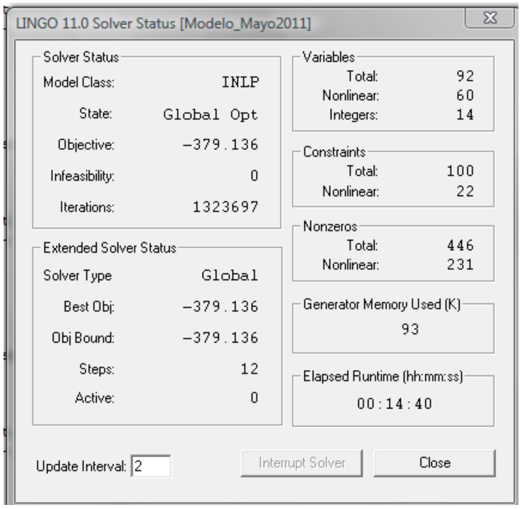


**Figure 18. Lingo's solver status for fourteen food items**

| Food item | i | Daily Offer (Decision) | Expected demand | Standard deviation | Expected excess | Q(i) |
|---|---|---|---|---|---|---|
| Turkey stew | 1 | 0 | 234.65 | 9.88 | 13.12948 | 247.3117294 |
| White rice | 2 | 1 | 234.65 | 12.35 | 16.41185 | 250.4771618 |
| Pinto beans | 3 | 1 | 172.9 | 49.4 | 65.6474 | 236.2086471 |
| Carrots | 4 | 1 | 148.2 | 32.11 | 42.67081 | 189.3506206 |
| Peaches | 5 | 0 | 227.24 | 29.64 | 39.38844 | 265.2251883 |
| Rice w/ sausage | 6 | 0 | 234.65 | 24.7 | 32.8237 | 266.3043236 |
| Pink beans | 7 | 0 | 209.95 | 2.47 | 3.28237 | 213.1154324 |
| Green bean salad /w carrots | 8 | 0 | 160.55 | 14.82 | 19.69422 | 179.5425941 |
| Pears | 9 | 0 | 209.95 | 12.35 | 16.41185 | 225.7771618 |
| Spaghetti w/ chicken | 10 | 0 | 234.65 | 22.23 | 29.54133 | 263.1388912 |
| Rice w/ red beans | 11 | 0 | 222.3 | 7.41 | 9.84711 | 231.7962971 |
| Meatballs sandwish | 12 | 0 | 247 | 9.88 | 13.12948 | 259.6617294 |
| Chicken nuggets | 13 | 1 | 247 | 22.23 | 29.54133 | 275.4888912 |
| Chicken stew | 14 | 0 | 242.06 | 24.7 | 32.8237 | 273.7143236 |
| Spanish tortilla | 15 | 0 | 209.95 | 7.41 | 9.84711 | 219.4462971 |
| Banana | 16 | 1 | 242.06 | 4.94 | 6.56474 | 248.3908647 |

| Input/Output data | |
|---|---|
| Expected Menu demand | 247 |
| Total items offered | 5 |
| g (salvage value) | 0.05 |
| Probability of find | 0.66339 |
| in-stock rate | 0.9 |
| Probability of choose | 0.99126 |

| Costs/Revenues | |
|---|---|
| Purchasing and cooking | 153.5777035 |
| Inventory | 0 |
| Salvage Value | 17.39574041 |
| Funds | 527.881955 |

| | |
|---|---|
| Objective value | -391.699992 |

**Figure 19. Results for sixteen food items**

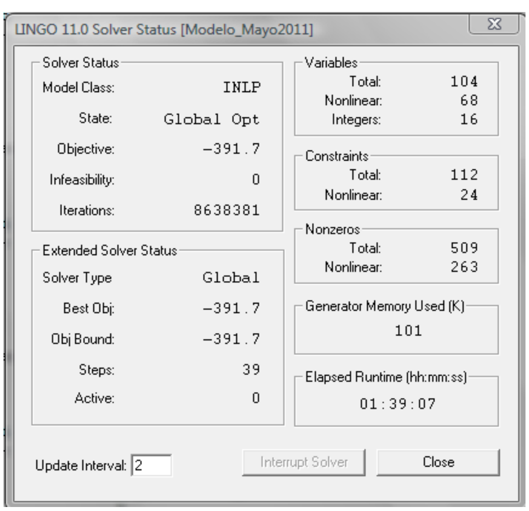


**Figure 20. Lingo's solver status for sixteen food items**

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | | | | | | | |
| 2 | Food item | i | Daily Offer (Decision) | Expected demand | Standard deviation | Expected excess | Q(i) |
| 3 | Turkey stew | 1 | 0 | 126.35 | 5.32 | 2.122373 | 126.3500001 |
| 4 | White rice | 2 | 1 | 126.35 | 6.65 | 2.652966 | 126.3500001 |
| 5 | Pinto beans | 3 | 1 | 93.1 | 26.6 | 10.61186 | 93.1000004 |
| 6 | Carrots | 4 | 0 | 79.8 | 17.29 | 6.897712 | 79.80000026 |
| 7 | Peaches | 5 | 0 | 122.36 | 15.96 | 6.367119 | 122.3600002 |
| 8 | Rice w/ sausage | 6 | 0 | 126.35 | 13.3 | 5.305932 | 126.3500002 |
| 9 | Pink beans | 7 | 0 | 113.05 | 1.33 | 0.530593 | 113.05 |
| 10 | Green bean salad /w carrots | 8 | 0 | 86.45 | 7.98 | 3.183559 | 86.45000012 |
| 11 | Pears | 9 | 0 | 113.05 | 6.65 | 2.652966 | 113.0500001 |
| 12 | Spaghetti w/ chicken | 10 | 0 | 126.35 | 11.97 | 4.775339 | 126.3500002 |
| 13 | Rice w/ red beans | 11 | 0 | 119.7 | 3.99 | 1.59178 | 119.7000001 |
| 14 | Meatballs sandwish | 12 | 0 | 133 | 5.32 | 2.122373 | 133.0000001 |
| 15 | Chicken nuggets | 13 | 0 | 133 | 11.97 | 4.775339 | 133.0000002 |
| 16 | Chicken stew | 14 | 0 | 130.34 | 13.3 | 5.305932 | 130.3400002 |
| 17 | Spanish tortilla | 15 | 1 | 113.05 | 3.99 | 1.59178 | 113.0500001 |
| 18 | Banana | 16 | 0 | 130.34 | 2.66 | 1.061186 | 130.34 |
| 19 | Applesauce | 17 | 1 | 131.67 | 14.63 | 5.836526 | 131.6700002 |
| 20 | Watermelon | 18 | 0 | 119.7 | 3.99 | 1.59178 | 119.7000001 |

| I | J |
|---|---|
| **Input/Output data** | |
| Expected Menu demand | 133 |
| Total items offered | 4 |
| g (salvage value) | 0.05 |
| Probability of find | 0.125 |
| in-stock rate | 0.5 |
| Probability of choose | 0.933305 |
| **Costs/Revenues** | |
| Purchasing and cooking | 42.53891134 |
| Inventory | 0 |
| Salvage Value | 2.693158652 |
| Funds | 50.42763578 |
| **Objective value** | **-10.5818831** |

**Figure 21. Results for eighteen food items**

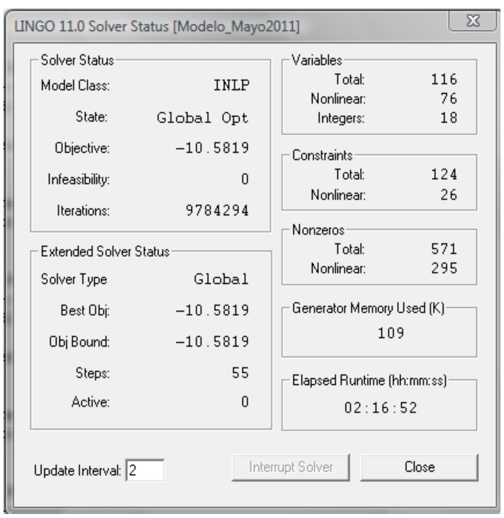


**Figure 22. Lingo's solver status for eighteen food items**

| Food item | i | Daily Offer (Decision) | Expected demand | Standard deviation | Expected excess | Q(i) |
|---|---|---|---|---|---|---|
| Turkey stew | 1 | 0 | 234.65 | 9.88 | 13.12948 | 247.3117294 |
| White rice | 2 | 1 | 234.65 | 12.35 | 16.41185 | 250.4771618 |
| Pinto beans | 3 | 1 | 172.9 | 49.4 | 65.6474 | 236.2086471 |
| Carrots | 4 | 1 | 148.2 | 32.11 | 42.67081 | 189.3506206 |
| Peaches | 5 | 0 | 227.24 | 29.64 | 39.38844 | 265.2251883 |
| Rice w/ sausage | 6 | 0 | 234.65 | 24.7 | 32.8237 | 266.3043236 |
| Pink beans | 7 | 0 | 209.95 | 2.47 | 3.28237 | 213.1154324 |
| Green bean salad /w carrots | 8 | 0 | 160.55 | 14.82 | 19.69422 | 179.5425941 |
| Pears | 9 | 0 | 209.95 | 12.35 | 16.41185 | 225.7771618 |
| Spaghetti w/ chicken | 10 | 0 | 234.65 | 22.23 | 29.54133 | 263.1388912 |
| Rice w/ red beans | 11 | 0 | 222.3 | 7.41 | 9.84711 | 231.7962971 |
| Meatballs sandwish | 12 | 0 | 247 | 9.88 | 13.12948 | 259.6617294 |
| Chicken nuggets | 13 | 1 | 247 | 22.23 | 29.54133 | 275.4888912 |
| Chicken stew | 14 | 0 | 242.06 | 24.7 | 32.8237 | 273.7143236 |
| Spanish tortilla | 15 | 0 | 209.95 | 7.41 | 9.84711 | 219.4462971 |
| Banana | 16 | 1 | 242.06 | 4.94 | 6.56474 | 248.3908647 |
| Applesauce | 17 | 0 | 244.53 | 27.17 | 36.10607 | 279.3497559 |
| Watermelon | 18 | 0 | 222.3 | 7.41 | 9.84711 | 231.7962971 |
| Mixed Vegetables | 19 | 0 | 217.36 | 12.35 | 16.41185 | 233.1871618 |
| Lettuce and tomatoes | 20 | 0 | 219.83 | 17.29 | 22.97659 | 241.9880265 |

| Input/Output data | |
|---|---|
| Expected Menu demand | 247 |
| Total items offered | 5 |
| g (salvage value) | 0.05 |
| Probability of find | 0.66339 |
| in-stock rate | 0.9 |
| Probability of choose | 0.99126 |

| Costs/Revenues | |
|---|---|
| Purchasing and cooking | 153.5777035 |
| Inventory | 0 |
| Salvage Value | 17.39574041 |
| Funds | 527.881955 |

| | |
|---|---|
| **Objective value** | -391.699992 |

**Figure 23. Results for twenty food items**

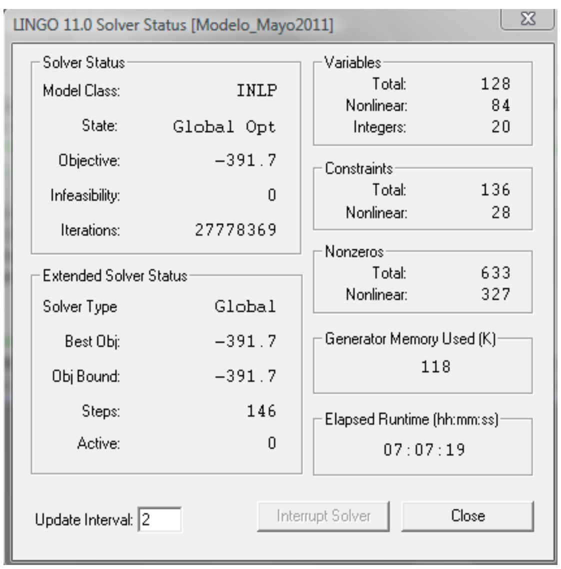


**Figure 24. Lingo's solver status for twenty food items**

# E. Input data used for the numerical study

| Color's Caption | |
|---|---|
| | Meats |
| | Cereals |
| | Grains |
| | Vegetables |
| | Fruits |

| | Cp | Cc | Ci |
|---|---|---|---|
| 1 | 0.375 | 0.125 | 0.5 |
| 2 | 0.1125 | 0.038 | 0.15 |
| 3 | 0.0375 | 0.013 | 0.05 |
| 4 | 0.075 | 0.025 | 0.1 |
| 5 | 0.15 | 0.05 | 0.2 |
| 6 | 0.1125 | 0.038 | 0.15 |
| 7 | 0.0375 | 0.013 | 0.05 |
| 8 | 0.075 | 0.025 | 0.1 |
| 9 | 0.15 | 0.05 | 0.2 |
| 10 | 0.375 | 0.125 | 0.5 |

Gamma

| | Avg | Std |
|---|---|---|
| 1 | 0.95 | 0.04 |
| 2 | 0.93 | 0.05 |
| 3 | 0.7 | 0.2 |
| 4 | 0.85 | 0.1 |
| 5 | 0.9 | 0.12 |
| 6 | 0.93 | 0.05 |
| 7 | 0.7 | 0.2 |
| 8 | 0.85 | 0.1 |
| 9 | 0.9 | 0.12 |
| 10 | 0.95 | 0.04 |

| | Holding |
|---|---|
| 1 | 0.00026 |
| 2 | 0.00008 |
| 3 | 0.00003 |
| 4 | 0.00005 |
| 5 | 0.00010 |
| 6 | 0.00008 |
| 7 | 0.00003 |
| 8 | 0.00005 |
| 9 | 0.00010 |
| 10 | 0.00026 |

| R | oz |
|---|---|
| 1 | 2 |
| 2 | 1.5 |
| 3 | 0 |
| 4 | 0 |
| 5 | 0 |

λik

| | Meats | Cereals | Vegetables | Grains | Fruits | Ui |
|---|---|---|---|---|---|---|
| 1 | 2.01 | 0 | 0 | 0 | 0 | 2.01 |
| 2 | 0 | 3.015 | 0 | 0 | 0 | 3.015 |
| 3 | 0 | 0 | 0 | 2.16 | 0 | 2.16 |
| 4 | 0 | 0 | 2.4 | 0 | 0 | 2.4 |
| 5 | 0 | 0 | 0 | 0 | 3.36 | 3.36 |
| 6 | 0 | 3.015 | 0 | 0 | 0 | 3.015 |
| 7 | 0 | 0 | 0 | 2.16 | 0 | 2.16 |
| 8 | 0 | 0 | 2.4 | 0 | 0 | 2.4 |
| 9 | 0 | 0 | 0 | 0 | 3.36 | 3.36 |
| 10 | 2.01 | 0 | 0 | 0 | 0 | 2.01 |

| Fik | Meats | Cereals | Vegetables | Grains | Fruits |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 1 | 0 |
| 4 | 0 | 0 | 1 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 1 |
| 6 | 0 | 1 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 1 | 0 |
| 8 | 0 | 0 | 1 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 1 |
| 10 | 1 | 0 | 0 | 0 | 0 |

| | Beta_i |
|---|---|
| 1 | 9 |
| 2 | 19 |
| 3 | 13 |
| 4 | 10 |
| 5 | 12 |
| 6 | 19 |
| 7 | 13 |
| 8 | 10 |
| 9 | 12 |
| 10 | 9 |

| BetaZero | 10 |
|---|---|

BetaInter

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 10 | 19 | 17 | 12 | 10 | 19 | 17 | 12 | 0 |
| 2 | 10 | 0 | 13 | 20 | 13 | 0 | 13 | 20 | 13 | 10 |
| 3 | 19 | 13 | 0 | 18 | 20 | 13 | 0 | 18 | 20 | 19 |
| 4 | 17 | 20 | 18 | 0 | 14 | 20 | 18 | 0 | 14 | 17 |
| 5 | 12 | 13 | 20 | 14 | 0 | 13 | 20 | 14 | 0 | 12 |
| 6 | 10 | 0 | 13 | 20 | 13 | 0 | 13 | 20 | 13 | 10 |
| 7 | 19 | 13 | 0 | 18 | 20 | 13 | 0 | 18 | 20 | 19 |
| 8 | 17 | 20 | 18 | 0 | 14 | 20 | 18 | 0 | 14 | 17 |
| 9 | 12 | 13 | 20 | 14 | 0 | 13 | 20 | 14 | 0 | 12 |
| 21 | 19 | 13 | 0 | 18 | 20 | 13 | 0 | 18 | 20 | 13 |

**Figure 25. Input data used to perform the numerical analysis**

## F. Decision variable results for a food item *i* changing the number of consumer participation parameter

Considering lower and higher funding revenues

| | $X_i$ | | | | |
|---|---|---|---|---|---|
| $\beta_i$ \ **Funds ($)** | **0.75** | **3.25** | **10** | **20** | **30** |
| **100** | 0 | 0 | 1 | 1 | 1 |
| **45** | 0 | 0 | 1 | 1 | 1 |
| **18** | 0 | 0 | 1 | 1 | 1 |
| **4.5** | 1 | 1 | 0 | 0 | 0 |
| **2** | 1 | 1 | 0 | 0 | 0 |
| **0** | 1 | 1 | 0 | 0 | 0 |

Considering funding revenue $f = \$3.25$

| $i$ | $\beta_i$ | $X_i$ |
|---|---|---|
| **1** | 100 | 0 |
| **1** | 45 | 0 |
| **1** | 18 | 0 |
| **1** | 4.5 | 1 |
| **1** | 2 | 1 |
| **1** | 0 | 1 |
| **2** | 500 | 0 |
| **2** | 38 | 0 |
| **2** | 9.5 | 1 |
| **3** | 65 | 0 |
| **3** | 6.5 | 1 |
| **4** | 20 | 0 |
| **4** | 1 | 1 |
| **5** | 36 | 0 |
| **5** | 6 | 1 |

**G. Decision variable results for different food items $i$ changing the standard deviation $\sigma_{\gamma_i}$ parameter together with the purchasing and cooking cost $C_i$ parameter**

| $i$ | $C_i$ | $\sigma_{\gamma_i}$ | $X_i$ |
|---|---|---|---|
| **1** | 2.00 | 0.900 | 0 |
| **1** | 2.00 | 0.010 | 0 |
| **1** | 0.50 | 0.060 | 0 |
| **1** | 0.50 | 0.020 | 1 |
| **1** | 0.02 | 0.090 | 1 |
| **1** | 0.02 | 0.010 | 1 |
| **2** | 0.80 | 0.900 | 0 |
| **2** | 0.80 | 0.010 | 0 |
| **2** | 0.15 | 0.075 | 0 |
| **2** | 0.15 | 0.025 | 1 |
| **2** | 0.05 | 0.900 | 1 |
| **2** | 0.05 | 0.010 | 1 |
| **3** | 0.70 | 0.900 | 0 |
| **3** | 0.70 | 0.001 | 0 |
| **3** | 0.05 | 0.300 | 1 |
| **3** | 0.05 | 0.100 | 0 |
| **3** | 0.05 | 0.150 | 0 |
| **3** | 0.01 | 0.900 | 1 |
| **3** | 0.01 | 0.001 | 1 |
| **4** | 0.50 | 0.900 | 0 |
| **4** | 0.50 | 0.010 | 0 |
| **4** | 0.10 | 0.150 | 0 |
| **4** | 0.10 | 0.050 | 1 |
| **4** | 0.04 | 0.900 | 1 |
| **4** | 0.04 | 0.010 | 1 |
| **5** | 1.00 | 0.900 | 0 |
| **5** | 1.00 | 0.010 | 0 |
| **5** | 0.20 | 0.180 | 0 |
| **5** | 0.20 | 0.060 | 1 |
| **5** | 0.03 | 0.900 | 1 |
| **5** | 0.03 | 0.010 | 1 |

## H. Decision variable results for different food items *i* changing the expected rate $\mu_{\gamma_i}$ that food item *i* is requested by the consumers

*Note that the values of the table are $X_i$.*

| $i$ | $\mu_{\gamma_i}$ | Funds ($) | | | | |
|---|---|---|---|---|---|---|
| | | **3.25** | **10** | **15** | **20** | **30** |
| **1** | 1.00 | 0 | 0 | 0 | 1 | 1 |
| | 0.98 | 0 | 0 | 0 | 1 | 1 |
| | 0.96 | 0 | 0 | 0 | 1 | 1 |
| | 0.90 | 1 | 1 | 1 | 0 | 0 |
| | 0.70 | 1 | 1 | 1 | 0 | 0 |
| | 0.40 | 1 | 1 | 1 | 0 | 0 |
| **2** | 1.00 | 0 | 1 | 1 | 1 | 1 |
| **3** | 1.00 | 0 | 1 | 1 | 1 | 1 |
| **4** | 1.00 | 0 | 1 | 1 | 1 | 1 |
| **5** | 1.00 | 0 | 0 | 1 | 1 | 1 |

## I. Decision variable results for different food items $i$ changing the holding cost $h_i$ parameter

| $i$ | $c_{p_i}$ | $h_i$ (x $10^{-5}$) | $X_i$ |
|---|---|---|---|
| **1** | 0.71 | 49 | 0 |
| **1** | 0.56 | 38 | 0 |
| **1** | 0.34 | 23 | 1 |
| **1** | 0.19 | 13 | 1 |
| **2** | 0.21 | 14 | 0 |
| **2** | 0.17 | 12 | 0 |
| **2** | 0.10 | 7 | 1 |
| **2** | 0.06 | 4 | 1 |
| **3** | 0.07 | 5 | 0 |
| **3** | 0.06 | 4 | 0 |
| **3** | 0.03 | 2 | 1 |
| **3** | 0.02 | 1 | 1 |
| **4** | 0.14 | 10 | 0 |
| **4** | 0.11 | 8 | 0 |
| **4** | 0.07 | 5 | 1 |
| **4** | 0.03 | 2 | 1 |
| **5** | 0.29 | 20 | 0 |
| **5** | 0.23 | 16 | 0 |
| **5** | 0.08 | 5 | 1 |
| **5** | 0.07 | 4 | 1 |

## J. Objective value vs. In-stock rate for funding revenue $f = 3.25$ & $f \geq 0$

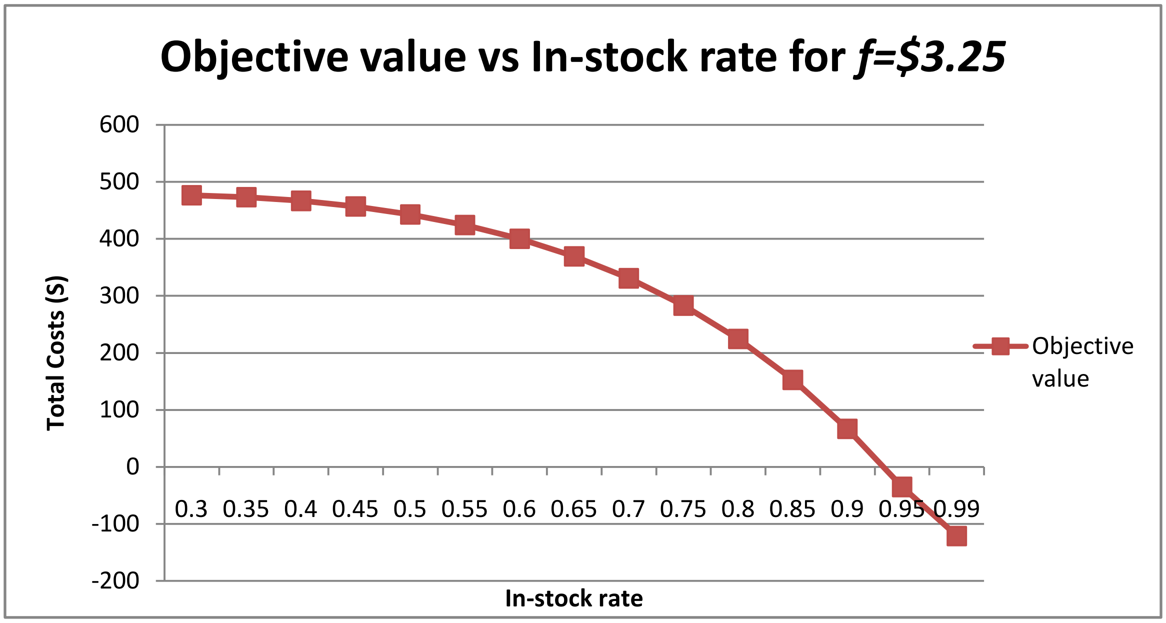


**Figure 26. Objective value for different in-stock rate values for revenues funding equals *f*=$3.25**

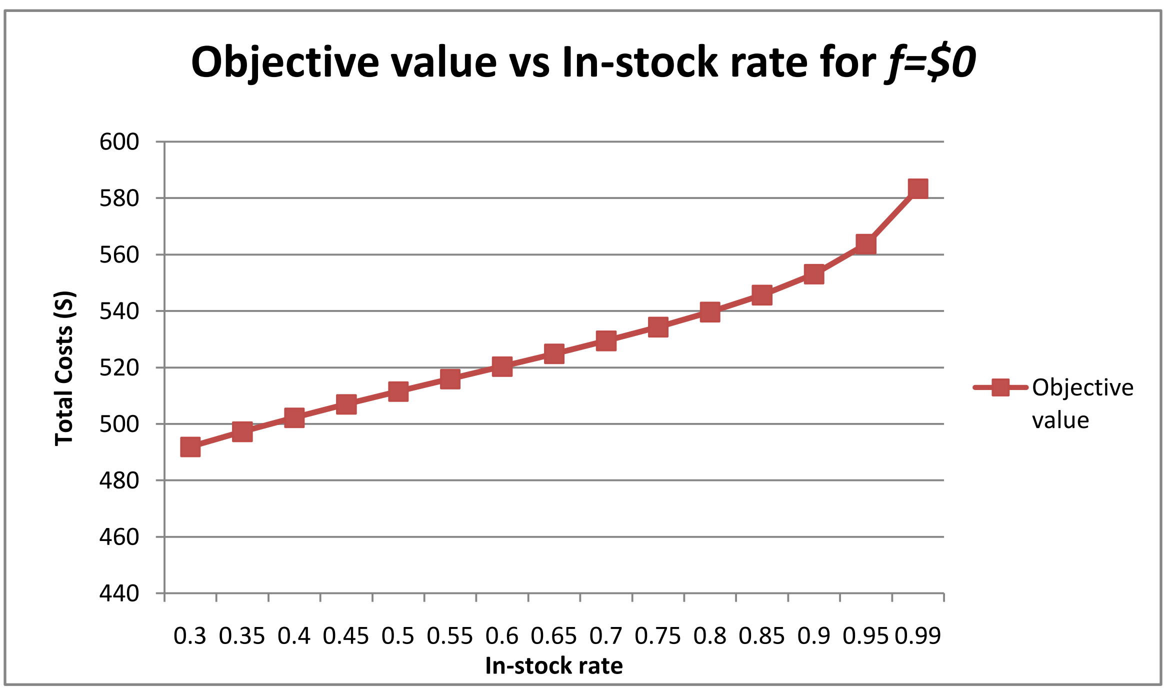


**Figure 27. Objective value for different in-stock rate values for revenues funding equals *f*=$0**

## K. Decision variable results for different food items $i$ changing the number of consumer participation parameter assuming $I_i > Q_i U_i \ \ \forall\, i$

| $i$ | $\beta_i$ | $X_i$ |
|---|---|---|
| **1** | 18 | 0 |
| **1** | 4.5 | 1 |
| **2** | 38 | 0 |
| **2** | 9.5 | 1 |
| **3** | 26 | 0 |
| **3** | 6.5 | 1 |
| **4** | 20 | 0 |
| **4** | 1 | 1 |
| **5** | 24 | 0 |
| **5** | 6 | 1 |

**L. Decision variable results for food items *i=4* and *i=5* changing the standard deviation $\sigma_{\gamma_i}$ parameter in parallel with the purchasing and cooking cost $C_i$ parameter**

| $i$ | $C_i$ | $\sigma_{\gamma_i}$ | $X_i$ |
|---|---|---|---|
| **4** | 0.50 | 0.90 | 0 |
| **4** | 0.50 | 0.01 | 0 |
| **4** | 0.10 | 0.15 | 0 |
| **4** | 0.10 | 0.05 | 1 |
| **4** | 0.04 | 0.90 | 1 |
| **4** | 0.04 | 0.01 | 1 |
| **5** | 1.00 | 0.90 | 0 |
| **5** | 1.00 | 0.01 | 0 |
| **5** | 0.20 | 0.18 | 0 |
| **5** | 0.20 | 0.06 | 1 |
| **5** | 0.03 | 0.90 | 1 |
| **5** | 0.03 | 0.01 | 1 |