A VARIABLE DAMPING SEMIACTIVE DEVICE FOR CONTROL OF THE SEISMIC RESPONSE OF BUILDINGS

By

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ABSTRACT

This dissertation proposes a new semiactive device, composed of two fixed-orifice viscous fluid dampers installed in the form of **V** whose top ends are attached to the upper floor and their lower ends to a point that can move along a vertical rod. The mechanism is termed the Variable-Damping-Semi-Active (**VDSA**) device. By varying its position, one can provide optimal instantaneous damping to the structure to minimize its seismic response. To determine the proper position of the moving end of the device, two algorithms based on a variation of the Instantaneous Optimal control theory were developed. The modified algorithms, referred to as ***Qv***, are used to compute the response of structures standing alone and of coupled structures. Single and multi degree of freedom models are considered. The responses of the structural systems with the **VDSA** device subjected to different earthquakes are numerically studied and compared with those obtained with fixed dampers.

COMPENDIO

En esta disertación se propone un nuevo equipo de control semiactivo. El equipo consiste de dos amortiguadores viscosos de orificio fijo, instalados en forma de **V** y cuyos extremos inferiores se pueden mover verticalmente hacia arriba y hacia abajo a lo largo de una barra vertical. El equipo se denomina **VDSA** (*Variable Damping Semi-Active*). Debido a la variación de la posición de los amortiguadores se puede suministrar un amortiguamiento óptimo a la estructura minimizando su respuesta. Para determinar la posición en cada instante de tiempo, se desarrollaron dos algoritmos de control basados en la Teoría de Control Óptimo. Los algoritmos, que se denominaron algoritmos ***Qv*** modificados, se usan para computar la respuesta sísmica de estructuras solas y acopladas de uno y múltiples grados de libertad. Se estudio en forma numérica la respuesta de los sistemas estructurales con el equipo **VDSA** sometidos a diferentes terremotos y se comparó con la respuesta con amortiguadores fijos.

DEDICATION

To my mother's memory, to my dear wife Julieta, to my children Rossy Lorena, Juan Sebastián, and Valentina, to my father Juan, to my brothers Juan, Luis Fernando and Maria Elenis.

ACKNOWLEDGMENTS

Firstly to God. Second, to my advisor Dr. Luis E. Suárez for his unconditional support from the beginning to the end of this work. Also to Dr. Ricardo López, Dr. José Antonio Martínez Cruzado, Dr. Luis Godoy and Dr. Ali Saffar, members of my committee, for their valuable contribution in the revision of this work.

To my family for the understanding and the support that take to a happy term this objective layout.

To my study partners for the atmosphere of friendship given during the elaboration of this work.

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CHAPTER I:

INTRODUCTION

1.1 JUSTIFICATION

Historically, natural phenomena such as hurricanes, tsunamis, earthquakes, and floods, have produced disasters of great magnitude, leaving in their path collapsed structures, extensive desolated natural regions and more importantly, a great loss of human lives. For instance, on the average, 10,000 people die each year from earthquakes (see Figure 1). A **UNESCO** study [1] estimates that the damage losses due to earthquakes from 1926 to 1950 amount to $10,000,000,000.

The study of the behavior of structures during earthquake events has been and still is of great interest to those involved in earthquake engineering and structural dynamics. These dynamic loads can cause severe and/or sustained vibratory motion, both of which can be detrimental to the structures and their contents.

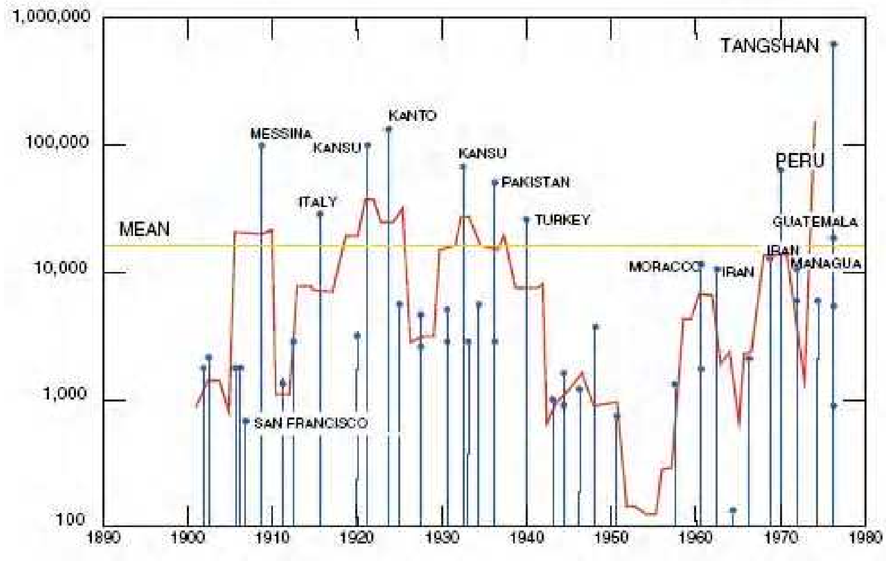


Figure 1.1: Loss of life caused by major earthquakes (Naeim, F. (2001))

Table 1.1 present the number of deaths reported in the earthquakes occurred between 1991 and November from 2005 (USGS-Earthquake Hazards Program).

There are several reasons why the earthquake hazard is more perilous than other natural disasters. One is the unpredictability of these events, at least up to the present. Other natural phenomena, such as hurricanes and floods, can be predicted with some advance and the population can (hopefully) take preventive measures. Another factor that contributes to the perils of earthquakes is the level of uncertainty associated wit these excitations. Although the accuracy in the prediction of the ground motion intensity expected at a site is increasing with more research, still there is a certain level of uncertainty in the estimation of the seismic loads. Moreover, by properly changing the construction materials or structural systems and paying attention to the connections, components, claddings, etc, it is possible to design structures than can resist hurricanes. The design for seismic loads is more involved and it is still the object of research.

Table 1.1: Loss of life caused by earthquakes between 1991 and November (2005)

Year	Total Deaths
1991	3,210
1992	3,920
1993	10,096
1994	1,634
1995	7,980
1996	589
1997	3,069
1998	9,430
1999	22,662
2000	231
2001	21,357
2002	1,686
2003	33,819
2004	284,010
2005	89,313
Total	**493,006**
Average	**32,867**

1.2 INTRODUCTION

Usually civil engineering structures rely on its strength to withstand the large forces imposed on them by strong earthquakes. With current design procedures, the structures are expected to suffer significant damage but no collapse if the earthquake scenario that was considered for its design occurs. Although this philosophy has been the standard for many decades, new design procedures and novel devices are changing the traditional approach. An example of the new procedures is the one known as performance-based design. This methodology will provide the structural engineer with the tools to predetermine the amount of damage that the user is willing to tolerate and design the structure accordingly. On the other hand, a number of modern mechanical systems have been proposed in the last two decades to reduce the structural response (and thus the amount of damage). They are known collectively as “protective devices” and they include added viscoelastic dampers, viscous fluid dampers, frictional dampers, hysteretic dampers, tuned-mass dampers, and base isolation systems. The devices themselves and their design methodology are referred to as “passive control systems”. At the highest level of sophistication for seismic protection are the so called “active control systems”. Although there are a variety of active systems proposed, they have in common that in one way or another, carefully defined external forces are applied to the structure so that they counteract the effects of earthquakes or winds. So far, and albeit there are already some demonstration installations (mostly in Japan), they did not wide the acceptance within the structural engineering community. There are several valid reasons for this opposition, in addition to a reluctance to abandon design methods that serve reasonably well for along time. Among the most drawbacks are reliability issues dealing with entrusting the safety of a structure to sophisticated devices that may fail to work as planned. Secondly, the energy required to apply these control forces is significant. Moreover, under certain conditions some control schemes, i.e. the methodology used to calculate the forces, can introduce instabilities in an otherwise perfectly stable structure.

There is an intermediate alternative between passive and fully active control systems: they are referred to as “semi-active systems”, and one of them is the topic of this dissertation. As its name indicates, a semi-active control combines the features of active and passive systems to reduce the dynamic response of structures. The technique uses the same passive devices mentioned before, but changes their properties in a beneficial way for the structure. To modify the operational characteristics of the passive devices some energy is required, but the amount is negligible compared to fully active systems. Moreover, in case of failure of the sensors or the computer hardware that control the semiactive device, the system becomes a passive one. In other words, the protective action of the semiactive system still remains in place, albeit with a decreased efficiency.

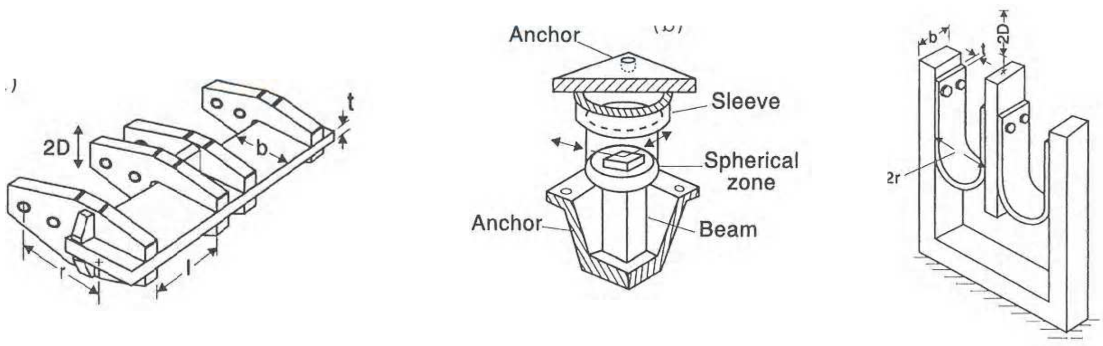
To calculate the control forces that operate the active as semiactive devices, it is necessary to know the response of the structure by measuring it with sensors. A proper numerical algorithm processes this information and calculates how the properties of the semiactive device should be modified. The semiactive control systems can be divided into two types: active variable stiffness and active variable damping devices. In the first category, the stiffness of elements of the structure (special joints, bracings, added springs, etc) is adjusted to drive the structure away from a resonant condition with the base motion. In the second category, the geometry or mechanical properties of supplemental energy dissipating devices are modified in order to enhance the response reduction due to the added damping.

1.3 PREVIOUS WORKS

1.3.1 PASSIVE CONTROL

Passive energy dissipation devices include friction dampers, metallic-yielding dampers, viscous fluid dampers, viscoelastic dampers, tuned mass dampers (TMD), tuned liquid dampers (TLD) and base isolation systems.

One of the most effective mechanisms for the dissipation of the energy input into a structure during an earthquake is through the inelastic deformation of metallic substances. The idea of utilizing separate metallic hysteretic dampers within a structure to absorb a large portion of the seismic energy began with the conceptual and experimental work by Kelly et al. [2] and Skinner et al. [3]. Some of the devices considered by those researchers are shown in Figure 1.2.



(a) (b) (c)

Figure 1.2: Metallic dampers - a) Torsional beam b) Flexural beam c) U-strip

Other metallic yield damper is the **ADAS** (Added Damping and Stiffness) designed by *Bechtel Power Corporation* and tested by Whittaker et al. in 1991 [4]. Figure 1.3 shows a photograph of a typical **ADAS** device on a test rig.

The application of viscoelastic dampers to civil engineering structures appears to have begun in 1969 when 10,000 of them were installed in each of the twin towers of the World Trade Center in New York to reduce the oscillations due to wind loads [20]. Figure 1.4(a) shown a photo of a typical viscoelastic damper.

Friction dampers utilize the mechanism of solid friction that develops between two solid bodies sliding relative to one another to provide the desired energy dissipation. Several types of friction dampers have been developed for the purpose of improving the seismic response of structures. One of the most well known friction dampers is that developed by Pall and Marsh [12] began their development of friction dampers by conducting static and dynamic test on a variety of simple sliding elements having different surface treatments.

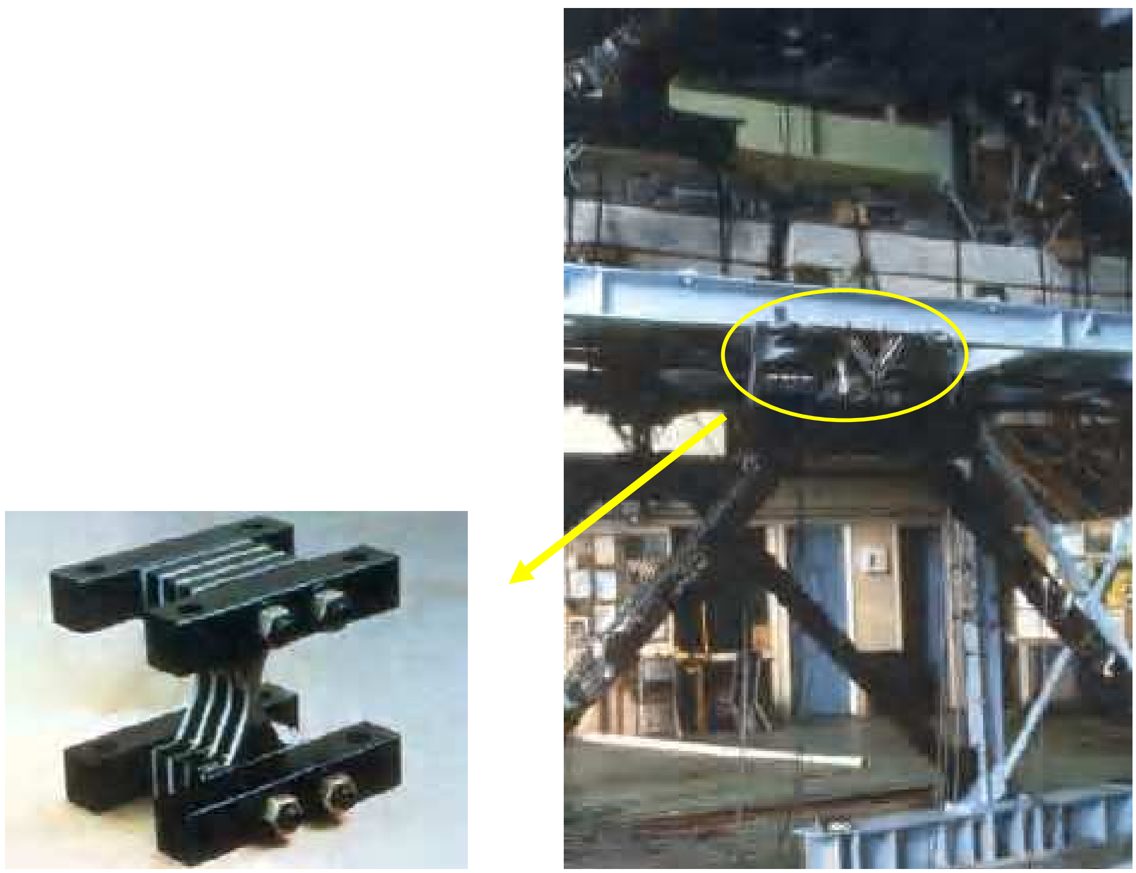


Figure 1.3: ADAS device (Reference [19])

Figure 1.4(b) shows the Pall friction damper. The theory and applications of passive dampers can be consulted in references [5] to [29].

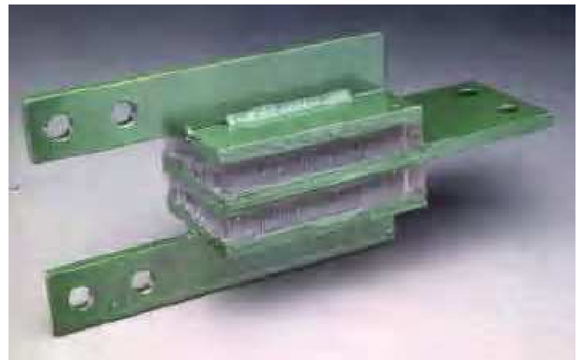


Figure 1.4: (a) Pall friction damper, (b) Viscoelastic damper (Reference [19])

Passive control strategies are popular and have been widely implemented. However, the performance of optimal passive control is sometimes limited, in that they are typically designed protect the structure from one particular dynamic loading.

1.3.2 SEMIACTIVE CONTROL

Semiactive control devices, also called "*smart*" control devices, retain the positive aspects of both the passive and active control systems. However, here the control actuator does not directly apply a force to the structure, but instead it is used to control the properties of a passive energy device.

To change the properties of semiactive devices it is necessary to perform some action, and thus one needs to have a low or a set of rules to operate the mechanism in an optimal way to maximize the response reduction. This law or set the rules is referred to as a control scheme or control algorithm. A semiactive control scheme is similar to the algorithm used in active control. Thus, semiactive control strategies can be used in many of the same civil applications as active control. Figure 1.5 displays some of the ways in which a semiactive system can be applied to a structure.

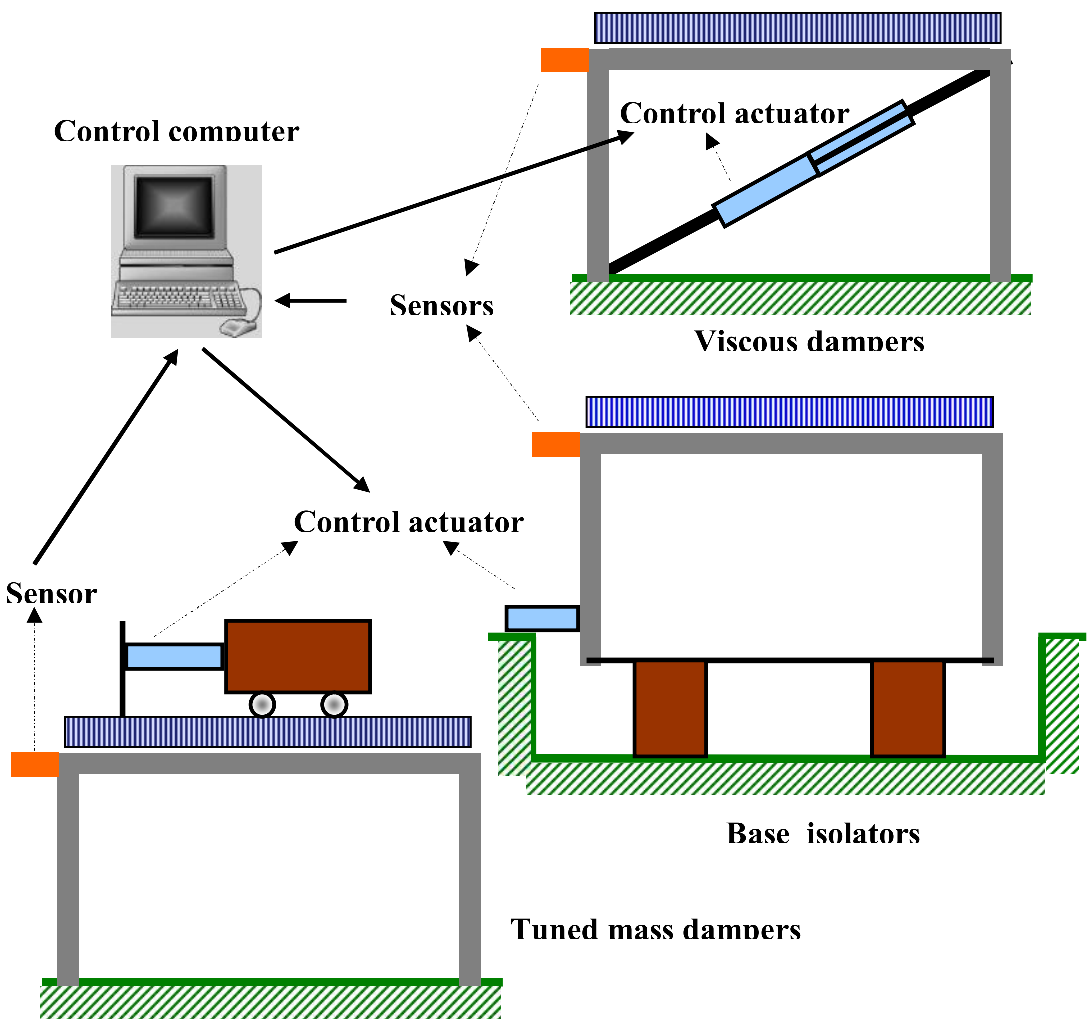


Figure 1.5: Models of semiactive control

Semiactive control systems have only recently been considered for applications to large civil structures. Probably the first application of these systems to civil engineering structures was reported by Hrovat et al. [29]. Since then, several damping devices with properties that change with time have been proposed. For instance, variable orifice dampers have been proposed by Symans and Constantinou [30], and Kurata et al. [33], [34]. Hydraulics dampers were developed and presented by Kawashima et al. [35], and Patten et al. [36], [37]. Variable stiffness devices have been proposed by Kobori et al. [38], Nagarajaiah et al. [39], Gluck et al. [40]. Furthermore, numerous algorithms have been developed for selecting the appropriate damping coefficient during an earthquake excitation (Sadeck and Mohraz [32], Soong [41], Yang et al. [42], etc). The list of references is just to provide relevant examples.

1.3.3 COUPLED STRUCTURES

There are a few analytic and experimental studies on the implementation of control systems in adjacent structures. The first work dealing with the application of controls for coupled civil engineering structures was published in 1972 by Klein et al. [44] in the United States. Four years later further investigations on this topic were carried out in Japan by Kunieda [45].

Mitsuta et al. [43] performed experimental tests on two adjacent single degree of freedom structural models and on two adjacent single and two degrees of freedom models. The floor masses were coupled with an active control actuator using displacement sensors for the feedback measurement. Yamada et al. [46] coupled a pair of two and three-story structural models at the second story with a negative stiffness active control device. They were able to effectively reduce the displacements of these low-rise models. Kageyama et al. [47] proposed a passive vibration control method in which two separated structures were connected with dampers. Active control strategies have been studied extensively for flexible structures by Seto et al. [50]. A semiactive coupled structures control has been proposed by Christenson et al. [51]. They studied various coupled structural configurations and experimentally verified the active coupled structural control employing acceleration feedback.

Examples of two coupled building control applications, both located in Japan, are shown in Figures 1.6(a) and 1.6(b).

Figure 1.6: (a) Kajima Intelligent Building, (b) Konoike Headquarter Building

1.4 OBJECTIVES

The objectives of the present dissertation is to propose a new Variable Damping Semi-Active control (**VDSA**) device, to develop the necessary analytical framework for its implementation and to investigate it's to reduce seismic response performance with numerical simulations. Contrary to semi-active dampers described in the technical literature, the damper coefficient ***c*** is not controlled by modifying the size of an orifice in the piston, but by changing the position of the damper. The optimal damping coefficient is calculated by means of two algorithms, *Closed-loop control* and *Closed-open-loop control* based on the Instantaneous Optimal Control theory.

The damping provided to the structure by the proposed device will vary from a minimum value to a maximum value that will depend on the different response quantities measured in the structure in two consecutive instants of time t_i and t_{i+1}. Depending on the amount of damping required, to drive the structure to the position of static equilibrium the proper vertical position $\boldsymbol{w(t)}$ of the lower end of the **VDSA** device is calculated.

The application of the **VDSA** device to structures standing alone and to the structures coupled by the device it self is examined. The single and coupled structures will be modeled as one degree of freedom and multiple degree of freedom systems.

Figures 1.7 and 1.9 show the analytical model used for the stand-alone structures fitted with the **VDSA** device. Figures 1.8 and 1.10 display the models of the structures with the proposed semiactive system installed between them.

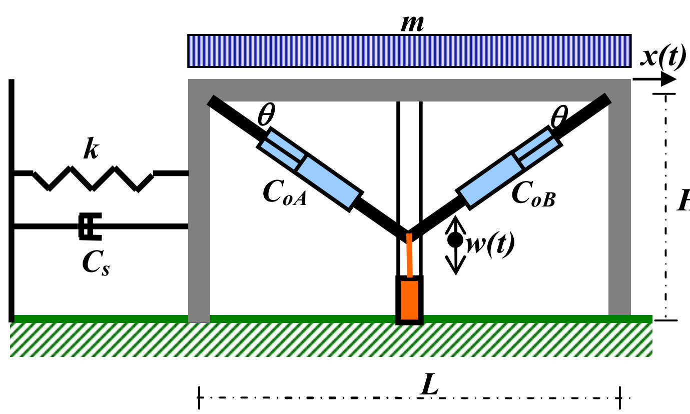


Figure 1.7: SDOF Structure with the VDSA device

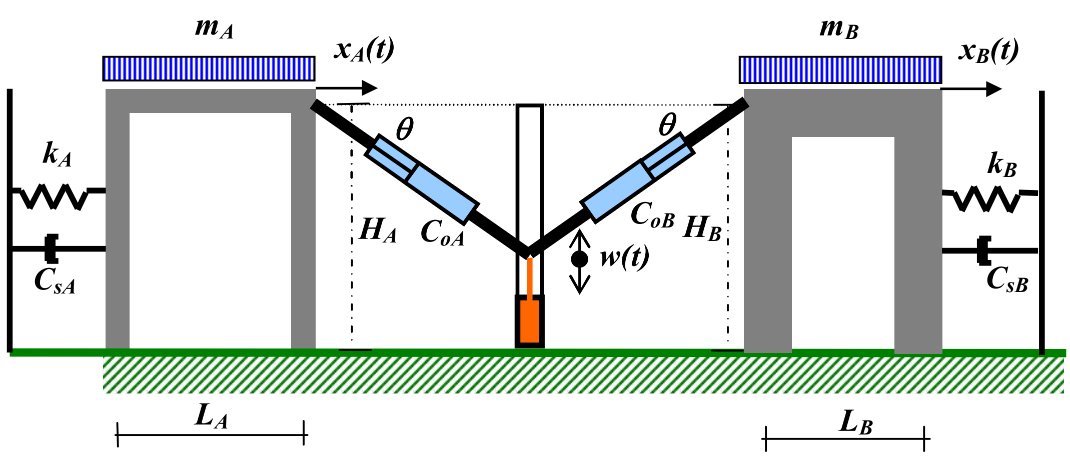


Figure 1.8: SDOF Coupled Structures with the VDSA device

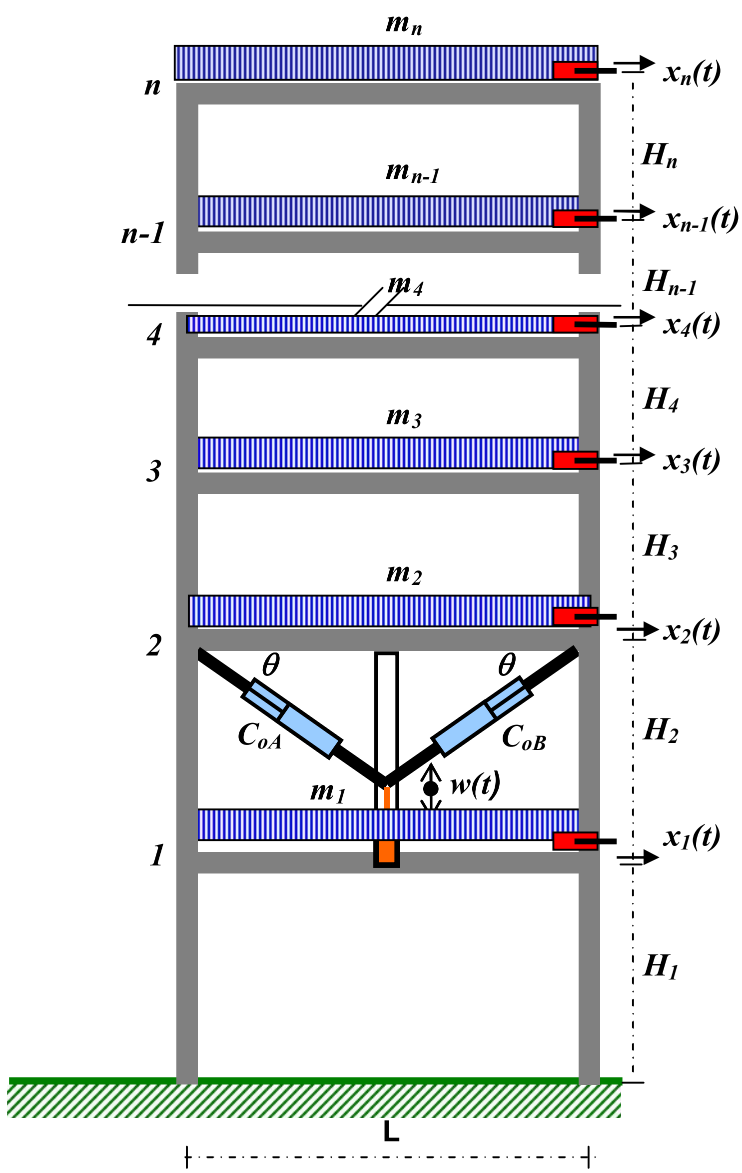


Figure 1.9: MDOF Structure with the VDSA device

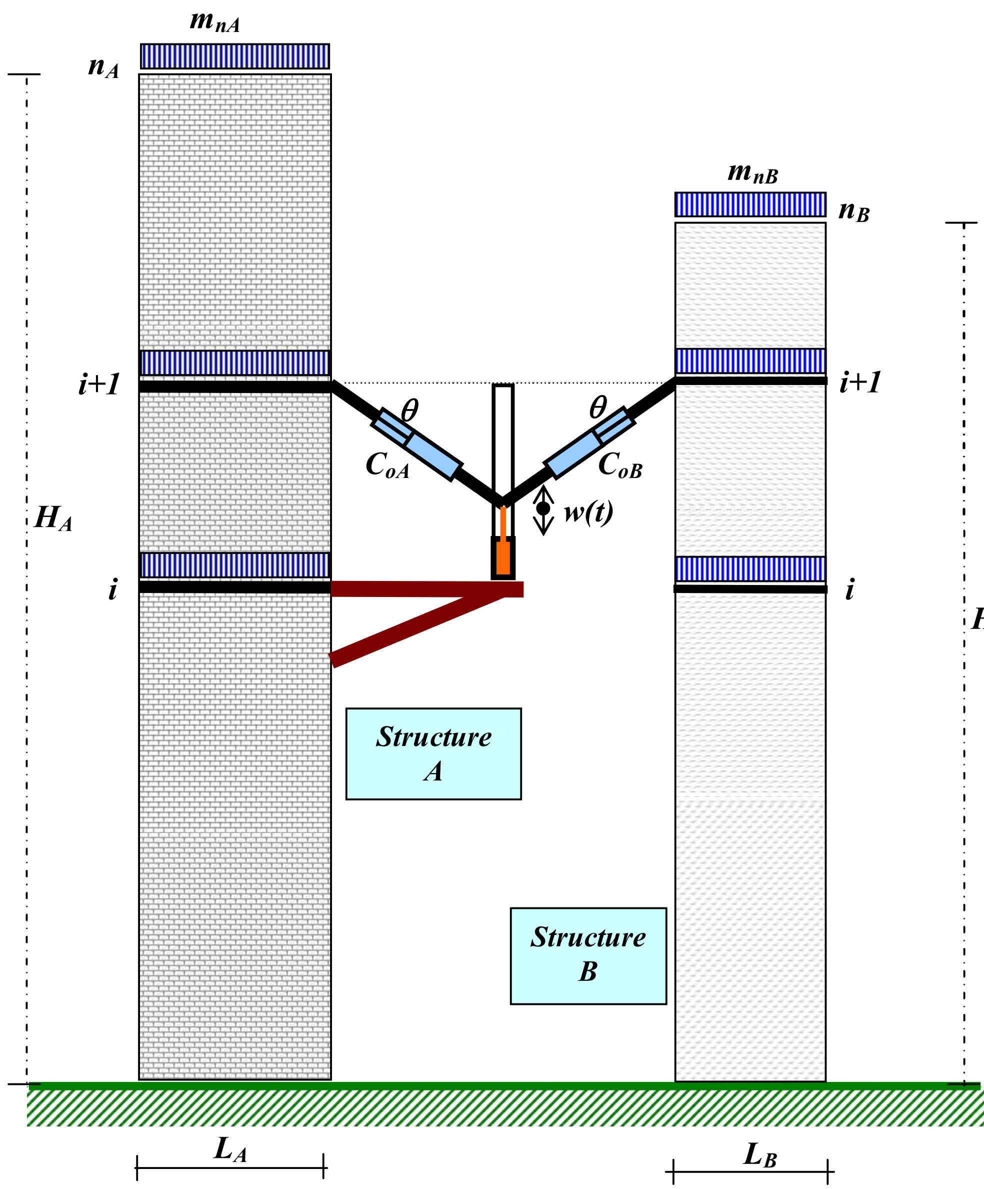


Figure 1.10: MDOF Coupled Structures with the VDSA device

1.4 ORGANIZATION OF THE THESIS

The dissertation consists of nine chapters. A brief description of the contents of each chapter is provided next.

Chapter I presents the reasons that motivated the present investigation. Subsequently, a literature review of the history and current status of passive control, semiactive control and coupled building control is provided. The chapter concludes with a description of the objectives of the investigation.

The analytical derivation of the two modified *Closed-loop control* algorithms ***Qv*** and modified *Closed-open-loop control* algorithms ***Qv*** is presented in Chapter II.

Chapters III and IV deal with the formulation of the equations of motion in single and coupled structures with the **VDSA** device, respectively. For each of these two cases, simulation of the response of the equations of motion are derived for SDOF and MDOF structures.

Chapter V, VI, VII and VIII present the numerical simulation of the response structural models developed in Chapters III and IV when they are subjected to acceleration time histories of real earthquakes. Additionally, the same responses obtained are compared with the corresponding quantities for the original structures and for the structures with passive dampers.

Chapter IX provides a summary and the conclusions that can be drawn for the numerical examples for the stand-alone and coupled structures controlled with the **VDSA** device and the new algorithms developed. Finally, this chapter concludes by suggesting a number of research areas for future studies.

The programs developed during the course of the study to perform the numerical simulations are included in Appendices. Those programs were developed in MATLAB.

CHAPTER II:

THEORETICAL DERIVATION OF MODIFIED ALGORITHMS *Qv*

2.1 INTRODUCTION

In this chapter, two algorithms of control (*Closed-loop* control and *Closed-open-loop* control) have been developed with base in the theory of Instantaneous Optimal control. This modified algorithm referred here as ***Qv***, can be used for active control and semi-active control. In this modified algorithm, the performance index ***J*** is quadratic in state vector $\{z(t)\}$, the control force $\{u(t)\}$, and the absolute velocity $\{x_a(t)\}$. The algorithms include a generalized **LQR** algorithm where a penalty is imposed through the matrix **Q** on the state vector, through the matrix **R** on the control vector and through the matrix **Qv** on the absolute velocity vector. The algorithms are applied to control the seismic response of a single and a multi-degree of freedom structural model. The algorithms also are used to control the seismic response in coupled structures of a SDOF and a MDOF system. Both algorithms are effective in reducing of the seismic response in the structures.

2.2 INSTANTANEOUS OPTIMAL CONTROL

It is well known that the equation of motion of a structure modeled as a multi-degree of freedom system and subjected to a base acceleration $\ddot{x}_g(t)$ at all its supports is:

$$\left[M\right]_{nxn}\{\ddot{x}(t)\}+\left[C\right]_{nxn}\{\dot{x}(t)\}+\left[K\right]_{nxn}\{x(t)\}=-\left[M\right]_{nxn}\{E\}\ddot{x}_g(t) \qquad (2.1)$$

where [M], [C] y [K] are the mass, damping and stiffness matrix respectively, the vectors $\{\ddot{x}(t)\}$, $\{\dot{x}(t)\}$ and $\{x(t)\}$ are the acceleration, velocity and displacement of each dynamic degree of freedom of the structure, $\{E\}$ is the vector of influence coefficients, and ***n*** is the number of degrees of freedom. Although it is not required by the formulation, for simplicity a shear building model will be used to model the structure.

If the structure is outfitted with ***r*** semi-active dampers, the previous equation of motion must be changed as follows:

$$\left[M\right]_{nxn}\{\ddot{x}(t)\}+\left[C\right]_{nxn}\{\dot{x}(t)\}+\left[K\right]_{nxn}\{x(t)\}=-\left[M\right]_{nxn}\{E\}\ddot{x}_g(t)+\left[D\right]_{nxr}\{u(t)\} \qquad (2.2)$$

The matrix [D] defines the locations of controllers, ***r*** is the number of controllers and $\{u(t)\}$ is the ***r***-dimensional control force vector.

To solve the system of motion equations 2.2 by decomposing into a set of uncoupled equations, it is convenient to change it into a system of $2n$ first order differential equations. In Linear System Theory this method is referred to as the state-space representation (see Appendix A). Introducing the following response vector and matrices,

$$\{z(t)\} = \begin{Bmatrix} \{x(t)\} \\ \{x(t)\} \end{Bmatrix} \quad [A] = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \quad [B] = \begin{bmatrix} 0 \\ M^{-1}D \end{bmatrix} \quad [H] = \begin{bmatrix} 0 \\ -E \end{bmatrix} \qquad (2.3)$$

Using the state-space representation, Equation 2.3 takes the form:

$$\{z(t)\}_{2nx1} = [A]_{2nx2n}\{z(t)\} + [B]_{2nx2n}\{u(t)\} + [H]_{2nx1}\{x_g(t)\} \qquad (2.4)$$

where $\{z(t)\}$ is state vector, the system matrix $[A]$, and the matrices $[B]$ and $[H]$ were proposed by Soong [34].

To define the variation of the control forces in $\{u(t)\}$ one needs to select a control algorithm. In this study the Optimal Control Theory will be used to define these forces. In particular, in the algorithm known as the linear quadratic regulator (or **LQR**) the optimal control forces $\{u(t)\}$ are selected by minimizing a performance index ***J***. The performance index is defined as the integral over time of quadratic forms defined in terms of the response and control vectors:

$$J = \int_0^{t_f} \left[z^T(t)\mathbf{Q}z(t) + u^T(t)\mathbf{R}u(t) \right] dt \qquad (2.5)$$

where t_f is the duration of excitation, $[Q]$ is a $2n$ x $2n$ symmetric positive semi-definite weighting matrix and $[R]$ is an m x m positive definite weighting matrix. By altering the coefficients in the weighting matrices $[Q]$ and $[R]$ one can increase the reduction in the response at the expense of having to apply higher control forces, or vice versa.

From this point on, the estimation of the control forces $\{u(t)\}$ provided by the device and the response state vector $\{z(t)\}$ depends of the specific control configuration of sensors and actuators used, and its corresponding algorithm. There are two possible configurations: *Closed-loop control* and *Closed-open-loop control*. The *Open-loop* case is the structure without any active or semi-active control system. The theory to define the control and response vectors can be found in many textbooks and monographs (Soong 1990, Meirovitch 1990, Brogan 1991, Connor 2003). Here only the final results are reported.

Considering first a *Closed-loop control*, the variables $\{u(t)\}$ and $\{z(t)\}$ can be obtained as follows:

$$u(t) = -\frac{\Delta t}{2} R^{-1} B^T Q z(t) \tag{2.6}$$

$$z(t) = \left[I + \frac{\Delta t^2}{4} B R^{-1} B^T Q \right]^{-1} \left[T d(t - \Delta t) + \frac{\Delta t}{2} H \ddot{x}_g(t) \right] \tag{2.7}$$

For *Closed-open-loop Control,* {$u(t)$} and {$z(t)$} is calculated with the equations 3.8 and 3.9 respectively:

$$u(t) = \frac{\Delta t}{4} R^{-1} B^T \left[P z(t) + p(t) \right] \tag{2.8}$$

$$z(t) = \left[I + \frac{\Delta t^2}{8} B R^{-1} B^T P \right]^{-1} \left[T d(t - \Delta t) + \frac{\Delta t^2}{8} B R^{-1} B^T p(t) + \frac{\Delta t}{2} H \ddot{x}_g(t) \right] \tag{2.9}$$

where [T] is the ($2n$ x $2n$) modal matrix whose columns are the eigenvectors of [A], and {$d(t\text{-}\Delta t)$} is a vector containing the response and control vectors and the excitation at time $t\text{-}\Delta t$, defined as:

$$d(t - \Delta t) = e^{\Lambda \Delta t} T^{-1} \left\{ z(t - \Delta t) + \frac{\Delta t}{2} \left[B u(t - \Delta t) + H \ddot{x}_g(t - \Delta t) \right] \right\} \quad \wedge \quad \Lambda = T^{-1} A T \tag{2.10}$$

The matrix [Λ] is a diagonal matrix with the eigenvalues of [A].

In the equations 2.8 and 2.9, ***P*** is the matrix Riccati equation and ***p*(*t*)** represents the *Open-loop control*.

$$P = -\left[I + \frac{\Delta t^2}{8} QBR^{-1}B^T \right]^{-1} Q \tag{2.11}$$

$$p(t) = P\left[Td(t - \Delta t) + \frac{\Delta t}{2} H\ddot{x}_g(t) \right] \tag{2.12}$$

2.3 THEORETICAL DERIVATION OF MODIFIED ALGORITHMS Qv

This modified algorithm referred here as ***Qv***, can be used for active control and semi-active control. In this modified algorithm, the performance index ***J*** is quadratic in state vector $\{z(t)\}$, the control force $\{u(t)\}$, and the absolute velocity $\{\dot{x}_a(t)\}$. The cost function takes the form:

$$J = \int_0^{t_f} \left[z^T(t)\mathbf{Q}z(t) + \dot{x}_a^T(t)\mathbf{Q_v}\dot{x}_a(t) + u^T(t)\mathbf{R}u(t) \right] dt \tag{2.13}$$

where **Qv** is a *nxn* symmetric positive semi-definite weighting matrix.

To solve the optimal control problem with ***J*** defined by equation 2.13 subject to the constraint represented by equation 2.3, the Lagrangian ***L*** is first formed by adjoining these two equations with a time-varying Lagrange multiplier $\lambda(t)$, see reference [34].

$$L = \int_0^{t_f} \begin{bmatrix} z^T(t)Qz(t) + x_a{}^T(t)Q_v x_a(t) + u^T(t)Ru(t) + \\ \lambda^T(t)\left[[A]\{z(t)\} + [B]\{u(t)\} + [H]\{x_g(t)\} - \{z(t)\}\right] \end{bmatrix} dt \tag{2.14}$$

The Hamiltonian $\mathcal{H}$ is defined as the integrand of equation 2.14. In this case, the Hamiltonian is:

$$\begin{aligned} \mathcal{H} &= z^T(t)Qz(t) + x_a{}^T(t)Q_v x_a(t) + u^T(t)Ru(t) + \\ &\lambda^T(t)\left[\{z(t)\} - Td(t-\Delta t) - \frac{\Delta t}{2}\left[[B]\{u(t)\} + [H]\{x_g(t)\}\right]\right] \end{aligned} \tag{2.15}$$

The absolute velocity vector $\{x_a(t)\}$ is computed from 2.3 as

$$\{x_a(t)\} = [A_v]_{nx2n}\{z(t)\} + [S_v]_{nx1}\{x_g(t)\} \tag{2.16}$$

where $[A_v] = [0 \;\vdots\; I]$, $[S_v] = [I]$ and $\{x_g(t)\}$ is the ground velocity vector.

The necessary conditions for minimization from 2.15 are:

$$\frac{\partial \mathcal{H}}{\partial z(t)} = 0 \;\rightarrow\; 2\,Qz(t) + 2A_v^T Q_v x_a(t) + \lambda(t) = 0 \tag{2.17}$$

$$\frac{\partial \mathcal{H}}{\partial u(t)} = 0 \;\rightarrow\; 2\,Ru(t) - \frac{\Delta t}{2}\left[B\right]^T \lambda(t) = 0 \tag{2.18}$$

$$\frac{\partial \mathcal{H}}{\partial \lambda(t)} = 0 \;\rightarrow\; \{z(t)\} - Td\left(t - \Delta t\right) - \frac{\Delta t}{2}\Big[\left[B\right]\{u(t)\} + \left[H\right]\{x_g(t)\}\Big] = 0 \tag{2.19}$$

2.3.1 CASE I: MODIFIED *CLOSED-LOOP CONTROL* ALGORITHM *Qv*

In the *Closed-loop control*, the control vector is regulated by the state vector as.

$$\lambda(t) = P\{z(t)\} \tag{2.20}$$

With the equations 2.16 and 2.17, $\lambda(t)$ can be expressed as:

$$\begin{aligned}\lambda(t) &= -2\,Qz(t) - 2A_v^T Q_v x_a(t) \\ &= -2\,Qz(t) - 2A_v^T Q_v\left[A_v z(t) + S_v x_g(t)\right] \\ &= -2\,Qz(t) - 2A_v^T Q_v A_v z(t) - 2A_v^T Q_v S_v x_g(t) \\ &= -2\left[Q + A_v^T Q_v A_v\right] z(t) - 2A_v^T Q_v S_v x_g(t)\end{aligned} \tag{2.21}$$

Comparing the equations 2.20 and 2.21, for this case P is calculated as

$$P = -2\left[Q + A_v^T Q_v A_v\right] \tag{2.22}$$

Of the equation 2.18, the control force vector can be obtained in function of $\lambda(t)$

$$u(t) = \frac{\Delta t}{4} R^{-1} B^T \lambda\left(t\right) \tag{2.23}$$

Replacing $\lambda(t)$ of equation 2.21 in 2.23, results in a control vector $u(t)$ given by

$$\begin{aligned} u(t) &= -\frac{\Delta t}{4} R^{-1} B^T \left[2\left[Q + A_v^T Q_v A_v\right] z(t) + 2A_v^T Q_v S_v x_g(t)\right] \\ &= -\frac{\Delta t}{2} R^{-1} B^T \left[A_2 z(t) + A_3 x_g(t)\right] \end{aligned} \tag{2.24}$$

where $A_2 = Q + A_v^T Q_v A_v$ and $A_3 = A_v^T Q_v S_v$

Of the equation 2.19

$$z(t) = Td\left(t - \Delta t\right)\frac{\Delta t}{2}\left[\left[B\right]\{u(t)\} + \left[H\right]\{x_g(t)\}\right] \tag{2.25}$$

Substituting equation 2.24 in 2.25

$$z(t) = Td\left(t-\Delta t\right)+\frac{\Delta t}{2}\left[-\frac{\Delta t}{2}BR^{-1}B^{T}\left[A_2 z(t)+A_3 x_g(t)\right]+H\ddot{x}_g(t)\right]$$
$$= Td\left(t-\Delta t\right)-\frac{\Delta t^2}{4}BR^{-1}B^{T}\left[A_2 z(t)+A_3 x_g(t)\right]+\frac{\Delta t}{2}H\ddot{x}_g(t)$$

$$\left[I+\frac{\Delta t^2}{4}BR^{-1}B^{T}A_2\right]z(t) = Td\left(t-\Delta t\right)-\frac{\Delta t^2}{4}BR^{-1}B^{T}A_3 x_g(t)+\frac{\Delta t}{2}H\ddot{x}_g(t)$$

Finally the state vector $z(t)$ for *Closed-loop control* in the modified algorithm ***Qv*** is calculated as,

$$z(t) = \left[I+\frac{\Delta t^2}{4}BR^{-1}B^{T}A_2\right]^{-1}\left[Td\left(t-\Delta t\right)-\frac{\Delta t^2}{4}BR^{-1}B^{T}A_3 x_g(t)+\frac{\Delta t}{2}H\ddot{x}_g(t)\right] \quad (2.26)$$

2.3.2 CASE II: MODIFIED *OPEN-CLOSED-LOOP CONTROL* ALGORITHM *Qv*

For the *Open-closed-loop control*, the control vector is regulated by the state vector and the external excitation.

$$\lambda\left(t\right) = P\{z(t)\}+p\left(t\right) \quad (2.27)$$

Rewriting the equation 2.21

$$\lambda(t)+2\left[Q+A_v^T Q_v A_v\right]z(t)+2A_v^T Q_v S_v x_g(t)=0$$
$$\lambda(t)+2Qz(t)+2A_v^T Q_v A_v z(t)+2A_v^T Q_v S_v x_g(t)=0 \tag{2.28}$$
$$\lambda(t)+Q\left[z(t)+z(t)\right]+2A_v^T Q_v A_v z(t)+2A_v^T Q_v S_v x_g(t)=0$$

Replacing the equations 2.23 and 2.25 in a state vector *z*(*t*) from the equation 2.28.

$$\begin{aligned}\lambda(t)+Q\left[z(t)+Td\left(t-\Delta t\right)+\frac{\Delta t}{2}\left[\frac{\Delta t}{4}BR^{-1}B^T\lambda(t)+H\ddot{x}_g(t)\right]\right]+\\ 2A_v^T Q_v A_v z(t)+2A_v^T Q_v S_v x_g(t)=0\end{aligned} \tag{2.29}$$

Substituting equation 2.27 in 2.29 and grouping for *z*(*t*) and *p*(*t*), it is obtained as

$$\begin{aligned}Pz(t)+p(t)+Q\left[z(t)+Td\left(t-\Delta t\right)+\frac{\Delta t}{2}\left[\frac{\Delta t}{4}BR^{-1}B^T\left(Pz(t)+p(t)\right)+H\ddot{x}_g(t)\right]\right]+\\ 2A_v^T Q_v A_v z(t)+2A_v^T Q_v S_v x_g(t)=0\end{aligned}$$

$$\begin{aligned}\left[Q+\frac{\Delta t^2}{8}QBR^{-1}B^T P+2A_v^T Q_v A_v+P\right]z(t)+\left[\frac{\Delta t^2}{8}QBR^{-1}B^T+I\right]p(t)+\\ \left[\left[Td\left(t-\Delta t\right)+\frac{\Delta t}{2}H\ddot{x}_g(t)\right]Q+2A_v^T Q_v S_v x_g(t)\right]=0\end{aligned}$$

$$\left[Q+2A_v^TQ_vA_v+\left[I+\frac{\Delta t^2}{8}QBR^{-1}B^T\right]P\right]z(t)+\left[\frac{\Delta t^2}{8}QBR^{-1}B^T+I\right]p(t)+$$
$$\left[\left[Td(t-\Delta t)+\frac{\Delta t}{2}H\ddot{x}_g(t)\right]Q+2A_v^TQ_vS_v\ddot{x}_g(t)\right]=0 \tag{2.30}$$

The solution for the unknown quantities P and $p(t)$ can be found from the above equation. Since $z(t)\neq 0$ and $p(t)\neq 0$, the desired results are obtained solving the equation 2.30, as follows:

$$P=-\left[Q+2A_v^TQ_vA_v\right]\left[I+\frac{\Delta t^2}{8}QBR^{-1}B^T\right]^{-1} \tag{2.31}$$

$$\left[\frac{\Delta t^2}{8}QBR^{-1}B^T+I\right]p(t)+\left[\left[Td(t-\Delta t)+\frac{\Delta t}{2}H\ddot{x}_g(t)\right]Q+2A_v^TQ_vS_v\ddot{x}_g(t)\right]=0$$
$$p(t)=-\left[\frac{\Delta t^2}{8}QBR^{-1}B^T+I\right]^{-1}\left[Q\left[Td(t-\Delta t)+\frac{\Delta t}{2}H\ddot{x}_g(t)\right]+2A_v^TQ_vS_v\ddot{x}_g(t)\right] \tag{2.32}$$

Replacing the equations 2.23 and 2.27 in 2.19, and factoring

$$z(t)=Td(t-\Delta t)+\frac{\Delta t}{2}\left[\frac{\Delta t}{4}BR^{-1}B^T\left(Pz(t)+p(t)\right)+H\ddot{x}_g(t)\right]$$

$$\left[I-\frac{\Delta t^2}{8}BR^{-1}B^TP\right]z(t)=Td\left(t-\Delta t\right)+\frac{\Delta t^2}{8}BR^{-1}B^Tp\left(t\right)+\frac{\Delta t}{2}H\ddot{x}_g(t)$$

The state vector $z(t)$ for *Open-closed-loop control* in the modified algorithm ***Qv*** is obtained with the next equation,

$$z(t)=\left[I-\frac{\Delta t^2}{8}BR^{-1}B^TP\right]^{-1}\left[Td\left(t-\Delta t\right)+\frac{\Delta t^2}{8}BR^{-1}B^Tp\left(t\right)+\frac{\Delta t}{2}H\ddot{x}_g(t)\right] \quad (2.33)$$

where the solution of the Riccati matrix P and the vector *open-loop control* $p(t)$, can be found from the equations 2.31 and 2.32, respectively.

The programs developed of instantaneous optimal control modified algorithms ***Qv*** (*Closed-loop control* and *Closed-open-loop control*), for SDOF and MDOF are presented in Appendices B, C, D, E, F, G, H and I.

CHAPTER III:

EQUATIONS OF MOTION OF SINGLE STRUCTURES CONTROLLED WITH THE VDSA DEVICE

3.1 INTRODUCTION

This chapter presents the mathematical formulation required to calculate the seismic response of a structure with a set of two fixed-orifice viscous fluid dampers at one floor. The dampers are installed in the form of ***V*** with a central rod between the two arms and with the upper ends attached to the top floor. The bottom end of the two arms of the ***V*** can move vertically up or down, with its movement governed by the damping required by the structure at each sampled time. This motion changes the forces applied by the dampers to the structure and thus they effectively modify its damping in a continuous fashion. The device is shown in Figure 3.3 for a single degree of freedom structure. The mechanism is termed the **V**ariable **D**amping **S**emi-**A**ctive (**VDSA**) device. The damping coefficient required by the structure is calculated by means of the two new instantaneous optimal control algorithms ***Qv*** (*Closed-loop control* and *Closed-open-loop control*) developed in Chapter II. The damping coefficient *c(t)* during the response can be adjusted between upper and lower limits, c_{max} and c_{min}.

3.2 PASSIVE ENERGY DISSIPATION

In October 1997, the U.S. Federal Emergency Management Agency published a new version of the NEHRP Guidelines and Commentary for the Seismic Rehabilitation of Buildings, also known as FEMA 273 and 274 (see reference [53]). The guidelines have more documentation on the seismic design of structures with passive energy dissipation devices than any other earlier published design codes. This section specifies analysis methods and design criteria for passive energy dissipation systems with fluid viscous devices according to FEMA 273 and 274. The procedure presented is valid for a structure with a lineal behavior. Linear procedures are only permitted if it can be demonstrated that the framing system, exclusive of the energy dissipation devices, remains essentially linearly elastic for the level of earthquake demand of interest after the effects of added damping are considered. Further more, the effective damping ratio afforded by the energy dissipation shall not exceed 30% of critical in the fundamental mode.

3.2.1 VISCOUS FLUID DEVICES

The force-displacement response of a velocity-dependent dashpot-type device is primarily a function of the relative velocity between each end of the device, and is generally dependent on the operating temperature (including temperature rise due to excitation). Figure 3.1 displays a typical longitudinal cross section of a viscous fluid damper.

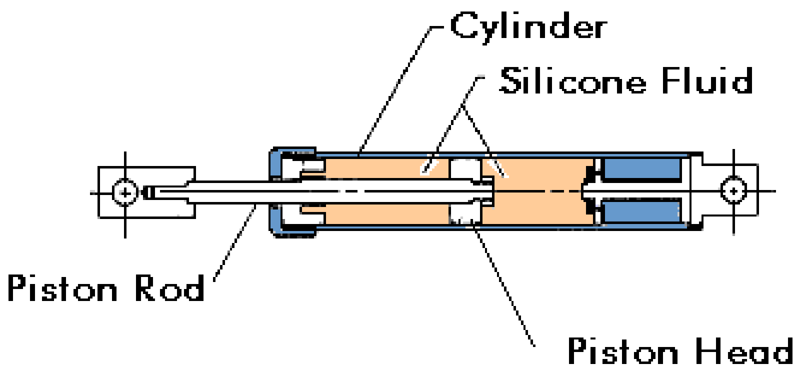


Figure 3.1: Longitudinal cross section of a viscous fluid damper

The ideal force output of a viscous fluid damper can be expressed as:

$$F_d = C_o \left|\dot{x}(t)\right|^{\alpha} \operatorname{sgn}\left(\dot{x}(t)\right) \tag{3.1}$$

where F_d is the damper force, C_0 is the damping coefficient for the device, $\dot{x}(t)$ is the relative velocity between each end of the device, *sgn* is the signum function that in this case defines the sign of the relative velocity term, and α is the velocity exponent for the device which may take values in the range of about 0.25 to 2 (see reference [10]). The case α=1 correspond to a linear viscous damper in which the damper force is proportional to the relative velocity. When α smaller than 1 the device is called a nonlinear viscous damper. Values of α larger than 1 (also a nonlinear damper) have not been used often in practical applications.

Figure 3.2 shows the force-velocity relationship for the three different types of viscous dampers. This figure demonstrates the efficiency of nonlinear dampers with α>1 in minimizing high velocity shocks. For a small relative velocity, the damper with a α value less than 1 can provide a larger damping force than the other types of dampers.

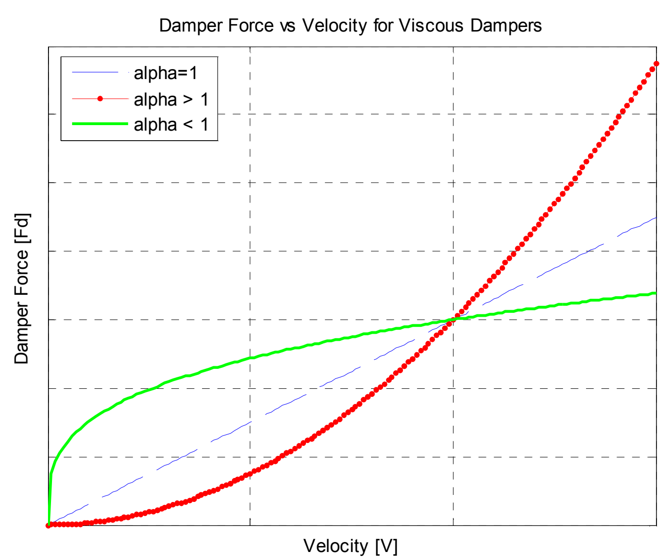


Figure 3.2: Force – Velocity relationship of viscous dampers

The calculation of the damping effect should be estimated as:

$$\beta_{eff} = \beta + \frac{\sum_j W_j}{4\pi W_k} \tag{3.2}$$

where β is the inherent damping in the structural frame, W_j is the work done (energy dissipated) by the device j in one complete cycle corresponding to floor displacements δ_i, the summation extends over all devices j, and W_k is the maximum strain energy stored in the frame, determined as:

$$W_k = \frac{1}{2}\sum_i F_i \delta_i \tag{3.3}$$

where F_i is the inertia force at floor level i and the summation extends over all floor levels.

The work done by the linear viscous device j in one complete cycle of loading can be calculated as:

$$W_j = \frac{2\pi^2}{T} C_j \delta_{rj}^2 \tag{3.4}$$

where T is the fundamental period of the building with the dampers including the stiffness of the velocity dependent devices, C_j is the damping constant for the device j, and δ_{rj} is the relative displacement between the ends of device j along the axis of the device j. An alternative equation for calculating the effective damping ratio defined in equation 3.2 is (see reference [53]):

$$\beta_{eff} = \beta + \frac{T \sum_j C_j \cos^2 \theta_j \phi_{rj}^2}{\pi \sum_i \left(\frac{W_i}{g} \right) \phi_i^2} \tag{3.5}$$

where θ_j is the angle of inclination of device j to the horizontal, ϕ_{rj} is the first mode relative displacement between the ends of device j in the horizontal direction, W_i is the reactive weight of floor level i, ϕ_i is the first mode displacement at floor level i, and the other terms are as defined above. Equation 3.5 applies to linear viscous devices only.

3.3 THE VDSA IN SDOF STRUCTURES

The system considered is shown schematically in Figure 3.3. It consists of a single degree of freedom structure (SDOF) with the proposed variable damping system installed. As mentioned before, the device consists of two viscous fluid dampers installed at a variable inclined position. The dampers have fixed-constant damping coefficient C_o.

The structure consists of a mass m distributed at the roof level, a massless frame that provides stiffness k to the system, and the natural (inherent) damping of the structure is represented by a damper with constant C_s (=$2\zeta m\omega$). This model may be considered as an idealization of a one-story structure. Each structural member (column, beam) of the structure contributes to the inertial (mass), elastic (stiffness or flexibility), and energy dissipation (damping) properties of the structure. In the idealized system, however, each of these properties is concentrated in three separate, pure components: a mass, a spring component, and a damper component.

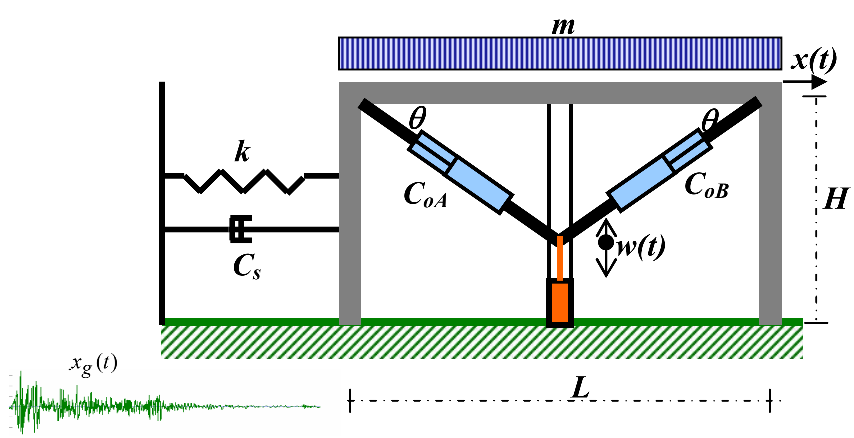


Figure 3.3: SDOF structure with the VDSA device

The governing differential equation of motion for a non-linear 1-story shear building with a semi-active damper is:

$$F_i(t)+F_d(t)+F_s(t)+F_c(t)=-m\ddot{x}_g(t) \tag{3.6}$$

where F_i is the inertia force, F_d is the damping force, F_s is the restoring force, m is the lumped mass and F_c denotes the control force.

The inertia, damping and elastic force are

$$F_i(t) = m\ddot{x}(t) \quad ; \quad F_d(t) = C_s\dot{x}(t) \quad ; \quad F_s(t) = kx(t) \tag{3.7}$$

The relative displacement *x(t)* of the mass at some instant of time is shown in Figure 3.4. The lower ends of the dampers is assumed to be fixed in this Figure. The displacement, velocity and acceleration are positive in the direction of the positive *x*-axis.

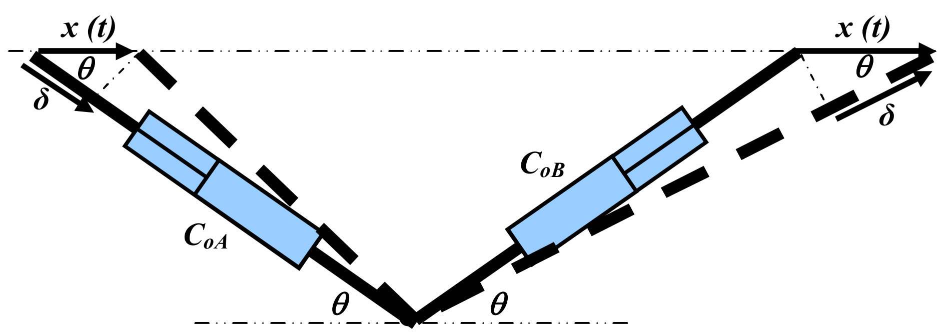


Figure 3.4: Deformation of the VDSA device installed in a SDOF structures

The displacement δ along the axis of the damper when the bottom of the damper is stationary is

$$\delta = x(t)\cos\theta(t) \tag{3.8}$$

Figure 3.5 displays the relative velocity $\dot{x}(t)$ of the floor and the velocity $\dot{w}(t)$ of the bottom end of the dampers. As shown in the figure, the damping force F_c proportional to the difference between the components of the velocities $\dot{x}(t)$ and $\dot{w}(t)$ along the axis of the dampers **A** and **B** (the **VDSA** device):

$$\begin{aligned}
F_{C_A} &= C_{O_A}\cos\theta(t)\left(\dot{\delta}-\dot{w}(t)\sin\theta(t)\right)\\
&= C_{O_A}\cos\theta(t)\left(\dot{x}(t)\cos\theta(t)-\dot{w}(t)\sin\theta(t)\right)\\
&= C_{O_A}\cos^2\theta(t)\dot{x}(t)-C_{O_A}\sin\theta(t)\cos\theta(t)\dot{w}(t)\\
F_{C_B} &= C_{O_B}\cos\theta(t)\left(\dot{\delta}+\dot{w}(t)\sin\theta(t)\right)\\
&= C_{O_B}\cos\theta(t)\left(\dot{x}(t)\cos\theta(t)+\dot{w}(t)\sin\theta(t)\right)\\
&= C_{O_B}\cos^2\theta(t)\dot{x}(t)+C_{O_B}\sin\theta(t)\cos\theta(t)\dot{w}(t)
\end{aligned} \tag{3.9}$$

$F_c(t)$ expressed in the equation 3.6 is obtained as,

$$\begin{aligned}
F_C(t) &= F_{C_A}+F_{C_B}\\
&= \left(C_{O_A}+C_{O_B}\right)\cos^2\theta(t)\dot{x}(t)-\left(C_{O_A}-C_{O_B}\right)\sin\theta(t)\cos\theta(t)\dot{w}(t)
\end{aligned} \tag{3.10}$$

According to equations 3.9, the force F_c is made of two contributions: ($F_C(t)=F_{C_1}(t)+F_{C_2}(t)$), where F_{c1} is the natural damping force which will provide a constant damping ratio

$$F_{C_1}=\left(C_{O_A}+C_{O_B}\right)\cos^2\theta(t)\dot{x}(t) \tag{3.11}$$

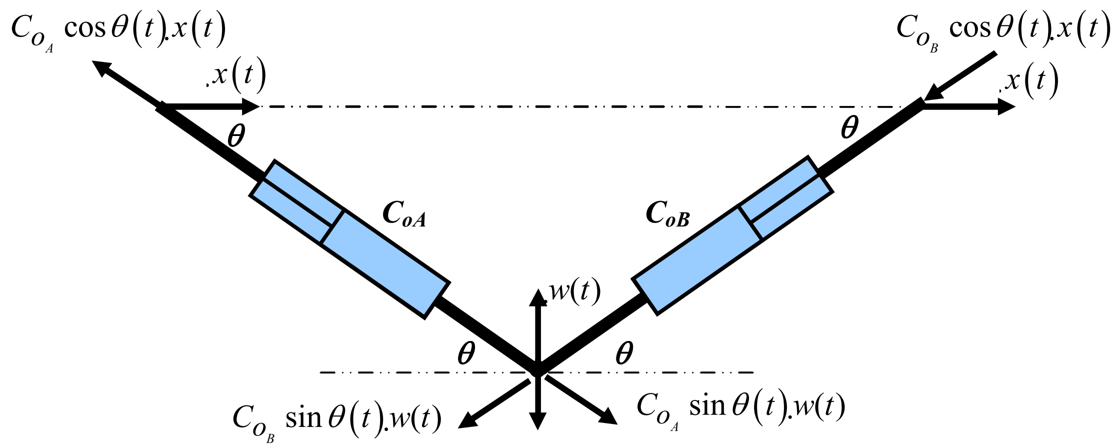


Figure 3.5: End velocities of the VDSA device installed in a SDOF structures

and F_{c2} is the variable damping force, which will provide an additional damping ratio variable in time:

$$F_{c_2} = -\left(C_{O_A} - C_{O_B}\right)\sin\theta(t)\cos\theta(t).w(t) = -\frac{1}{2}\left(C_{O_A} - C_{O_B}\right)\sin 2\theta(t).w(t) \qquad (3.12)$$

The forces $F_{c2}(t)$ can also be regarded as a control force, i.e., a time-varying force whose magnitude is determined by a control algorithm.

Substituting equations 3.7, 3.11 and 3.12 into equation 3.6, can be shown that the equation of motion for the single dof structure subjected to the horizontal component of an earthquake - induced ground acceleration is

$$m\ddot{x}(t)+\left(C_S+\left(C_{O_A}+C_{O_B}\right)\cos^2\theta(t)\right)\dot{x}(t)+kx(t)=-m\ddot{x}_g(t)+\frac{1}{2}\left(C_{O_A}-C_{O_B}\right)\sin 2\theta(t)\dot{w}(t) \quad (3.13)$$

For a structure with two dampers in a fixed configuration, the second term in the right hand side of the equation of motion (3.13) vanishes. This term arises due to the motion of the lower end of the dampers along the vertical guiding rod.

Equation 3.13, can also be written as:

$$\begin{aligned}\ddot{x}(t)=-m^{-1}\left(C_S+\left(C_{O_A}+C_{O_B}\right)\cos^2\theta(t)\right)\dot{x}(t)-m^{-1}kx(t)-\ddot{x}_g(t)\\+\frac{1}{2}m^{-1}\left(C_{O_A}-C_{O_B}\right)\sin 2\theta(t)\dot{w}(t)\end{aligned} \quad (3.14)$$

Rewriting equation 3.13 using the space-state representation leads to

$$\begin{Bmatrix}\dot{z}_1(t)\\ \dot{z}_2(t)\end{Bmatrix}=\begin{bmatrix}0 & 1\\ -m^{-1}k & -m^{-1}\left(C_S+\left(C_{O_A}+C_{O_B}\right)\cos^2\theta(t)\right)\end{bmatrix}\begin{Bmatrix}z_1(t)\\ z_2(t)\end{Bmatrix}+\begin{bmatrix}0\\ \frac{1}{2}m^{-1}\left(C_{O_A}-C_{O_B}\right)\sin 2\theta(t)\end{bmatrix}\dot{w}(t)+\begin{bmatrix}0\\ -1\end{bmatrix}\ddot{x}_g(t) \quad (3.15)$$

where,

$$z_1(t)=x(t) \quad ; \quad z_2(t)=\dot{x}(t)$$

$$\cos^2\theta(t)=\frac{a^2}{a^2+\left[H-w(t)\right]^2} \quad ; \quad \sin 2\theta(t)=\frac{a\left[H-w(t)\right]}{a^2+\left[H-w(t)\right]^2} \quad ; \quad a=\frac{L}{2}$$

Equation 3.15 can be easily solved by decoupling it with the eigenvectors of the matrix in the right hand side, provided that the velocity displacement $\dot{w}(t)$ of the bottom support of the dampers is known. The term $\dot{w}(t)$ must be determined by using one of the two control algorithms specified in Chapter II.

3.4 THE VDSA IN MDOF STRUCTURES

The application of the **VDSA** device too multi-degree of freedom systems is similar to the SDOF case. Figure 3.6 shows the model a multi-story building with a **VDSA** device installed at the first floor.

In this case the forces applied by the mechanism to the first floor is still given by equation 3.10.

Figure 3.7 shows a multiple degree of freedom system with the **VDSA** device installed in the second level. The equation of motion of the a structures joined by the **VDSA** device placed between the $\boldsymbol{i}^{\text{th}}$ and $(\boldsymbol{i+1})^{\text{th}}$ floors and subjected to horizontal base acceleration is

$$[M]\{\ddot{x}(t)\}+[C_s]\{\dot{x}(t)\}+[K]\{x(t)\}+\{F_c(t)\}=-[M]\{r\}\ddot{x}_g(t) \qquad (3.16)$$

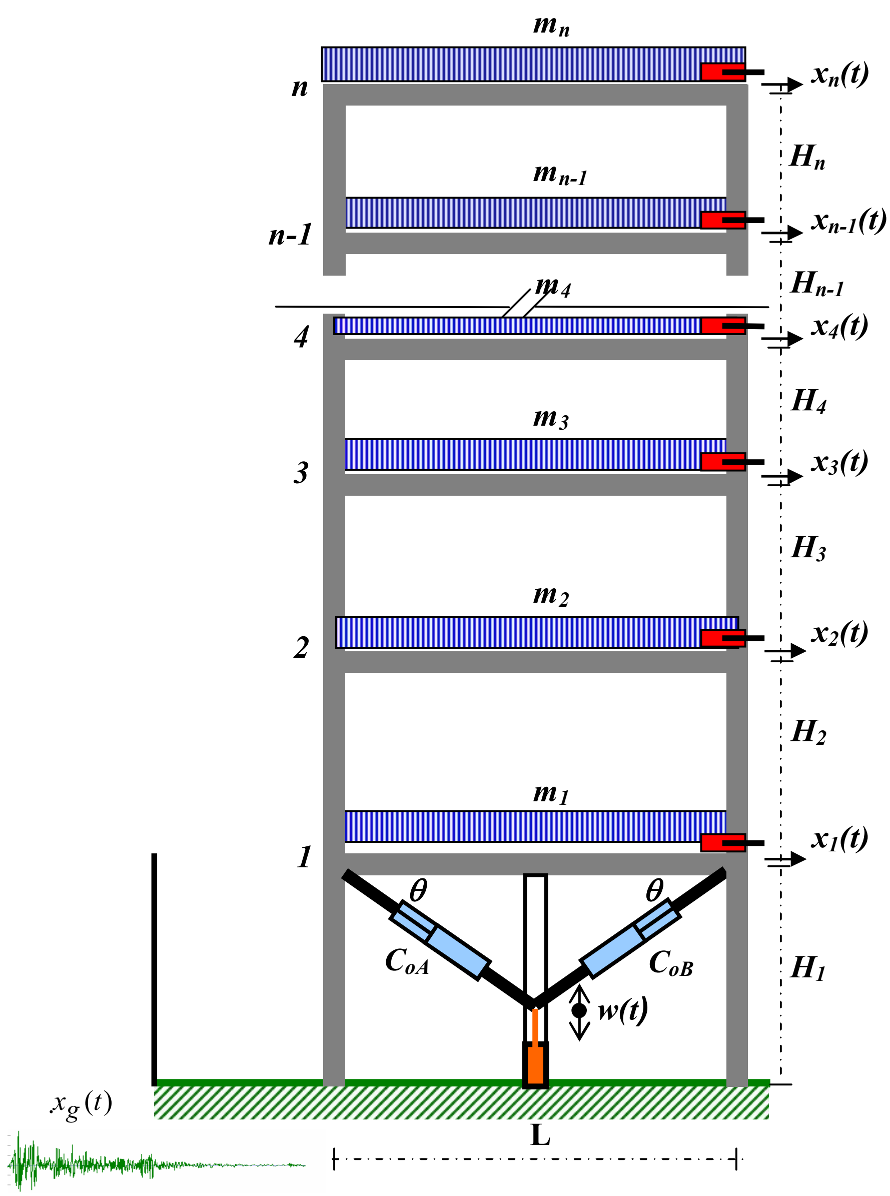


Figure 3.6: MDOF structure with the VDSA device installed in the first level

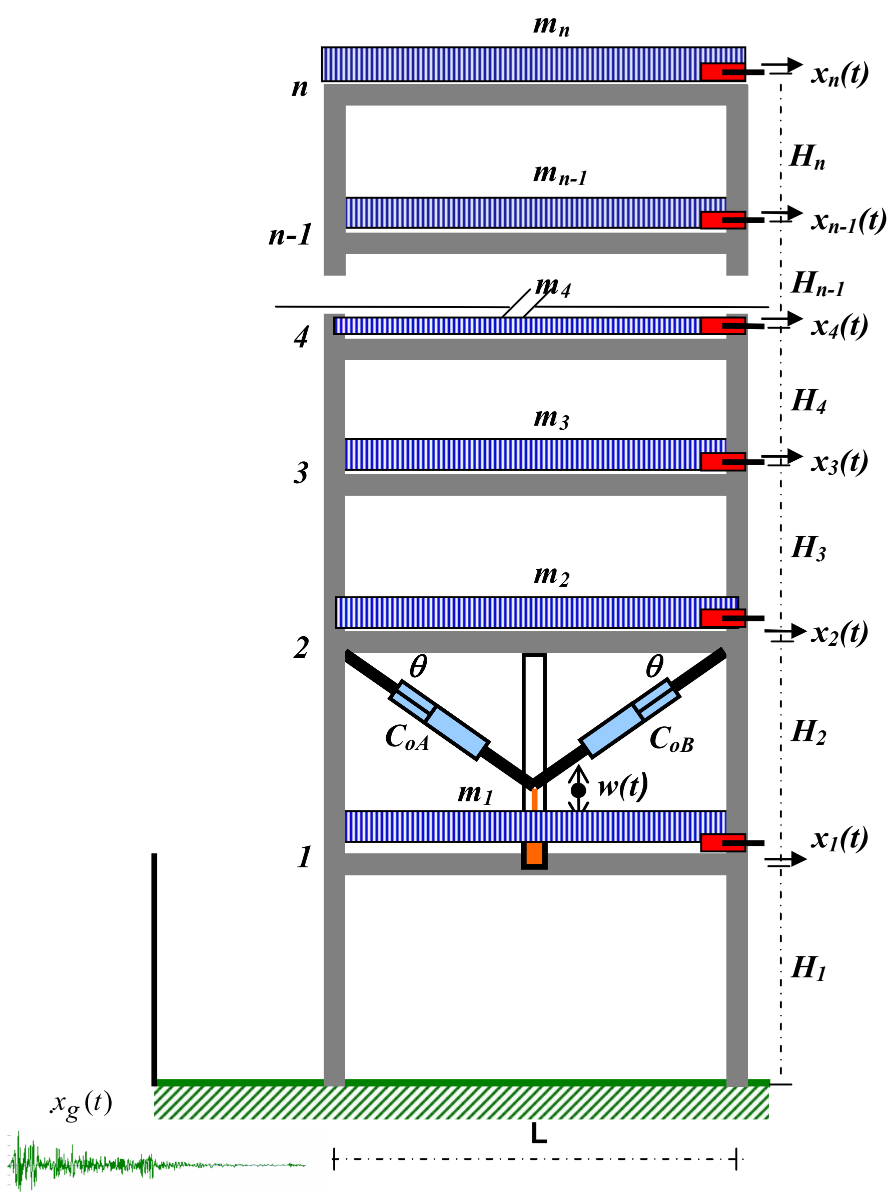


Figure 3.7: MDOF structure with the VDSA device installed in the second level

where $\{x(t)\}$, $\{\dot{x}(t)\}$ and $\{\ddot{x}(t)\}$ are, respectively, the vectors of relative displacements, velocities and accelerations, $[M]$, $[C]$ and $[K]$ are, respectively, the mass, damping and stiffness matrices of the structure, $\{r\}$ is the vector of displacement influence coefficients; $\{F_c(t)\}$ is the control forces vector. The dimensions of the vectors $\{x(t)\}$, $\{\dot{x}(t)\}$, $\{\ddot{x}(t)\}$, $\{F_c(t)\}$ and $\{r\}$, is ***n*x*1***.

The control force vector contains the forces applied by the **VDSA** device to the structure. The explicit form of the control forces in the dampers **A** and **B** will be determined later.

When the **VDSA** device this installed in the 2nd or superior level, the damping force $F_c(t)$ is related to the velocities $\dot{x}_i(t)$, $\dot{x}_{i+1}(t)$ and $\dot{w}(t)$ across the linear viscous damper (**VDSA** device) as shows in the Figure 3.8. The damping force F_d is related to the floor velocity $\dot{x}_{i+1}(t)$ through of C_s ($F_d(t) = C_s\dot{x}_{i+1}(t)$). The damping forces in the dampers F_{cA} and F_{cB} are calculates as:

$$\begin{aligned}
F_{c_A}(t) &= C_{O_A}\cos\theta(t)\left(\dot{x}_{i+1}(t)\cos\theta(t) - \dot{x}_i(t)\cos\theta(t) + \dot{w}(t)\sin\theta(t)\right) \\
&= C_{O_A}\cos^2\theta(t)\left(\dot{x}_{i+1} - \dot{x}(t)\right) + C_{O_A}\sin\theta(t)\cos\theta(t)\dot{w}(t) \\
F_{c_B}(t) &= C_{O_B}\cos\theta(t)\left(\dot{x}_{i+1}(t)\cos\theta(t) - \dot{x}_i(t)\cos\theta(t) - \dot{w}(t)\sin\theta(t)\right) \\
&= C_{O_B}\cos^2\theta(t)\left(\dot{x}_{i+1}(t) - \dot{x}(t)\right) - C_{O_B}\sin\theta(t)\cos\theta(t)\dot{w}(t)
\end{aligned} \tag{3.17}$$

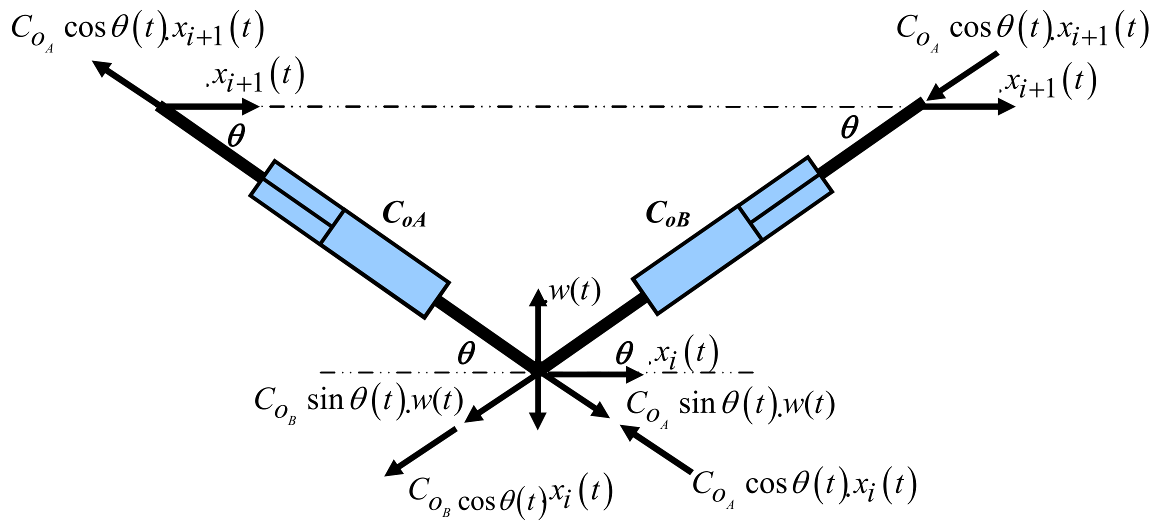


Figure 3.8: End velocities of the VDSA device installed in a MDOF structures in a level *i*

The expressions found for the control forces in the equations 3.17, define the vector $\{F_c(t)\}$ of the equation of motion (3.16).

$$
\begin{aligned}
F_c(t) &= F_{c_A}(t) + F_{c_B}(t) \\
&= \left(C_{O_A} + C_{O_B}\right)\cos^2\theta(t).x_{i+1}(t) - \left(C_{O_A} + C_{O_B}\right)\cos^2\theta(t).x_i(t) \\
&\qquad + \left(C_{O_A} - C_{O_B}\right)\sin\theta(t)\cos\theta(t).w(t) \qquad (3.18) \\
&= \left(C_{O_A} + C_{O_B}\right)\cos^2\theta(t).x_{i+1}(t) - \left(C_{O_A} + C_{O_B}\right)\cos^2\theta(t).x_i(t) \\
&\qquad + \frac{1}{2}\left(C_{O_A} - C_{O_B}\right)\sin 2\theta(t).w(t)
\end{aligned}
$$

The expressions found for the control force in the equations 3.18, must be written in matrix form, the vector $\{F_c(t)\}$ is function of $\{x_{i+1}(t)\}$, $\{x_i(t)\}$ and $.w(t)$:

$$\{F_c\} = [C_1]\{x_{i+1}(t)\} + [C_2]\{x_i(t)\} + \{D\}.w(t) \tag{3.19}$$

The matrices $[C_1]$ and $[C_2]$, and the vector $\{D\}$, are defined with three vectors with only one or two non-zero elements. The vectors $\{e_1\}$, $\{e_2\}$ and $\{e_3\}$, with length $\boldsymbol{n}$, are:

$$\begin{aligned} \{e_1\}^T &= [0,0,...,0,1,0,...,0] \text{ with 1 at column "}i+1\text{"} \\ \{e_2\}^T &= [0,0,...,0,-1,0,...,0] \text{ with -1 at column "}i\text{"} \\ \{e_3\}^T &= [0,0,...,0,-1,1,...,0] \text{ with -1 at column "}i\text{", 1 at column "}i+1\text{"} \end{aligned} \tag{3.20}$$

Using the three vectors of the equations 3.20, the matrices $[C_1]$ and $[C_2]$, and the vector $\{D\}$, can be written as:

$$\begin{gathered} [C_1] = \left(C_{o_A} + C_{o_B}\right)\cos^2\theta(t)\{e_1\}\{e_3\}^T \quad ; \quad [C_2] = \left(C_{o_A} + C_{o_B}\right)\cos^2\theta(t)\{e_2\}\{e_1\}^T \\ \{D\} = \frac{1}{2}\left(C_{o_A} - C_{o_B}\right)\sin 2\theta(t)\{e_1\} \end{gathered} \tag{3.21}$$

Substituting equations 3.19 and 3.21 into equation 3.16 and solving for $\{x(t)\}$ leads to

$$\{x(t)\} = -[M]^{-1}\left([C_s]+[C_1]+[C_2]\right)\{x(t)\} - [M]^{-1}[K]\{x(t)\} - [M]^{-1}\{D\}.w(t) - \{r\}.x_g(t) \tag{3.22}$$

Defining four matrices $[A_c]$, $[B_c]$, $[D_c]$ and $[E_c]$ as follows

$$[A_c] = -\left[[M]^{-1}[K]\right]$$

$$[B_c] = -\left[[M]^{-1}\left([C_s]+[C_1]+[C_2]\right)\right] \quad ; \quad [D_c] = -\left[[M]^{-1}\{D\}\right] \quad ; \quad [E_c] = \left[\{r\}\right] \tag{3.23}$$

And introducing the components of a state vector

$$\{z_1(t)\} = \{x(t)\} \qquad ; \qquad \{z_2(t)\} = \{x(t)\} \tag{3.24}$$

The equation 3.22, can be write as:

$$\dot{z}_2(t) = [A_c]\{z_1(t)\} + [B_c]\{z_2(t)\} + [D_c].w(t) - [E_c].x_g(t) \tag{3.25}$$

Finally, when equations 3.25 is supplemented with the following identity,

$$\{z_1(t)\} = \{z_2(t)\} \tag{3.26}$$

With the equations 3.25 and 3.26, is obtained the state-space form for the equation of motion 3.16:

$$\begin{Bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{Bmatrix} = \begin{bmatrix} [O_c] & [I_c] \\ [A_c] & [B_c] \end{bmatrix} \begin{Bmatrix} z_1(t) \\ z_2(t) \end{Bmatrix} + \begin{bmatrix} [O_c] \\ [D_c] \end{bmatrix} \dot{w}(t) - \begin{bmatrix} [O_c] \\ [E_c] \end{bmatrix} \left\{ \ddot{x}_g(t) \right\} \tag{3.27}$$

3.5 THE DAMPING COEFFICIENT SELECTED FOR THE STRUCTURE

The position ***w(t)*** required in each instant of time, can be computed of:

$$\begin{aligned} &\left(C_{O_A} + C_{O_B}\right)\cos^2\theta(t)\,\dot{x}(t) = u(t) \\ &\left(C_{O_A} + C_{O_B}\right)\frac{a^2}{a^2 + \left[H - w(t)\right]^2}\,\dot{x}(t) = u(t) \end{aligned} \tag{3.28}$$

where the force $u(t)$ is obtained with one of the two control algorithms (*Closed-loop control* and *Open-closed-loop control*) developed in Chapter II.

Solving equation 3.28 for ***w(t)***,

$$w(t) = H - a\sqrt{\left(C_{O_A} + C_{O_B}\right)\left|\frac{\dot{x}(t)}{u(t)}\right| - 1} \tag{3.29}$$

where $\dot{x}(t)$ is the relative velocity in the end of damper.

The damping coefficient for the structure is selected as,

$$C(t)=\begin{cases} C_S+\left(C_{O_A}+C_{O_B}\right)\dfrac{a^2}{a^2+\left[H-w_{\min}\right]^2} & \text{for } w(t)<w_{\min} \\ C_S+\left(C_{O_A}+C_{O_B}\right)\dfrac{a^2}{a^2+\left[H-w(t)\right]^2} & \text{for } w_{\min}<w(t)<w_{\max} \\ C_S+\left(C_{O_A}+C_{O_B}\right)\dfrac{a^2}{a^2+\left[H-w_{\max}\right]^2} & \text{for } w(t)>w_{\max} \end{cases} \tag{3.30}$$

3.6 SIMPLIFIED METHOD FOR CALCULATE THE VELOCITY IN EACH LEVEL

With the purpose of not install velocimeters in all the floors of the structure, a simple methodology is presented. This methodology consist in obtain the velocities between the levels two and *n-1*, knowing the speed of the first and last floor. Of the method of modal analysis we know that:

$$\begin{aligned} \dot{x}_n(t) &= \sum_{j=1}^{n}\phi_{n,j}\dot{\eta}_j(t) \\ \dot{x}_1(t) &= \sum_{j=1}^{n}\phi_{1,j}\dot{\eta}_j(t) \end{aligned} \tag{3.31}$$

where $\phi_{1,j}$ and $\phi_{n,j}$ are the modes vibration vector in the first and last floor respectively, $\dot{\eta}_j(t)$ is the "*modal coordinate*" or "*modal displacement*" of the mode *j*, $\dot{x}_1(t)$ and $\dot{x}_n(t)$ are the velocities in the first and last floor, respectively. Expanding the equations 3.31 for *j=2*

$$\begin{aligned} \dot{x}_n(t) &= \begin{bmatrix} \phi_{n,1} & \phi_{n,2} \end{bmatrix} \begin{bmatrix} \dot{\eta}_1(t) \\ \dot{\eta}_2(t) \end{bmatrix} \\ \dot{x}_1(t) &= \begin{bmatrix} \phi_{1,1} & \phi_{1,2} \end{bmatrix} \begin{bmatrix} \dot{\eta}_1(t) \\ \dot{\eta}_2(t) \end{bmatrix} \end{aligned} \tag{3.32}$$

Writing in matrix form the equations 3.32, takes the form

$$\begin{bmatrix} \dot{x}_n(t) \\ \dot{x}_1(t) \end{bmatrix} = \begin{bmatrix} \phi_{n,1} & \phi_{n,2} \\ \phi_{1,1} & \phi_{1,2} \end{bmatrix} \begin{bmatrix} \dot{\eta}_1(t) \\ \dot{\eta}_2(t) \end{bmatrix} \tag{3.33}$$

Solving 3.33 for $\dot{\eta}_1(t)$ and $\dot{\eta}_2(t)$

$$\begin{bmatrix} \dot{\eta}_1(t) \\ \dot{\eta}_2(t) \end{bmatrix} = \begin{bmatrix} \phi_{n,1} & \phi_{n,2} \\ \phi_{1,1} & \phi_{1,2} \end{bmatrix}^{-1} \begin{bmatrix} \dot{x}_n(t) \\ \dot{x}_1(t) \end{bmatrix} \tag{3.34}$$

The velocity in the level ***i***, can be calculated as

$$\dot{x}_i(t) = \begin{bmatrix} \phi_{i,1} & \phi_{i,2} \end{bmatrix} \begin{bmatrix} \dot{\eta}_1(t) \\ \dot{\eta}_2(t) \end{bmatrix} \tag{3.35}$$

Substituting the equation 3.34 in 3.35

$$\dot{x}_i(t) = \begin{bmatrix} \phi_{i,1} & \vdots & \phi_{i,2} \end{bmatrix} \begin{bmatrix} \phi_{n,1} & \vdots & \phi_{n,2} \\ \cdots & & \cdots \\ \phi_{1,1} & \vdots & \phi_{1,2} \end{bmatrix}^{-1} \begin{bmatrix} \dot{x}_n(t) \\ \cdots \\ \dot{x}_1(t) \end{bmatrix} \tag{3.36}$$

With the equation 3.36, can be calculated the velocity in the level ***i***, knowing the velocity in the first and ***n*** level. The equation 3.36 can be extended depending of the number of floors instrumented.

3.7 VDSA DEVICE COMPARED WITH PASSIVE SYSTEM CONTROL

Finally, the response for three cases: uncontrolled, controlled with passive dampers and controlled with **VDSA** device are compared. In the case for the structure controlled with passive dampers, the effective damping ratio contributed by dampers is calculated with the equation 3.5.

For the purpose of comparison with the structure controlled with passive dampers, two configurations I and II are presented in the Figure 3.9 and 3.10, respectively.

The numerical examples of SDOF and MDOF system on single and coupled structure are presented in Chapter V and VI, respectively. These examples are for the three analyses: uncontrolled, controlled with passive dampers and controlled with the **VDSA** device using the modified algorithm ***Qv*** (*Closed-loop control* and *Open-closed-loop control*).

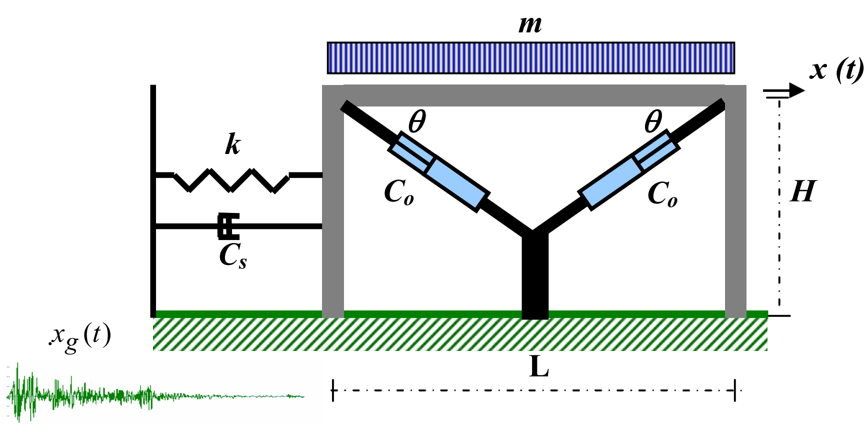


Figure 3.9: Configuration I for SDOF or MDOF structure with passive dampers installed

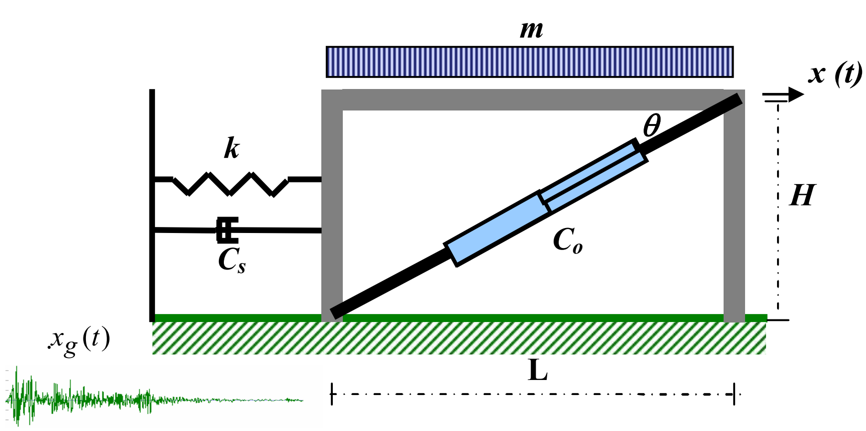


Figure 3.10: Configuration II for SDOF or MDOF structure with passive dampers installed

CHAPTER IV:

EQUATIONS OF MOTION FOR COUPLED STRUCTURES WITH THE VDSA DEVICE

4.1 INTRODUCTION

Coupling adjacent structures with supplemental damping devices is one of the methods that have been proposed to mitigate structural responses due to wind and seismic excitations. This chapter presents the formulation of the equations of motion for adjacent structures coupled with the **VDSA** device. The structures and the proposed **VDSA** device are now combined to form the coupled structure design model. The coupled building system is subjected to a ground excitation to simulate a seismic event. Again, the damping coefficient required by the structures is calculated with the instantaneous optimal control modified algorithms ***Qv*** (*Closed-loop control* and *Closed-open-loop control* versions), presented in Chapter II. The computer programs created for the simulation are provided in Appendix F to I. The algorithms are applied to control the seismic response of coupled structures represented by a single and a multi-degree of freedom structural model. The variable damping coefficient *c(t)* provided by the **VDSA** device can be adjusted during the earthquake between an upper and lower limit, c_{max} and c_{min}.

4.2 TWO COUPLED SDOF STRUCTURES

Figure 4.1 shows the analytical model of the two structures identified as **A** and **B** and modeled as single degree of freedom system. The structures are coupled through the **VDSA** device. The SDOF structures have masses m_A and m_B, a stiffness coefficients k_A and k_B, and the natural (inherent) damping of each structure is represented by dampers with constants C_{sA} and C_{sB}. These damping constants can be related to their natural frequencies ω and damping ratios ξ. The motion of the two structures will be described by the relative displacement (with respect to the ground) $x_A(t)$ and $x_B(t)$. A **VDSA** device is attached to the lumped mass of each structure and to a vertical rod connected to the ground. The dampers of the **VDSA** device have fixed damping coefficients C_{oA} and C_{oB}.

Both structures are assumed to be subjected to the same ground acceleration $\ddot{x}_g(t) = \ddot{x}_{g_A}(t) = \ddot{x}_{g_B}(t)$ and any effects due to spatial variations of the ground is neglected. The equations of motion for each of the structures with the semiactive control device system can be expressed as.

$$
\begin{aligned}
F_{i_A}(t)+F_{d_A}(t)+F_{S_A}(t)+F_{c_A}(t) &= -m_A\ddot{x}_g(t)\\
F_{i_B}(t)+F_{d_B}(t)+F_{S_B}(t)+F_{c_B}(t) &= -m_B\ddot{x}_g(t)
\end{aligned}
\tag{4.1}
$$

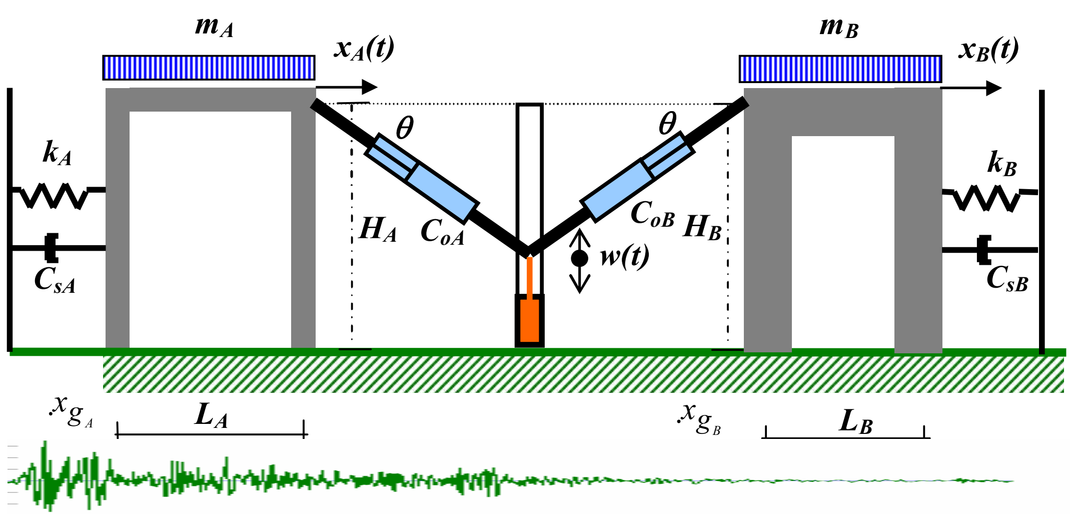


Figure 4.1: SDOF structures coupled through the VDSA device

$F_{cA}(t)$ and $F_{cB}(t)$ are the control forces applied to the two structures by the **VDSA** device. The specific form of the control forces is not known yet but they will become apparent later. Note that the two structures are coupled by these two forces. The subscripted forces $F_i(t)$, $F_d(t)$ and $F_s(t)$ are, respectively, the inertia, damping and elastic forces. The subscript identifies the structure as ***A*** or ***B***. The inertia, damping and elastic forces are:

$$\begin{aligned} F_{i_A} &= m_A \ddot{x}_A(t) & \qquad F_{i_B} &= m_B \ddot{x}_B(t) \\ F_{d_A} &= C_{s_A} \dot{x}_A(t) & \qquad F_{d_B} &= C_{s_B} \dot{x}_B(t) \\ F_{s_A} &= k_A x_A(t) & \qquad F_{s_B} &= k_B x_B(t) \end{aligned} \tag{4.2}$$

The dampers with constants C_{sA} and C_{sB} are not physical devices; rather they are used to represent the original energy dissipation characteristics of the individual structures. By assuming damping ratios ξ_A and ξ_B for the two individual structures, the damper constants can be determined as:

$$C_{s_A} = 2\xi_A m_A \omega_A \quad ; \quad C_{s_B} = 2\xi_B m_B \omega_B \tag{4.3}$$

where ω_A and ω_B are the natural frequencies of the uncoupled systems.

Figure 4.2 shows the deformed shape of the **VDSA** device due to the relative motion between the two adjacent structures. It is assumed in this figure that the lower end of the device is stationary.

The velocities $\dot{x}(t)$ of the masses of the two SDOF structures at a given instant of time are shown in Figure 4.3. This figure also shows the velocity $\dot{w}(t)$ of the lower end of the dampers which is used to control the structures.

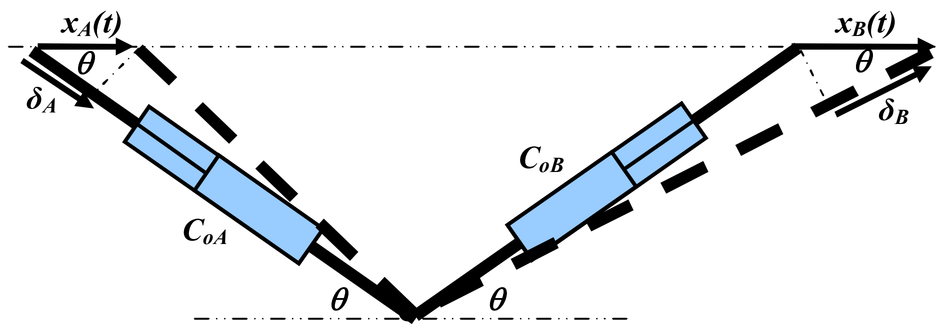


Figure 4.2: Deformation of the VDSA device installed between two SDOF structures

If the lower end of the dampers is at rest, the displacement δ along the axes in each of the viscous fluid dampers that form the **VDSA** is:

$$\delta_A = x_A(t)\cos\theta(t) \quad ; \quad \delta_B = x_B(t)\cos\theta(t) \tag{4.4}$$

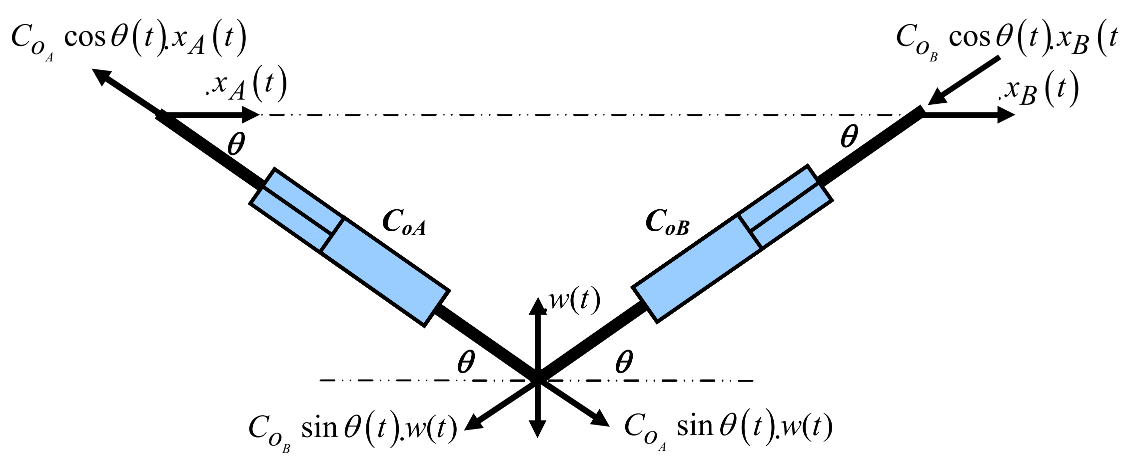


Figure 4.3: End velocities of the VDSA device installed between two SDOF structures

The forces $F_{cA}(t)$ and $F_{cB}(t)$ provided by the dampers to the two structures in the horizontal direction is then obtained by multiplying the relative velocity between the two ends of the dampers by the respective damping constant and then obtaining the horizontal components of the forces, i.e.,

$$\begin{aligned} F_{c_A} &= C_{o_A} \cos\theta(t)\left(\dot{\delta}_A - \dot{w}(t)\sin\theta(t)\right) \\ &= C_{o_A} \cos\theta(t)\left(\dot{x}_A(t)\cos\theta(t) - \dot{w}(t)\sin\theta(t)\right) \\ F_{c_B} &= C_{o_B} \cos\theta(t)\left(\dot{\delta}_B + \dot{w}(t)\sin\theta(t)\right) \\ &= C_{o_B} \cos\theta(t)\left(\dot{x}_B(t)\cos\theta(t) + \dot{w}(t)\sin\theta(t)\right) \end{aligned} \tag{4.5}$$

Substituting equations 4.2 and 4.5 into equations 4.1 leads to the equations of motion for each of the structures subjected to the horizontal component of a ground acceleration:

$$\begin{aligned} & m_A \ddot{x}_A(t) + C_{S_A}\dot{x}_A(t) + C_{o_A}\cos\theta(t)\left(\dot{x}_A(t)\cos\theta(t) - \dot{w}(t)\sin\theta(t)\right) \\ & \qquad + k_A x_A(t) = -m_A \ddot{x}_g(t) \\ & m_B \ddot{x}_B(t) + C_{S_B}\dot{x}_B(t) + C_{o_B}\cos\theta(t)\left(\dot{x}_B(t)\cos\theta(t) + \dot{w}(t)\sin\theta(t)\right) \\ & \qquad + k_B x_B(t) = -m_B \ddot{x}_g(t) \end{aligned} \tag{4.6}$$

Equations 4.6 can also be written as:

$$
\begin{aligned}
\ddot{x}_A(t) = & -m_A^{-1}\left(C_{S_A} + C_{O_A}\cos^2\theta(t)\right)\dot{x}_A(t) - m_A^{-1}k_A x_A(t) \\
& + \frac{1}{2}m_A^{-1}C_{O_A}\sin 2\theta(t)\dot{w}(t) - \ddot{x}_g(t) \\
\ddot{x}_B(t) = & -m_B^{-1}\left(C_{S_B} + C_{O_B}\cos^2\theta(t)\right)\dot{x}_B(t) - m_B^{-1}k_B x_B(t) \\
& - \frac{1}{2}m_B^{-1}C_{O_B}\sin 2\theta(t)\dot{w}(t) - \ddot{x}_g(t)
\end{aligned}
\tag{4.7}
$$

Note that the equations of motion of the two structures are coupled by the velocity $\dot{w}(t)$ at the lower end of the dampers.

Introducing two vectors $\{z_1(t)\}$ and $\{z_2(t)\}$ defined as

$$
\{z_1(t)\} = \begin{Bmatrix} x_A(t) \\ x_B(t) \end{Bmatrix} \quad ; \quad \{z_2(t)\} = \begin{Bmatrix} \dot{x}_A(t) \\ \dot{x}_B(t) \end{Bmatrix} \tag{4.8}
$$

the two equations 4.7 can be written as

$$
\dot{z}_2(t) = [B_c]\{z_1(t)\} + [A_c]\{z_2(t)\} + [D_c]\dot{w}(t) - [E_c]\ddot{x}_g(t) \tag{4.9}
$$

where the matrices $[A_c]$, $[B_c]$, $[D_c]$ and $[E_c]$ are:

$$[A_c] = -\left[\begin{array}{c:c} m_A^{-1}\left(C_{S_A} + C_{O_A}\cos^2\theta(t)\right) & 0 \\ \hdashline 0 & m_B^{-1}\left(C_{S_b} + C_{O_B}\cos^2\theta(t)\right) \end{array}\right]$$

(4.10)

$$[B_c] = -\left[\begin{array}{c:c} m_A^{-1}k_A & 0 \\ \hdashline 0 & m_B^{-1}k_B \end{array}\right] \quad ; \quad [D_c] = \left[\begin{array}{c} \frac{1}{2}m_A^{-1}C_{O_A}\sin 2\theta(t) \\ \hdashline -\frac{1}{2}m_B^{-1}C_{O_B}\sin 2\theta(t) \end{array}\right] \quad ; \quad [E_c] = \left[\begin{array}{c} 1 \\ \hdashline 1 \end{array}\right]$$

The set of equations 4.9 is supplemented with an identity expression, $\dot{z}_1(t) = z_2(t)$, written as follows:

$$\dot{z}_1(t) = [O_c]\{z_1(t)\} + [I_c]\{z_2(t)\} \tag{4.11}$$

where $[I_c]$ and $[O_c]$ are $2n$x$2n$ identity and null matrices, respectively.

Equations 4.9 and 4.11 form a set of four first order differential equations of the form

$$\left\{\begin{array}{c} \dot{z}_1(t) \\ \hdashline \dot{z}_2(t) \end{array}\right\} = \left[\begin{array}{c:c} [O_c] & [I_c] \\ \hdashline [A_c] & [B_c] \end{array}\right]\left\{\begin{array}{c} z_1(t) \\ \hdashline z_2(t) \end{array}\right\} + \left[\begin{array}{c} [O_c] \\ \hdashline [D_c] \end{array}\right]\dot{w}(t) - \left[\begin{array}{c} [O_c] \\ \hdashline [E_c] \end{array}\right]\{\ddot{x}_g(t)\} \tag{4.12}$$

This is the state space form of the equations of motion 4.6.

The study of the case of two SDOF systems coupled by the **VDSA** device was selected because due to its relative simplicity it allows to introduce the concept in a straight forward way and to obtain the equations of the motion of the coupled system without complex derivations. However, in most real applications MDOF models will be used to represent the uncoupled structures. This case is examined in the following section.

4.2 TWO COUPLED MDOF STRUCTURES

The structural systems to be coupled consist of two dissimilar buildings with, in general, different height, mass, stiffness and damping properties. As it was done before, the two structures will be identified as **A** and **B**. For simplicity, only horizontal displacements will be used to describe the dynamic behavior of the two structures. The number of degrees of freedom of the structures is n_A and n_B for the buildings **A** and **B**, respectively. The structures are connected with a **VDSA** device in an upper floor as shown in Figure 4.4.

The equation of motion of the two structures joined by the **VDSA** device placed between the i^{th} and $(i+1)^{th}$ floors and subjected to the same horizontal base acceleration are

$$\begin{aligned} [M_A]\{\ddot{x}_A(t)\}+\left[C_{s_A}\right]\{\dot{x}_A(t)\}+[K_A]\{x_A(t)\}+\left\{F_{c_A}(t)\right\} &= -[M_A]\{r_A\}\ddot{x}_g(t) \\ [M_B]\{\ddot{x}_B(t)\}+\left[C_{s_B}\right]\{\dot{x}_B(t)\}+[K_B]\{x_B(t)\}+\left\{F_{c_B}(t)\right\} &= -[M_B]\{r_B\}\ddot{x}_g(t) \end{aligned} \tag{4.13}$$

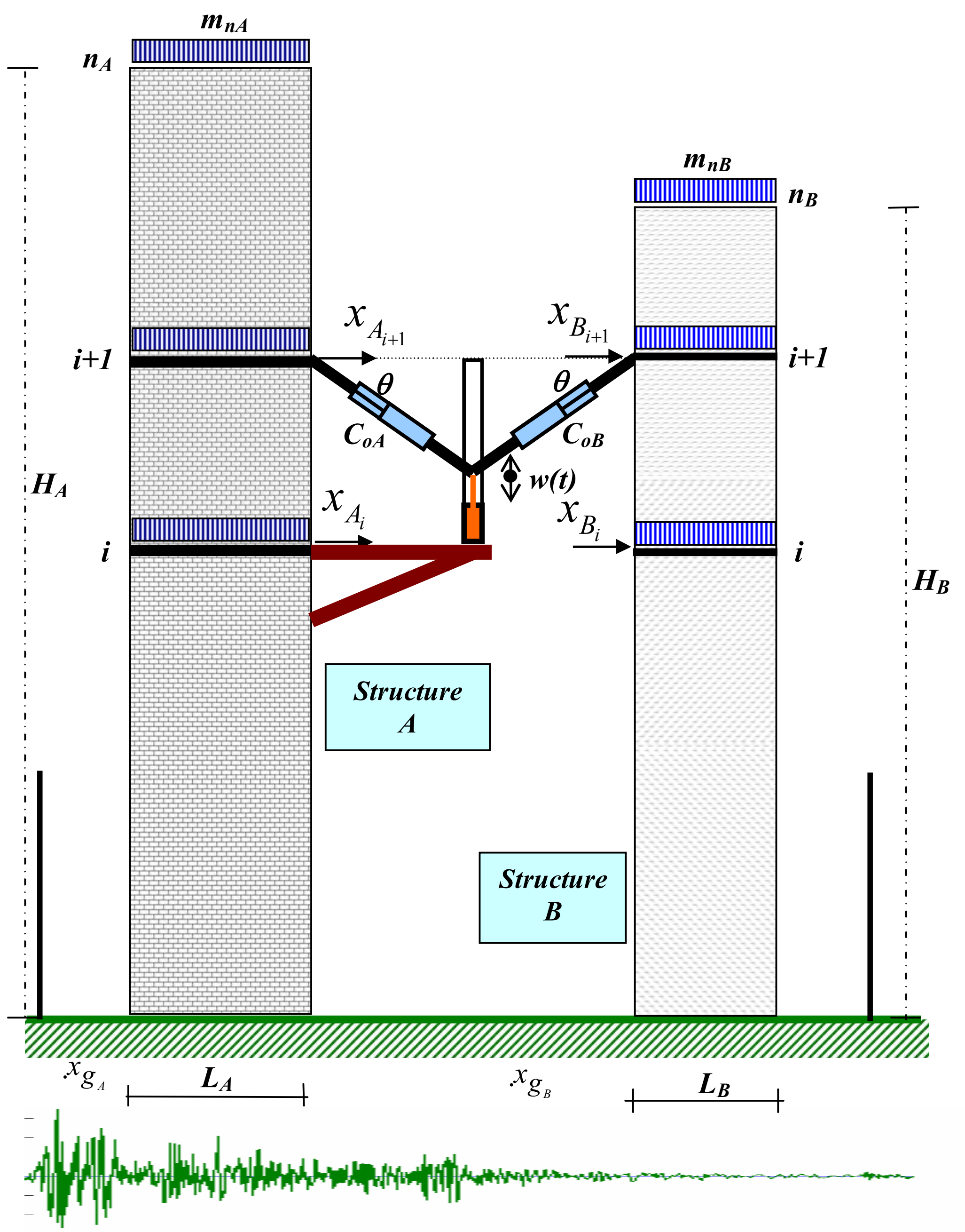


Figure 4.4: MDOF structures coupled through the VDSA device

where $\{x_i(t)\}$, $\{\dot{x}_i(t)\}$ and $\{\ddot{x}_i(t)\}$, $i = A$ and B, are, respectively, the vectors of relative displacements, velocities and accelerations. The matrices $[M_i]$, $[C_{S_i}]$ and $[K_i]$, $i = A$ and B, are, respectively, the mass, damping and stiffness matrices of the structure "i", $\{r_i\}$, is the vector of displacement influence coefficients and $\{F_{ci}(t)\}$, is the control forces vector. The dimensions of the vectors $\{x_A(t)\}$, $\{\dot{x}_A(t)\}$, $\{\ddot{x}_A(t)\}$, $\{F_{cA}(t)\}$ and $\{r_A\}$, are n_Ax1 whereas those of the corresponding vectors subscript B is n_Bx1.

The control force vectors contain the forces applied by the **VDSA** mechanism to each structure. The explicit form of the control forces $\{F_{cA}(t)\}$ and $\{F_{cB}(t)\}$ will be determined later. Note from equations 4.13 that the two structures are coupled by the control forces vectors.

The damping matrices in the left hand side of the two equations of motion represent the original (i.e. natural) damping present in the structures when they are subject to small amplitude deformations.

Figure 4.5 shows the end velocities of their dampers and the components along the dampers axes for the **VDSA** device installed between the floors ***i*** and ***i+1***. The components of the velocities along the axes of the dampers are shown with their signs changed, i.e. in the direction required to define the reactive forces.

The lower end of the **VDSA** device will be assumed to be attached to the structure to the left (**A**). Moreover, it will the considered that the bracket or support structure of the **VDSA** is rigid. Thus, the horizontal motion of the **VDSA** vertical guiding rod will be the same as that of the building, i.e. $x_{A_i}(t)$.

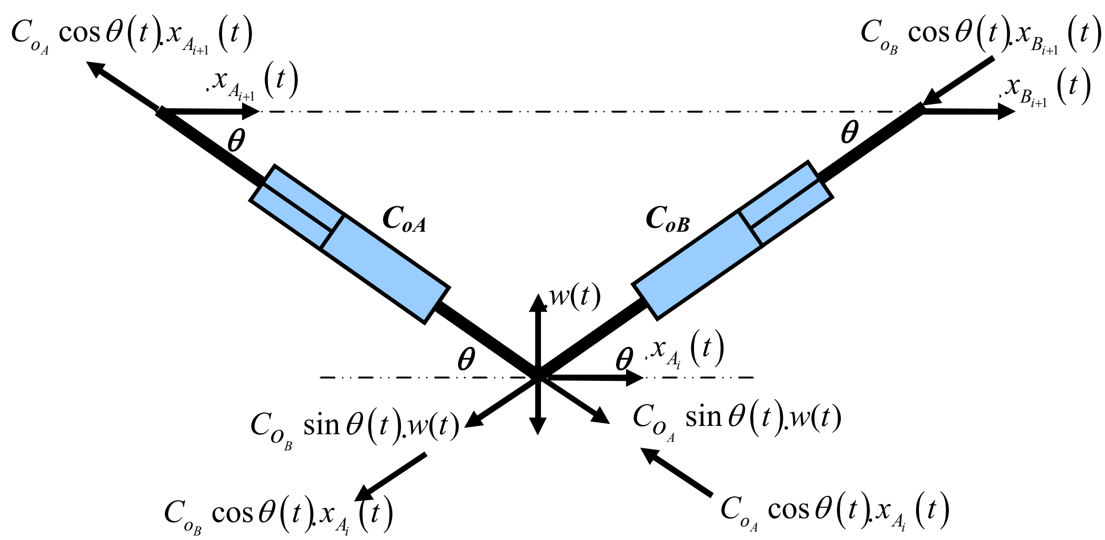


Figure 4.5: End velocities of the VDSA device installed between two MDOF structures

The damping force $F_{cA}(t)$ and $F_{cB}(t)$ that act on the two structures are related to the velocities of the building $\dot{x}_{A_i}(t)$, $\dot{x}_{A_{i+1}}(t)$, $\dot{x}_{B_{i+1}}$ and to the control velocity $\dot{w}(t)$ as follows:

$$
\begin{aligned}
F_{c_A}(t) &= \left(C_{O_A}\cos\theta(t)\right)\left(\dot{x}_{A_{i+1}}(t)\cos\theta(t) - \dot{x}_{A_i}(t)\cos\theta(t) + \dot{w}(t)\sin\theta(t)\right) \\
&= C_{O_A}\cos^2\theta(t)\dot{x}_{A_{i+1}}(t) - C_{O_A}\cos^2\theta(t)\dot{x}_{A_i}(t) + \frac{1}{2}C_{O_A}\sin 2\theta(t)\dot{w}(t) \\
F_{c_B}(t) &= \left(C_{O_B}\cos\theta(t)\right)\left(\dot{x}_{B_{i+1}}(t)\cos\theta(t) - \dot{x}_{A_i}(t)\cos\theta(t) - \dot{w}(t)\sin\theta(t)\right) \\
&= C_{O_B}\cos^2\theta(t)\dot{x}_{B_{i+1}}(t) - C_{O_B}\cos^2\theta(t)\dot{x}_{A_i}(t) - \frac{1}{2}C_{O_B}\sin 2\theta(t)\dot{w}(t)
\end{aligned}
\tag{4.14}
$$

The expressions for the control forces, equations 4.14, must be written in matrix form to define the vectors $\{Fc_A(t)\}$ and $\{Fc_B(t)\}$ in the equations of motion (4.13). According to equations 4.14, the vectors $\{Fc_A(t)\}$ and $\{Fc_B(t)\}$ will be function of $\{\dot{x}_A(t)\}$, $\{\dot{x}_B\}$ and $\dot{w}(t)$:

$$
\begin{aligned}
\left\{F_{c_A}\right\} &= [C_{AA}]\{\dot{x}_A(t)\} + [C_{AB}]\{\dot{x}_B(t)\} + \{D_A\}\dot{w}(t) \\
\left\{F_{c_B}\right\} &= [C_{BA}]\{\dot{x}_A(t)\} + [C_{BB}]\{\dot{x}_B(t)\} + \{D_B\}\dot{w}(t)
\end{aligned}
\tag{4.15}
$$

To define matrices $[C_{AA}]$, $[C_{AB}]$, etc., and the vectors $\{D_A\}$ and $\{D_B\}$, four vectors with only one or two non-zero elements are defined. The vectors $\{e_{1A}\}$, $\{e_2\}$ and $\{e_3\}$, with length n_A, are:

$$
\begin{aligned}
\{e_{1A}\}^T &= [0,0,\ldots,0,1,0,\ldots,0] \text{ with 1 at column "}i+1\text{"} \\
\{e_2\}^T &= [0,0,\ldots,0,-1,0,\ldots,0] \text{ with -1 at column "}i\text{"} \\
\{e_3\}^T &= [0,0,\ldots,0,-1,1,\ldots,0] \text{ with -1 at column "}i\text{", 1 at column "}i+1\text{"}
\end{aligned}
\tag{4.16}
$$

The fourth vector, $\{e_{1B}\}$, is similar to vector $\{e_{1A}\}$ except that it has $\boldsymbol{n_B}$ elements.

With these four vectors, and examining equations 4.14 and 4.15 it is not difficult to verify that the matrices $[C_{AA}]$, $[C_{AB}]$, etc., can be written as:

$$\begin{aligned}
&[C_{AA}] = C_{o_A}\cos^2\theta(t)\{e_{1A}\}\{e_3\}^T \quad ; \quad [C_{AB}] = [0] \\
&[C_{BA}] = C_{o_A}\cos^2\theta(t)\{e_{1B}\}\{e_2\}^T \quad ; \quad [C_{BB}] = C_{o_B}\cos^2\theta(t)\{e_{1B}\}\{e_{1B}\}^T
\end{aligned} \tag{4.17}$$

Similarly, examining equations and using the definition in equation 4.16 the vectors $\{D_A\}$ and $\{D_B\}$ can be expressed as:

$$\{D_A\} = \frac{1}{2}C_{o_A}\sin 2\theta(t)\{e_{1A}\} \quad ; \quad \{D_B\} = -\frac{1}{2}C_{o_B}\sin 2\theta(t)\{e_{1B}\} \tag{4.18}$$

Substituting equations 4.15 in 4.13 and solving for $\{\ddot{x}_A(t)\}$ and $\{\ddot{x}_B(t)\}$ leads to

$$\begin{aligned}
\{\ddot{x}_A(t)\} = &-[M_A]^{-1}\left([C_{s_A}]+[C_{AA}]\right)\{\dot{x}_A(t)\} - [M_A]^{-1}[K_A]\{x_A(t)\} \\
&-[M_A]^{-1}[C_{AB}]\{\dot{x}_B(t)\} - [M_A]^{-1}\{D_A\}.w(t) - \{r_A\}.\ddot{x}_g(t) \\
\{\ddot{x}_B(t)\} = &-[M_B]^{-1}\left([C_{s_B}]+[C_{BB}]\right)\{\dot{x}_B(t)\} - [M_B]^{-1}[K_B]\{x_B(t)\} \\
&-[M_B]^{-1}[C_{BA}]\{\dot{x}_A(t)\} - [M_B]^{-1}\{D_B\}.w(t) - \{r_B\}.\ddot{x}_g(t)
\end{aligned} \tag{4.19}$$

Defining four matrices $[A_c]$, $[B_c]$, $[D_c]$ and $[E_c]$ as follows

$$[A_c] = -\left[\begin{array}{c|c} [M_A]^{-1}\left([C_{S_A}]+[C_{AA}]\right) & [M_A]^{-1}[C_{AB}] \\ \hline [M_B]^{-1}[C_{BA}] & [M_B]^{-1}\left([C_{S_B}]+[C_{BB}]\right) \end{array}\right]$$

$$[B_c] = -\left[\begin{array}{c|c} [M_A]^{-1}[K_A] & 0 \\ \hline 0 & [M_B]^{-1}[K_B] \end{array}\right] \quad ; \quad [D_c] = -\left[\begin{array}{c} [M_A]^{-1}\{D_A\} \\ \hline [M_B]^{-1}\{D_B\} \end{array}\right] \quad ; \quad [E_c] = \left[\begin{array}{c} \{r_A\} \\ \hline \{r_B\} \end{array}\right] \tag{4.20}$$

and introducing the components of a state vector

$$\{z_1(t)\} = \left\{\begin{array}{c} \{x_A(t)\} \\ \hline \{x_B(t)\} \end{array}\right\} \quad ; \quad \{z_2(t)\} = \left\{\begin{array}{c} \{\dot{x}_A(t)\} \\ \hline \{\dot{x}_B(t)\} \end{array}\right\} \tag{4.21}$$

one can write equations 4.19 as

$$\{\dot{z}_2(t)\} = [A_c]\{z_1(t)\} + [B_c]\{z_2(t)\} + [D_c].w(t) - [E_c].\ddot{x}_g(t) \tag{4.22}$$

Finally, when equation 4.22 is supplemented with the following identity,

$$\{\dot{z}_1(t)\} = \{z_2(t)\} \tag{4.23}$$

one obtains the state-space form of the equations of motion 4.13:

$$\left\{\begin{array}{c}\dot{z}_1(t)\\ \hdashline \dot{z}_2(t)\end{array}\right\}=\left[\begin{array}{c:c}[O_c] & [I_c]\\ \hdashline -[A_c] & -[B_c]\end{array}\right]\left\{\begin{array}{c}z_1(t)\\ \hdashline z_2(t)\end{array}\right\}+\left[\begin{array}{c}[O_c]\\ \hdashline [D_c]\end{array}\right].w(t)-\left[\begin{array}{c}[O_c]\\ \hdashline [E_c]\end{array}\right]\left\{\ddot{x}_g(t)\right\} \tag{4.24}$$

where $[O_c]$ and $[I_c]$ are, respectively, a null and identify matrix of dimension (n_A+n_B)x(n_A+n_B).

The vertical position of the lower end of the **VDSA** device *w(t)* required in each instant of time and the damping coefficient for the coupled system can be computed with equations 3.29 and 3.30, respectively.

The reduction in the seismic response of coupled SDOF and MDOF structures with a **VDSA** device will be examined via numerical examples in Chapter VII and VIII. The *Closed-loop control* and *Open-closed-loop control* algorithms ***Qv*** will be used to calculate the optimal position $w(t)$ of the **VDSA** device. The response of the uncontrolled structures and controlled with passive dampers will be compared with that of the structures with the proposed semiactive device.

CHAPTER V:

APPLICATION OF THE VDSA DEVICE AND MODIFIED ALGORITHMS Q_V TO SDOF STRUCTURES

5.1 INTRODUCTION

To illustrate the effectiveness of the **VDSA** device in reducing the seismic response, two examples are presented in this chapter: (1) A SDOF model of a stiffness structure. (2) A SDOF model of a flexible structure. The classification of the structures as stiff or flexible was realized keeping in mind the value of the period. The structure with an inferior period to one second is considered as stiff (UBC 1997). The response obtained by applying the *Closed-loop control* modified algorithm ***Qv*** and the *Closed-open-loop control* modified algorithm ***Qv*** is compared against the response of the uncontrolled structures. In addition, the response of the structures are equipped with dampers passive are considered. Three earthquake accelerograms with different characteristics are applied to the one dof structures. The numerical response is calculated with programs specifically developed in MATLAB.

5.2 DESCRIPTION OF THE EARTHQUAKES

The structures were subjected to the horizontal component of three different earthquakes. First, the well known El Centro record of the Imperial Valley, California earthquake of May 18, 1940 is considered (see Figure 5.1). This record has a peak ground acceleration (PGA) of 0.348g. Next, the San Fernando, California record with a PGA of 1.007g of the February 9, 1971 earthquake is used (see Figure 5.2). Finally, the record of the Friuli, Italy, earthquake of May 6, 1976 with a PGA of 0.4788g is applied to the structures (see Figure 5.3). The accelerations are sampled at equal time intervals of 0.02 sec.

To apply the modified algorithm ***Qv*** it is necessary to know the ground velocity. Figures 5.4 to 5.6 present the ground velocity for El Centro, San Fernando and Friuli records. The peak ground velocities (PGV) for El Centro, San Fernando and Friuli record are 16.9113 in/sec, 88.6291 in/sec, and 18.6307 in/sec, respectively.

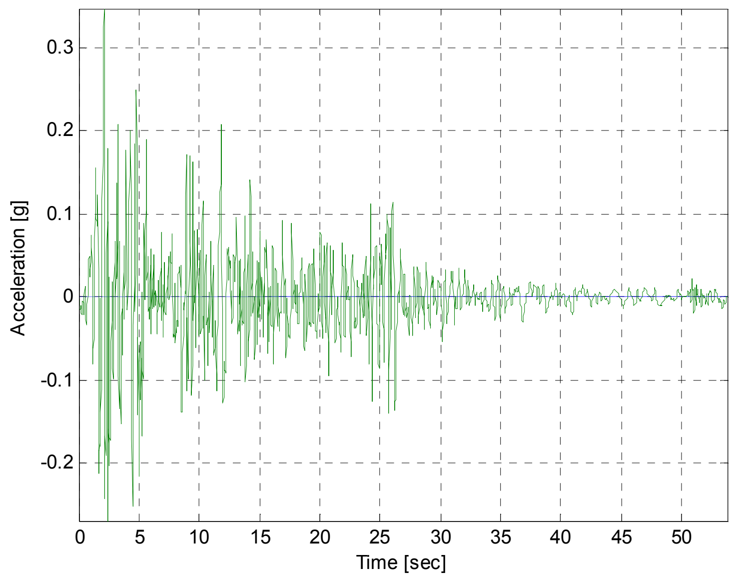


Figure 5.1: Ground acceleration of the 1940 El Centro record

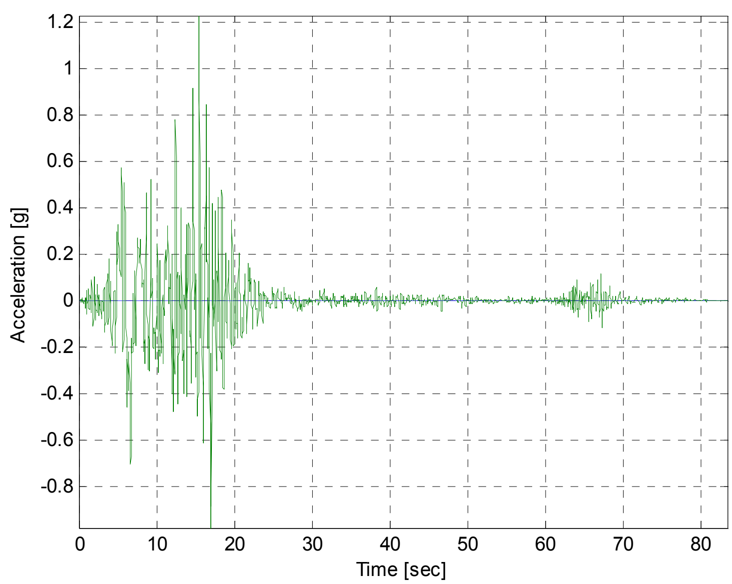


Figure 5.2: Ground acceleration of the 1971 San Fernando record

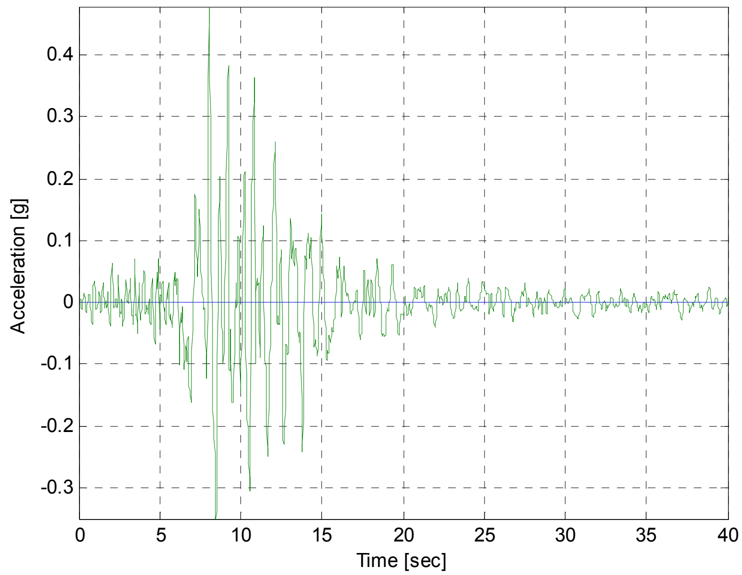


Figure 5.3: Ground acceleration of the 1976 Friuli record

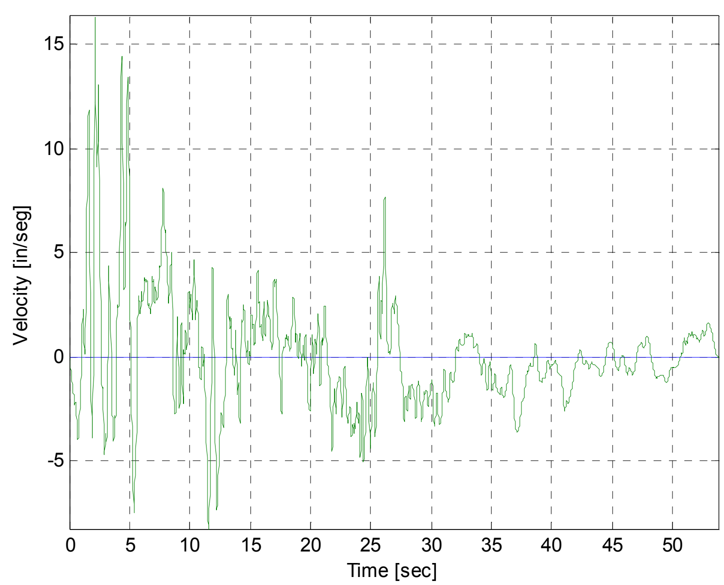


Figure 5.4: Ground velocity of the 1940 El Centro record

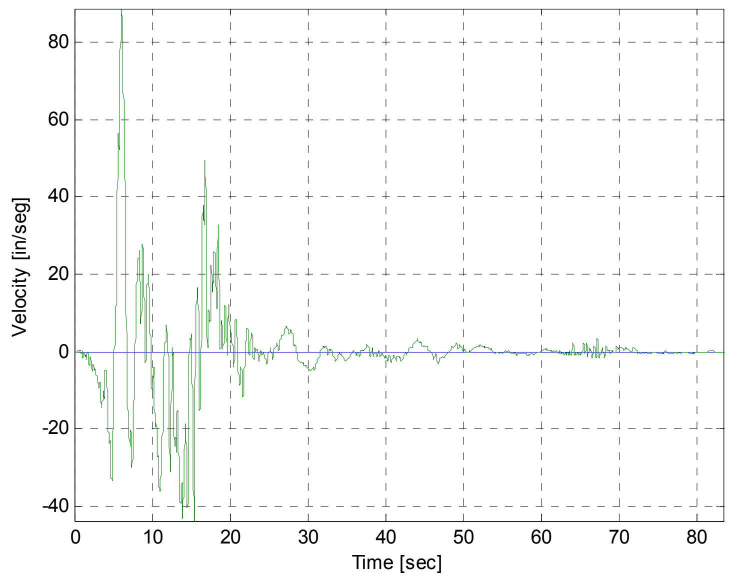


Figure 5.5: Ground velocity of the 1971 San Fernando record

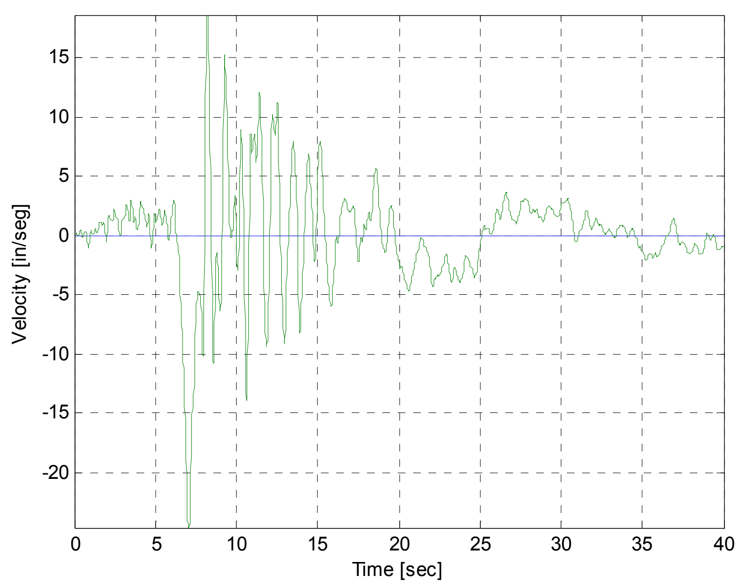


Figure 5.6: Ground velocity of the 1976 Friuli record

5.3 EXAMPLE 1: STIFF STRUCTURE

The first structural system selected to present numerical examples is a SDOF model with a weight of 13,630 kip, stiffness coefficient of 34,814 kip/in, damping ratio of 2% and natural period of 0.20 sec. The coefficients of the dampers **A** and **B** of the **VDSA** device are 20 kip.sec/in and 10 kip.sec/in, respectively. The results obtained are compared with those for the uncontrolled structure (with a damping ratio of 5%) and also with the case when the dampers are installed in a position fixed. For this case of passive damping, the damping ratio is set equal to 30%, and to achieve this damping constant C_o is calculated with equation 3.5. A summary of the properties for the passive damper case is presented in Table 5.1.

Table 5.1: Properties and damping constant for the passive damper in the stiff structure

Floor	$Cos^2\theta_1$	ϕ_1	ϕ_{r1}	ξ_{eff}	C_o
[#]				[%]	[kip.sec/in$^+$]
1	0.9686	0.0053	0.0053	30	40

*1 in = 2.54 cm, $^+$1 kip = 4.448 kN.

5.3.1 APPLICATION OF THE MODIFIED *CLOSED-LOOP CONTROL* ALGORITHM *Qv*

The weighting matrix **Q** is selected as [***I***]x10^2**,** where [***I***] is a 2x2 identity matrix, and the matrices **Qv** and **R**, are scalars whose values are 10^1 and 10^{-4}, respectively.

5.3.1.a RESPONSE TO THE EL CENTRO RECORD

The first results for the single degree of freedom structure of example 1, are the responses to the accelerogram of the El Centro earthquake. Figures 5.7 to 5.9 show a comparison of the relative displacement time histories for the uncontrolled structure, the structure with fixed dampers, and controlled with the **VDSA** device. Only the first twenty seven seconds of the response is shown. Figures 5.10 to 5.12 present similar results but the response displayed is the total base shear as a function of time. Figure 5.13 shows the control force. The maximum instantaneous force applied by the control system is 3.318 kip. The height variation of the lower end of the **VDSA** device is presented in Figure 5.14. The height minimum ($w_{\min}$) assumed for the control is 25 in.

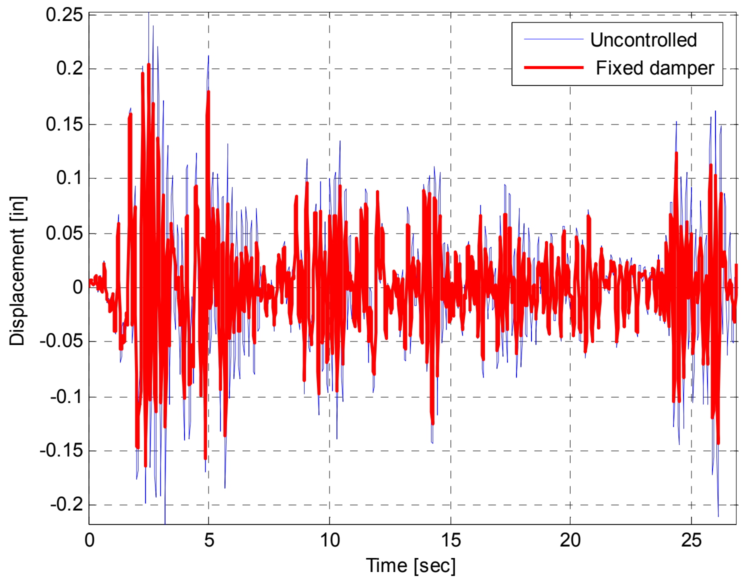


Figure 5.7: Relative displacement of the top stiff 1-dof system for the El Centro record - Uncontrolled vs. Fixed damper

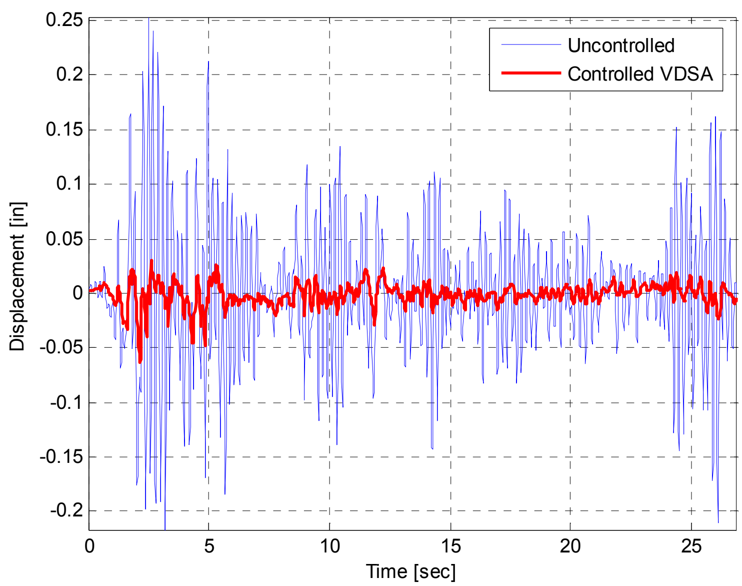


Figure 5.8: Relative displacement of the stiff 1-dof system for the El Centro record - Uncontrolled vs. VDSA: (*Closed-loop control*)

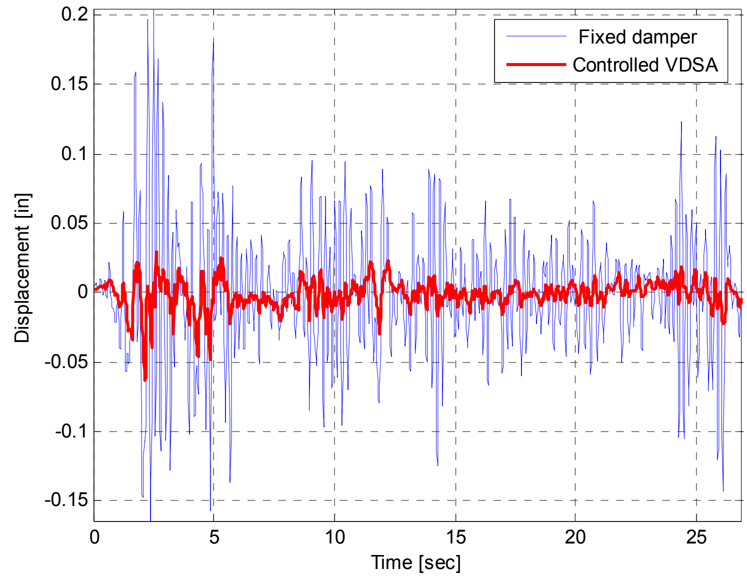


Figure 5.9: Relative displacement of the stiff 1-dof system for the El Centro record - Fixed damper vs. VDSA: El Centro record (*Closed-loop control*)

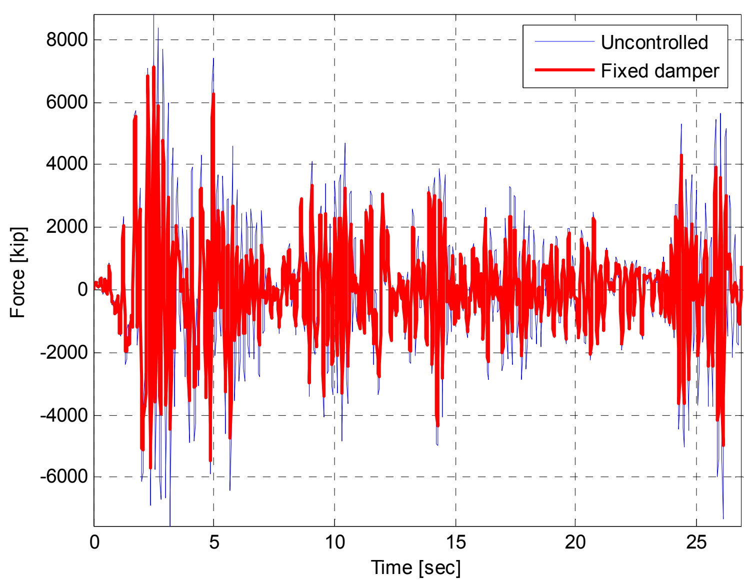


Figure 5.10: Base shear of the stiff 1-dof system for the El Centro record - Uncontrolled vs. Fixed damper (*Closed-loop control*)

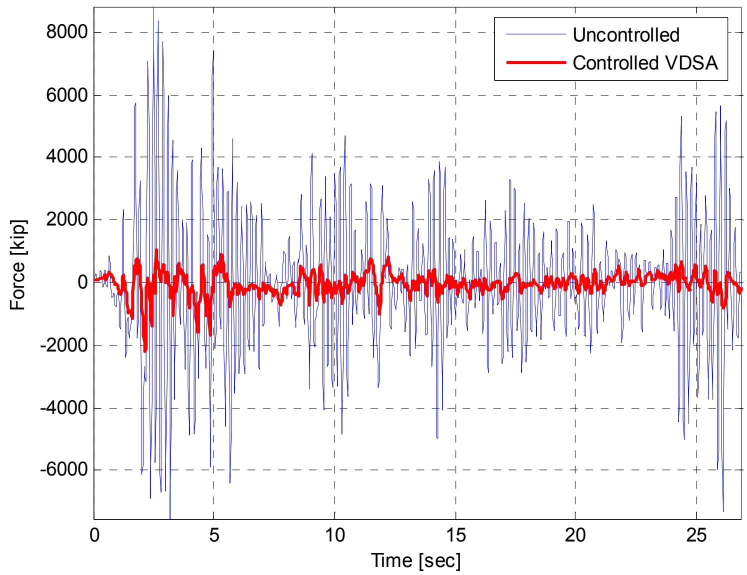


Figure 5.11: Base shear of the stiff 1-dof system for the El Centro record - Uncontrolled vs. VDSA (*Closed-loop control*)

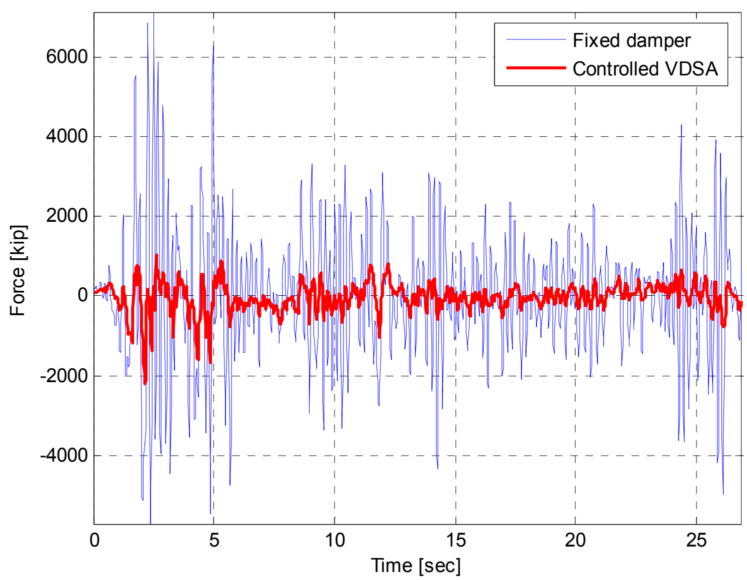


Figure 5.12: Base shear of the stiff 1-dof system for the El Centro record - Fixed damper vs. VDSA (*Closed-loop control*)

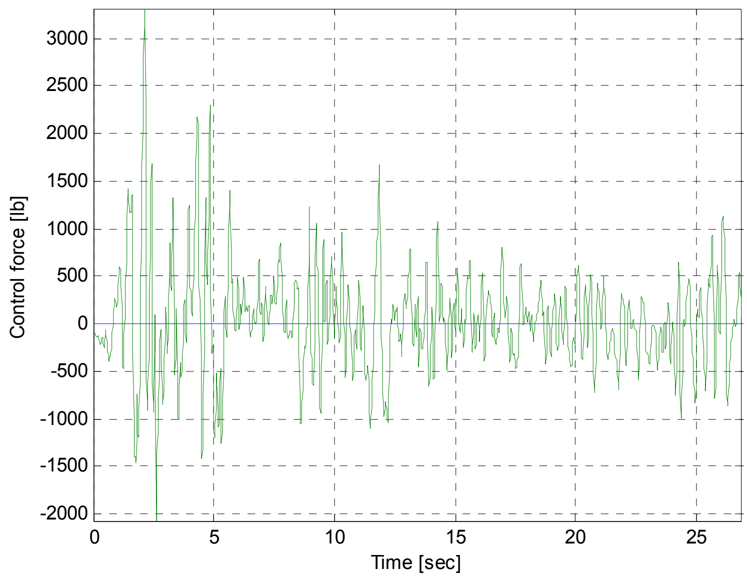


Figure 5.13: Control force of the stiff 1-dof system for the El Centro record (*Closed-loop control*)

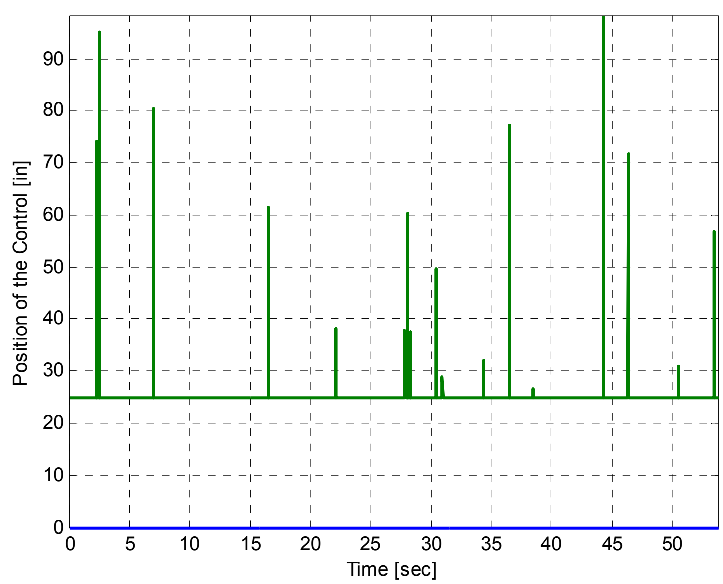


Figure 5.14: Variation of the position of the VDSA device in the stiff 1-dof system for the El Centro record (*Closed-loop control*)

5.3.1.b RESPONSE TO THE SAN FERNANDO RECORD

The previous response calculations were repeated this time with the San Fernando record. The responses compared are those obtained with the uncontrolled structure, with fixed dampers and with the **VDSA** device. This comparison was made for the relative displacement and total base shear response. Figures 5.15 to 5.17 show the time variation of the displacements for the various conditions whereas Figures 5.18 to 5.20 display the base shear time histories. The first forty two seconds of the response is shown. The control force time history is presented in Figure 5.21. The maximum force produced by control was 7.898 kip. Figure 5.22 shows the variation of the height of the lower end of the device with w_{min}=25 in.

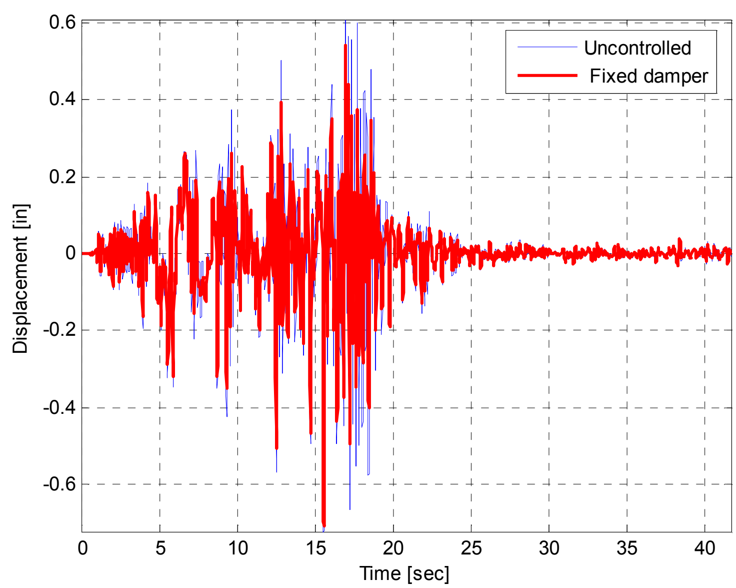


Figure 5.15: Relative displacement of the stiff 1-dof system for the San Fernando record - Uncontrolled vs. Fixed damper

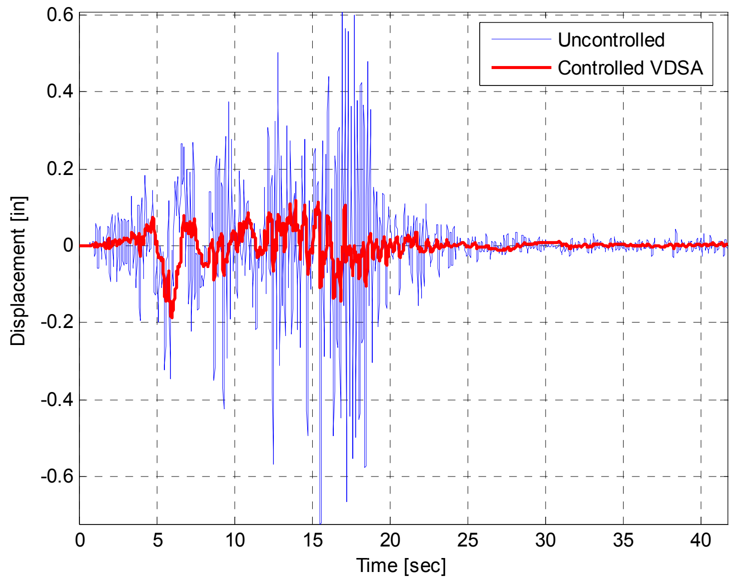


Figure 5.16: Relative displacement of the stiff 1-dof system for the San Fernando record - Uncontrolled vs. VDSA (*Closed-loop control*)

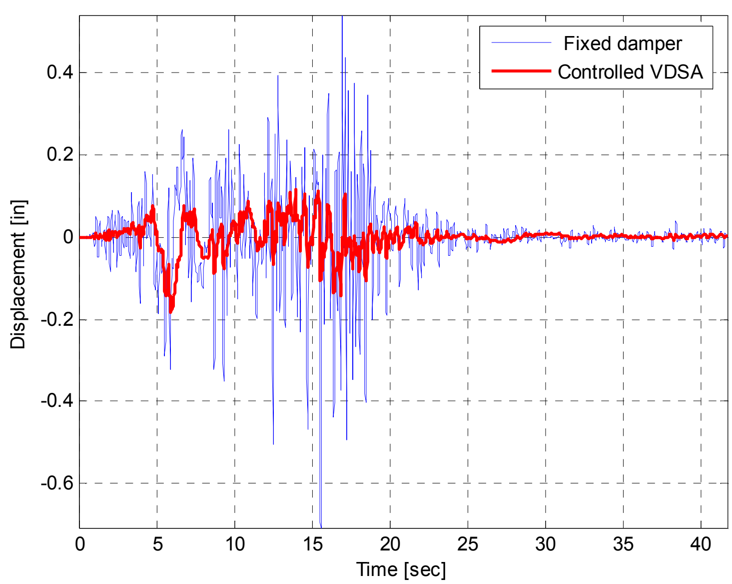


Figure 5.17: Relative displacement of the stiff 1-dof system for the San Fernando record - Fixed damper vs. VDSA (*Closed-loop control*)

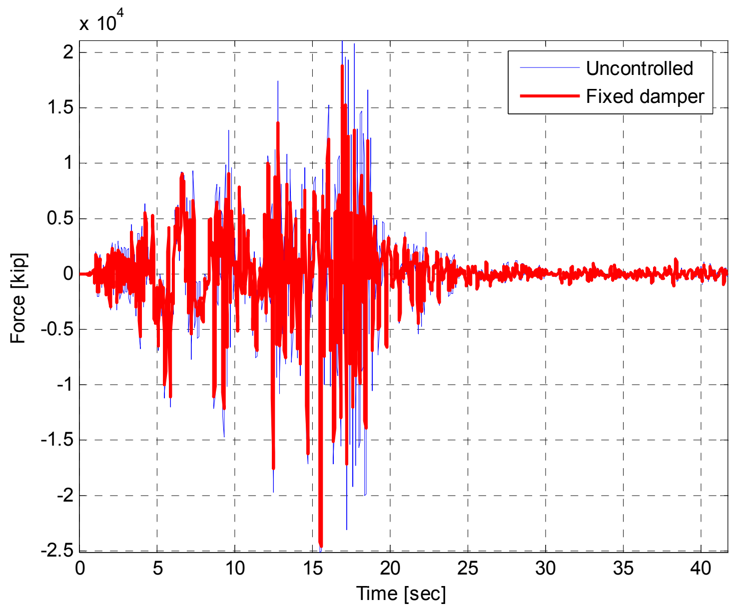


Figure 5.18: Base shear of the stiff 1-dof system for the San Fernando record - Uncontrolled vs. Fixed damper

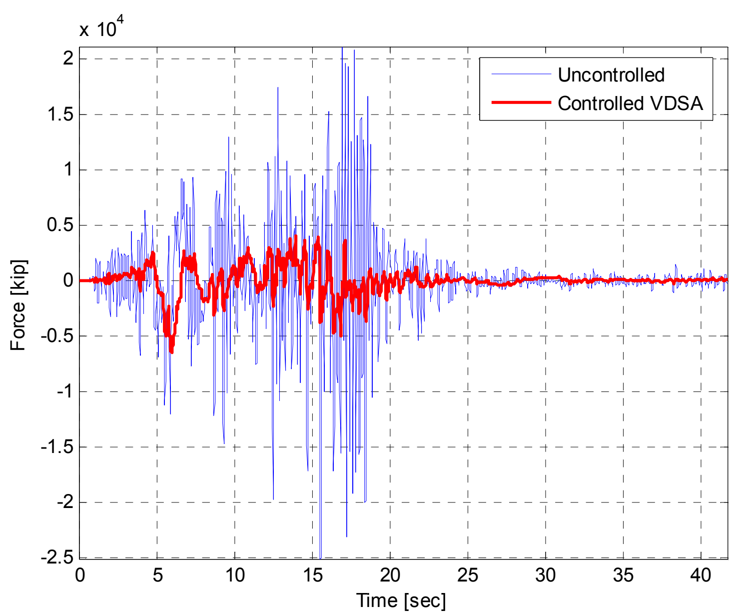


Figure 5.19: Base shear of the stiff 1-dof system for the San Fernando record - Uncontrolled vs. VDSA (*Closed-loop control*)

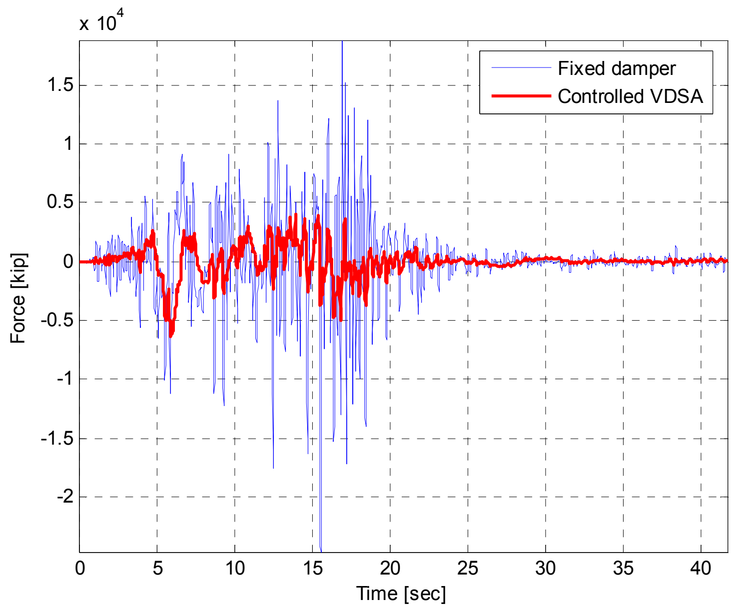


Figure 5.20: Base shear of the stiff 1-dof system for the San Fernando record - Fixed damper vs. VDSA (*Closed-loop control*)

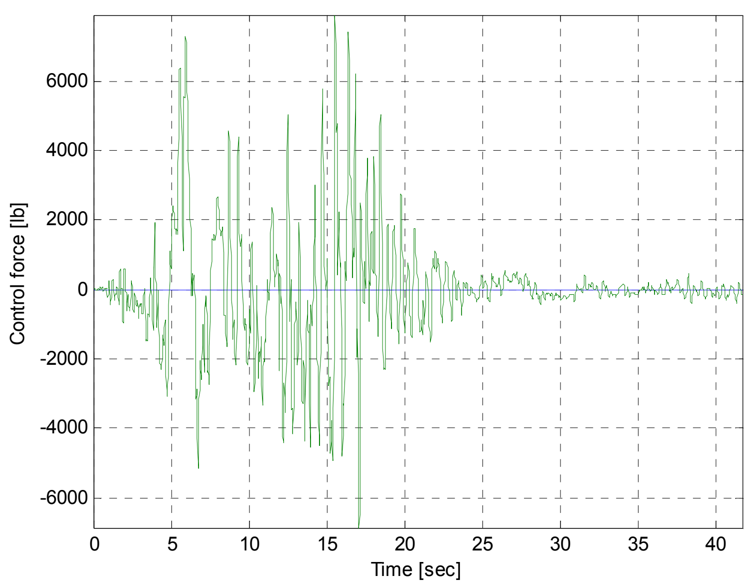


Figure 5.21: Control force of the stiff 1-dof system for the San Fernando record (*Closed-loop control*)

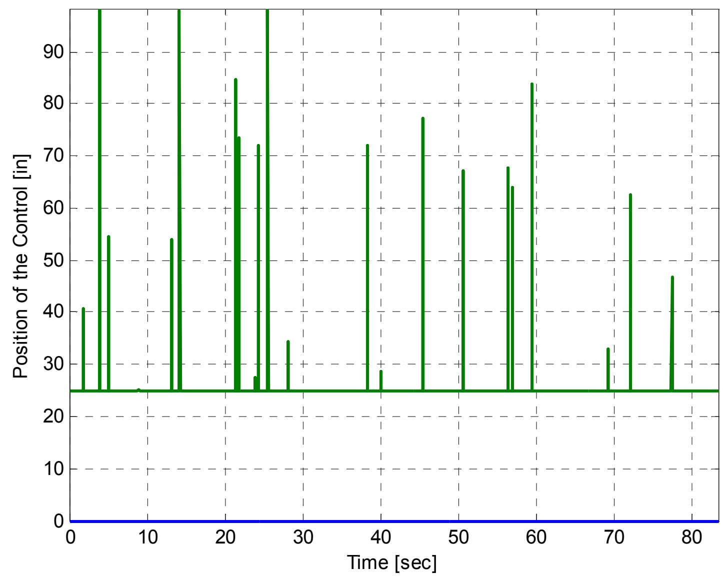


Figure 5.22: Variation of the position of the VDSA device in the stiff 1-dof system for the San Fernando record (*Closed-loop control*)

5.3.1.c RESPONSE TO THE FRIULI RECORD

The next set of results corresponds to the 1976 Friuli accelerogram. Again, the responses compared are the relative displacement of the mass and the total base shear for the structure in uncontrolled mode, and controlled with fixed dampers and with the **VDSA** device. The results are presented in Figures 5.23 to 5.28 for the first twenty seconds of the response. Figures 5.29 and 5.30 shows, respectively, the control force time history and the variation of the position of the **VDSA** device. The maximum force required from the control mechanism for the Friuli earthquake is 4.043 kip and $w_{\min}$ used was 25 in.

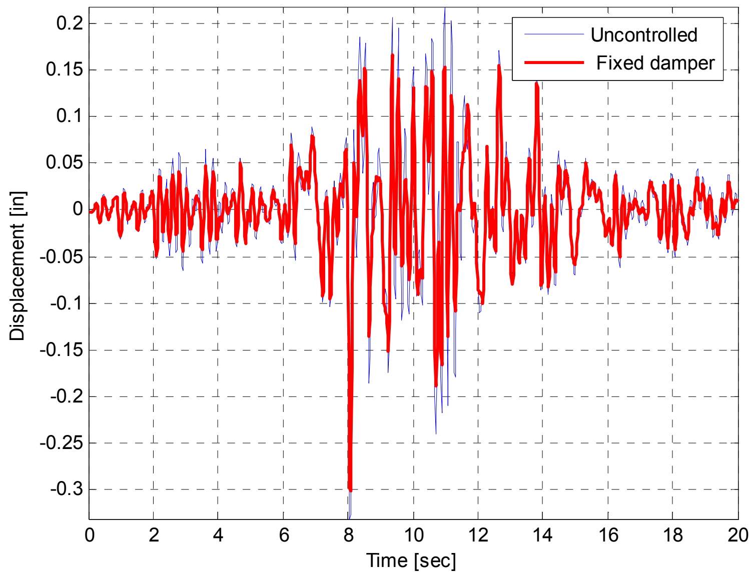


Figure 5.23: Relative displacement of the stiff 1-dof system for the Friuli record - Uncontrolled vs. Fixed damper

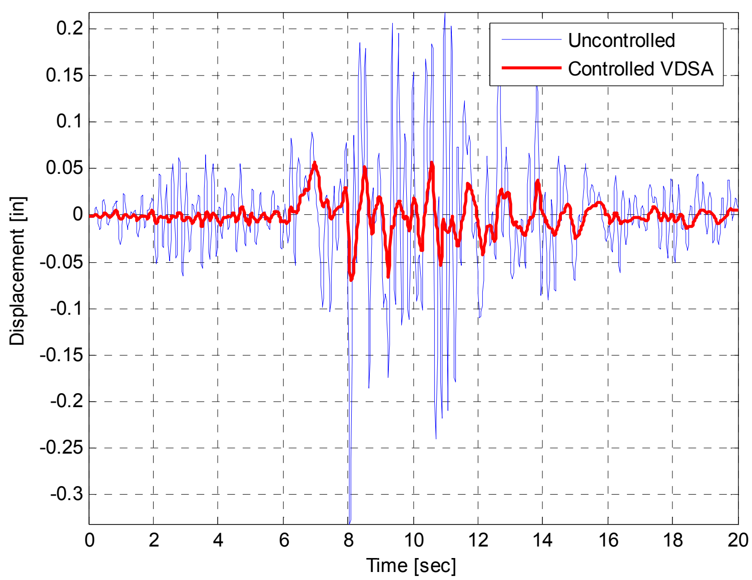


Figure 5.24: Relative displacement of the stiff 1-dof system for the Friuli record - Uncontrolled vs. VDSA (*Closed-loop control*)

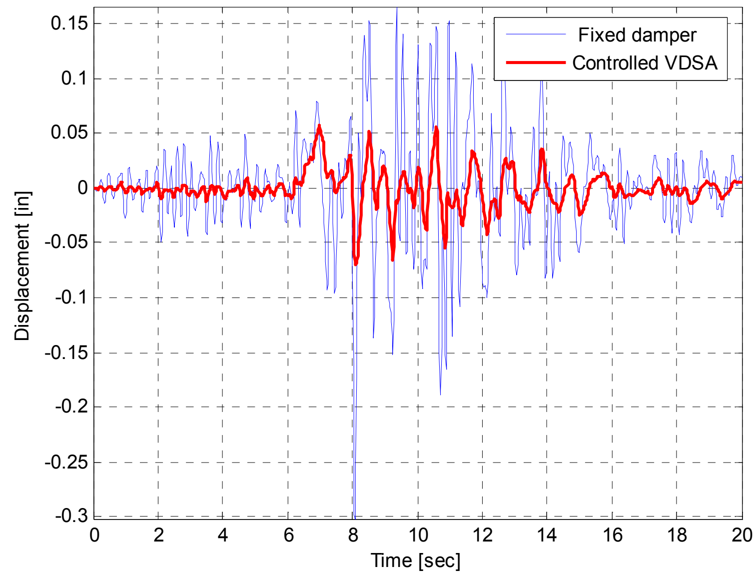


Figure 5.25: Relative displacement of the stiff 1-dof system for the Friuli record - Fixed damper vs. VDSA (*Closed-loop control*)

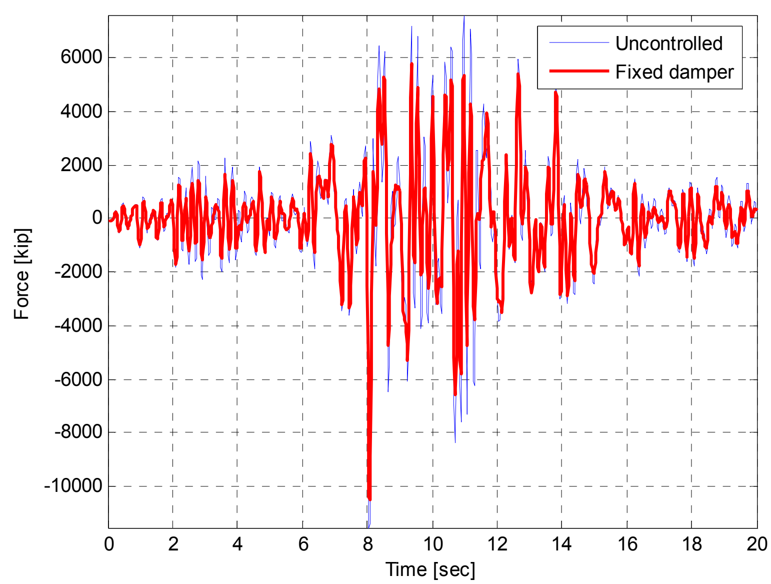


Figure 5.26: Base shear of the stiff 1-dof system for the Friuli record - Uncontrolled vs. Fixed damper (*Closed-loop control*)

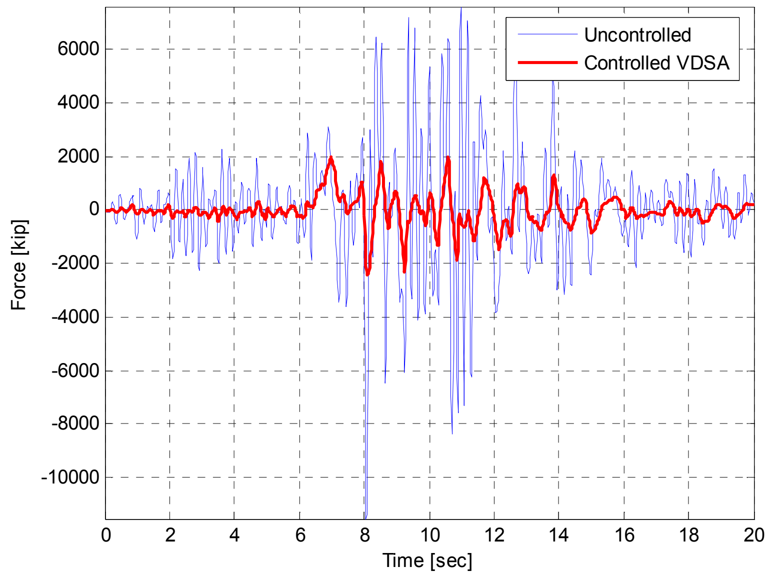


Figure 5.27: Base shear of the stiff 1-dof system for the Friuli record - Uncontrolled vs. VDSA (*Closed-loop control*)

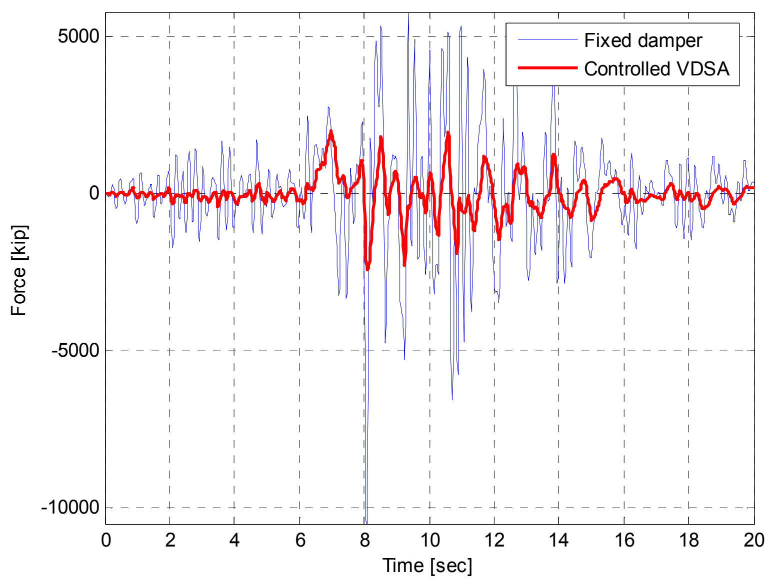


Figure 5.28: Base shear of the stiff 1-dof system for the Friuli record - Fixed damper vs. VDSA (*Closed-loop control*)

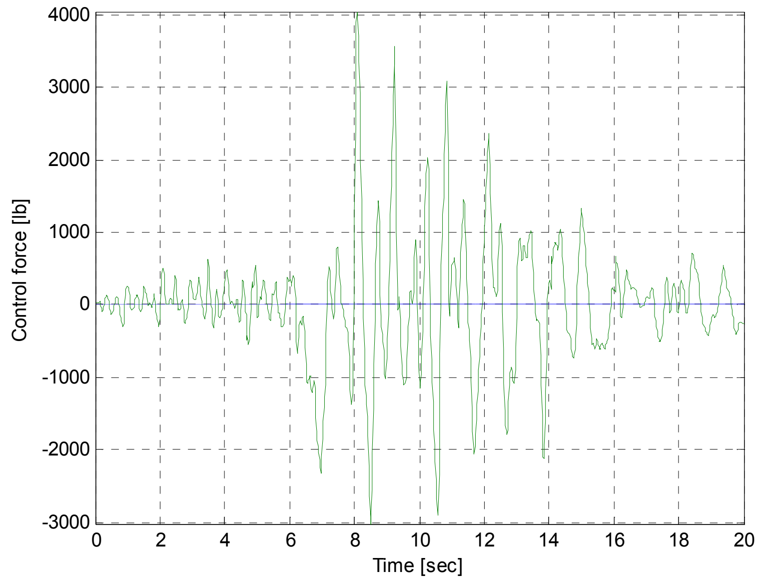


Figure 5.29: Control force in the stiff 1-dof system for the Friuli record (*Closed-loop control*)

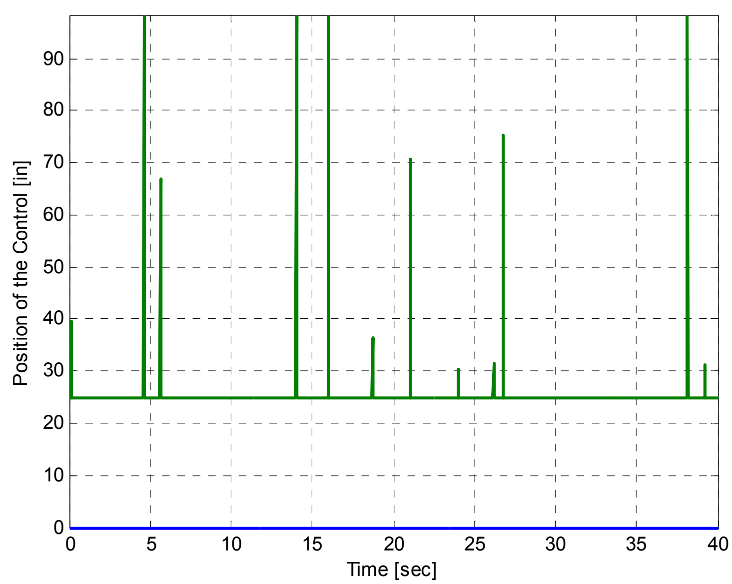


Figure 5.30: Variation of the position of the VDSA device in the stiff 1-dof system for the Friuli record (*Closed-loop control*)

Table 5.2 shows a summary of the maximum response quantities displayed in Figure 5.7 to 5.28, for the stiff single DOF structure of Example 1. To implement the semiactive control with the **VDSA** device the *Closed-loop control* modified algorithm ***Qv*** was used. Table 5.2 demonstrates the advantages of using the **VDSA** device with the *Closed-loop control* modified algorithm ***Qv*** and in the reduction of the seismic response of stiff structures.

Table 5.2: Maximum response of the stiff SDOF structure without control and with a passive and semiactive system (*Closed-loop control*)

Earthquake	Displacement			Total base shear		
	Uncont. [in*]	Fixed damper [in*]	VDSA [in*]	Uncont. [kip⁺]	Damper Fixed [kip⁺]	VDSA [kip⁺]
El Centro	0.2536	0.2049	0.0630	8828.0	7133.1	2192.1
San Fernando	0.7219	0.7072	0.1848	25132.0	24620.0	6435.3
Friuli	0.3315	0.3019	0.0696	11540.0	10509.0	2424.3

*1 in = 2.54 cm, ⁺1 kip = 4.448 kN.

5.3.2 APPLICATION OF THE MODIFIED *CLOSED-OPEN-LOOP CONTROL* ALGORITHM *Qv*

The same structure used before is subjected to the previously described seismic input but now the **VDSA** device is controlled with the alternative algorithm, i.e. the modified *Closed-open-loop control* algorithm ***Qv***. In this case the weighting matrix **Q** is [***I***]x10^2 and the values selected for **Qv** and **R** (which are scalars for single dof systems) are 10^1 and 10^{-4}, respectively.

5.3.2.a RESPONSE TO THE EL CENTRO RECORD

The relative displacement computed for the uncontrolled structure is compared with similar response for the single dof system controlled with the **VDSA** device in Figures 5.31. Figure 5.32 presents a similar comparison but for the structure controlled with fixed and variable dampers. As before, only the first twenty seven seconds of the response to the El Centro ground motion is presented. The total base shear time history is compared, for the two cases Figures 5.33 and 5.34. As it is shown in Figure 5.35 the maximum force realized by the control was 3.249 kip. The variation of the position control is shown in Figure 5.36. The parameter w_{min} was set equal to 20 in.

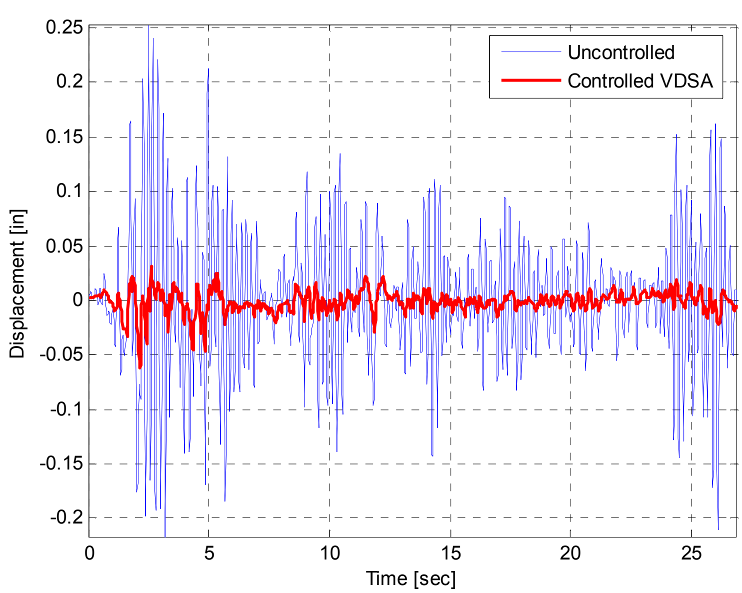


Figure 5.31: Relative displacement of the stiff 1-dof system for the El Centro record - Uncontrolled vs. VDSA (*Closed-open-loop control*)

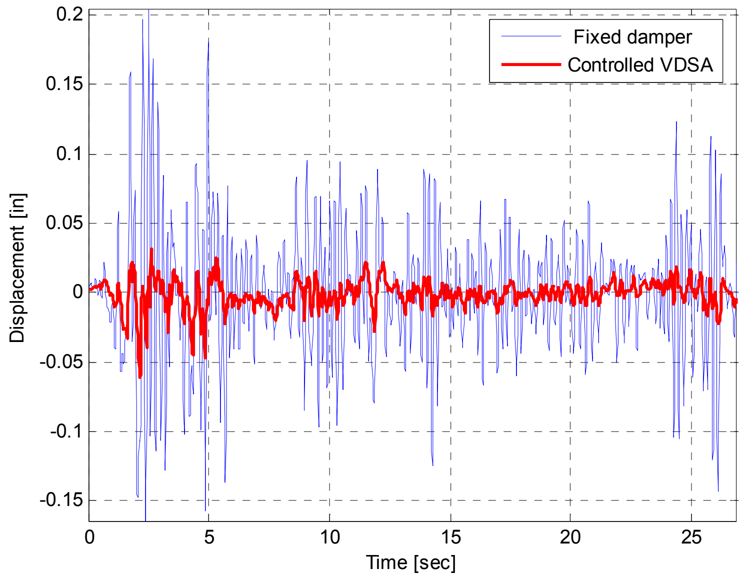


Figure 5.32: Relative displacement of the stiff 1-dof system for the El Centro record - Fixed damper vs. VDSA (*Closed-open-loop control*)

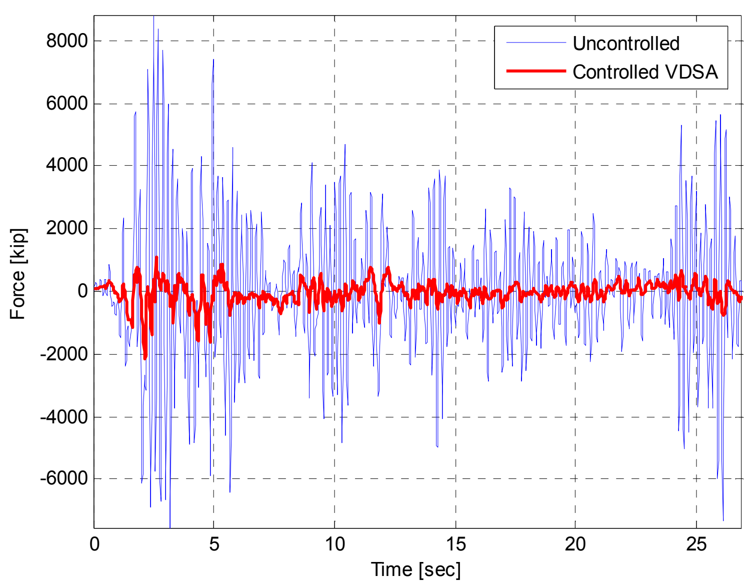


Figure 5.33: Base shear of the stiff 1-dof system for the El Centro record - Uncontrolled vs. VDSA (*Closed-open-loop control*)

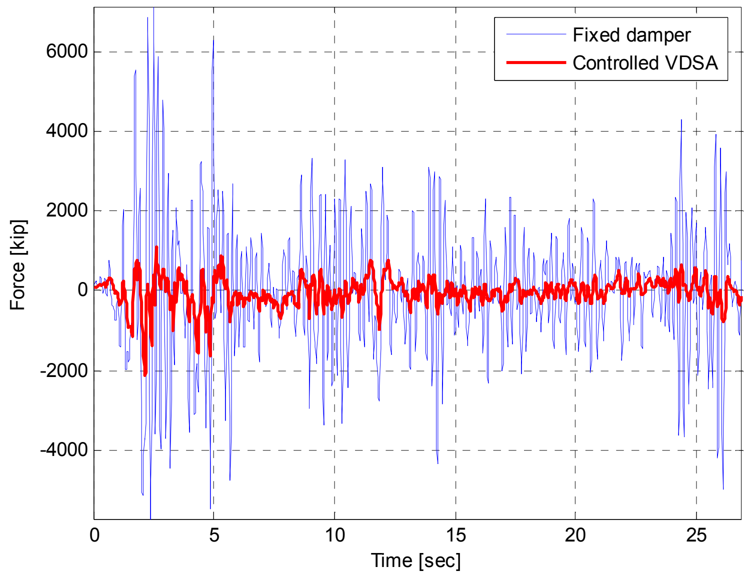


Figure 5.34: Base shear of the stiff 1-dof system for the El Centro record - Fixed damper vs. VDSA (*Closed-open-loop control*)

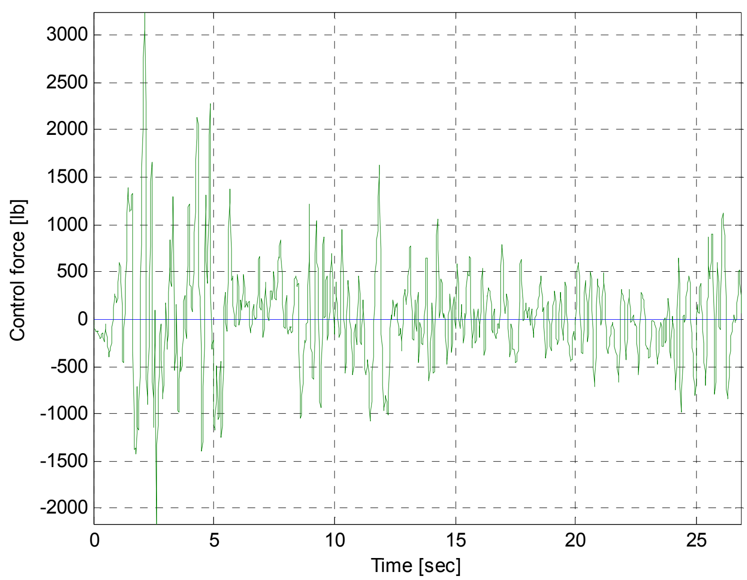


Figure 5.35: Control force in the stiff 1-dof system for the El Centro record (*Closed-open-loop control*)

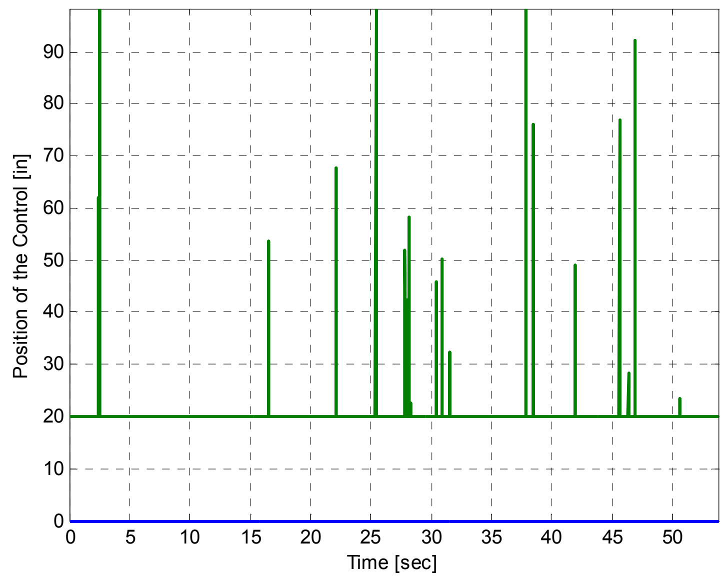


Figure 5.36: Variation of the position of the VDSA device in the stiff 1-dof system for the El Centro record (*Closed-open-loop control*)

5.3.2.b RESPONSE TO THE SAN FERNANDO RECORD

The previous analyses were repeated for the second earthquake signal, the 1971 San Fernando ground motion. Figures 5.37 to 5.40 display the time variation of the displacement and total base shear response in the uncontrolled structure and in the structure with fixed dampers and controlled with the **VDSA** device. The control force and the variation of the device's position with $w_{\min}$ = 20 in, are shown in Figure 5.41 and 5.42, respectively. The maximum force generated by the control device was 7.762 kip.

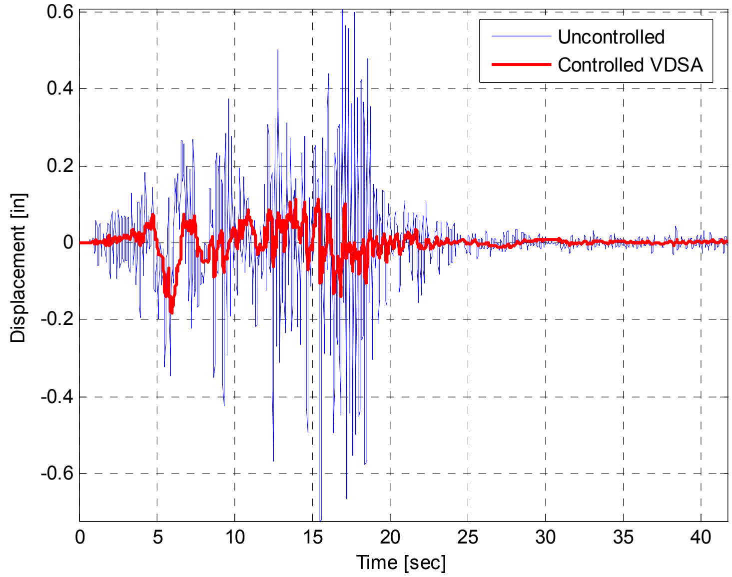


Figure 5.37: Relative displacement of the stiff 1-dof system for the San Fernando record - Uncontrolled vs. VDSA (*Closed-open-loop control*)

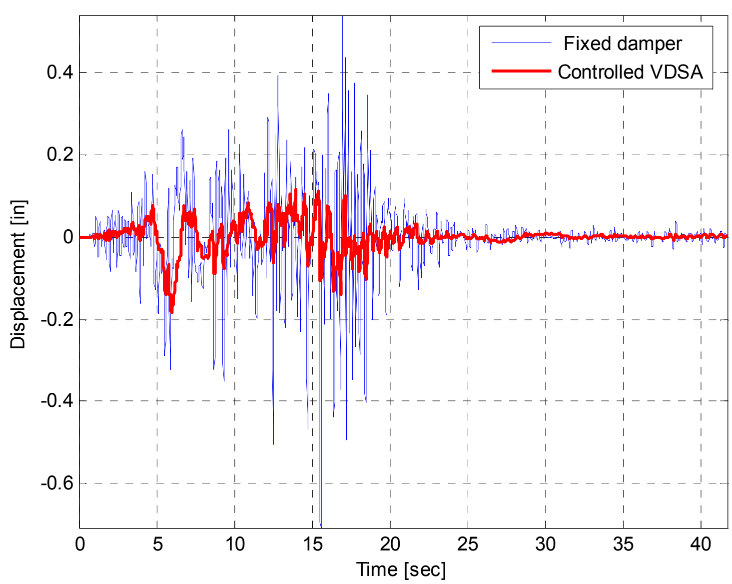


Figure 5.38: Relative displacement of the stiff 1-dof system for the San Fernando record - Fixed damper vs. VDSA (*Closed-open-loop control*)

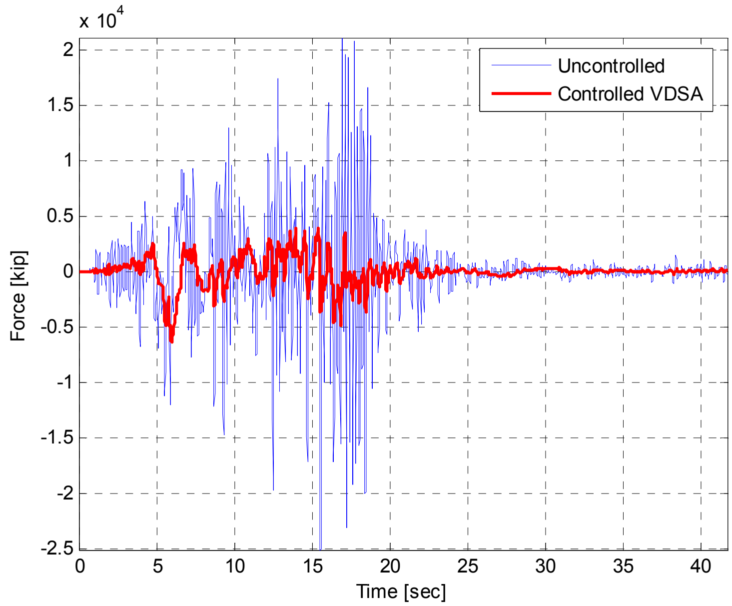


Figure 5.39: Base shear of the still 1-dof system for the San Fernando record - Uncontrolled vs. VDSA (*Closed-open-loop control*)

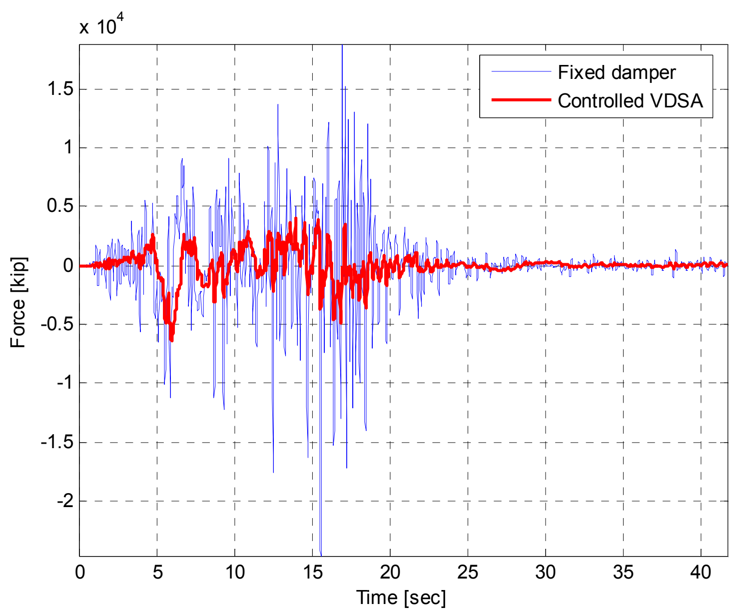


Figure 5.40: Base shear of the still 1-dof system for the San Fernando record - Fixed damper vs. VDSA (*Closed-open-loop control*)

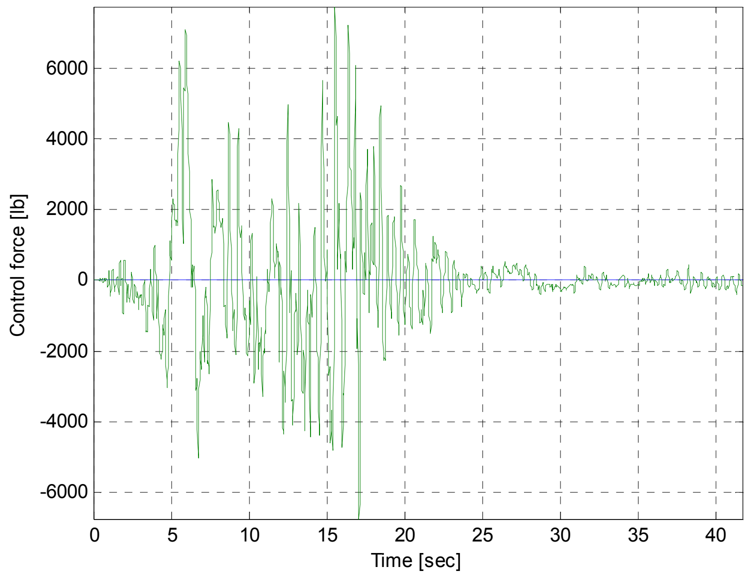


Figure 5.41: Control force in the stiff 1-dof system for the San Fernando record (*Closed-open-loop control*)

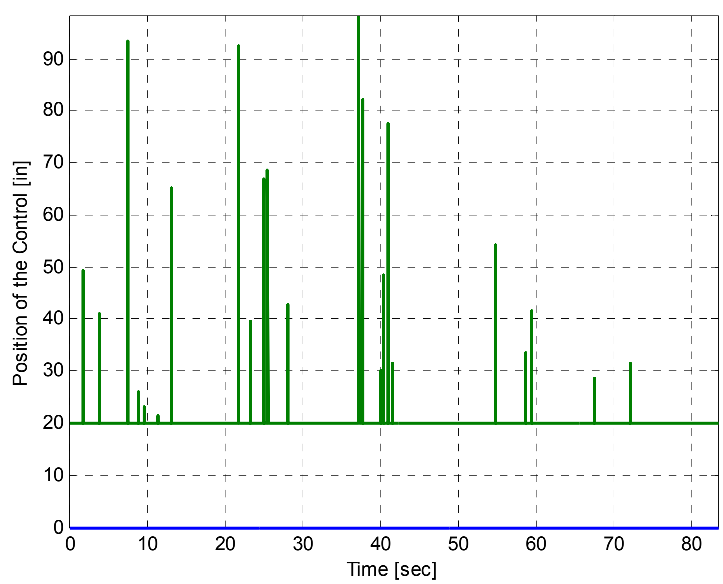


Figure 5.42: Variation of the position of the VDSA device in the stiff 1-dof system for the San Fernando record (*Closed-open-loop control*)

5.3.2.c RESPONSE TO THE FRIULI RECORD

Here again the responses compared are the relative displacement of the mass and the total base shear in the stiff 1-dof system. Figures 5.43 and 5.44 show the displacement time history whereas Figures 5.45 to 5.46 display the base shear variation. Figures 5.47 and 5.48 show the control force time history and the variation in the position of the **VDSA** device, respectively. The maximum value in the control force time history for the Friuli earthquake is 3.962 kip and $w_{\min} = 20$ in.

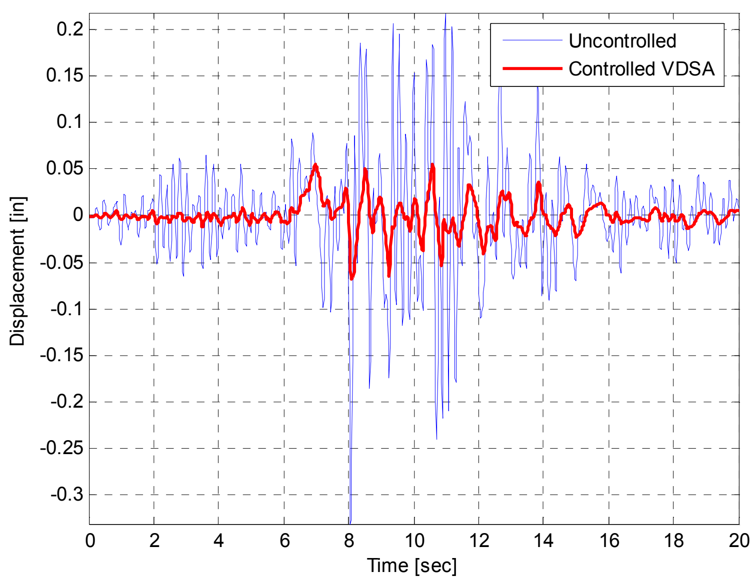


Figure 5.43: Relative displacement of the stiff 1-dof system for the Friuli record - Uncontrolled vs. VDSA (*Closed-open-loop control*)

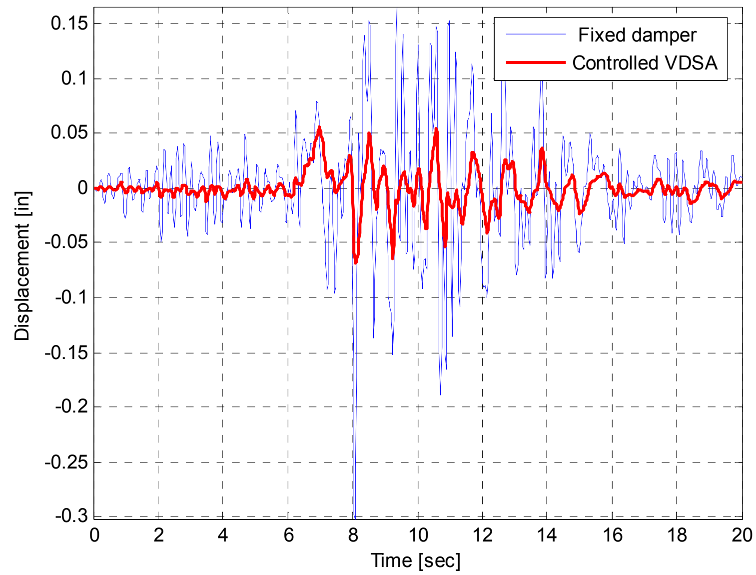


Figure 5.44: Relative displacement of the stiff 1-dof system for the Friuli record - Fixed damper vs. VDSA (*Closed-open-loop control*)

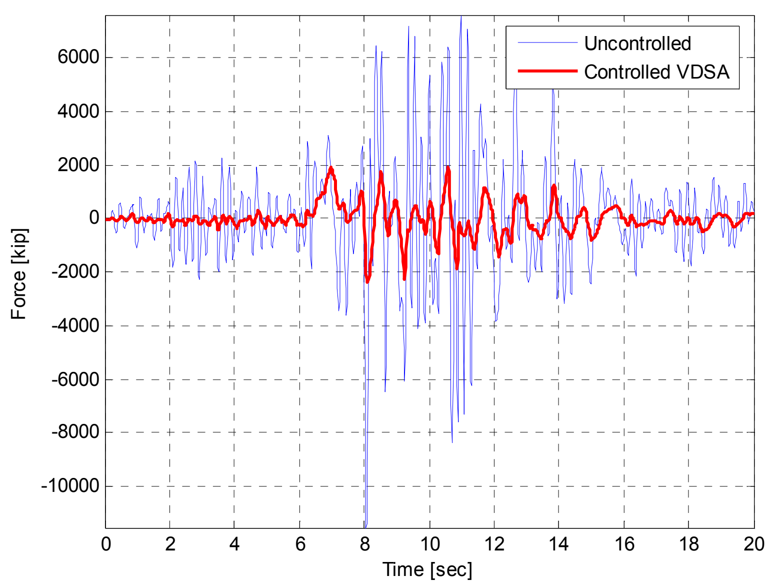


Figure 5.45: Base shear of the stiff 1-dof system for the Friuli record - Uncontrolled vs. VDSA (*Closed-open-loop control*)

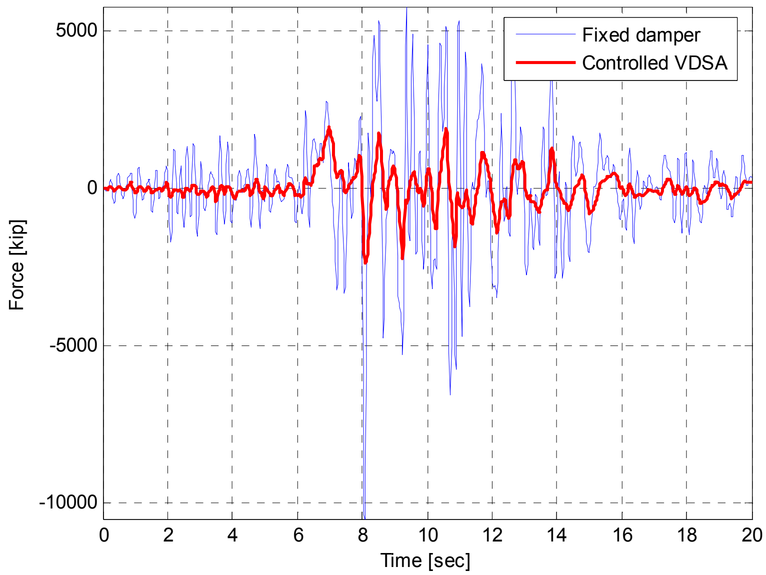


Figure 5.46: Base shear of the stiff 1-dof system for the Friuli record - Fixed damper vs. VDSA (*Closed-open-loop control*)

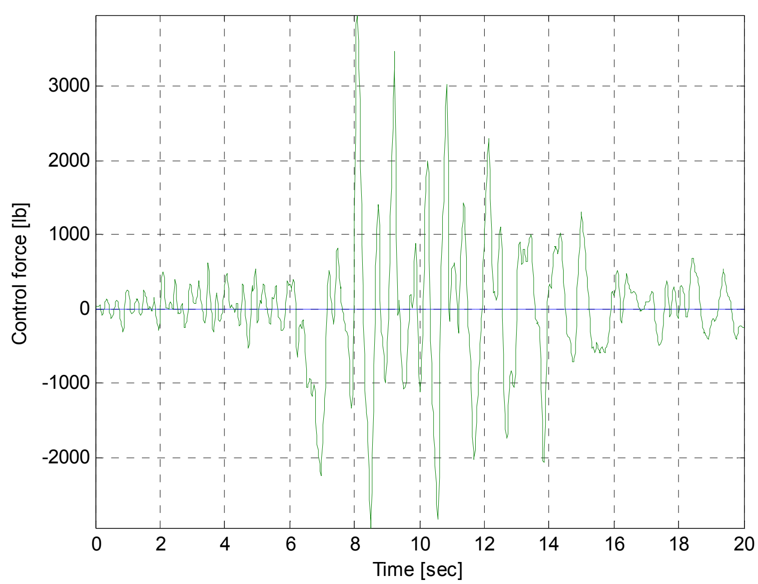


Figure 5.47: Control force in the stiff 1-dof system for the Friuli record (*Closed-open-loop control*)

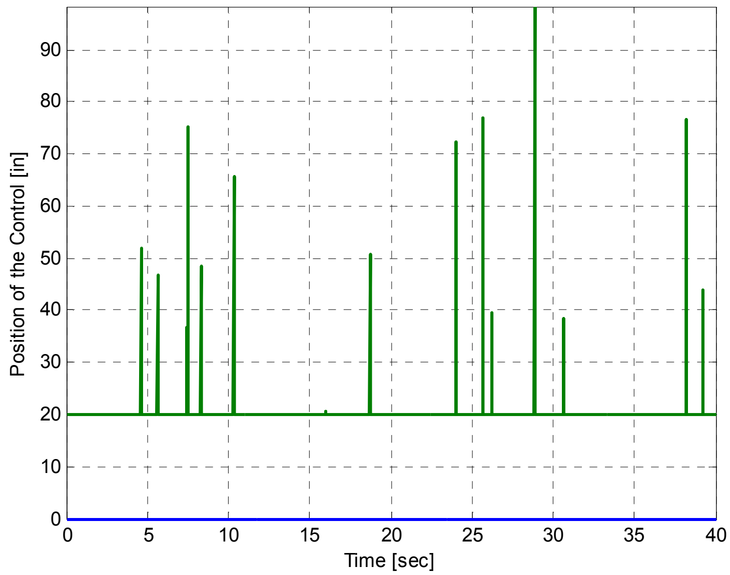


Figure 5.48: Variation of the position of the VDSA device in the stiff 1-dof system for the Friuli record (*Closed-open-loop control*)

A summary of the results for the three analyses cases carried out and for each of the three earthquakes is shown in Table 5.3.

Table 5.3: Maximum response of the SDOF stiff structure without control and with a passive and semiactive system (*Closed-open-loop control*)

Earthquake	Displacement			Total base shear		
	Uncont. [in*]	Fixed damper [in*]	VDSA [in*]	Uncont. [kip+]	Fixed damper [kip+]	VDSA [kip+]
El Centro	0.2536	0.2049	0.0616	8828.0	7133.1	2144.5
San Fernando	0.7219	0.7072	0.1816	25132.0	24620.0	6322.2
Friuli	0.3315	0.3019	0.0678	11540.0	10509.0	2361.6

*1 in = 2.54 cm, +1 kip = 4.448 kN.

5.3.2 DISCUSSION OF THE RESULTS FOR EXAMPLE 1

It is interesting to compare the results obtained by applying the two control algorithms to drive the **VDSA** device. The most important information drawn from Figures 5.7 through 5.48 is summarized in Table 5.4. This table shows the peak control force, maximum relative displacement and maximum base shear obtained with the *Closed-loop control* and *Closed-open-loop control* modified algorithms ***Qv***. It can be sum that when the control force is determined with the *Closed-open-loop control* scheme, the response is reduced more than with the *Closed-loop control*. Moreover, this is achieved by applying a slightly smaller control force. According to the data in Table 5.4, the differences in the maximum control force, peak relative displacement and maximum base shear between the *Closed-loop control* and *Closed-open-loop control* are smaller than one percent. By using *Closed-loop control* modified algorithm ***Qv*** the maximum displacement of the 1-dof structure due to the El Centro, San Fernando and Friuli accelerograms was reduced by 75.2, 74.4, and 79.0%, respectively. The corresponding reduction obtained with the *Closed-open-loop control* modified algorithm ***Qv*** were 75.7, 74.8, and 79.6%.

Table 5.4: Maximum response of the stiff SDOF structure
(*Closed-loop control* and *Closed-open-loop control* algorithms)

Earthquake	Control force		Displacement		Total base shear	
	Closed-loop	***Closed-open-loop***	***Closed-loop***	***Closed-open-loop***	***Closed-loop***	***Closed-open-loop***
	[kip⁺]	**[kip⁺]**	**[in*]**	**[in*]**	**[kip⁺]**	**[kip⁺]**
El Centro	3.318	3.249	0.0630	0.0616	2192.1	2144.5
S. Fernando	7.898	7.762	0.1848	0.1816	6435.3	6322.2
Friuli	4.043	3.962	0.0696	0.0678	2424.3	2361.6

*1 in = 2.54 cm, $^{+}$1 kip = 4.448 kN.

5.4 EXAMPLE 2: FLEXIBLE STRUCTURE

Another structure whose behavior can be approximated by a single dof system is used as Example 2. This system differs from that of Example 1 in its stiffness characteristics. The new system is more flexible than the previous one. The weight of the structure is 14.4 kip and the lateral stiffness is 800 kip/in, which gives a natural period of 1.36 sec. The coefficients of the dampers in the **VDSA** device are 25 kip.sec/in and 10 kip.sec/in for the dampers **A** and **B**, respectively. Table 5.5 shows the properties and damping coefficient of the passive damper.

Table 5.5: Properties and damping constant for the passive damper in the flexible structure

Floor	$Cos^2\theta_1$	ϕ_1	ϕ_{r1}	ξ_{eff}	C_o
[#]				**[%]**	**[kip.sec/in⁺]**
1	0.9686	0.0052	0.0052	30	25

*1 in = 2.54 cm, $^{+}$1 kip = 4.448 kN.

5.4.1 APPLICATION OF THE MODIFIED *CLOSED- LOOP CONTROL* ALGORITHM *Qv*

In the first set of results the *Close-loop* algorithm is used to choose the optimal variation of the position of the **VDSA** device. The three earthquakes records previously used are again considered here as the seismic input.

The weighting matrix **Q** is a 2x2 diagonal matrix equal to [***I***]x10^2. The matrices **Qv** and **R** are scalars and were taken equal to 10^1 and 0^{-4}, respectively. These are the same values used in Example 1 (stiff structure).

5.4.1.a RESPONSE TO THE EL CENTRO RECORD

Figures 5.49 through 5.52 show the relative displacement response of the three cases (uncontrolled, with fixed dampers, with **VDSA**) to the El Centro accelerogram. The time variation of the total base shear is compared in Figures 5.53 to 5.55. Figures 5.56 display only the base shear when the **VDSA** device is used in the structure. The vibration period is the same for the three cases, so that the differences in the response are only associated with their damping. The time variation of the force control and the position of the semiactive device are presented in Figures 5.57 and 5.58, respectively. The peak value of the control force is 2.290 kip.

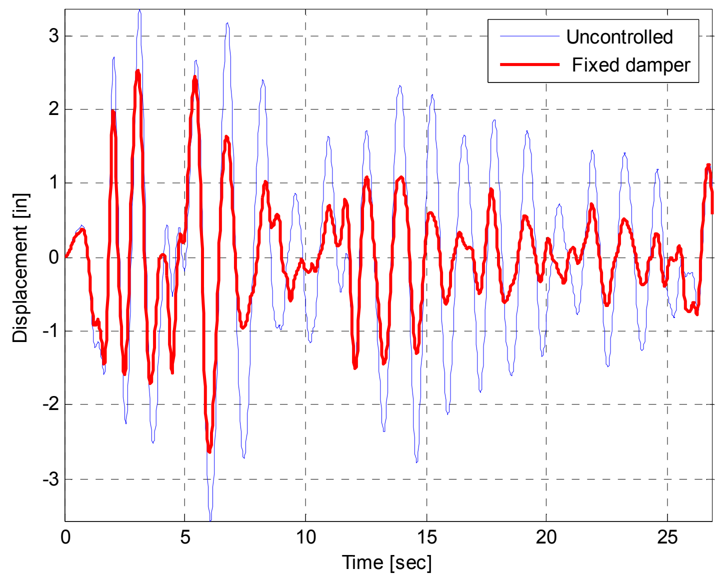


Figure 5.49: Relative displacement of the flexible 1-dof system for the El Centro record - Uncontrolled vs. Fixed damper

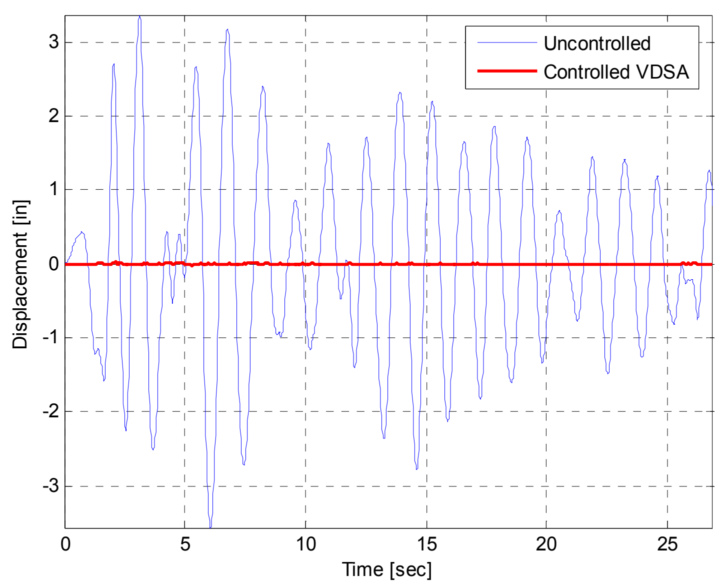


Figure 5.50: Relative displacement of the flexible 1-dof system for the El Centro record - Uncontrolled vs. VDSA (*Closed-loop control*)

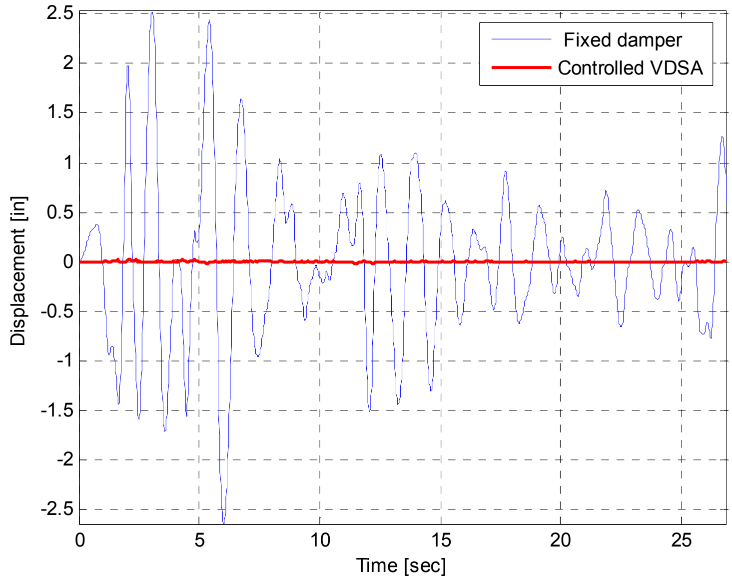


Figure 5.51: Relative displacement of the flexible 1-dof system for the El Centro record - Fixed damper vs. VDSA (*Closed-loop control*)

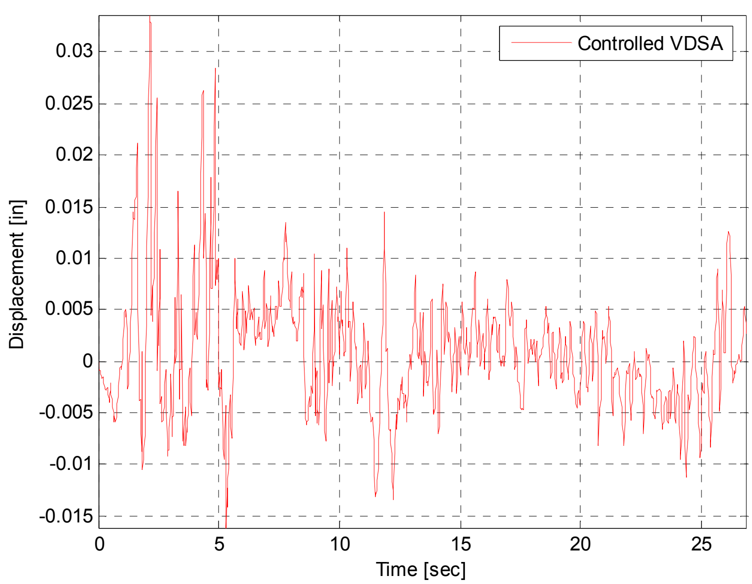


Figure 5.52: Relative displacement of the flexible 1-dof system for the El Centro record controlled with VDSA (*Closed-loop control*)

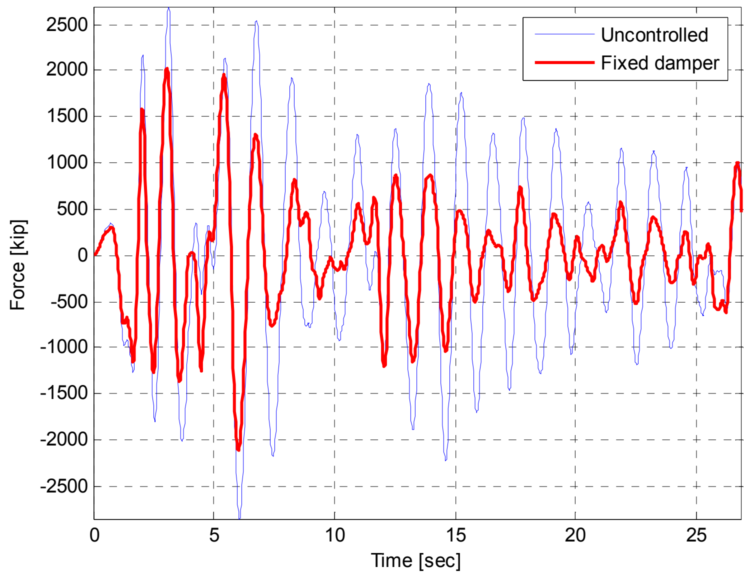


Figure 5.53: Base shear of the flexible 1-dof system for the El Centro record - Uncontrolled vs. Fixed damper

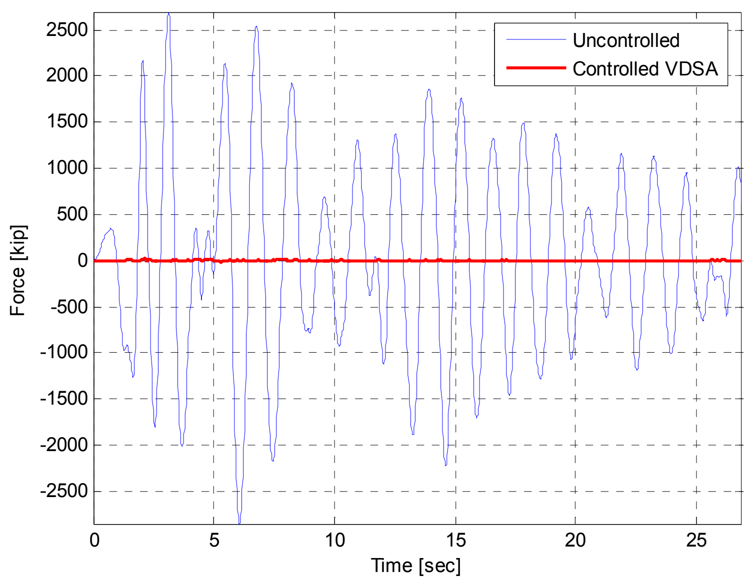


Figure 5.54: Base shear of the flexible 1-dof system for the El Centro record - Uncontrolled vs. VDSA (*Closed-loop control*)

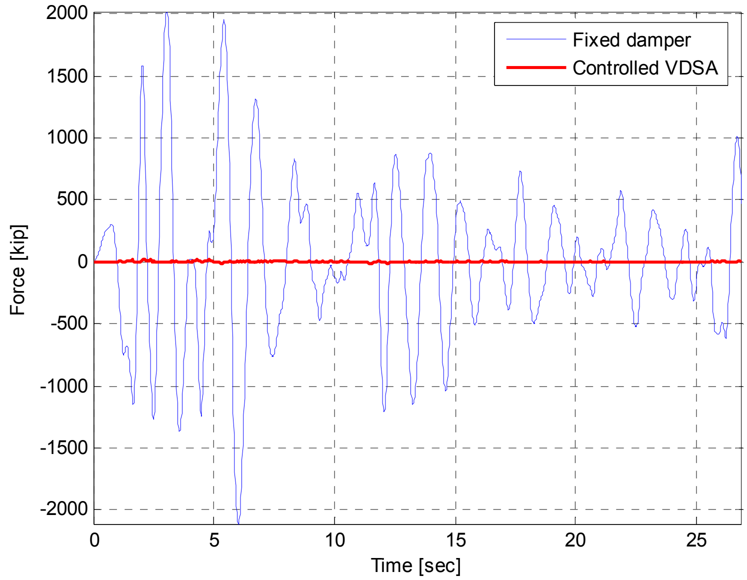


Figure 5.55: Base shear flexible 1-dof system for the El Centro record - Fixed damper vs. VDSA (*Closed-loop control*)

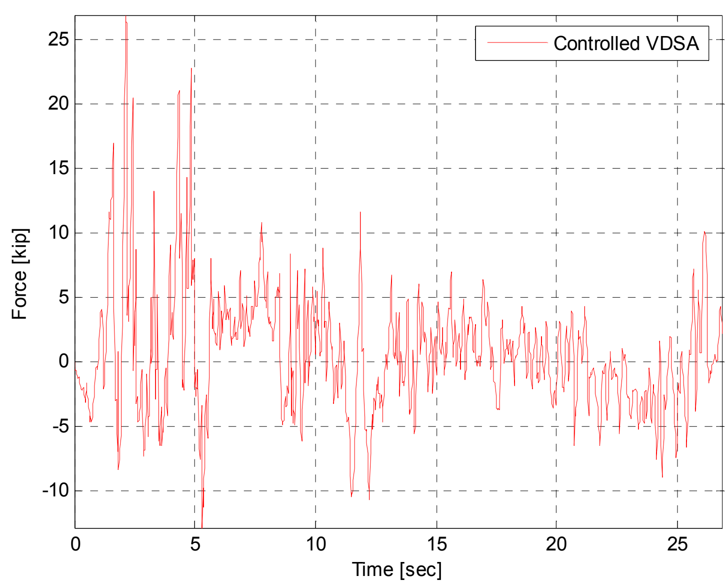


Figure 5.56: Base shear flexible 1-dof system for the El Centro record controlled with VDSA (*Closed-loop control*)

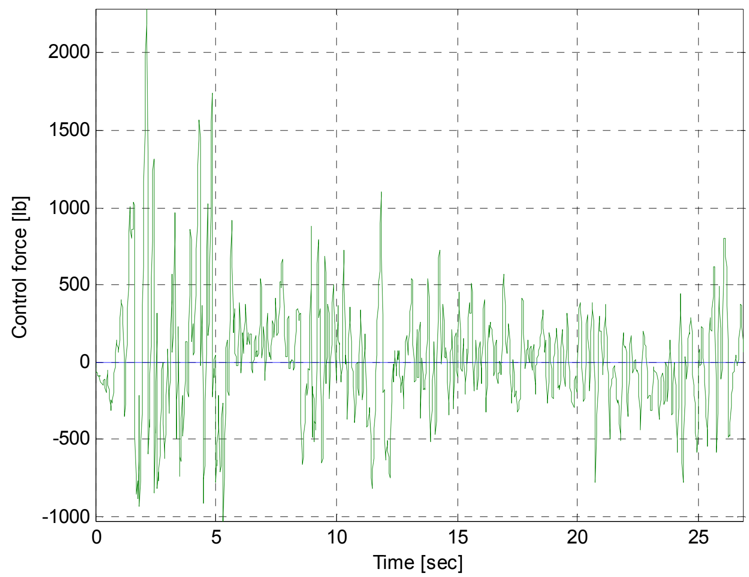


Figure 5.57: Control force in the flexible 1-dof system for the El Centro record (*Closed-loop control*)

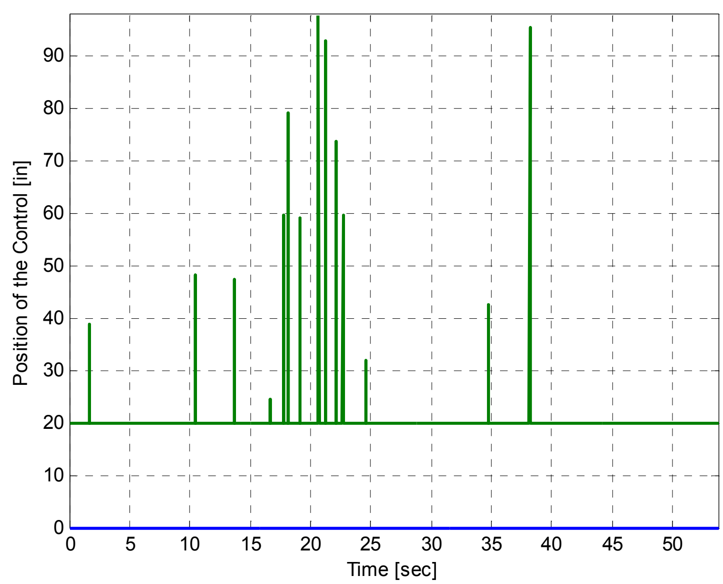


Figure 5.58: Variation of the position of the VDSA device in the flexible 1-dof system for the El Centro record (*Closed-loop control*)

5.4.1.b RESPONSE TO THE SAN FERNANDO RECORD

The variation of the relative displacement of the flexible structure with the **VDSA** device is shown and compared in Figures 5.60 and 5.61 with the responses of the uncontrolled structure and with fixed dampers. Figure 5.59 compares the reduction in displacement achieved with fixed dampers against the original response. Figure 5.62 shows the displacement of the structure with the **VDSA** device standing alone for a better visualization. Similar results in terms of the total base shear are also presented in Figures 5.63 through 5.66. Figures 5.67 and 5.68 show the time variation of the force control and the position of the control device, respectively. In this case the maximum force produced by the control system was 5.777 kip.

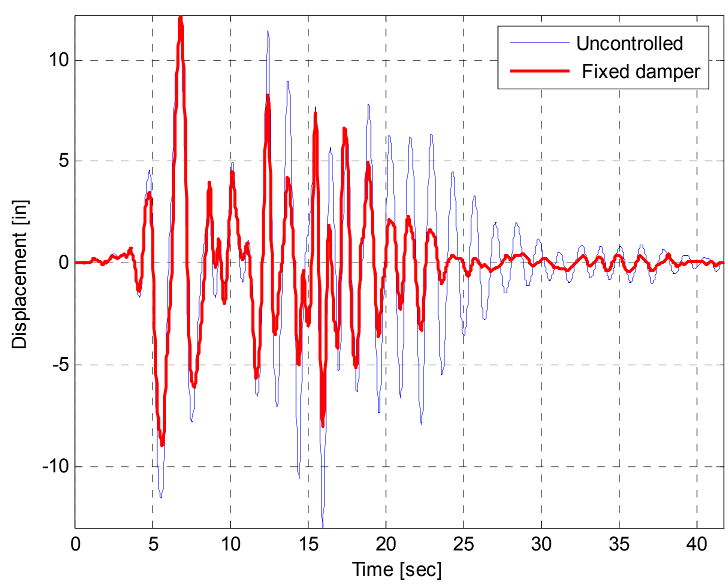


Figure 5.59: Relative displacement of the flexible 1-dof system for the San Fernando record - Uncontrolled vs. Fixed damper

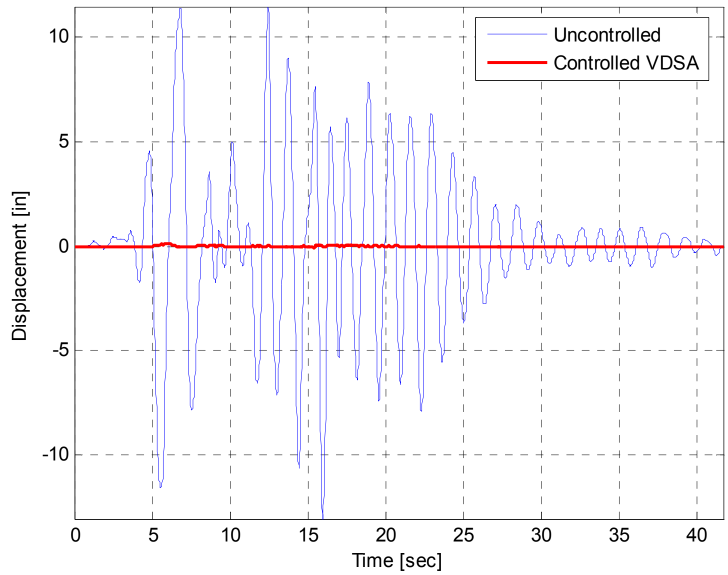


Figure 5.60: Relative displacement of the flexible 1-dof system for the San Fernando record - Uncontrolled vs. VDSA (*Closed-loop control*)

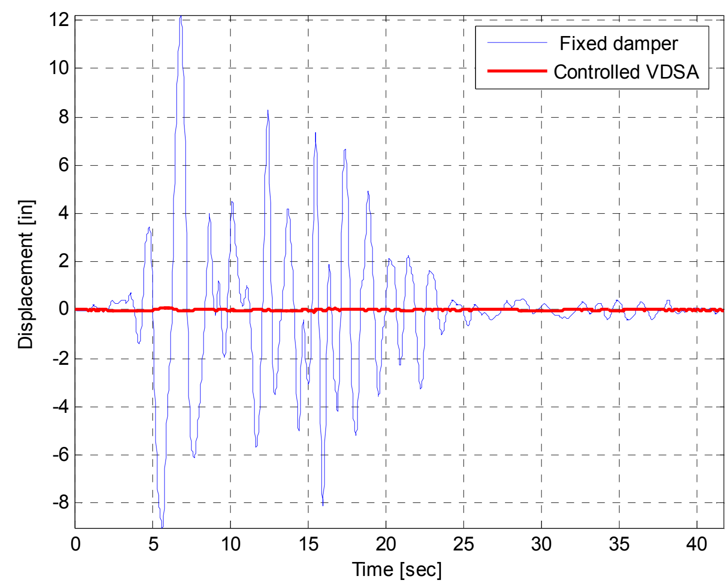


Figure 5.61: Relative displacement of the flexible 1-dof system for the San Fernando record - Fixed damper vs. VDSA (*Closed-loop control*)

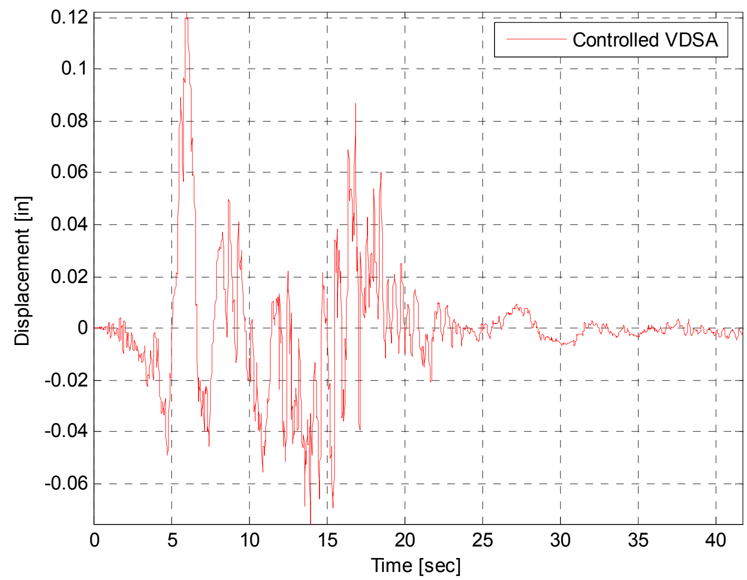


Figure 5.62: Relative displacement of the flexible 1-dof system for the San Fernando record controlled with VDSA (*Closed-loop control*)

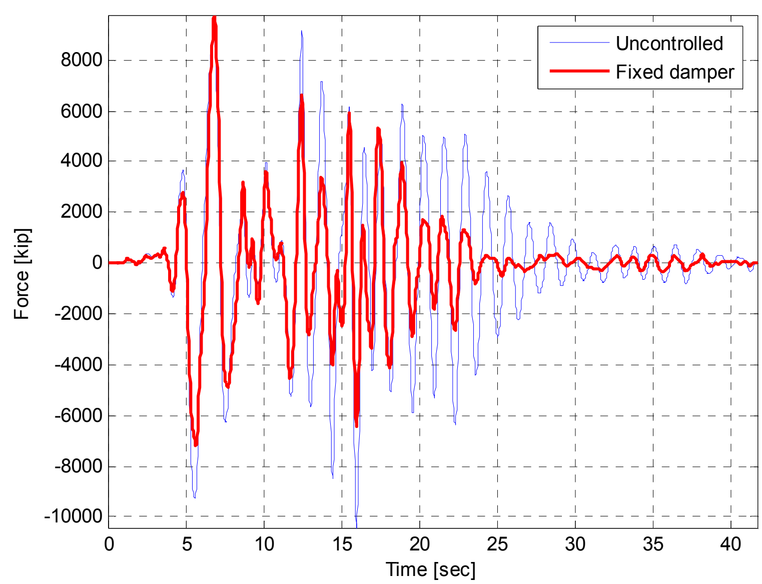


Figure 5.63: Base shear of the flexible 1-dof system for the San Fernando record - Uncontrolled vs. Fixed damper

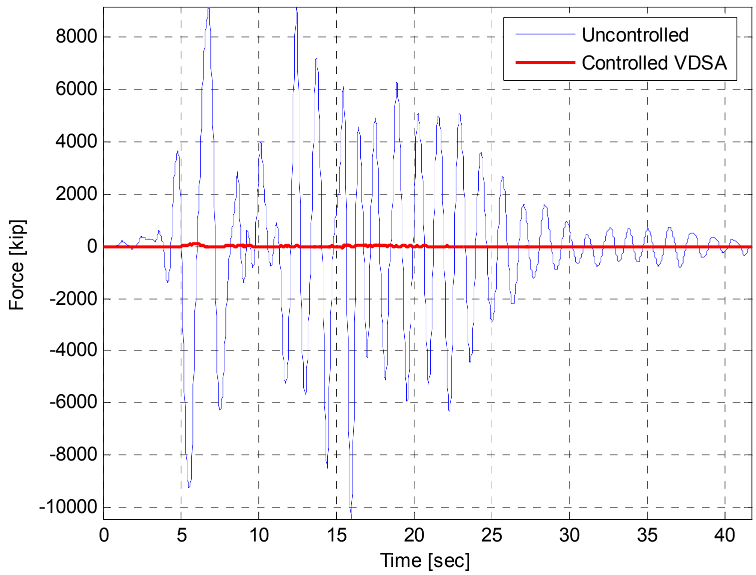


Figure 5.64: Base shear of the flexible 1-dof system for the San Fernando record - Uncontrolled vs. VDSA (*Closed-loop control*)

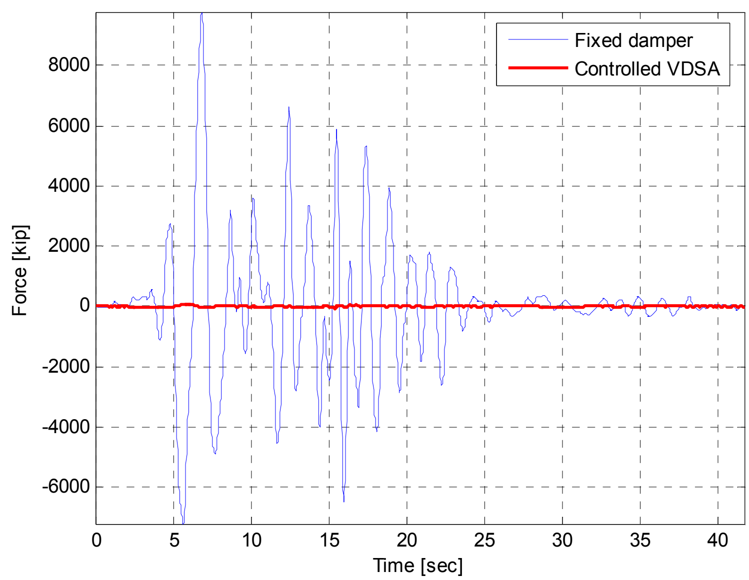


Figure 5.65: Base shear of the flexible 1-dof system for the San Fernando record - Fixed damper vs. VDSA (*Closed-loop control*)

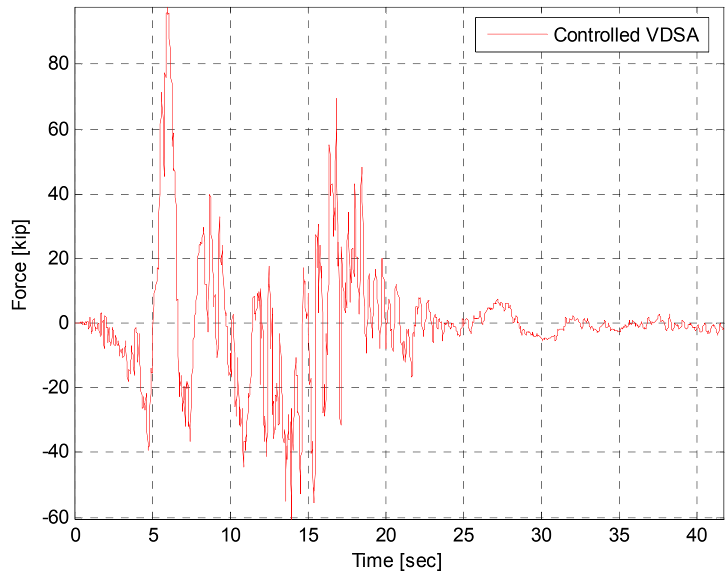


Figure 5.66: Base shear of the flexible 1-dof system for the San Fernando record controlled with VDSA (*Closed-loop control*)

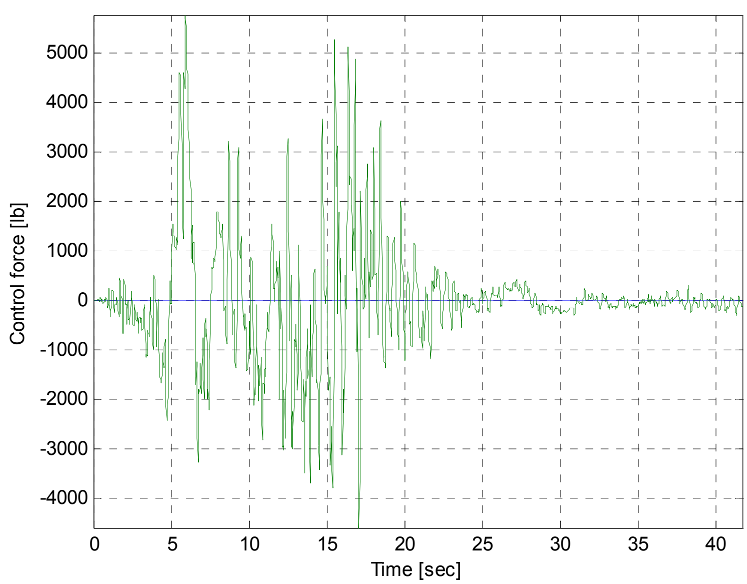


Figure 5.67: Control force of the flexible 1-dof system for the San Fernando record (*Closed-loop control*)

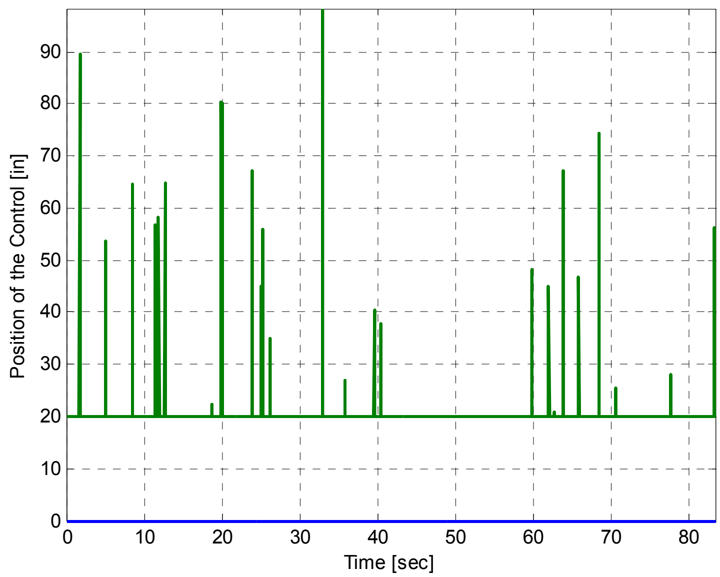


Figure 5.68: Variation of the position of the VDSA device in the flexible 1-dof system for the San Fernando record (*Closed-loop control*)

5.4.1.c RESPONSE TO THE FRIULI RECORD

The last set of results obtained with the *Closed-loop control* algorithm is the response to the 1976 Friuli earthquake record. The displacement and total base shear response of the flexible structure for the three cases (original, passive and semiactive control) are plotted as a function of time in Figures 5.69 to 5.76. The maximum force required of the control system for the Friuli earthquake was 2.612 kip. Its variation is shown in Figure 5.77. Finally the position of the lower end of the **VDSA** device at each instant of time is shown in Figure 5.78, where the lowest position is 20 in above the ground level.

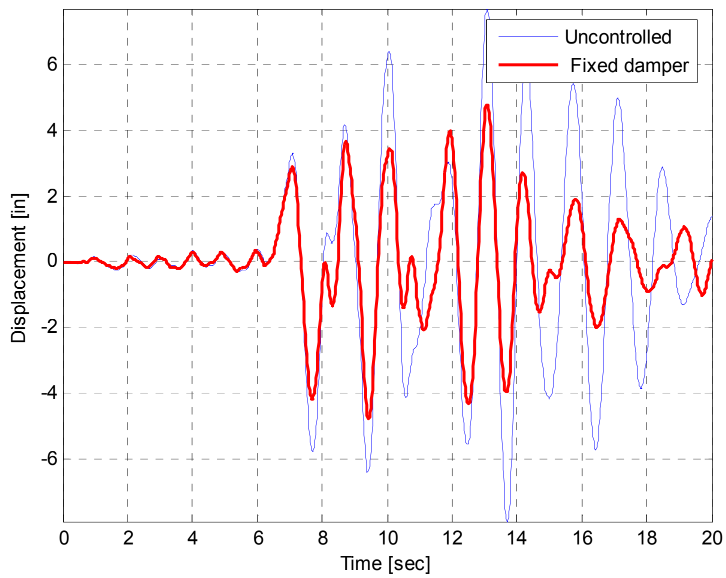


Figure 5.69: Relative displacement of the flexible 1-dof system for the Friuli record - Uncontrolled vs. Fixed damper

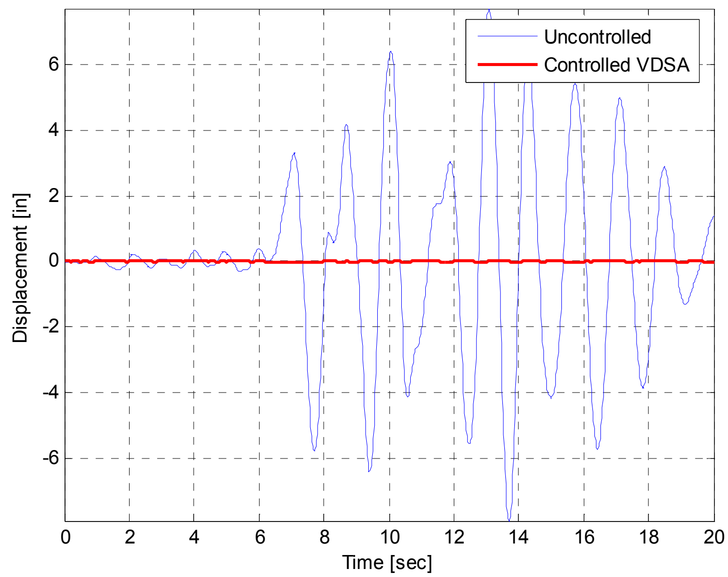


Figure 5.70: Relative displacement of the flexible 1-dof system for the Friuli record - Uncontrolled vs. VDSA (*Closed-loop control*)

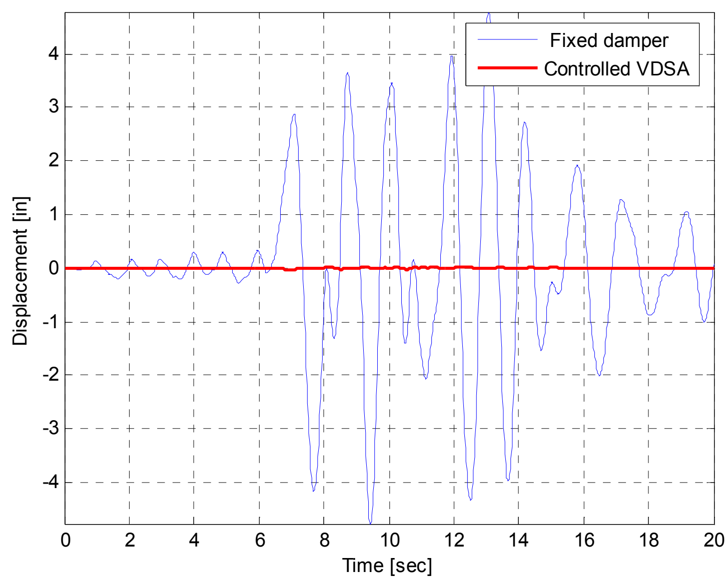


Figure 5.71: Relative displacement of the flexible 1-dof system for the Friuli record - Fixed damper vs. VDSA (*Closed-loop control*)

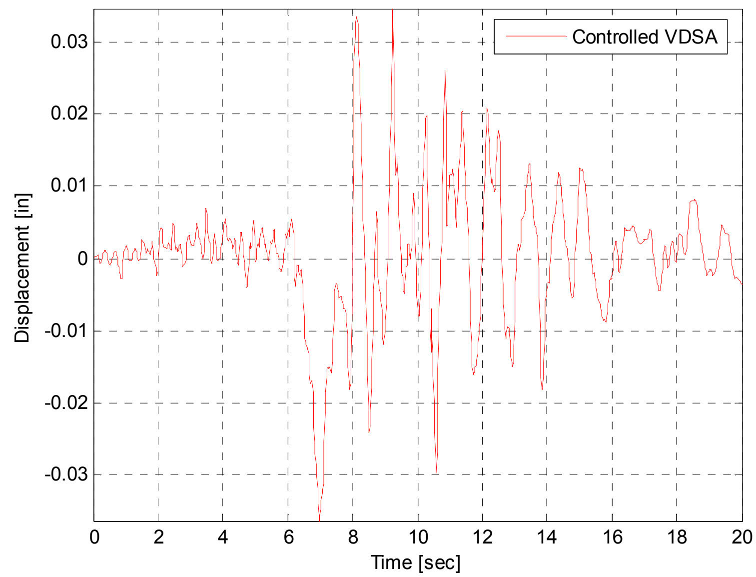


Figure 5.72: Relative displacement of the flexible 1-dof system for the Friuli record controlled with VDSA (*Closed-loop control*)

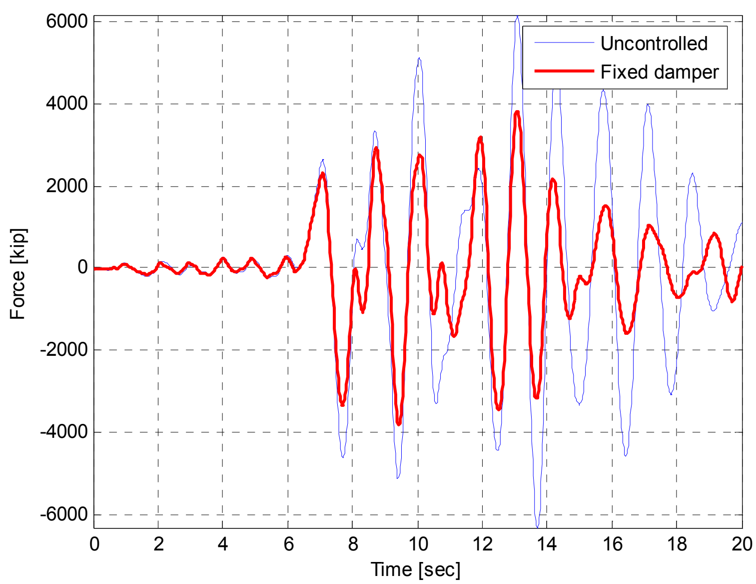


Figure 5.73: Base shear of the flexible 1-dof system for the Friuli record - Uncontrolled vs. Fixed damper

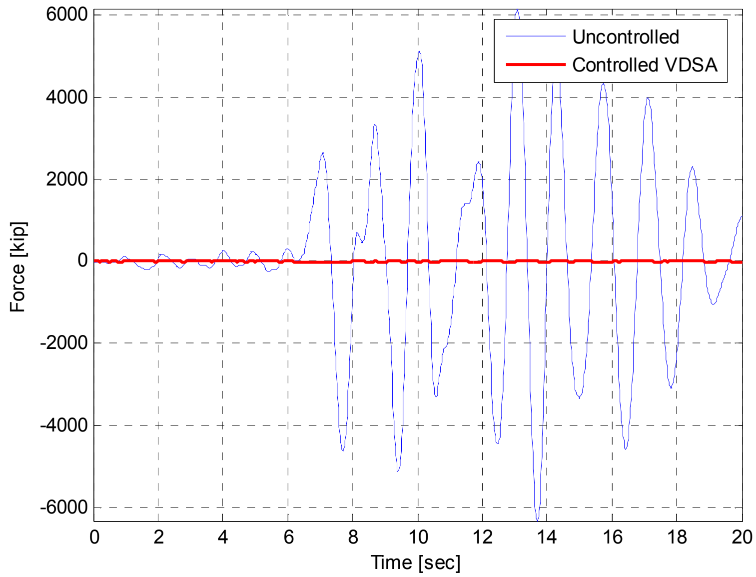


Figure 5.74: Base shear of the flexible 1-dof system for the Friuli record - Uncontrolled vs. VDSA (*Closed-loop control*)

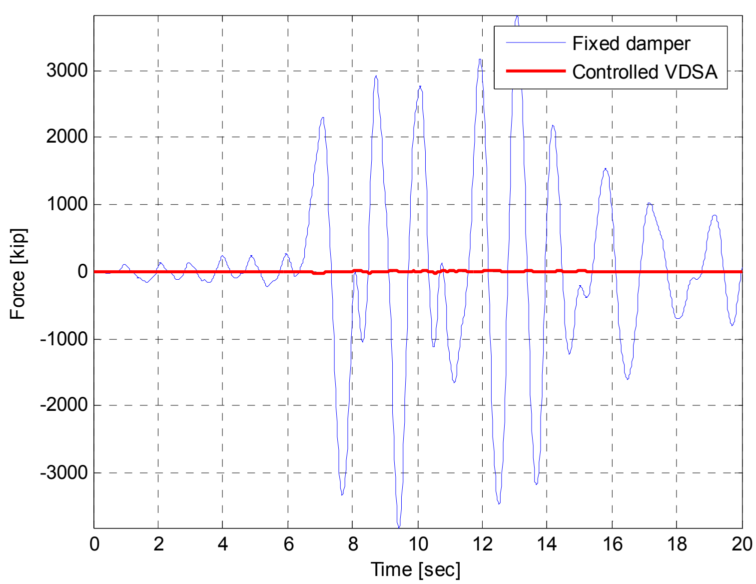


Figure 5.75: Base shear of the flexible 1-dof system for the Friuli record - Fixed damper vs. VDSA (*Closed-loop control*)

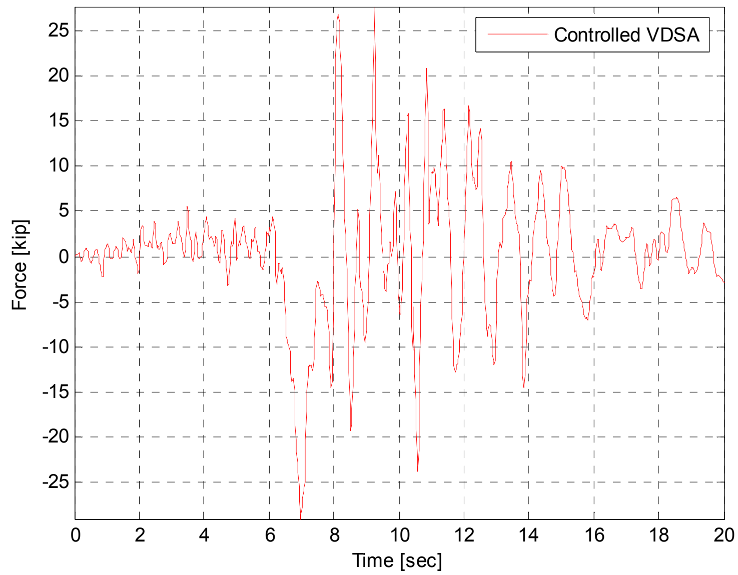


Figure 5.76: Base shear of the flexible 1-dof system for the Friuli record controlled with VDSA (*Closed-loop control*)

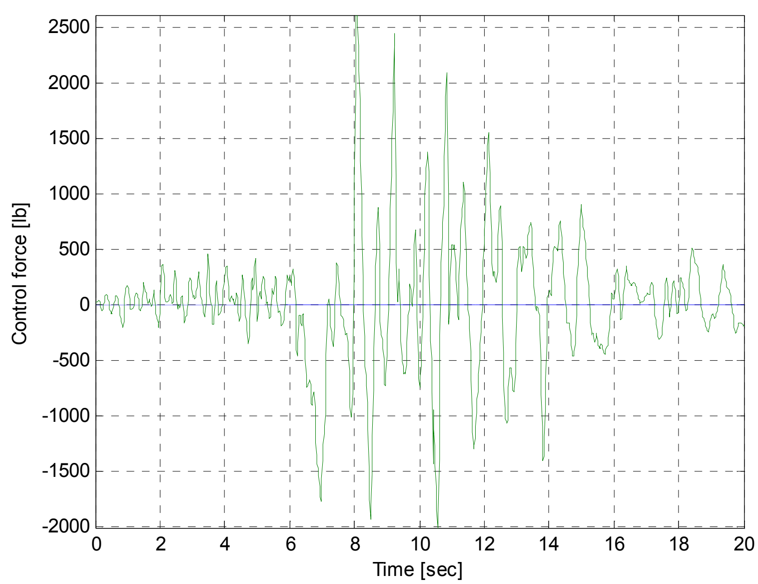


Figure 5.77: Control force of the flexible 1-dof system for the Friuli record (*Closed-loop control*)

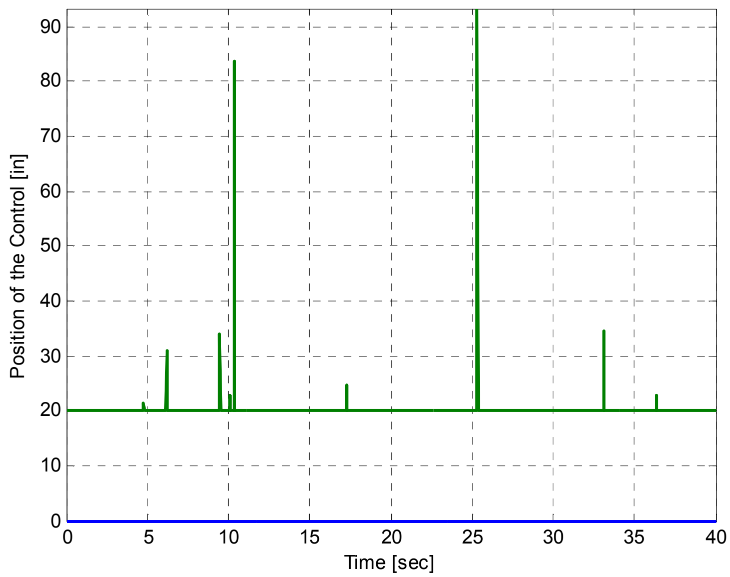


Figure 5.78: Variation of the position of the VDSA device in the flexible 1-dof system for the Friuli record (*Closed-loop control*)

Table 5.6 shows a summary of the maximum response of the flexible 1-dof structure obtained for the three different earthquakes and for each one of the cases. The results for the structure with the **VDSA** device were obtained by using the *Closed-loop control* modified algorithm ***Qv***.

Table 5.6: Maximum response of the SDOF flexible structure without control and with a passive and semiactive system (*Closed-loop control*)

Earthquake	Displacement			Total base shear		
	Uncont. [in*]	Fixed damper [in*]	VDSA [in*]	Uncont. [kip+]	Fixed damper [kip+]	VDSA [kip+]
El Centro	3.5606	2.6375	0.0336	2848.5	2110.0	26.8
San Fernando	13.0256	12.2307	0.1225	10420.0	9784.5	98.0
Friuli	7.8865	4.7842	0.0362	6309.2	3827.3	29.0

*1 in = 2.54 cm, +1 kip = 4.448 kN.

5.4.2 APPLICATION OF THE MODIFIED *CLOSED-OPEN-LOOP CONTROL* ALGORITHM *Qv*

The flexible 1-dof structure with the **VDSA** is now controlled with the *Closed-open loop* algorithm. The structure is subjected to the same three earthquake ground motions applied before.

The weighting matrix **Q** and the vectors **Qv** and **R** selected were exactly the same that the used in the *Closed-loop control* modified algorithm ***Qv*** (**Q** = [***I***]x10^2, **Qv** = 10^1 and **R** = 10^{-4}).

5.4.2.a RESPONSE TO THE EL CENTRO RECORD

The original El Centro record is applied to the flexible single dof structure with the **VDSA** device. The result is compared with the uncontrolled response and with the response achieved with fixed dampers. Figures 5.79 and 5.80 present the comparison of the relative displacement time history. Figure 5.81 is a plot of the displacement time history obtained with the **VDSA** device in action. It is plotted separately because otherwise it cannot be distinguished when it is presented with the other cases. Figures 5.82 to 5.84 show the total base shear time history. The maximum force applied by the control was 2.290 kip and its time variation is shown in Figure 5.85. The variation of the position of the lower end of the **VDSA** device is presented in Figure 5.86.

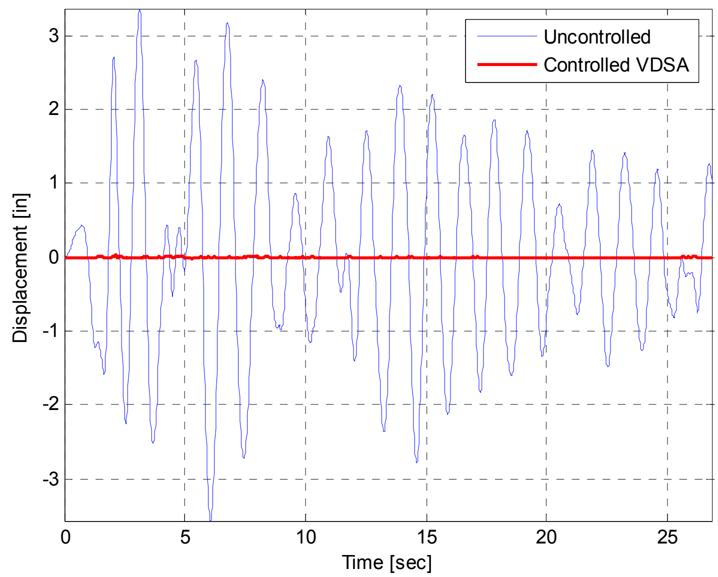


Figure 5.79: Relative displacement of the flexible 1-dof system for the El Centro record - Uncontrolled vs. VDSA (*Closed-open-loop control*)

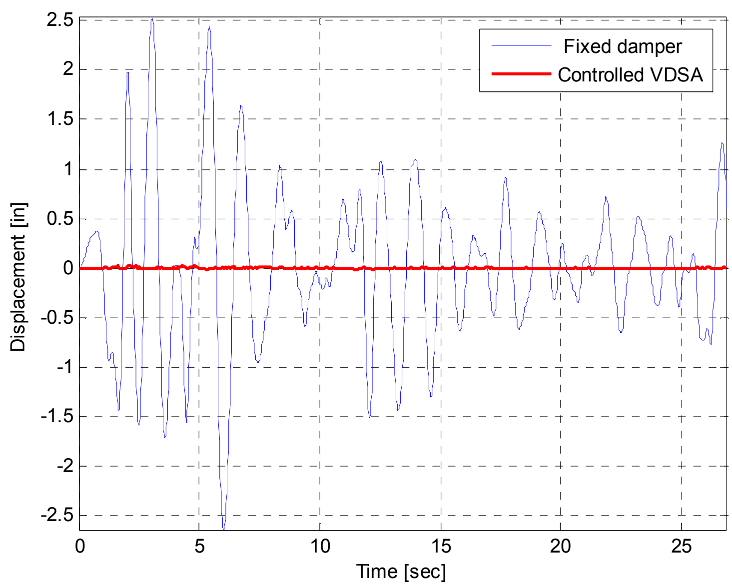


Figure 5.80: Relative displacement of the flexible 1-dof system for the El Centro record - Fixed damper vs. VDSA (*Closed-open-loop control*)

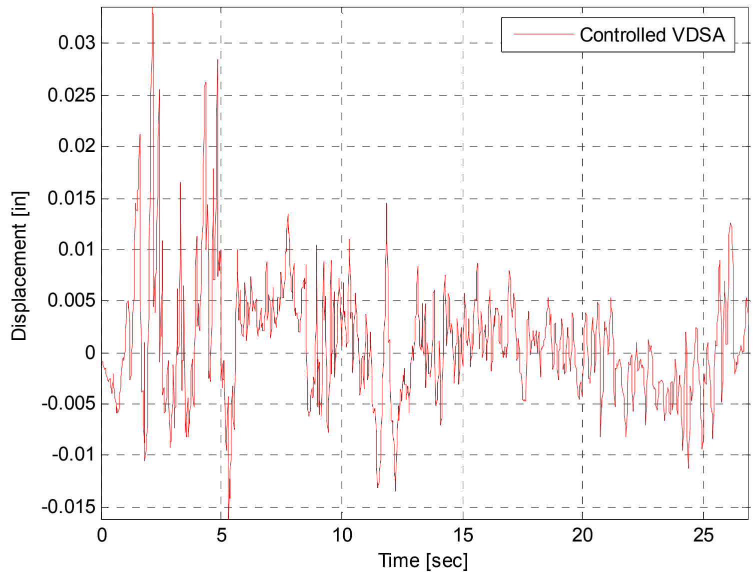


Figure 5.81: Relative displacement of the flexible 1-dof system for the El Centro record controlled with VDSA (*Closed-open-loop control*)

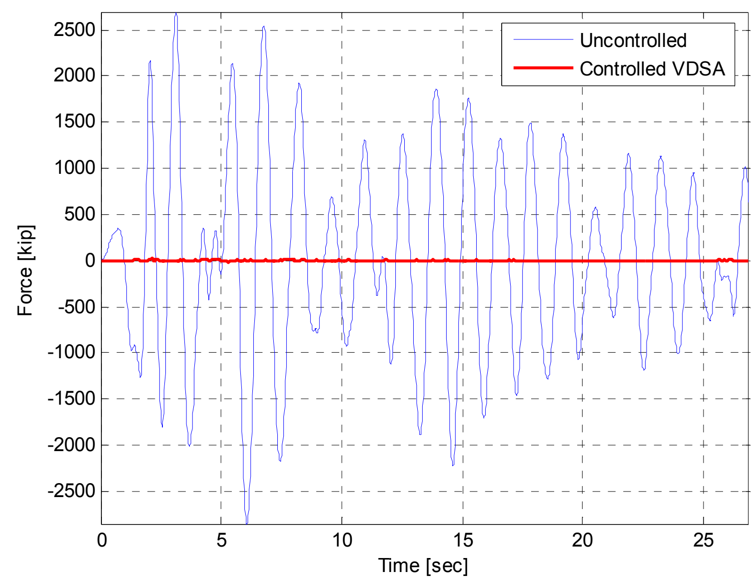


Figure 5.82: Base shear of the flexible 1-dof system for the El Centro record - Uncontrolled vs. VDSA (*Closed-open-loop control*)

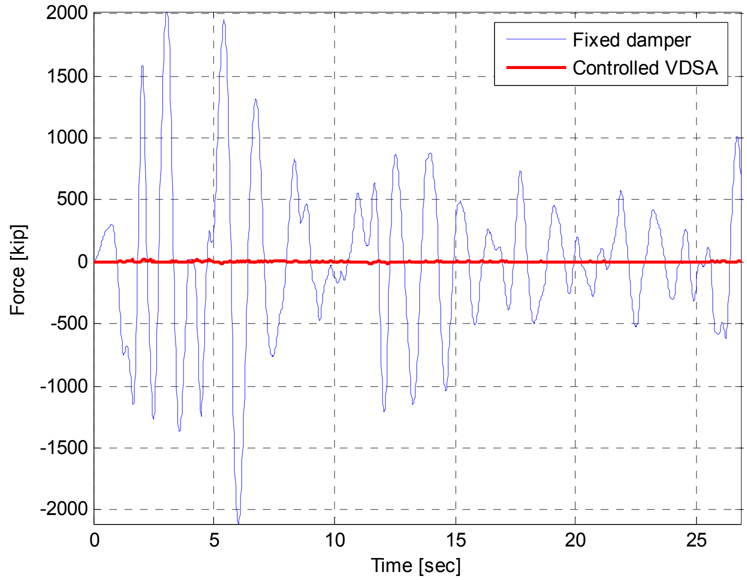


Figure 5.83: Base shear of the flexible 1-dof system for the El Centro record - Fixed Damper vs. VDSA (*Closed-open-loop control*)

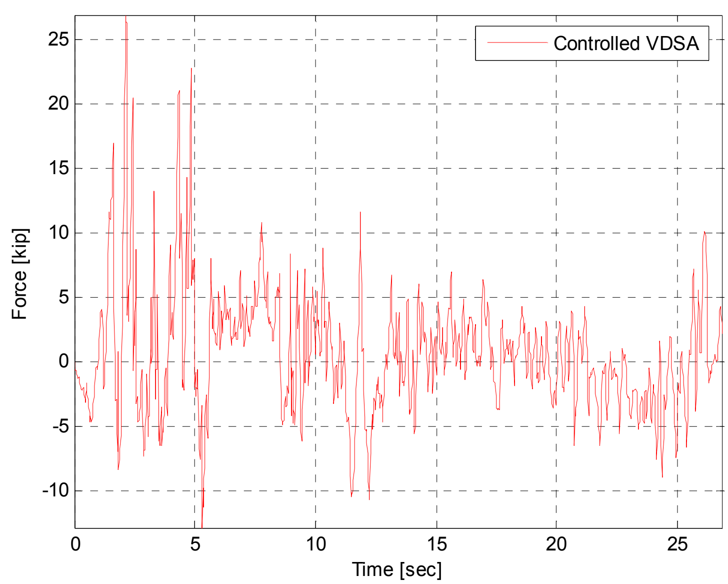


Figure 5.84: Base shear of the flexible 1-dof system for the El Centro record controlled with VDSA (*Closed-open-loop control*)

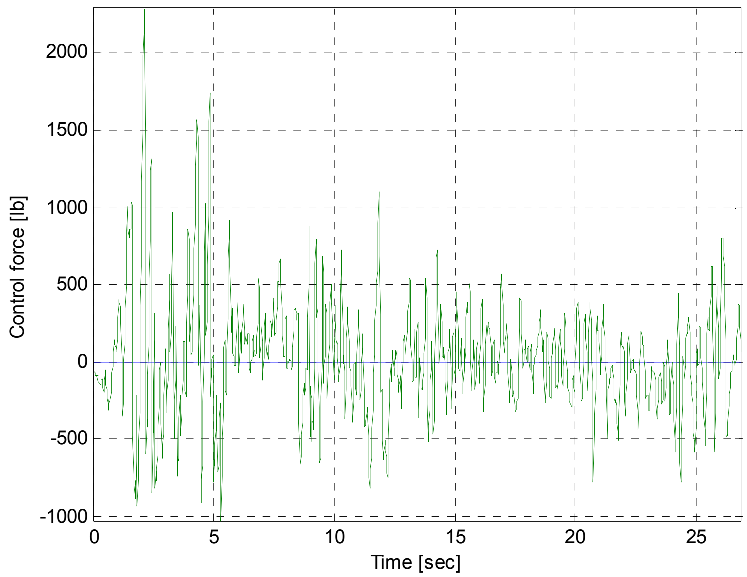


Figure 5.85: Control force of the flexible 1-dof system for the El Centro record (*Closed-open-loop control*)

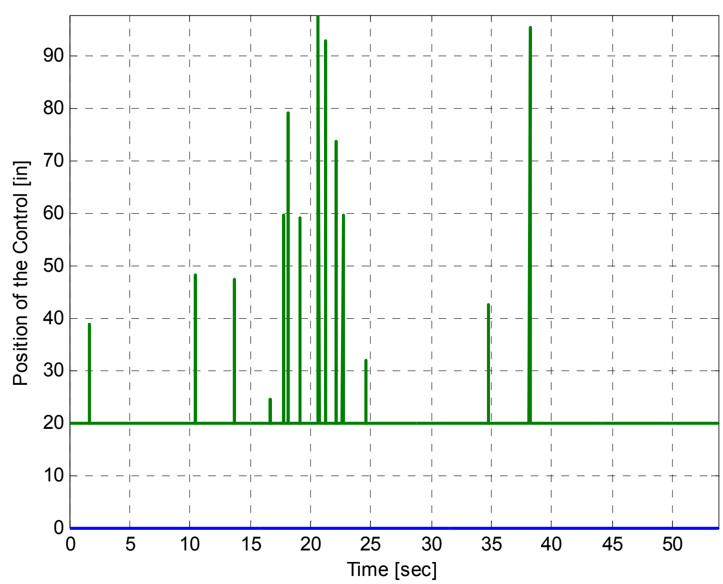


Figure 5.86: Variation of the position of the VDSA device in the flexible 1-dof system for the El Centro record (*Closed-open-loop control*)

5.4.2.b RESPONSE TO THE SAN FERNANDO RECORD

The control effect in the displacement provided by the **VDSA** can be observed in Figures 5.87 through 5.89. Figures 5-90 to 5.92 show reduction in the total base shear, in the structure with the **VDSA** device. Figure 5.93 shows the required control force (the maximum control force was 5.777 kip) in the **VDSA** device. The corresponding variation in the vertical position is shown in Figure 5.94 (w_{min} = 20 in).

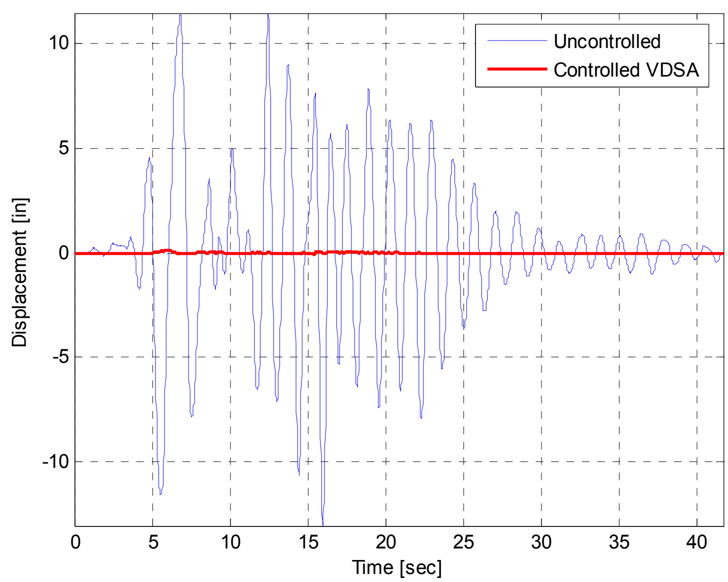


Figure 5.87: Relative displacement of the flexible 1-dof system for the San Fernando record - Uncontrolled vs. VDSA (*Closed-open-loop control*)

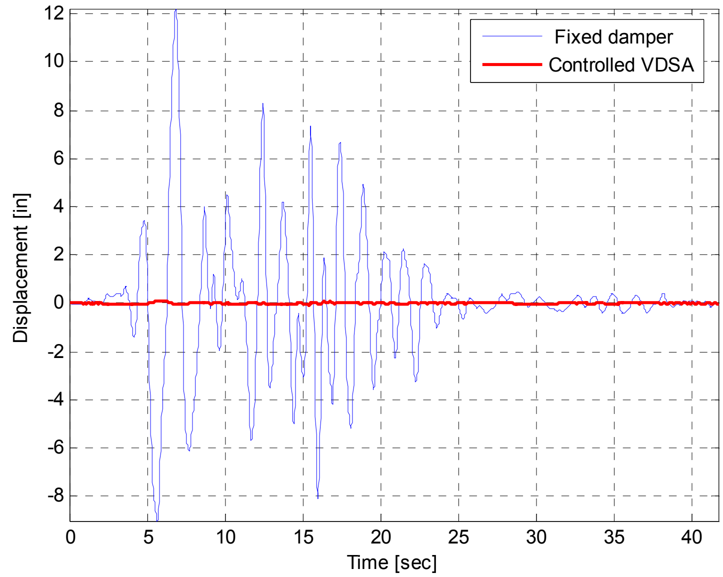


Figure 5.88: Relative displacement of the flexible 1-dof system for the San Fernando record - Fixed damper vs. VDSA (*Closed-open-loop control*)

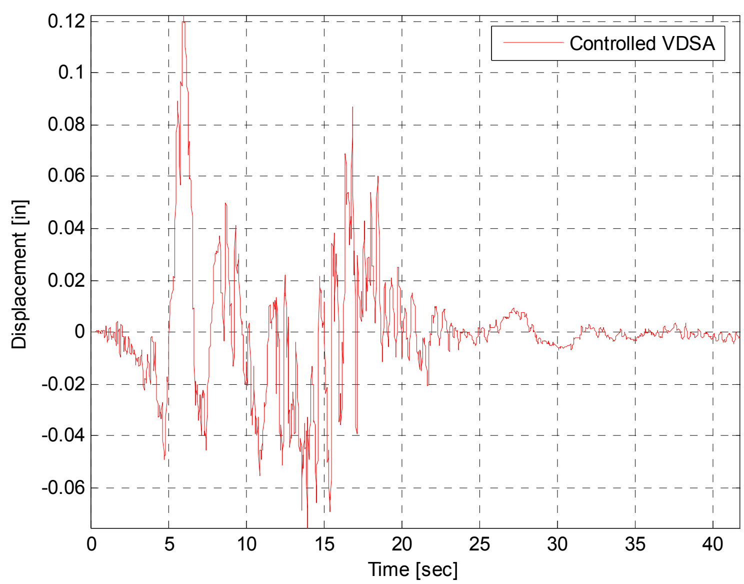


Figure 5.89: Relative displacement of the flexible 1-dof system for the San Fernando record controlled with VDSA (*Closed-open-loop control*)

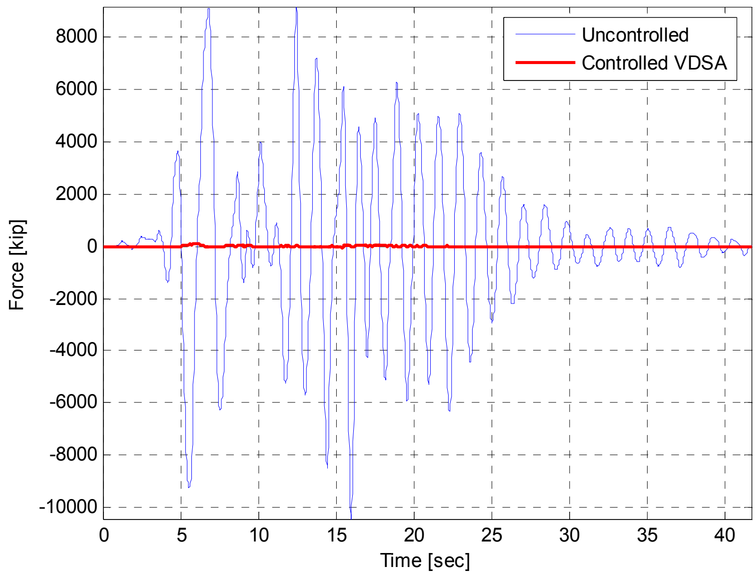


Figure 5.90: Base shear of the flexible 1-dof system for the San Fernando record - Uncontrolled vs. VDSA (*Closed-open-loop control*)

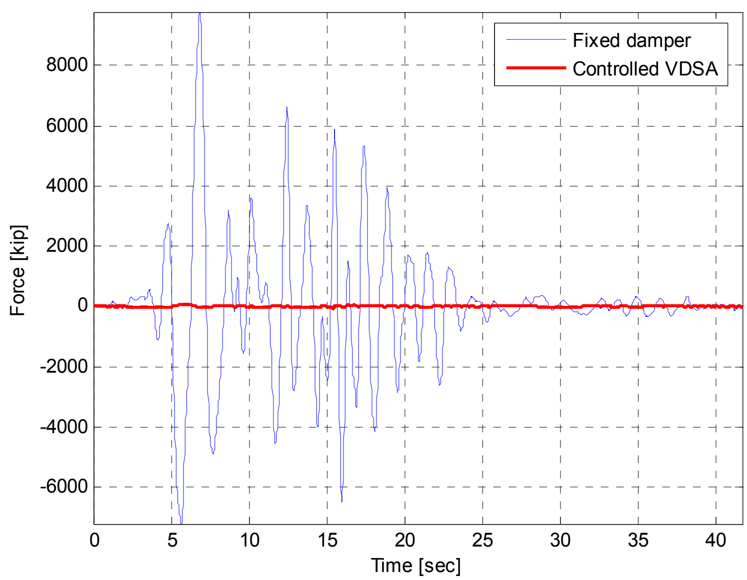


Figure 5.91: Base shear of the flexible 1-dof system for the San Fernando record - Fixed damper vs. VDSA (*Closed-open-loop control*)

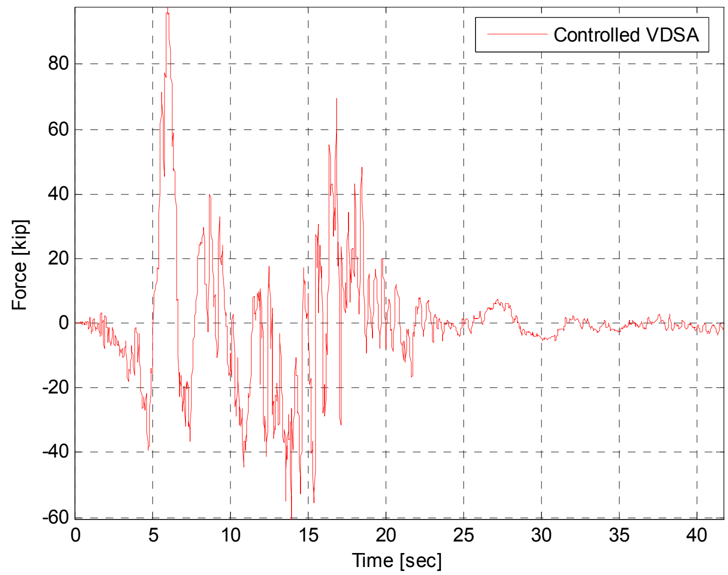


Figure 5.92: Base shear of the flexible 1-dof system for the San Fernando record controlled with VDSA (*Closed-open-loop control*)

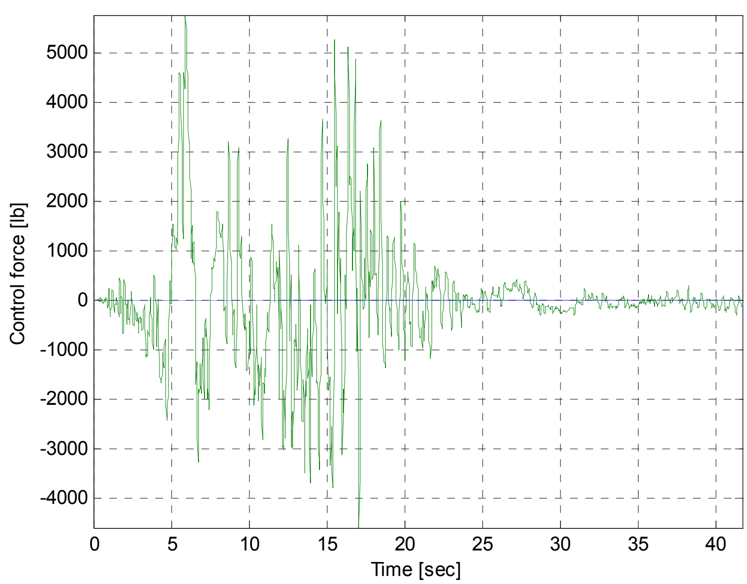


Figure 5.93: Control force of the flexible 1-dof system for the San Fernando record (*Closed-open-loop control*)

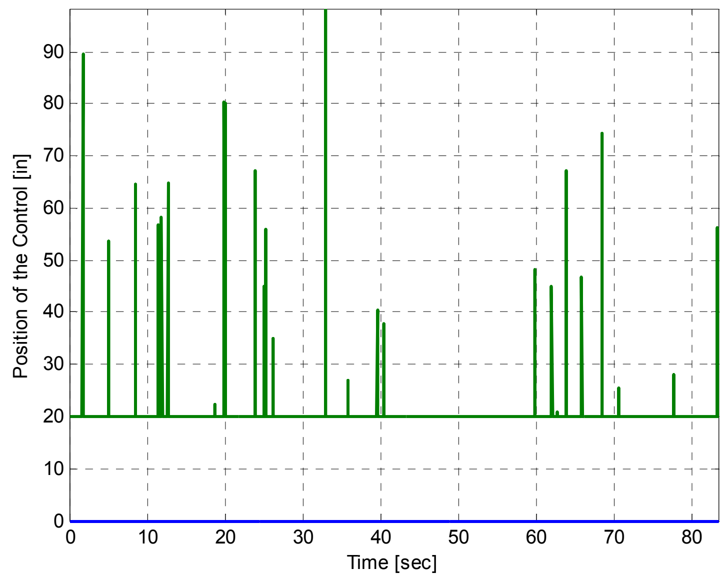


Figure 5.94: Variation of the position of the VDSA device in the flexible 1-dof system for the San Fernando record (*Closed-open-loop control*)

5.4.2.c RESPONSE TO THE FRIULI RECORD

The first set of results for the flexible 1-dof structure corresponds to the 1976 Friuli earthquake. Figures 5.95 to 5.100 shows the time histories of the relative displacements and the total base shears for the structure in uncontrolled condition, controlled with fixed dampers and controlled with **VDSA** device. Figures 5.101 and 5.102 display the control force time history and the variation of the vertical position of the **VDSA** mechanism, respectively. The maximum force generated by the control system is 2.612 kip and $w_{\min} = 20$ in.

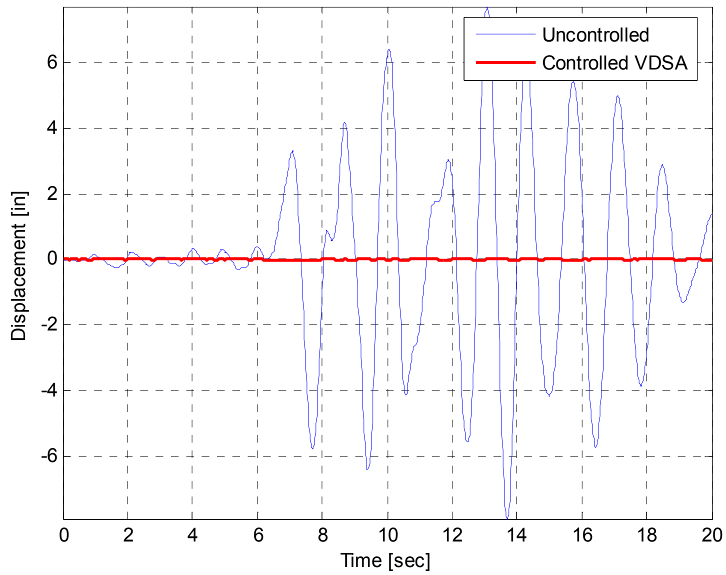


Figure 5.95: Relative displacement of the flexible 1-dof system for the Friuli record - Uncontrolled vs. VDSA (*Closed-open-loop control*)

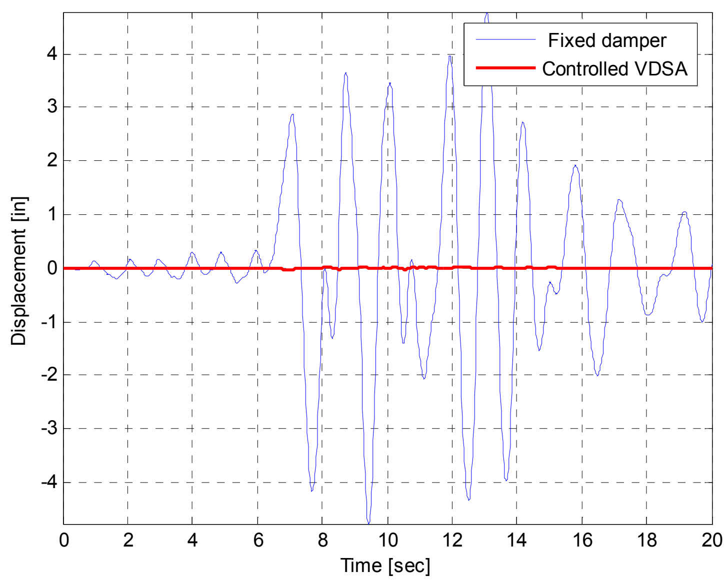


Figure 5.96: Relative displacement of the flexible 1-dof system for the Friuli record - Fixed damper vs. VDSA (*Closed-open-loop control*)

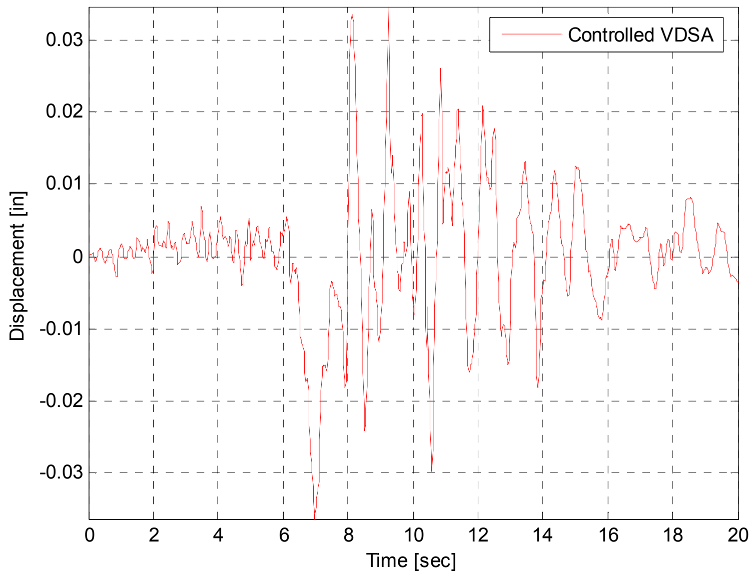


Figure 5.97: Relative displacement of the flexible 1-dof system for the Friuli record controlled with VDSA (*Closed-open-loop control*)

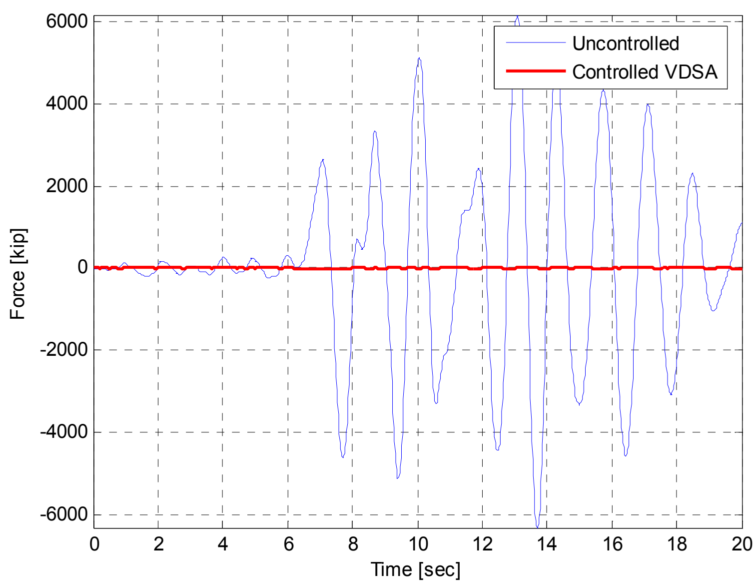


Figure 5.98: Base shear of the flexible 1-dof system for the Friuli record - Uncontrolled vs. VDSA (*Closed-open-loop control*)

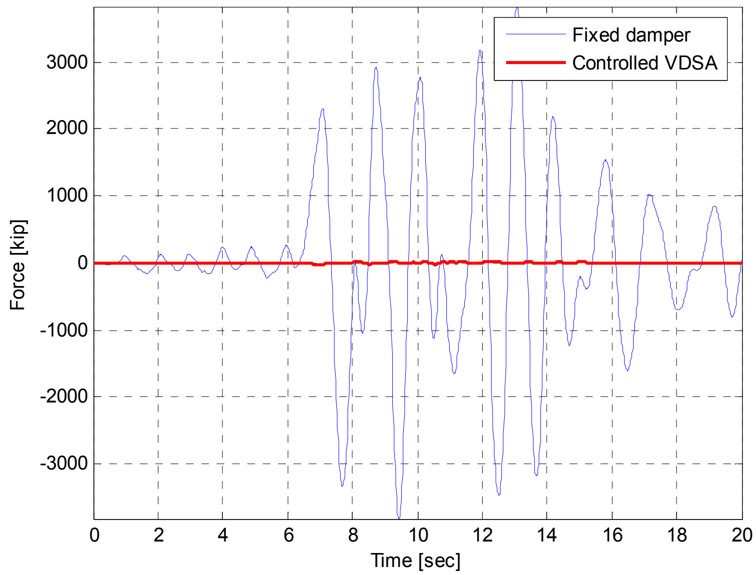


Figure 5.99: Base shear of the flexible 1-dof system for the Friuli record - Fixed damper vs. VDSA (*Closed-open-loop control*)

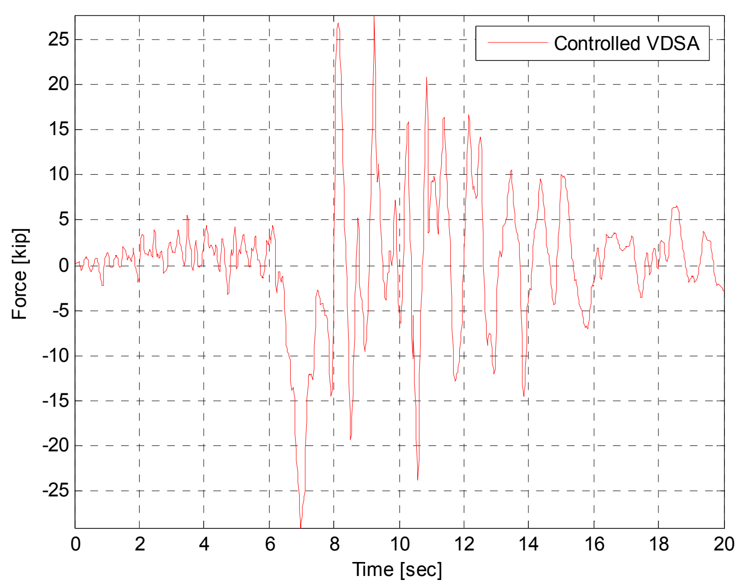


Figure 5.100: Base shear of the flexible 1-dof system for the Friuli record controlled with VDSA (*Closed-open-loop control*)

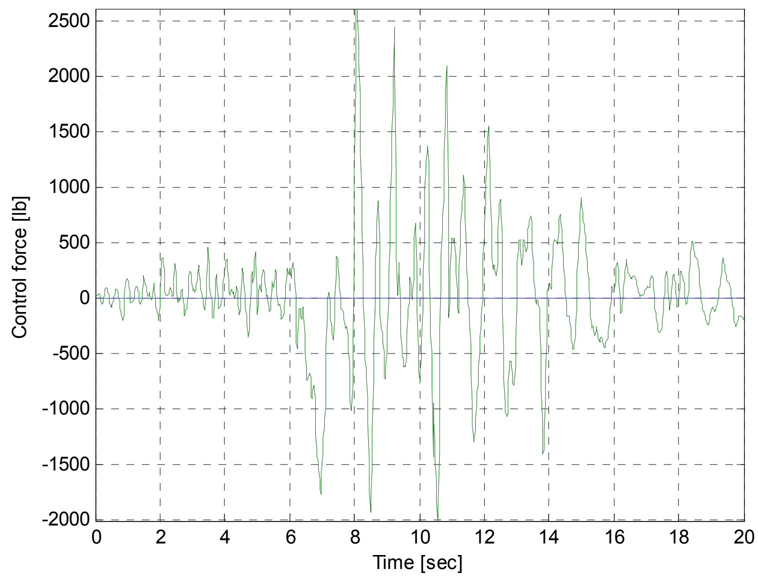


Figure 5.101: Control force of the flexible 1-dof system for the Friuli record (*Closed-open-loop control*)

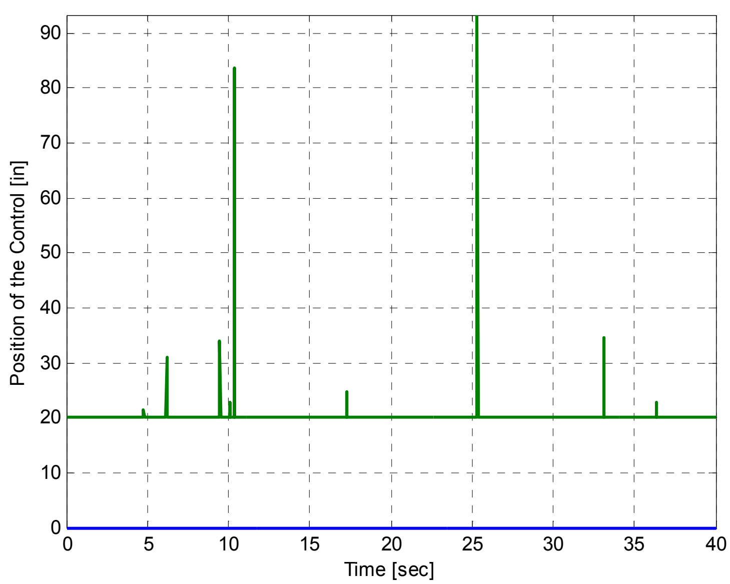


Figure 5.102: Variation of the position of the VDSA device in the flexible 1-dof system for the Friuli record (*Closed-open-loop control*)

Table 5.7 shows a summary of the results for the flexible single dof structure subjected to the three earthquake ground motion when the **VDSA** device is used, the *Closed-open-loop control* modified algorithm ***Qv*** is applied to determine the optimal position of the system. Table 5.6 clearly demonstrates the advantages of the **VDSA** device in the reduction of the response in flexible structures that can be modeled as single dof systems.

Table 5.7: Maximum response of the SDOF flexible structure without control and with a passive and semiactive system (*Closed-open-loop control*)

Earthquake	**Displacement**			**Total base shear**		
	Uncont. [in*]	**Damper Fixed [in*]**	**VDSA [in*]**	**Uncont. [kip+]**	**Damper Fixed [kip+]**	**VDSA [kip+]**
El Centro	3.5606	2.6375	0.0336	2848.5	2110.0	26.9
San Fernando	13.0256	12.2307	0.1225	10420.0	9784.5	98.0
Friuli	7.8865	4.7842	0.0362	6309.2	3827.3	29.0

*1 in = 2.54 cm, +1 kip = 4.448 kN.

5.4.3 DISCUSSION OF RESULTS FOR EXAMPLE 2

Note that when the **VDSA** device is activated, the response, either relative displacement or base shear, is about a hundred times smaller than the original response. Moreover, the level reduction is independent of the seismic excitation acting on the structure. This level of reduction is also significantly superior to the one observed in the stiff single dof structure.

Table 5.8 summarizes the maximum control force, relative displacement and base shear obtained when the *Close-loop* and *Close-open-loop* algorithms are used to calculate the motion of the **VDSA** device in a flexible structure. It is immediately clear that both algorithms perform practically equal. Therefore, it seems that for flexible structures there is no advantage in using one algorithm over the other, at least in terms of the reduction achieved and the maximum force required. Note also that the peak values of the control force are about 2500 to 6300 smaller than the structure's weight, depending on the earthquake shaking applied.

Table 5.8: Maximum response of the flexible SDOF structure (*Closed-loop control* and *Closed-open-loop control*)

Earthquake	Control force		Displacement		Total base shear	
	Closed-loop	*Closed-open-loop*	*Closed-loop*	*Closed-open-loop*	*Closed-loop*	*Closed-open-loop*
	[kip⁺]	[kip⁺]	[in*]	[in*]	[kip⁺]	[kip⁺]
El Centro	2.290	2.290	0.0336	0.0336	26.8	26.9
S. Fernando	5.777	5.777	0.1225	0.1225	98.0	98.0
Friuli	2.612	2.612	0.0362	0.0362	29.0	29.0

*1 in = 2.54 cm, $^{+}$1 kip = 4.448 kN.

5.5 TIME DELAY EFFECT

Therefore, it is relevant to study the effect in the performance of the system of the time delay between the response recorded and the application of the control force to move the semi-active device.

In a real system, it is impossible to apply the control action (in the present case, move or down the **VDSA** device) without an unavoidable time delay. Even when numerical simulations are performed, there is a minimum time delay equal to the time interval Δt used in the sampling. It is recalled that in the present study Δt was set equal to 0.02 sec.

5.5.1 STIFF STRUCTURE

Table 5.9 shows a summary of the maximum relative displacements for the structure of Example 1, using the modified *Closed-loop control* algorithm ***Qv*** and the **VDSA** device for El Centro, San Fernando and Friuli ground motion, obtained accounting for the time delay effect. The time delays considered goes from 0.02 seconds to 0.10 seconds. For a better visualization, these results are displayed in Figures 5.103 to 5.105. Table 5.10 presents the results obtained by using the modified *Closed-open-loop control* algorithm ***Qv.*** Note that the column for 0.02 sec. correspond to the ideal case (minimum possible time delay) presented in the previous numerical examples.

As expected the solution in the maximum displacements is affected by the time delay. If the time delay keeps increasing the action of the semiactive system may even be detrimental. However, it is felt that is a real implementation, provided that the proper case is taken, the time delay effect can be kept under control and it should not influence significantly the results. Of course, this statement should be validated with actual laboratory or field tests, which are beyond the scope of this study.

Table 5.9: Summary of the maximum response of the SDOF stiff structure controlled with the VDSA with time delay effects (*Closed-loop control*)

Earthquake	Displacement					
	Uncontrolled [in*]	0.02 s [in*]	0.04 s [in*]	0.06 s [in*]	0.08 s [in*]	0.10 s [in*]
El Centro	0.2536	0.0630	0.0630	0.0665	0.0672	0.0770
San Fernando	0.7219	0.7222	0.7237	0.7247	0.7258	0.7275
Friuli	0.3315	0.3315	0.3328	0.3340	0.3351	0.3376

*1 in = 2.54 cm.

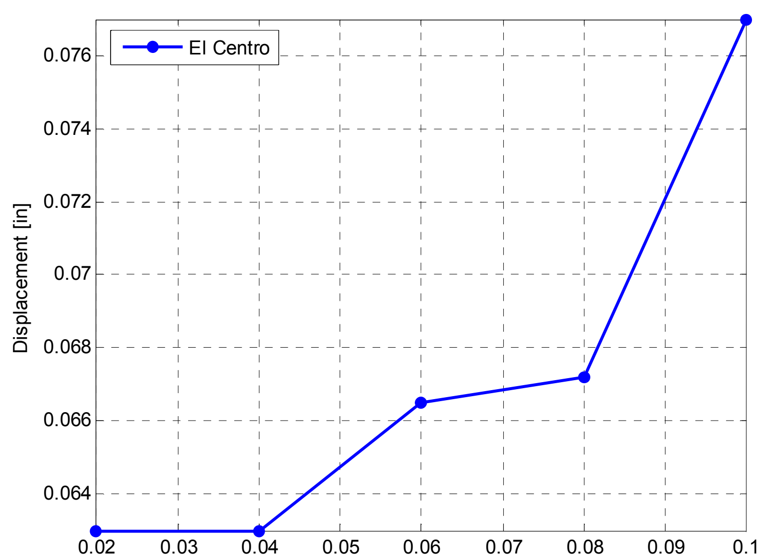


Figure 5.103: Maximum displacements with time delay effects for the stiff 1-dof system and the El Centro record (*Closed-loop control*)

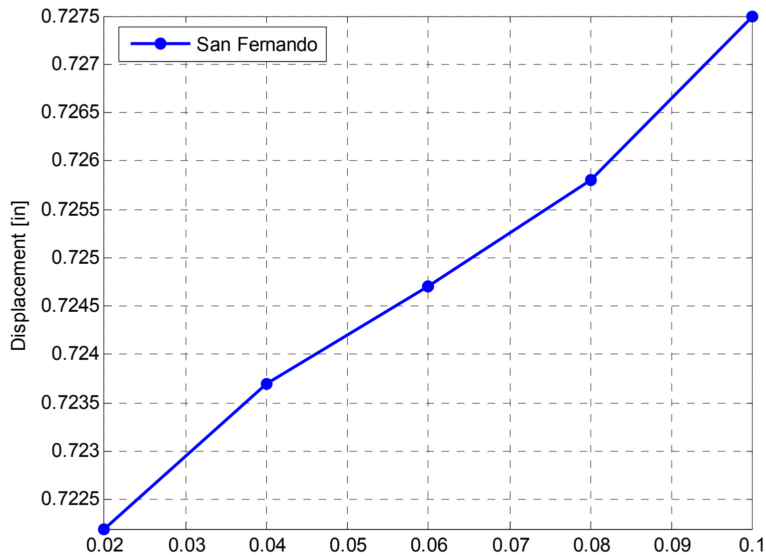


Figure 5.104: Maximum displacements with time delay effects for the stiff 1-dof system and the San Fernando record (*Closed-loop control*)

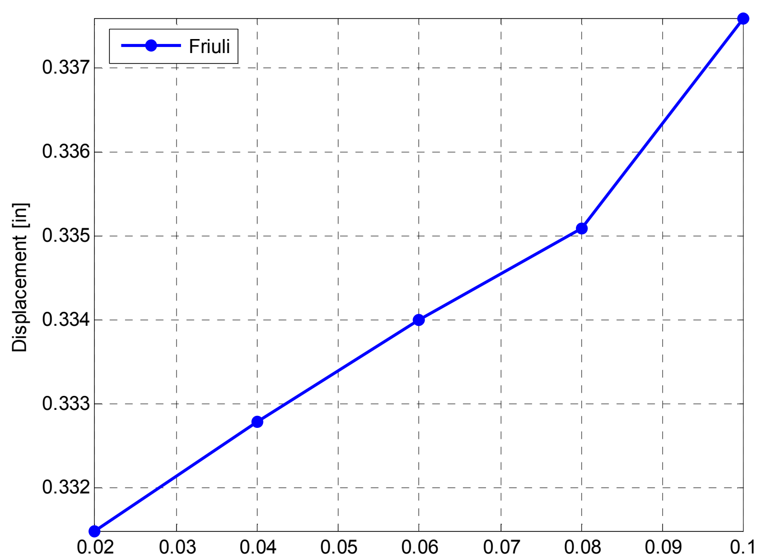


Figure 5.105: Maximum displacements with time delay effects for the stiff 1-dof system and the Friuli record (*Closed-loop control*)

Table 5.10: Summary of the maximum response of the SDOF stiff structure controlled with the VDSA with time delay effects (*Closed-open-loop control*)

Earthquake	Displacement					
	Uncontrolled	0.02 s	0.04 s	0.06 s	0.08 s	0.10 s
	[in*]	[in*]	[in*]	[in*]	[in*]	[in*]
El Centro	0.2536	0.0616	0.0617	0.0638	0.0647	0.0659
San Fernando	0.7219	0.7220	0.7234	0.7241	0.7253	0.7269
Friuli	0.3315	0.3315	0.3326	0.3337	0.3348	0.3357

*1 in = 2.54 cm.

5.5.2 FLEXIBLE STRUCTURE

Table 5.11 shows a summary of the time delay effect in the maximum relative displacement obtained for the stiff structure using the modified *Closed-loop control* algorithm ***Qv*** for each of the three ground motions (El Centro, San Fernando and Friuli). Table 5.12 contains similar results but for the modified *Closed-open-loop control* algorithm ***Qv***.

Table 5.11: Summary of the maximum response of the SDOF stiff structure controlled with the VDSA with time delay effect (*Closed-loop control*)

Earthquake	Displacement					
	Uncontrolled	0.02 s	0.04 s	0.06 s	0.08 s	0.10 s
	[in*]	[in*]	[in*]	[in*]	[in*]	[in*]
El Centro	3.5606	0.0336	0.0339	0.0349	0.0358	0.0370
San Fernando	13.0256	0.1225	0.1226	0.1235	0.1249	0.1259
Friuli	7.8865	0.0362	0.0363	0.0371	0.0380	0.0392

*1 in = 2.54 cm..

Table 5.12: Summary of the maximum response of the SDOF stiff structure controlled with the VDSA with time delay effect (*Closed-open-loop control*)

Earthquake	Displacement					
	Uncontrolled	0.02 s	0.04 s	0.06 s	0.08 s	0.10 s
	[in*]	[in*]	[in*]	[in*]	[in*]	[in*]
El Centro	3.5606	0.0336	0.0339	0.0349	0.0358	0.0370
San Fernando	13.0256	0.1225	0.1226	0.1235	0.1249	0.1259
Friuli	7.8865	0.0362	0.0363	0.0371	0.0380	0.0392

*1 in = 2.54 cm.

CHAPTER VI:

APPLICATION OF THE VDSA DEVICE AND MODIFIED ALGORITHMS Q_V TO MDOF STRUCTURES

6.1 INTRODUCTION

To illustrate the effectiveness of the **VDSA** device in reducing the seismic response of MDOF structures, a plane frame of a six-story building is considered. The structure is subjected to the horizontal component of the three earthquakes described in Chapter V. The responses obtained by applying the modified *Closed-loop control* algorithm ***Qv*** and the modified *Closed-open-loop control* algorithm ***Qv*** are compared against the response of the uncontrolled structures. In addition, the responses of the structures are compared against the results obtained when passive dampers are installed at three different locations in the building. The numerical response is calculated with computer programs specially developed in MATLAB. These programs are listed in Appendices D and E.

6.2 EXAMPLE: MDOF STRUCTURE

A six-story frame modeled as a shear building was used to show the implementation via a numerical simulation and to illustrate the effectiveness of the **VDSA** device in reducing the seismic response of a multi-degree of freedom structure. The total lateral stiffness coefficients of the columns are $\boldsymbol{k_i}$ = 5315 kip/in and the floor weights are $\boldsymbol{W_i}$ = 2205 kip. The damping ratio of the uncontrolled structure is assumed to be 5% for all modes. For the case where the dampers are installed in a position fixed, the damping ratio provided by them is assumed to be 30%. The damping constant C_o for this case is calculated with equation 3.5. Table 6.1 presents a summary of the properties of the passive dampers. The damping coefficients in the dampers of the **VDSA** device are 25 kip.sec/in and 10 kip.sec/in for the dampers **A** and **B**, respectively. The results obtained are compared with those obtained for the uncontrolled structure and also with the response calculated for each one of the fixed dampers configurations.

The three configurations identified as I, II and III were considered for the fixed dampers with increasing number of the devices. As indicated in Table 6.2, in case I a single damper was installed at the first floor, in case II three dampers are placed on the three lower floors or in case III a damper is installed at each of the six floors. The **VDSA** device was assumed to be installed in the fourth floor.

Table 6.1: Properties of the passive dampers in the MDOF structure

Floor [#]	$Cos^2\theta_1$	ϕ_1	ϕ_{r1}	ξ_{eff} [%]	C_o [kip.sec/in$^+$]
6	0.9686	0.0117	0.0007	30	6
5	0.9686	0.0110	0.0013	30	6
4	0.9686	0.0097	0.0019	30	6
3	0.9686	0.0078	0.0023	30	6
2	0.9686	0.0055	0.0027	30	6
1	0.9686	0.0028	0.0028	30	6

*1 in = 2.54 cm, $^+$1 kip = 4.448 kN.

Table 6.2: Damper locations for passive damper

Case	I	II	III
6th floor			X
5th floor			X
4th floor			X
3rd floor		X	X
2nd floor		X	X
1st floor	X	X	X

The natural periods of the structure are 0.5309, 0.1805, 0.1126, 0.0855, 0.0723 and 0.0659 sec. The control displacement at the bottom of the **VDSA** device was calculated with the two *Closed-loop control* and *Closed-open-loop control* algorithms ***Qv*** described in Chapter II.

6.2.1 APPLICATION OF THE MODIFIED *CLOSED-LOOP CONTROL* ALGORITHM *Qv*

For *Closed-loop control* and *Closed-open-loop control* algorithms ***Qv*** the weighting matrix **Q** and **Qv** were selected as [***I***]x10^4 and [***I***]x10^2**,** respectively, where [***I***] is a identity matrix. **R** is selected as a scalar with a value equal to 10^{-1}

6.2.1.a RESPONSE TO THE EL CENTRO RECORD

The first results for the multiple degree of freedom structure is the response to the record of the 1941 El Centro earthquake. The relative displacement computed for the uncontrolled structure is compared with a similar response but for the structure controlled with the **VDSA** device. This comparison in terms of the relative displacement of each of the six floors is presented for in Figures 6.1 to 6.6. Figures 6.7 and 6.8 show a comparison of the relative displacement time history in the first and top floor for the structure uncontrolled and with fixed dampers in all floors (case III). Figures 6.9 and 6.10 present a similar comparison but for the structure with fixed dampers in all floors and with variable dampers in the first and last floor. The maximum relative displacements in all floors and for all cases considered is shown in Figure 6.11. There are five cases considered: original structure, structure with a single **VDSA** device, and the three fixed dampers configurations. Clearly, the response reduction achieved by the **VDSA** system is remarkable, even when all floors are provided with viscous dampers at the is maximum practical range. The maximum force applied by the control was 0.492 kip and its time variation is shown in Figure 6.12. The variation of the control device position is presented in Figure 6.13, with $w_{\min} = 30$ in.

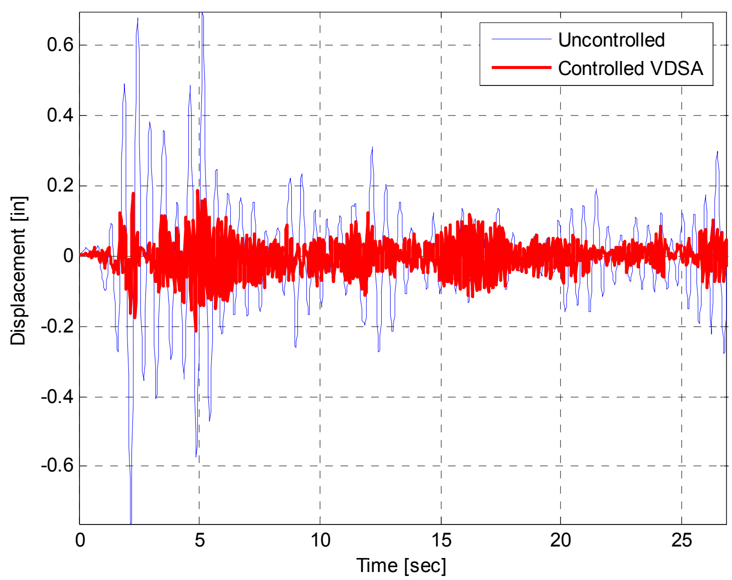


Figure 6.1: Relative displacement of the first floor for the El Centro record - Uncontrolled vs. VDSA (*Closed-loop control*)

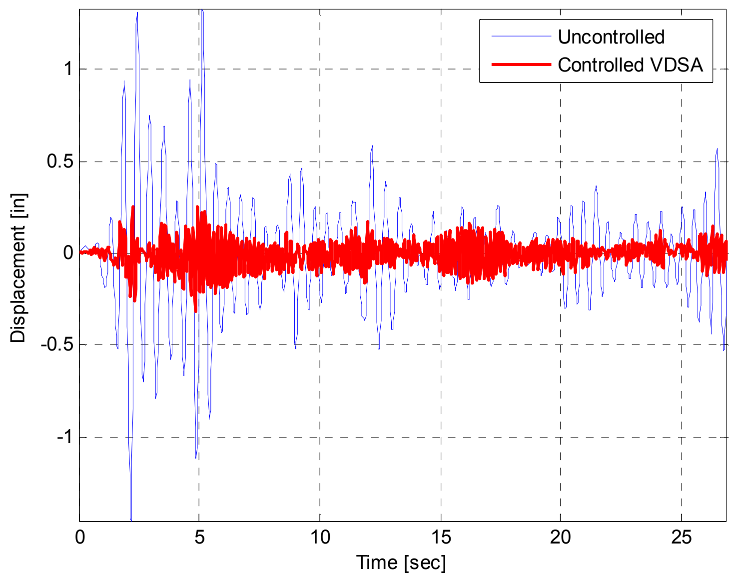


Figure 6.2: Relative displacement of the second floor for the El Centro record - Uncontrolled vs. VDSA (*Closed-loop control*)

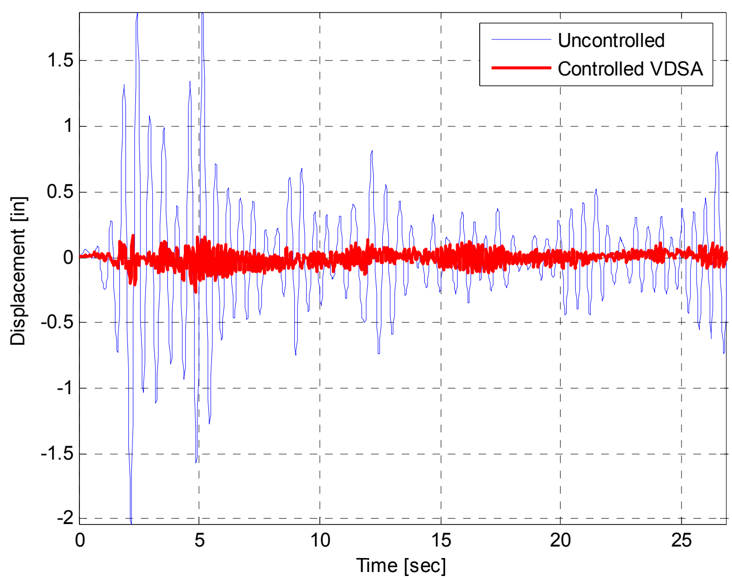


Figure 6.3: Relative displacement of the third floor for the El Centro record - Uncontrolled vs. VDSA (*Closed-loop control*)

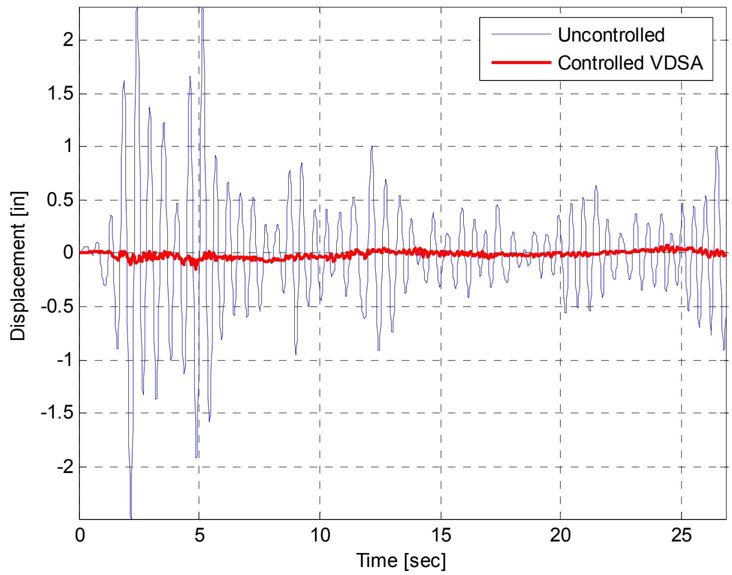


Figure 6.4: Relative displacement of the fourth floor for the El Centro record - Uncontrolled vs. VDSA (*Closed-loop control*)

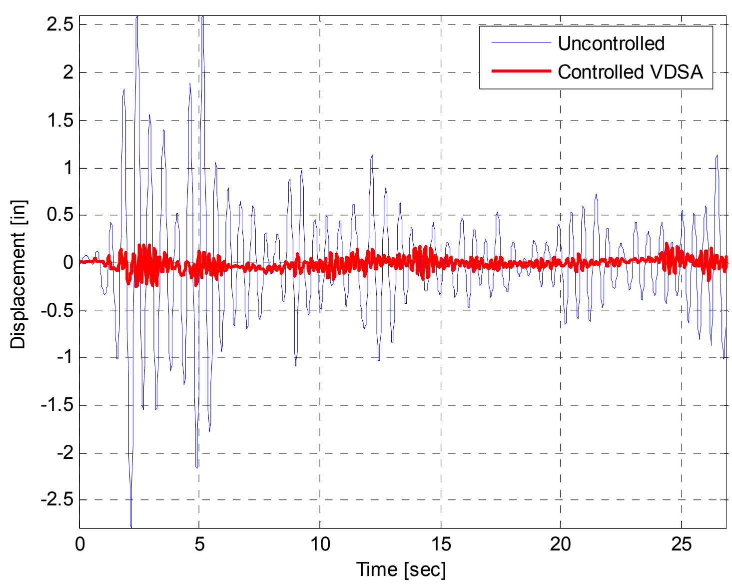


Figure 6.5: Relative displacement of the fifth floor for the El Centro record - Uncontrolled vs. VDSA (*Closed-loop control*)

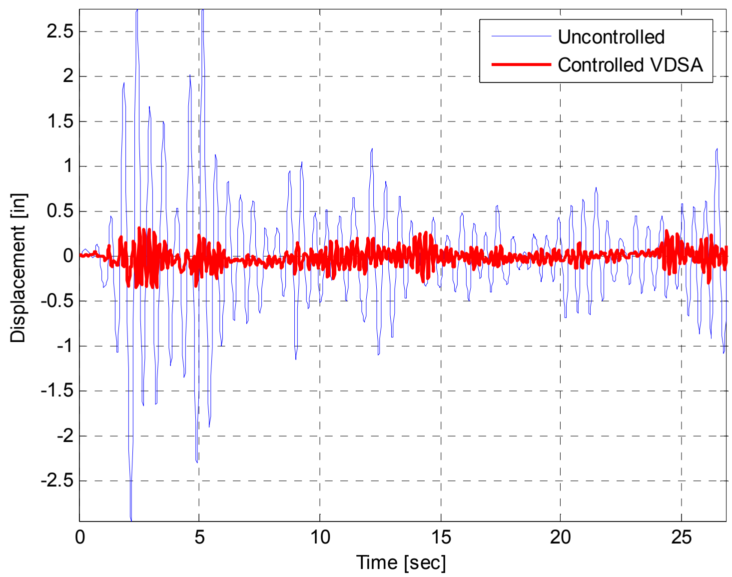


Figure 6.6: Relative displacement of the top floor for the El Centro record - Uncontrolled vs. VDSA (*Closed-loop control*)

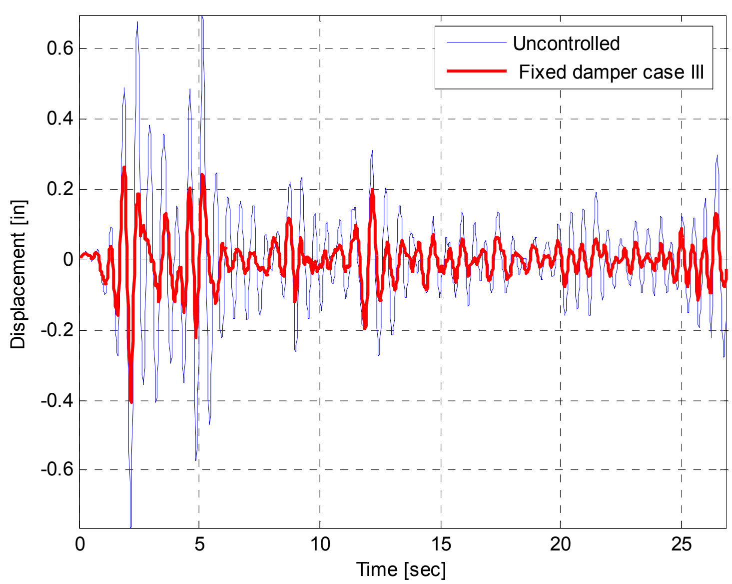


Figure 6.7: Relative displacement of the first floor for the El Centro record - Uncontrolled vs. Fixed dampers, case III (*Closed-loop control*)

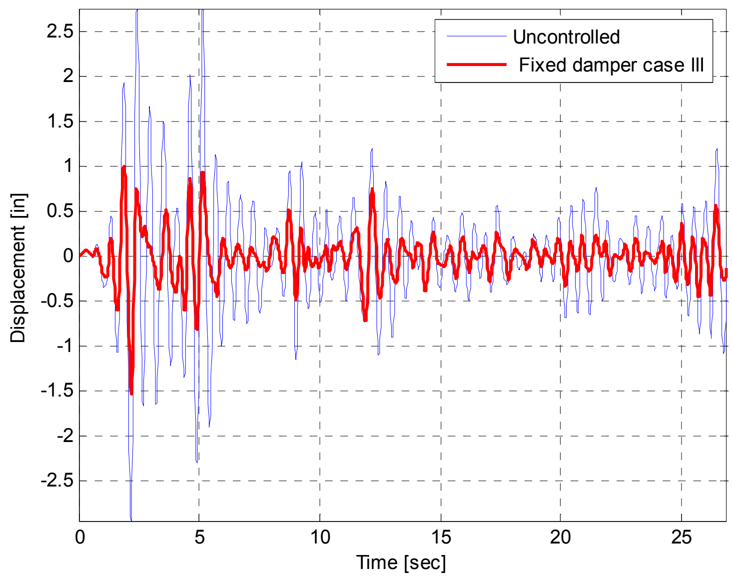


Figure 6.8: Relative displacement of the top floor for the El Centro record - Uncontrolled vs. Fixed dampers, case III (*Closed-loop control*)

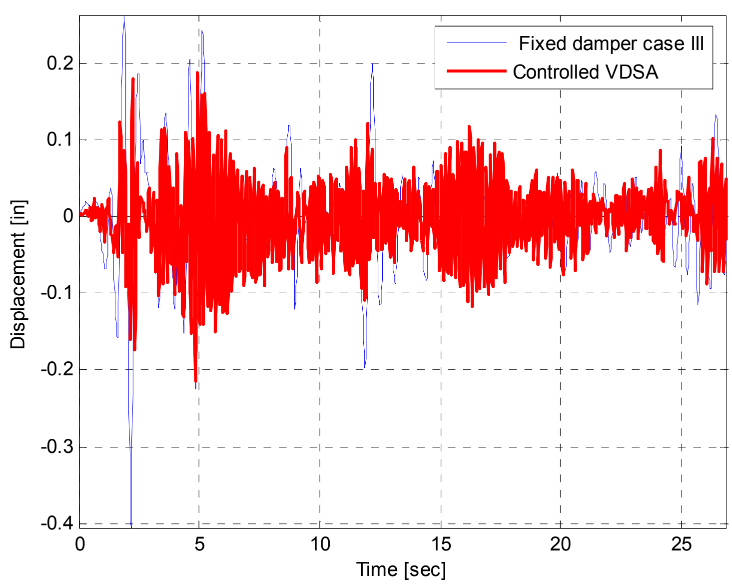


Figure 6.9: Relative displacement of the first floor for the El Centro record - Fixed dampers, case III vs. VDSA (*Closed-loop control*)

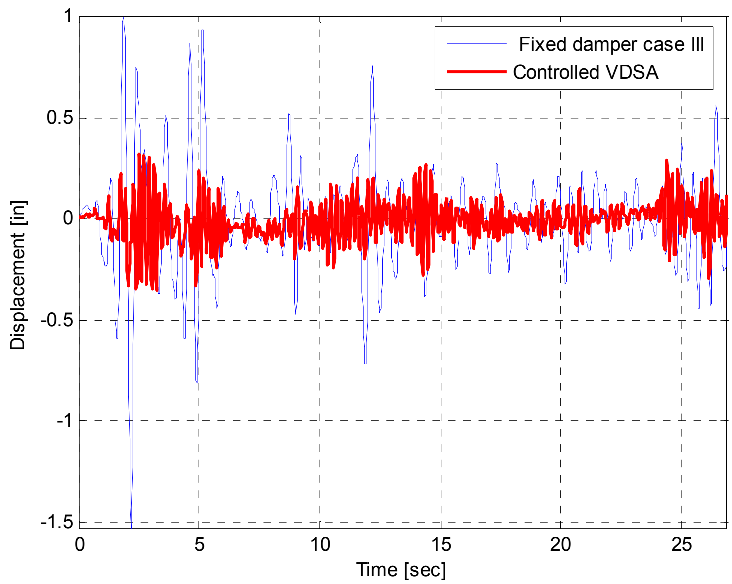


Figure 6.10: Relative displacement of the top floor for the El Centro record - Fixed dampers, case III vs. VDSA (*Closed-loop control*)

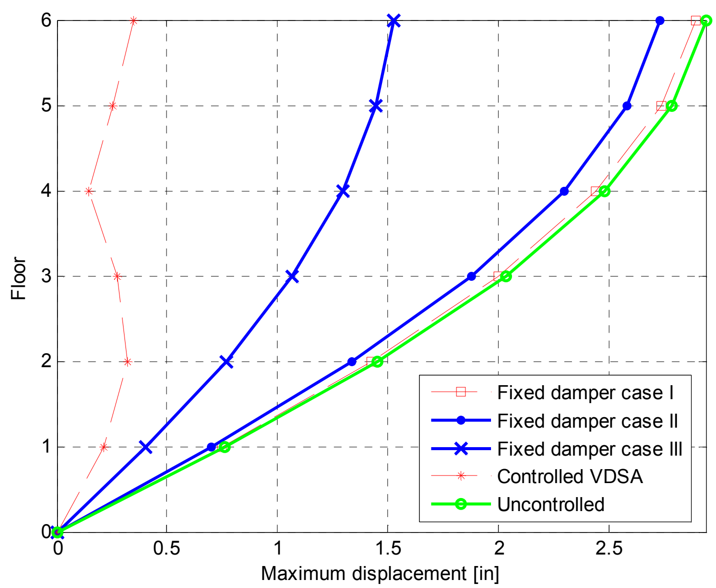


Figure 6.11: Maximum floor displacements for the El Centro record for all cases (*Closed-loop control*)

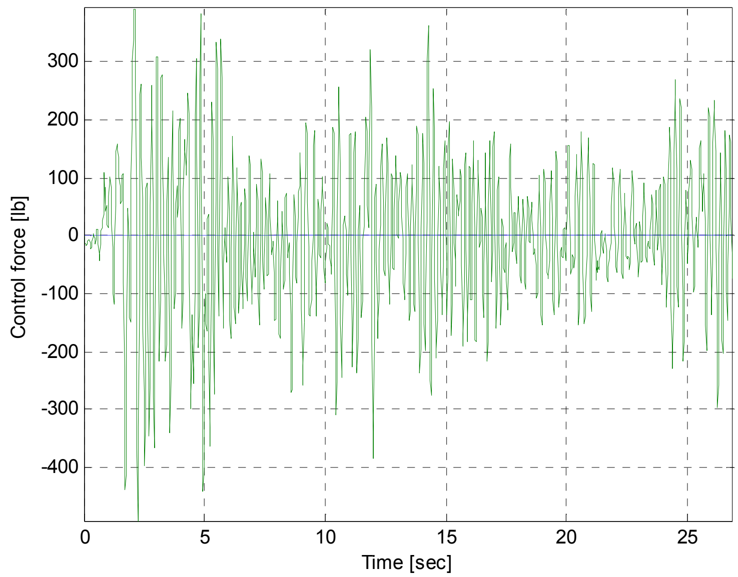


Figure 6.12: Control force at the VDSA device in the 4th floor for the El Centro record (*Closed-loop control*)

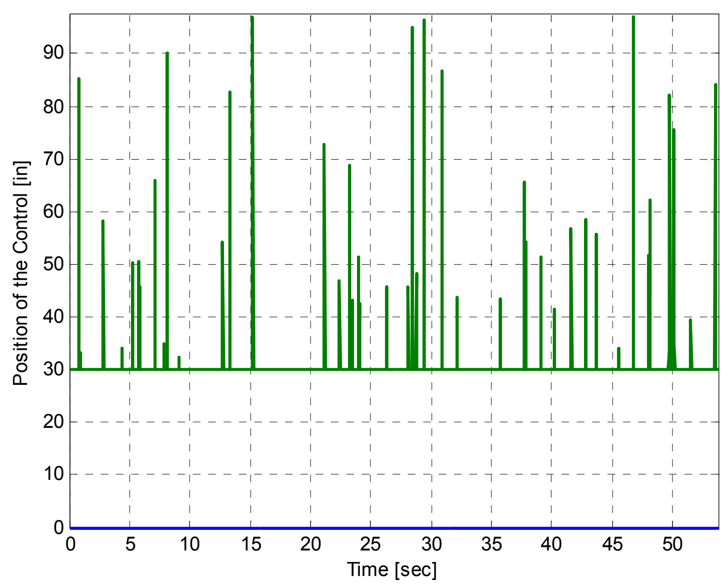


Figure 6.13: Variation of the position of the VDSA device in the 6-story building for the El Centro record (*Closed-loop control*)

Table 6.3 shows a summary of the results displayed in Figure 6.11, when the **VDSA** device is regulated with the modified *Closed-loop control* algorithm ***Qv***.

Table 6.3: Maximum displacements of the 6-story building without control and with a passive and semiactive system for the El Centro record (*Closed-loop control*)

Floor	Displacement				
	Uncont.	Fixed damper Case I	Fixed damper Case II	Fixed damper Case III	VDSA
	[in*]	[in*]	[in*]	[in*]	[in*]
6th	2.9392	2.8943	2.7316	1.5288	0.3519
5th	2.7857	2.7423	2.5863	1.4481	0.2523
4th	2.4832	2.4427	2.3005	1.2961	0.1415
3rd	2.0359	2.0004	1.8807	1.0689	0.2733
2nd	1.4544	1.4273	1.3400	0.7692	0.3198
1st	0.7622	0.7468	0.7001	0.4054	0.2154

*1 in = 2.54 cm.

The total base shear for all cases is presented in Table 6.4.

Table 6.4: Maximum base shear in the 6-story building without control and with a passive and semiactive system for the El Centro record (*Closed-loop control*)

Earthquake	Total base shear				
	Uncont.	Fixed damper Case I	Fixed damper Case II	Fixed damper Case III	VDSA
	[kip+]	[kip+]	[kip+]	[kip+]	[kip+]
El Centro	8102.4	7938.5	7441.8	4309.9	2290.2

+1 kip = 4.448 kN.

6.2.1.b RESPONSE TO THE SAN FERNANDO RECORD

The previous analyses were repeated for the San Fernando ground motion. Figures 6.14 to 6.19 display the time variation of the displacement response for each one of the floors of the uncontrolled structure and controlled with the **VDSA** device. The next set results in Figures 6.20 and 6.21 correspond to the comparison of the relative displacement of the first and top floor, respectively, for the uncontrolled structure and with fixed dampers installed in all floors. Figures 6.22 and 6.23 present a similar comparison but for the structure controlled with fixed dampers in all level and controlled with the **VDSA** device, for the first and top floor, respectively. Figure 6.24 shows the maximum relative displacement of the six floors for all cases studied. Figures 6.25 and 6.26 display, respectively, the control force time history (where the maximum force generated by the control was 2.127 kip) and the variation of the height of the lower end of the device (with $w_{\min}$=30 in).

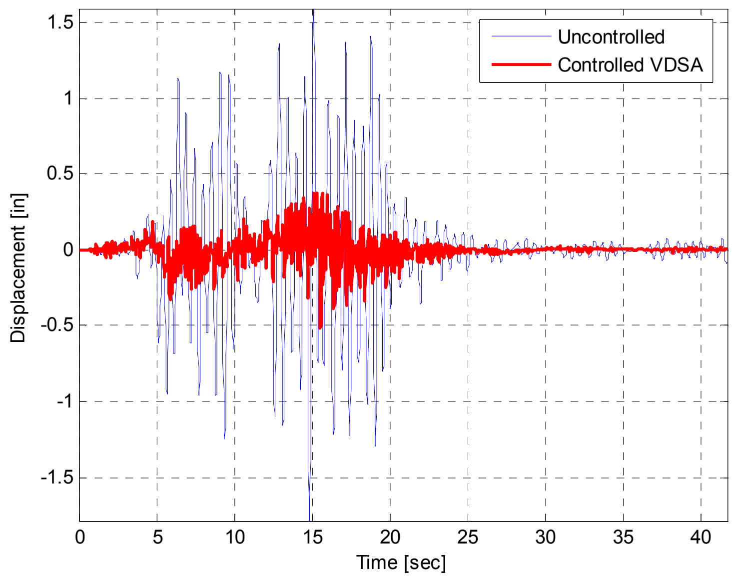


Figure 6.14: Relative displacement of the first floor for the San Fernando record - Uncontrolled vs. VDSA (*Closed-loop control*)

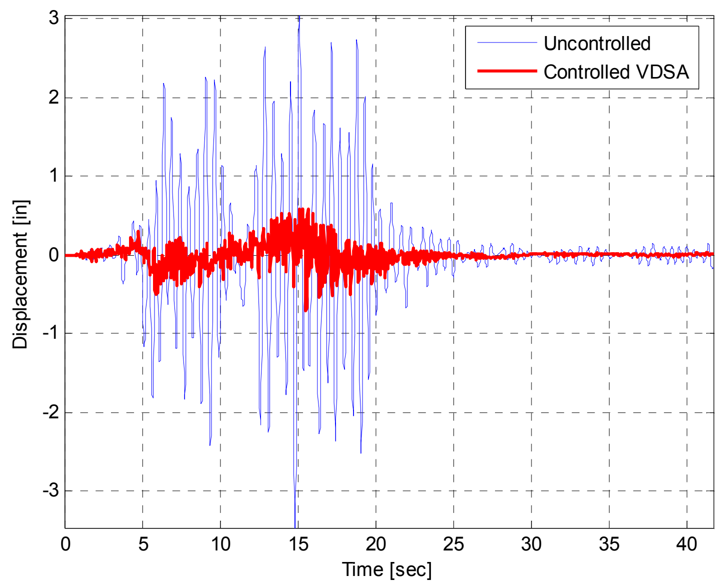


Figure 6.15: Relative displacement of the second floor for the San Fernando record - Uncontrolled vs. VDSA (*Closed-loop control*)

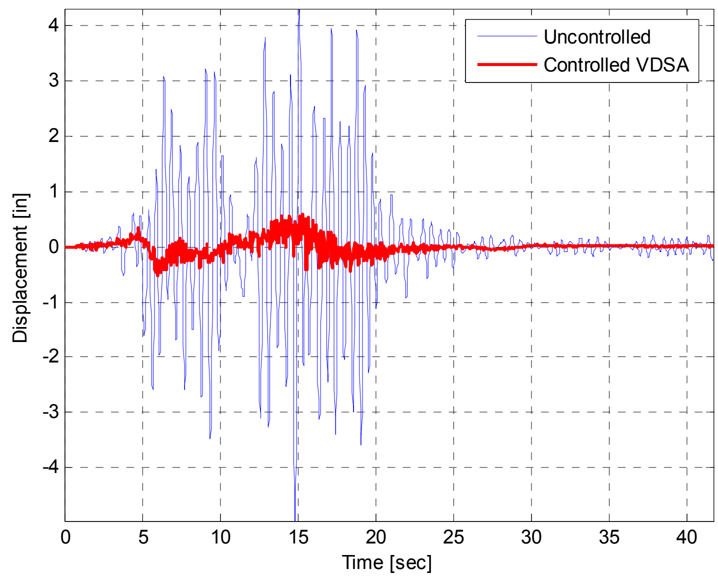


Figure 6.16: Relative displacement of the third floor for the San Fernando record - Uncontrolled vs. VDSA (*Closed-loop control*)

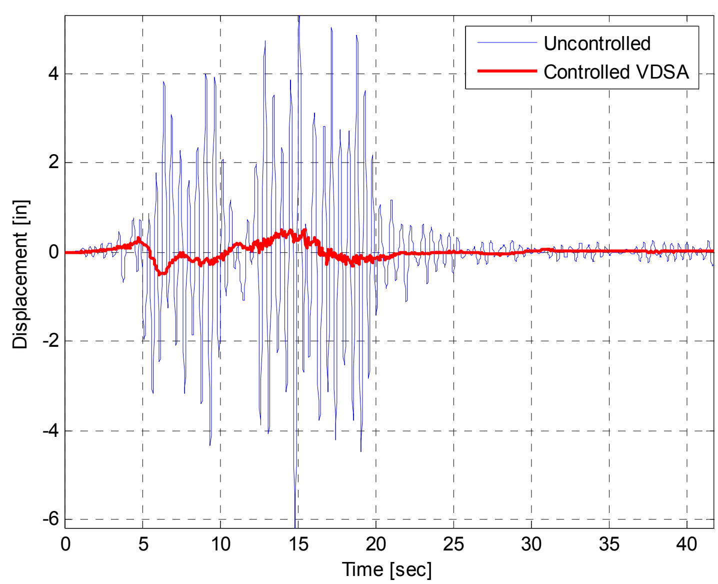


Figure 6.17: Relative displacement of the fourth floor for the San Fernando record - Uncontrolled vs. VDSA (Closed-loop control)

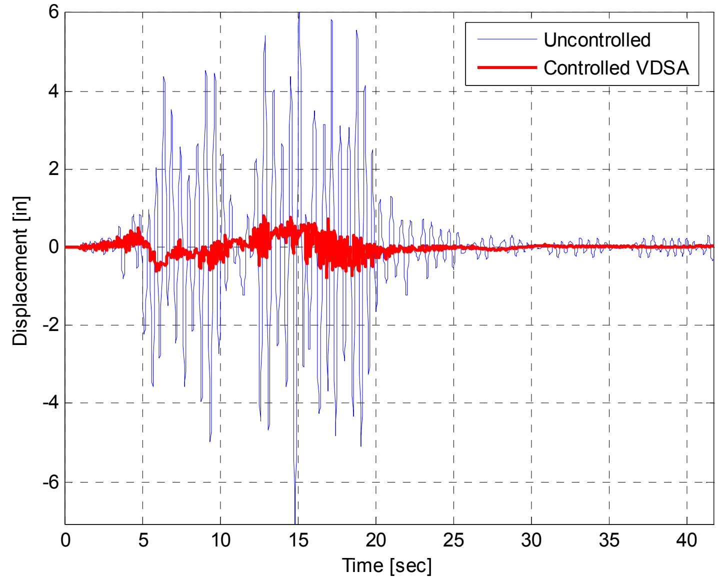


Figure 6.18: Relative displacement of the fifth floor for the San Fernando record - Uncontrolled vs. VDSA (*Closed-loop control*)

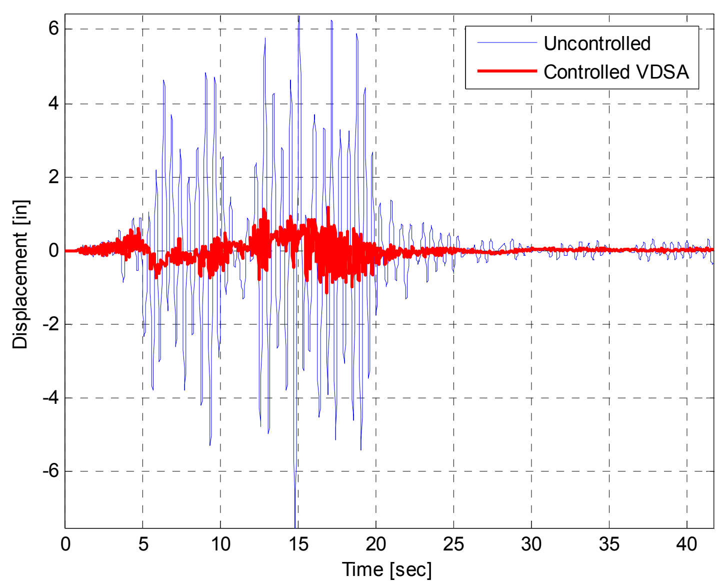


Figure 6.19: Relative displacement of the top floor for the San Fernando record - Uncontrolled vs. VDSA (*Closed-loop control*)

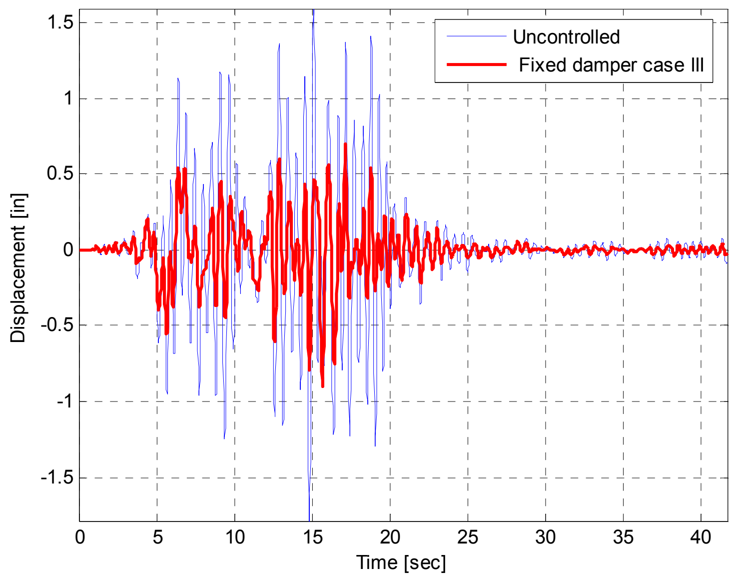


Figure 6.20: Relative displacement of the first floor for the San Fernando record - Uncontrolled vs. Fixed dampers, case III (*Closed-loop control*)

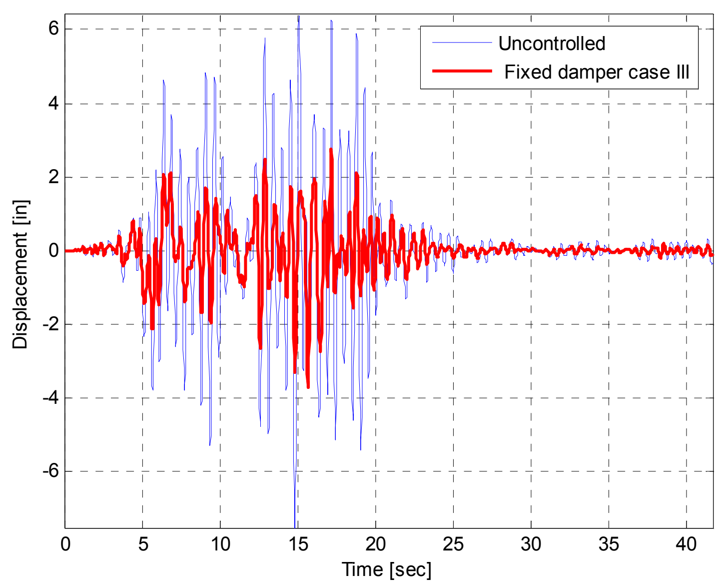


Figure 6.21: Relative displacement of the top floor for the San Fernando record - Uncontrolled vs. Fixed dampers, case III (*Closed-loop control*)

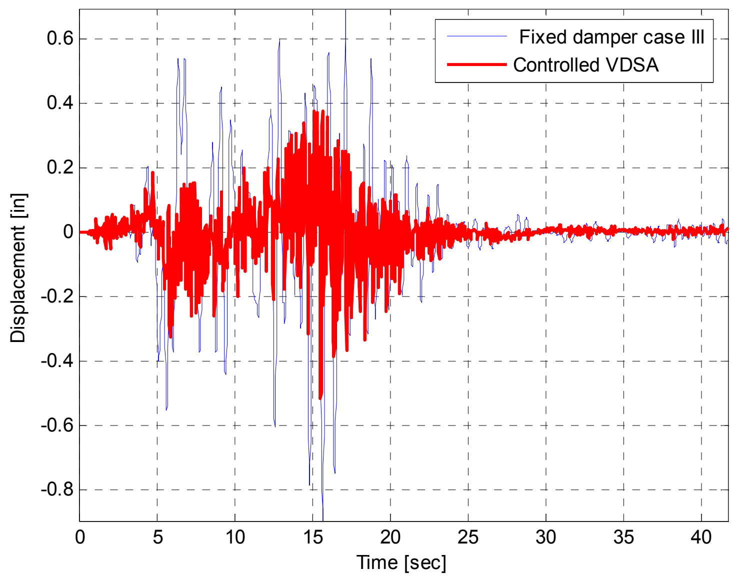


Figure 6.22: Relative displacement of the first floor for the San Fernando record - Fixed dampers, case III vs. VDSA (*Closed-loop control*)

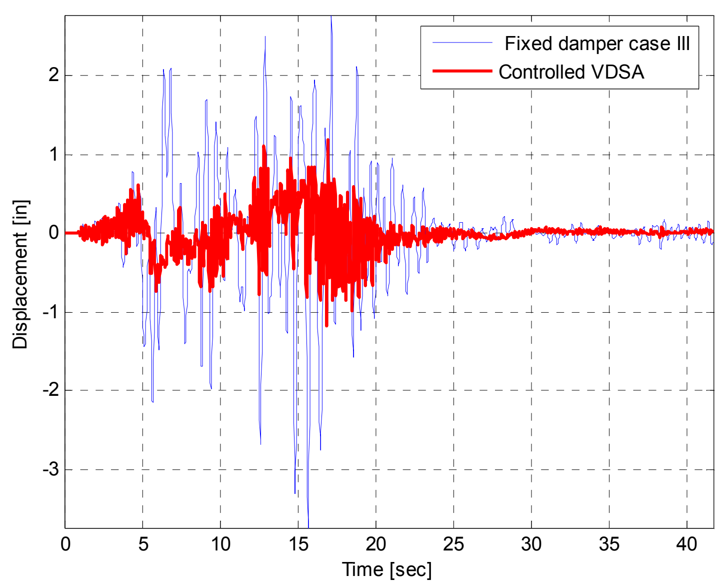


Figure 6.23: Relative displacement of the top floor for the San Fernando record - Fixed dampers, case III vs. VDSA (*Closed-loop control*)

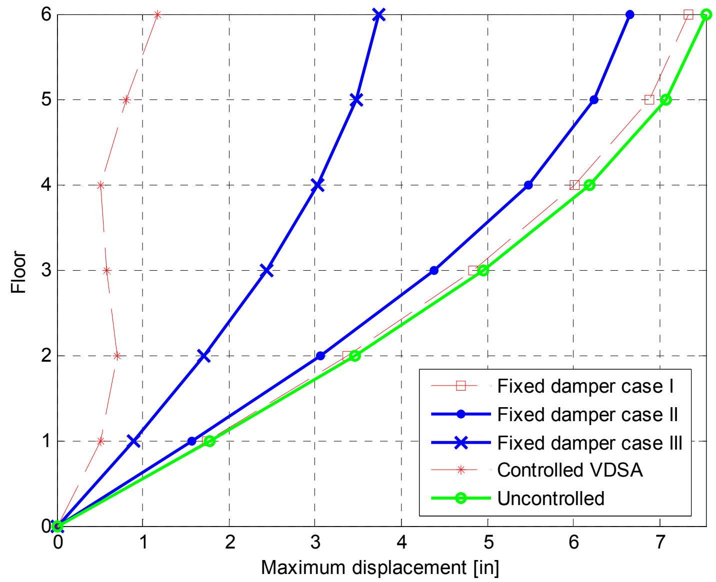


Figure 6.24: Maximum floor displacements for the San Fernando record for all cases (*Closed-loop control*)

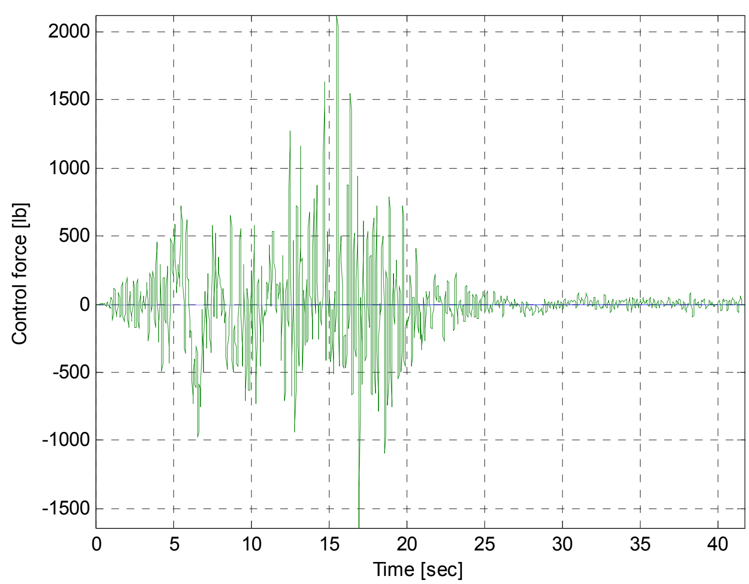


Figure 6.25: Control force of the VDSA device in the 4th floor for the San Fernando record (*Closed-loop control*)

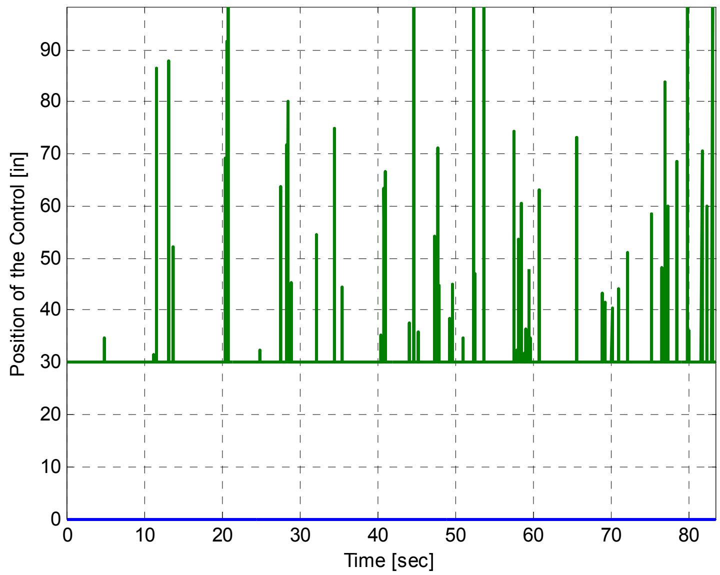


Figure 6.26: Variation of the position of the VDSA device in the 6-story building for the San Fernando record (*Closed-loop control*)

Table 6.5 shows the maximum displacement response for the uncontrolled structure, controlled for the **VDSA** device and with fixed dampers (cases I, II and III).

Table 6.5: Maximum displacements of the 6-story building without control and with a passive and semiactive system for the San Fernando record (*Closed-loop control*)

Floor	Displacement				
	Uncont.	Fixed damper Case I	Fixed damper Case II	Fixed damper Case III	VDSA
	[in*]	[in*]	[in*]	[in*]	[in*]
6th	7.5278	7.3209	6.6504	3.7308	1.1778
5th	7.0656	6.8728	6.2438	3.4850	0.8003
4th	6.1855	6.0177	5.4672	3.0347	0.5192
3rd	4.9567	4.8232	4.3803	2.4325	0.5854
2nd	3.4634	3.3711	3.0570	1.7149	0.7011
1st	1.7855	1.7377	1.5753	0.8971	0.5192

*1 in = 2.54 cm.

The maximum total base shear for the five cases studied is listed in Table 6.6.

Table 6.6: Maximum base shear in the 6-story building without control and with a passive and semiactive system for the San Fernando record (*Closed-loop control*)

Earthquake	Total base shear				
	Uncont.	Fixed damper Case I	Fixed damper Case II	Fixed damper Case III	VDSA
	[kip$^+$]	[kip$^+$]	[kip$^+$]	[kip$^+$]	[kip$^+$]
San Fernando	18980.0	18471.0	16746.0	9535.6	5519.3

$^+$1 kip = 4.448 kN.

6.2.1.c RESPONSE TO THE FRIULI RECORD

The last set of results for the six-story building is the response to the acceleration time history of the Friuli earthquake. Only the first twenty seconds of the response will be shown. Figures 6.27 to 6.32 display the relative displacement time histories of the structure in uncontrolled mode and controlled with the variable dampers for the six floors. Figures 6.33 and 6.34 shows, respectively, the relative displacement of the first and last floor for the original structure and when it is fitted with fixed dampers (case III). A comparison between the relative displacement of the first and top floor of the structure with fixed and variable dampers is presented in Figures 6.35 and 6.36. Figure 6.37 show the variation in height in the maximum relative displacements for the five cases analyzed. Figures 6.38 and 6.39 show, respectively, the control force time history and the variation in the position of the **VDSA** device. The maximum value in the control force time history for the Friuli earthquake was 0.740 kip and the minimum height w_{min} of the lower end of the **VDSA** device was 30 in.

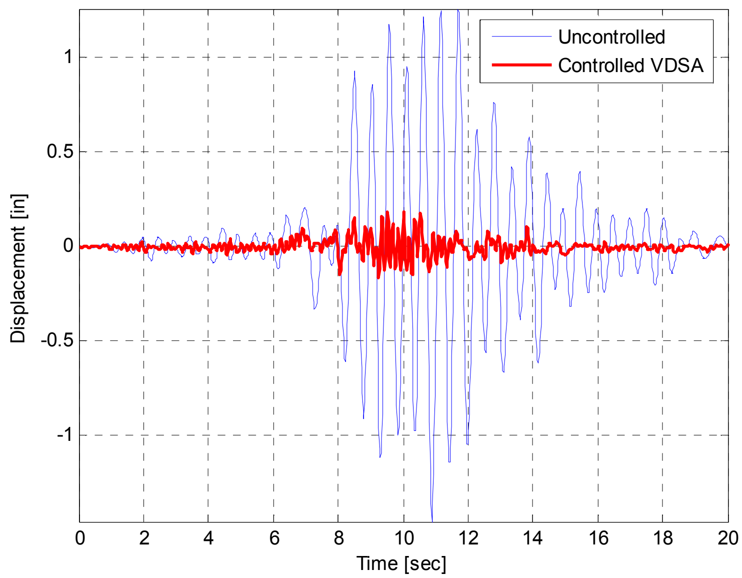


Figure 6.27: Relative displacement of the first floor for the Friuli record - Uncontrolled vs. VDSA (*Closed-loop control*)

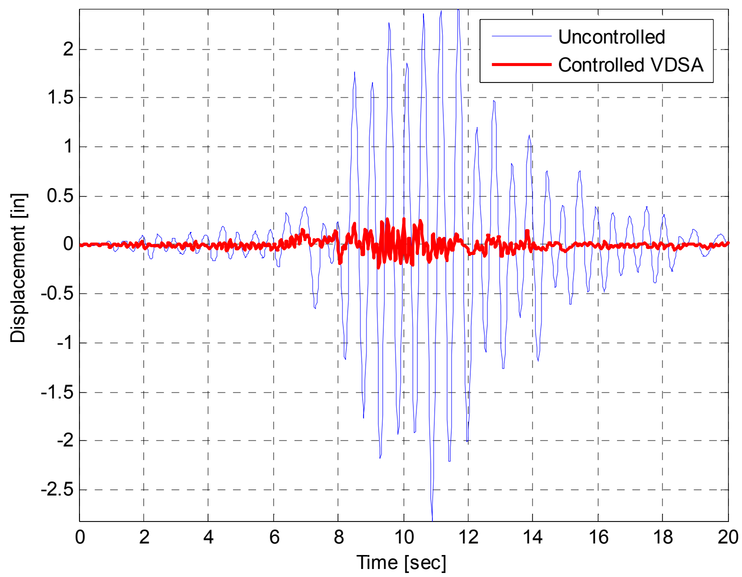


Figure 6.28: Relative displacement of the second floor for the Friuli record - Uncontrolled vs. VDSA (*Closed-loop control*)

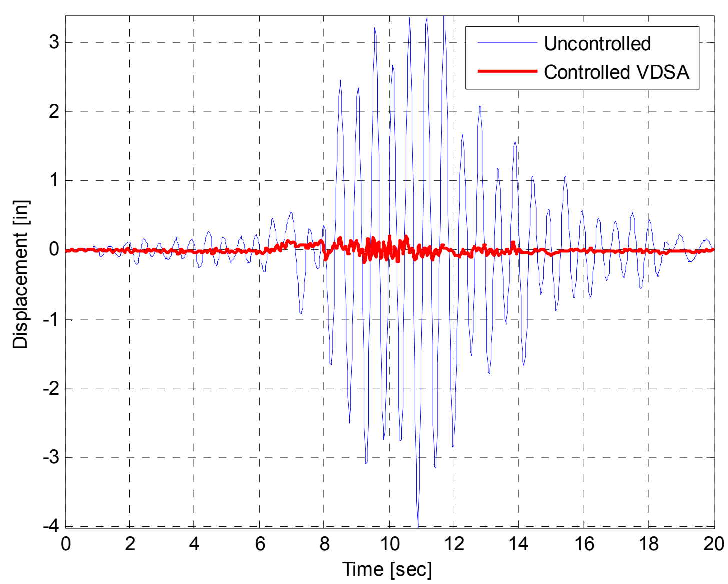


Figure 6.29: Relative displacement of the third floor for the Friuli record - Uncontrolled vs. VDSA (*Closed-loop control*)

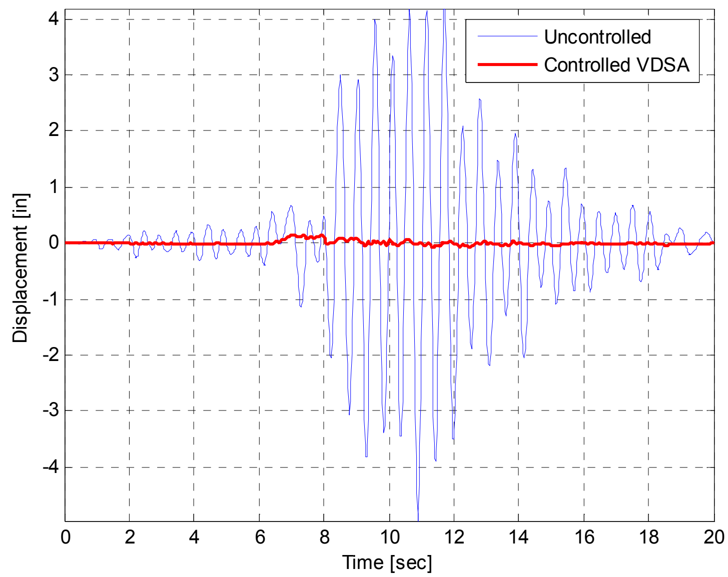


Figure 6.30: Relative displacement of the fourth floor for the Friuli record - Uncontrolled vs. VDSA (*Closed-loop control*)

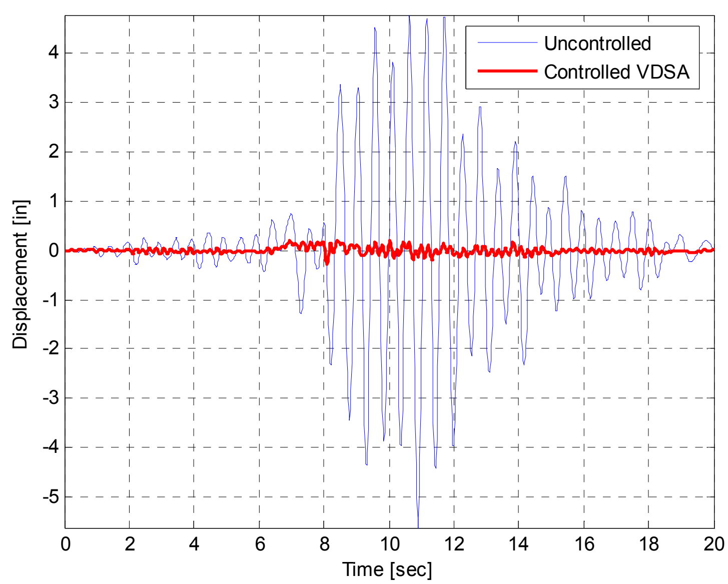


Figure 6.31: Relative displacement of the fifth floor for the Friuli record - Uncontrolled vs. VDSA (*Closed-loop control*)

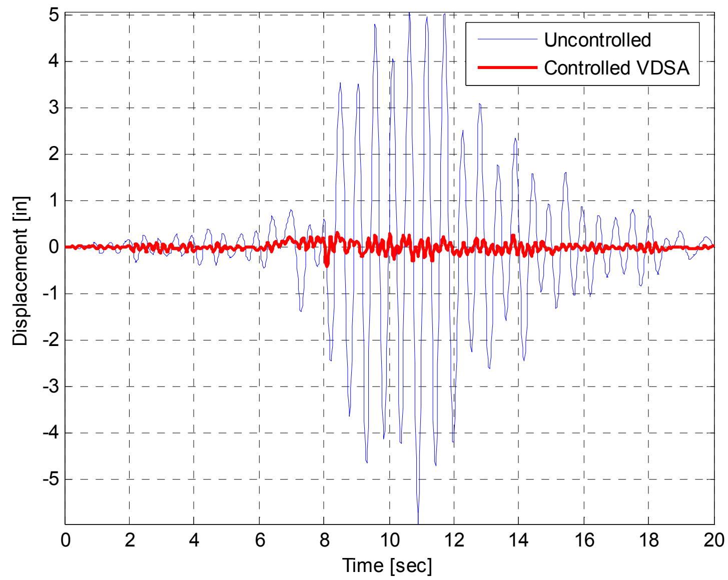


Figure 6.32: Relative displacement of the top floor for the Friuli record - Uncontrolled vs. VDSA (*Closed-loop control*)

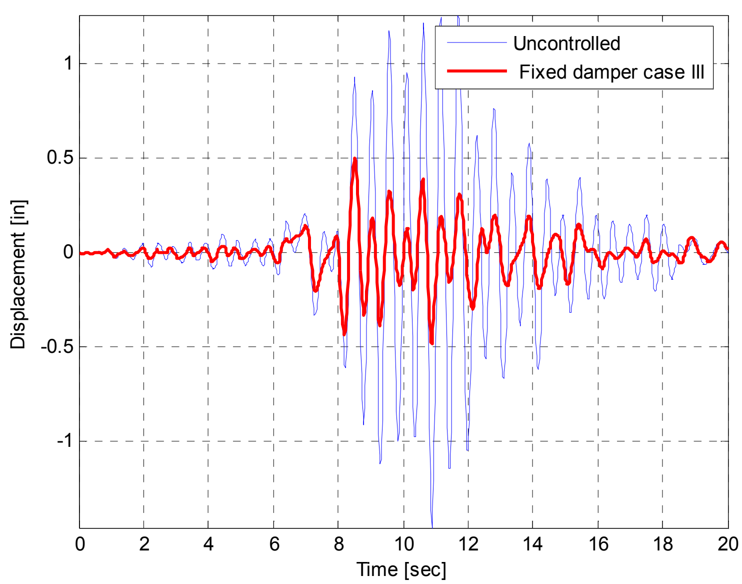


Figure 6.33: Relative displacement of the first floor for the Friuli record - Uncontrolled vs. Fixed dampers, case III (*Closed-loop control*)

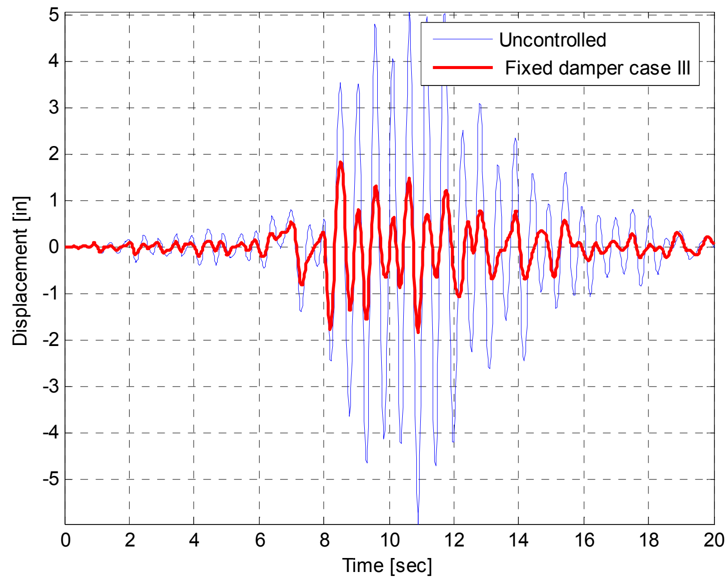


Figure 6.34: Relative displacement of the top floor for the Friuli record - Uncontrolled vs. Fixed dampers, case III (*Closed-loop control*)

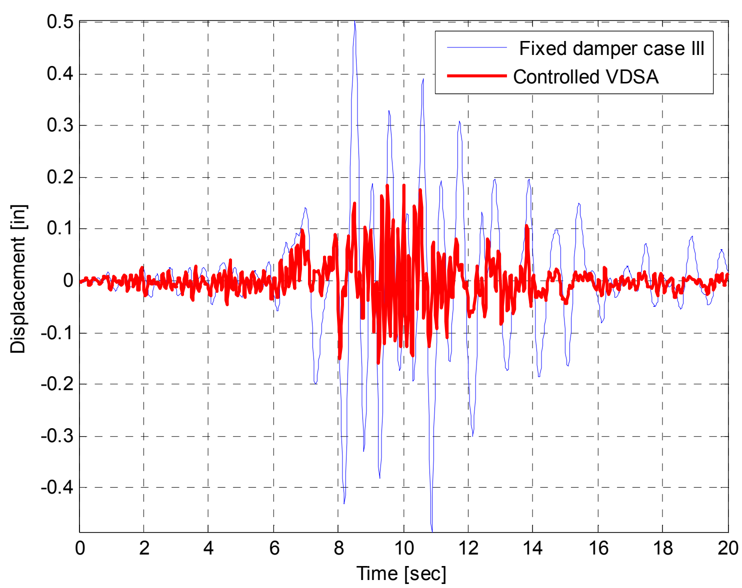


Figure 6.35: Relative displacement of the first floor for the Friuli record - Fixed dampers, case III vs. VDSA (*Closed-loop control*)

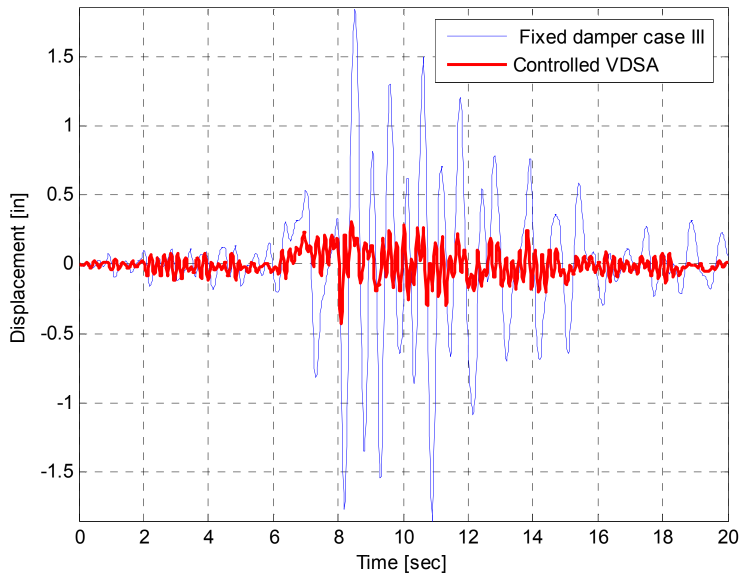


Figure 6.36: Relative displacement of the top floor for the Friuli record - Fixed dampers, case III vs. VDSA (*Closed-loop control*)

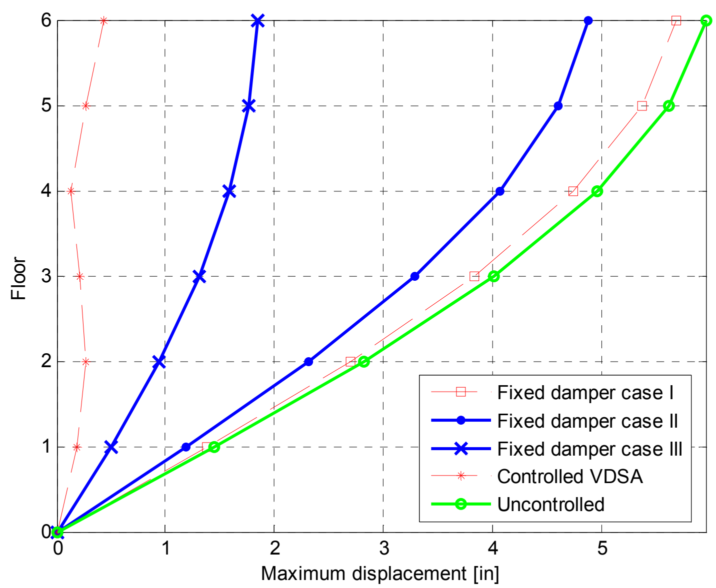


Figure 6.37: Maximum floor displacements for the Friuli record for all cases (*Closed-loop control*)

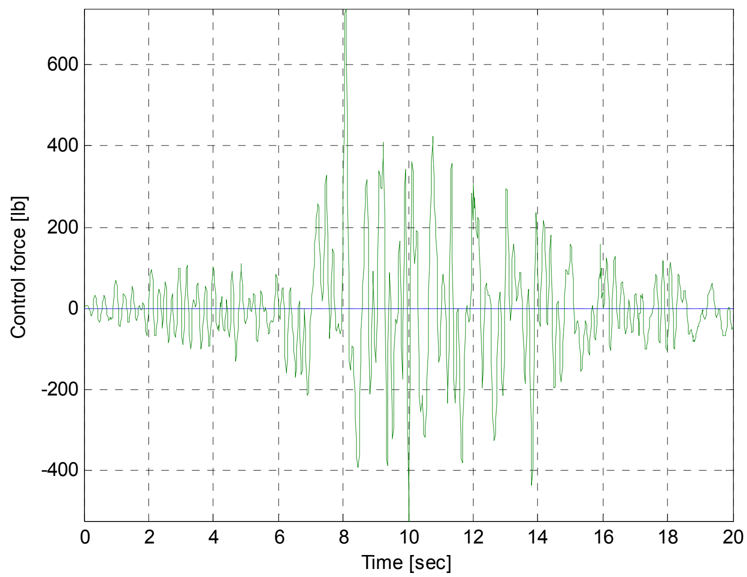


Figure 6.38: Control force of the VDSA device in the 4th floor for the Friuli record (*Closed-loop control*)

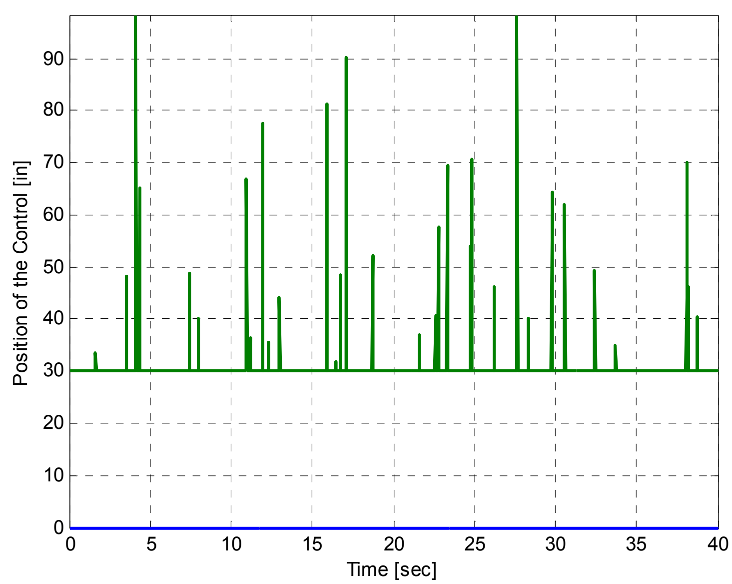


Figure 6.39: Variation of the position of the VDSA device in the 6-story building for the Friuli record (*Closed-loop control*)

Table 6.7 shows a summary of the maximum relative displacement obtained for the Friuli record.

Table 6.7: Maximum displacements of the 6-story building without control and with a passive and semiactive system for the Friuli record (*Closed-loop control*)

Floor	Displacement				
	Uncont.	Fixed damper Case I	Fixed damper Case II	Fixed damper Case III	VDSA
	[in*]	[in*]	[in*]	[in*]	[in*]
6th	5.9511	5.6891	4.8770	1.8504	0.4279
5th	5.6169	5.3690	4.6024	1.7586	0.2710
4th	4.9605	4.7409	4.0633	1.5787	0.1322
3rd	4.0097	3.8317	3.2834	1.3069	0.2100
2nd	2.8128	2.6882	2.3041	0.9457	0.2690
1st	1.4521	1.3829	1.1844	0.5051	0.1858

*1 in = 2.54 cm.

Table 6.8 shows a summary of the maximum total base shears found in the 6-story original building, equipped with the proposed device and with the three fixed dampers configurations.

Table 6.8: Maximum base shear in the 6-story buildings without control and with a passive and semiactive system for the Friuli record (*Closed-loop control*)

Earthquake	Displacement				
	Uncont.	Fixed damper Case I	Fixed damper Case II	Fixed damper Case III	VDSA
	[kip+]	[kip+]	[kip+]	[kip+]	[kip+]
Friuli	15436.0	14700.0	12590.0	5368.7	1975.4

+1 kip = 4.448 kN.

6.2.2 APPLICATION OF THE MODIFIED *CLOSED-OPEN-LOOP CONTROL* ALGORITHM *Qv*

The maximum relative displacements response and total base shears obtained for this structure using the modified *Closed-open-loop control* algorithm ***Qv*** were practically the same that those obtained with the *Close-loop control* presented in the previous section. Only small differences in the form that varies the position of the control device were found. The variation of the position of the lower end of the **VDSA** device in the cases of the El Centro, San Fernando and Friuli records are shown in Figures 6.40 to 6.42, respectively.

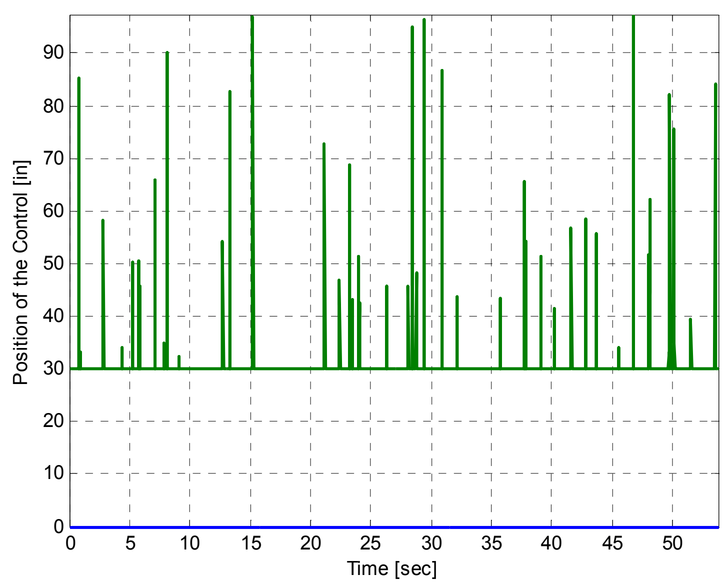


Figure 6.40: Variation of the position of the VDSA device in the 6-story building for the El Centro record (*Closed-open-loop control*)

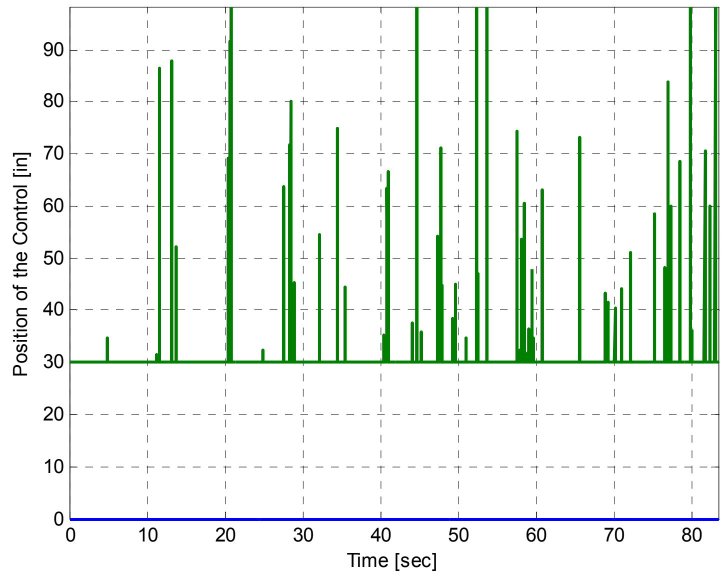


Figure 6.41: Variation of the position of the VDSA device in the 6-story building for the San Fernando record (*Closed-open-loop*)

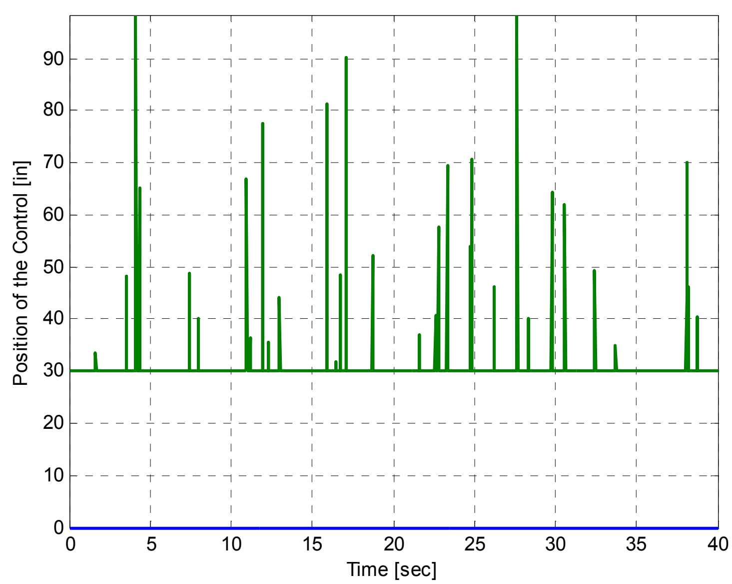


Figure 6.42: Variation of the position of the VDSA device in the 6-story building for the Friuli record (*Closed-open-loop control*)

6.2.3 DISCUSSION OF RESULTS

The response of the original structure and controlled with passive dampers and the proposed semiactive device were compared in Table 6.3 to 6.8. These tables summarized the maximum relative displacement for all floors and the base shear force, for El Centro, San Fernando and Friuli ground motions. It is noted that the **VDSA** device is effective in reducing both responses. Although the results are not presented here, it is recalled that the **VDSA** device was installed in the all the floors of the multi-story building and the response was calculated for the sit cases. The results presented in this Chapter correspond to the **VDSA** device was installed in the fourth floor, where the best response reduction was obtained. As expected, the best results obtained with passive control were for case III when the dampers are installed in all floors. It is recalled that in this passive case the dampers were assigned a damping coefficient equal to the recommended upper practical limit. The reduction in the maximum displacements of the building with the **VDSA** device compared to the passive control (case III) ranges from 50 to 80%. When the decrease in the values of the maximum displacements achieved with the proposed system is compared with the original structure, the reduction found very from 88 to 93% in the top floor.

Finally, the effectiveness in the response reduction in the multi degrees of freedom structures with the **VDSA** device controlled with the modified *Closed-loop* *control* algorithm ***Qv*** and the *Closed-open-loop control* algorithm was observed to be the same. In other words, in terms of performance, there was not found an advantage of one methodology over the other.

6.3 TIME DELAY EFFECT

As it was done for the single degree of freedom structure in Chapter V, a study of the effect in the response of the time delay between the measured of the required variables and the application of the control action is presented here.

Tables 6.9 to 6.11 present a summary of the time delay effect in the maximum relative displacement at each floor of the 6-story building. Each table shows the results for the El Centro, San Fernando and Friuli ground motion and the modified *Closed-loop control* algorithm ***Qv*** is used to control the **VDSA** system.

The minimum time delay records, which is the sampling time of the ground acceleration signal. This should be considered as the response without time delay since it cannot be avoided and it is presented in all the numerical examples, presented in the chapter previously.

Table 6.9: Summary of the maximum response of the MDOF structure controlled with the VDSA device with time delay effects for the El Centro record (*Closed-loop control*)

Floor	Displacement					
	Uncontrolled	0.02 s	0.04 s	0.06 s	0.08 s	0.1 s
	[in*]	[in*]	[in*]	[in*]	[in*]	[in*]
6th	2.9392	0.3519	0.3522	0.3538	0.3551	0.3556
5th	2.7857	0.2523	0.2524	0.2537	0.2549	0.2560
4th	2.4832	0.1415	0.1417	0.1429	0.1437	0.1448
3rd	2.0359	0.2733	0.2734	0.2741	0.2750	0.2762
2nd	1.4544	0.3198	0.3200	0.3209	0.3218	0.3224
1st	0.7622	0.2154	0.2154	0.2167	0.2179	0.2187

*1 in = 2.54 cm.

Table 6.10: Summary of the maximum response of the MDOF structure controlled with the VDSA device with time delay effects for the San Fernando record (*Closed-loop control*)

Floor	Displacement					
	Uncontrolled	0.02 s	0.04 s	0.06 s	0.08 s	0.1 s
	[in*]	[in*]	[in*]	[in*]	[in*]	[in*]
6th	7.5278	1.1778	1.1779	1.1793	1.1805	1.1819
5th	7.0656	0.8003	0.8004	0.8016	0.8027	0.8039
4th	6.1855	0.5192	0.5192	0.5203	0.5216	0.5229
3rd	4.9567	0.5854	0.5854	0.5865	0.5877	0.5888
2nd	3.4634	0.7011	0.7011	0.7018	0.7028	0.7037
1st	1.7855	0.5192	0.5194	0.5205	0.5219	0.5228

*1 in = 2.54 cm.

Table 6.11: Summary of the in the maximum response of the MDOF structure controlled with the VDSA device with time delay effects for the Friuli record (*Closed-loop control*)

Floor	Displacement					
	Uncontrolled	0.02 s	0.04 s	0.06 s	0.08 s	0.1 s
	[in*]	[in*]	[in*]	[in*]	[in*]	[in*]
6^{th}	5.9511	0.4279	0.4279	0.4290	0.4303	0.4317
5^{th}	5.6169	0.2710	0.2711	0.2722	0.2738	0.2751
4^{th}	4.9605	0.1322	0.1322	0.1334	0.1341	0.1349
3^{rd}	4.0097	0.2100	0.2103	0.2116	0.2128	0.2139
2^{nd}	2.8128	0.2690	0.2692	0.2703	0.2716	0.2729
1^{st}	1.4521	0.1858	0.1859	0.1872	0.1881	0.1895

*1 in = 2.54 cm.

The average in the displacement response for the maximum time delay considered (0.1 s) is 1.38%, 0.53% and 1.62% for the El Centro, San Fernando and Friuli earthquakes, respectively. These values are the averages of the increases in the response for the six floors of the building. It is considered that the increase due to time delay effects is acceptable, at lead when the delay is five times the sampling interval.

CHAPTER VII:

APPLICATION OF THE VDSA DEVICE AND MODIFIED ALGORITHMS Q_V TO SDOF COUPLED STRUCTUREs

7.1 INTRODUCTION

To illustrate the effectiveness of the **VDSA** device in reducing the response of coupled structures, this chapter examines the case of two single degree of freedom coupled structures. The response obtained by applying the modified *Closed-loop control* and *Closed-open-loop control* algorithms ***Qv*** the fitted is compared against the response of the uncontrolled structures and with passive dampers.

To perform the comparisons and show the performance of the variable damping device, the response of the coupled structures subjected to the earthquakes records of El Centro, San Fernando and Friuli is calculated.

7.2 EXAMPLE: SDOF STRUCTURES

This example consists of two one-story frames **A** and **B** with a weight of 14400 kip each one and a lateral stiffness of 800 kip/in and 1300 kip/in, respectively. This leads to natural periods of 1.4 sec for structures **A** and 1.6 sec for structure **B**. The original (uncontrolled) damping ratio is 2% for both structures. The structures are joined only by a single **VDSA** device installed between them.. The damping coefficients of the dampers **A** and **B** of the **VDSA** device are 15 kip.sec/in and 10 kip.sec/in, respectively. The results obtained are compared with these of the uncontrolled structures with damping ratio and with the case in which the dampers remain in a fixed position. The properties and damping coefficient of the passive damper in the structures **A** and **B** is presented in Table 7.1.

Table 7.1: Properties and damping constant for the passive damper in the coupled structures

Structure	$Cos^2\theta_1$	ϕ_1	ϕ_{r1}	ξ_{eff}	C_o
[#]				**[%]**	**[kip⁺.sec/in*]**
A	0.9686	0.0052	0.0052	30	15
B	0.9686	0.0051	0.0051	30	20

*1 in = 2.54 cm, +1 kip = 4.448 kN.

7.2.1 APPLICATION OF THE MODIFIED *CLOSED-LOOP CONTROL* ALGORITHM *QV*

The first set of results correspond to those obtained when the *Close-loop control* algorithm ***Qv*** is used to establish. This algorithm is used for obtained the optimal position of the **VDSA** device at each instant of time. The response to each of the three accelerograms is presented separately in the following sections.

For all the cases the weighting matrix **Q** is selected as $[\boldsymbol{I}]x10^5$**,** the matrix **Qv** and **R**, are scalars whose values are 10^2 and 10^{-3}, respectively.

7.2.1.a RESPONSE TO THE EL CENTRO RECORD

The first results for the single degree of freedom coupled structure are the response to the record of the El Centro earthquake.

Figure 7.1 shows the time variation of the relative displacement of the structure **A** and **B** with the **VDSA** device installed between the two structures. The two displacement time histories are shown separately because the reduction achieved by the **VDSA** device is so dramatic that they are difficult to appreciate when they are compared with the response for other cases. For instance, Figure 7.2 compares the reduction in the displacement of the structures **A** and **B** with the **VDSA** device and with the uncontrolled structures. From a visual comparison, it is evident the benefit of adding the **VDSA** device. The variation of the displacement of the two structures with the **VDSA** device is compared in Figure 7.3 with the corresponding response of the structure with fixed dampers. Similar results but in terms of the total base shear are presented in Figures 7.4 to 7.6. The base shear shown is the sum of the shear forces in the two structures. Figure 7.7 show the time variation of the force control required to move the **VDSA** device. The maximum force required by the control device for the two structures is 0.579 kip. The time variation of the vertical position of the lower end of the **VDSA** system is presented in Figure 7.8 where $w_{\min}$ is 25 in.

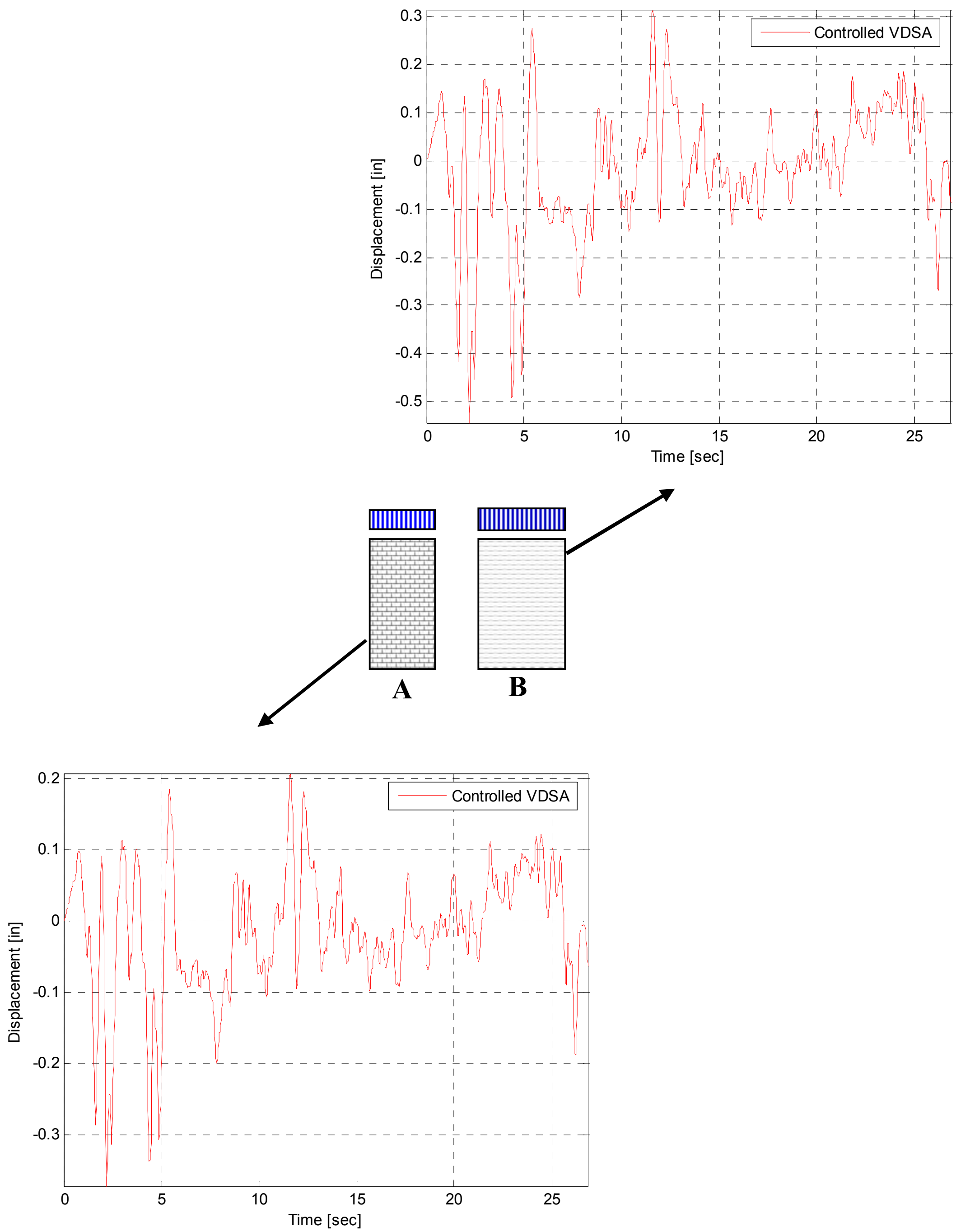


Figure 7.1: Relative displacement of the coupled sdof systems A and B for the El Centro record controlled with VDSA (*Closed-loop control*)

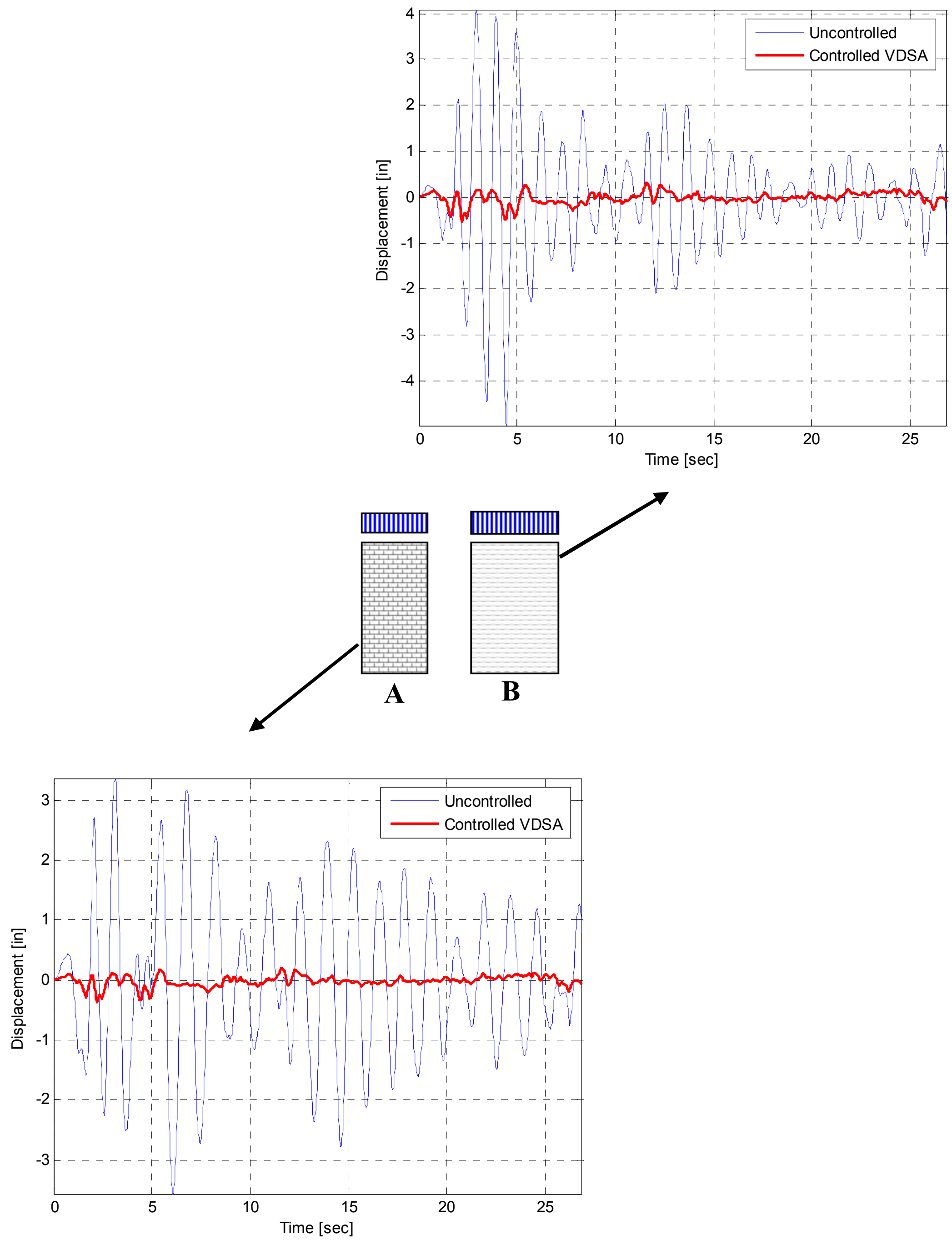


Figure 7.2: Relative displacement of the coupled sdof systems A and B for the El Centro record - Uncontrolled vs. VDSA (*Closed-loop control*)

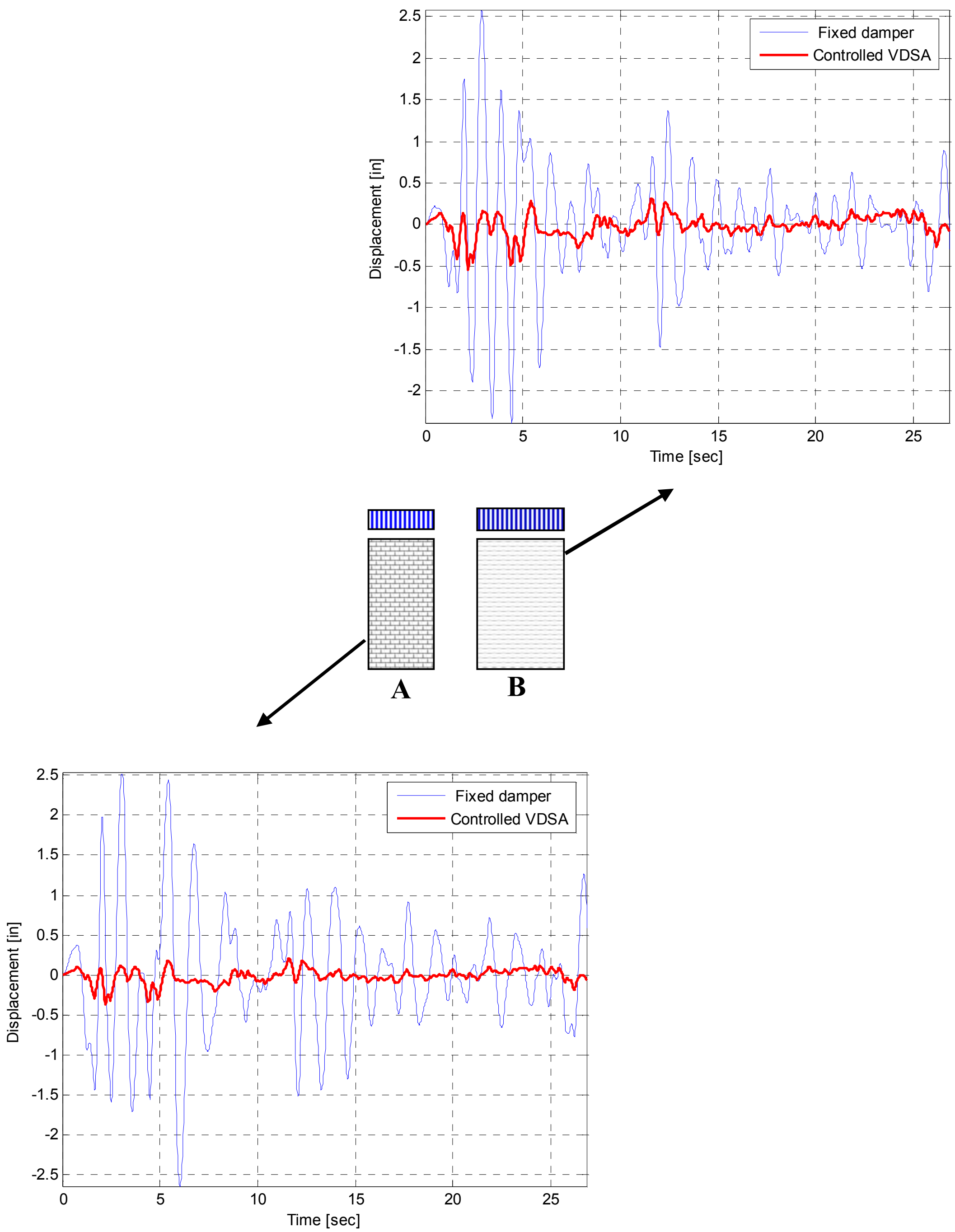


Figure 7.3: Relative displacement of the coupled sdof systems A and B for the El Centro record - Fixed damper vs. VDSA (*Closed-loop control*)

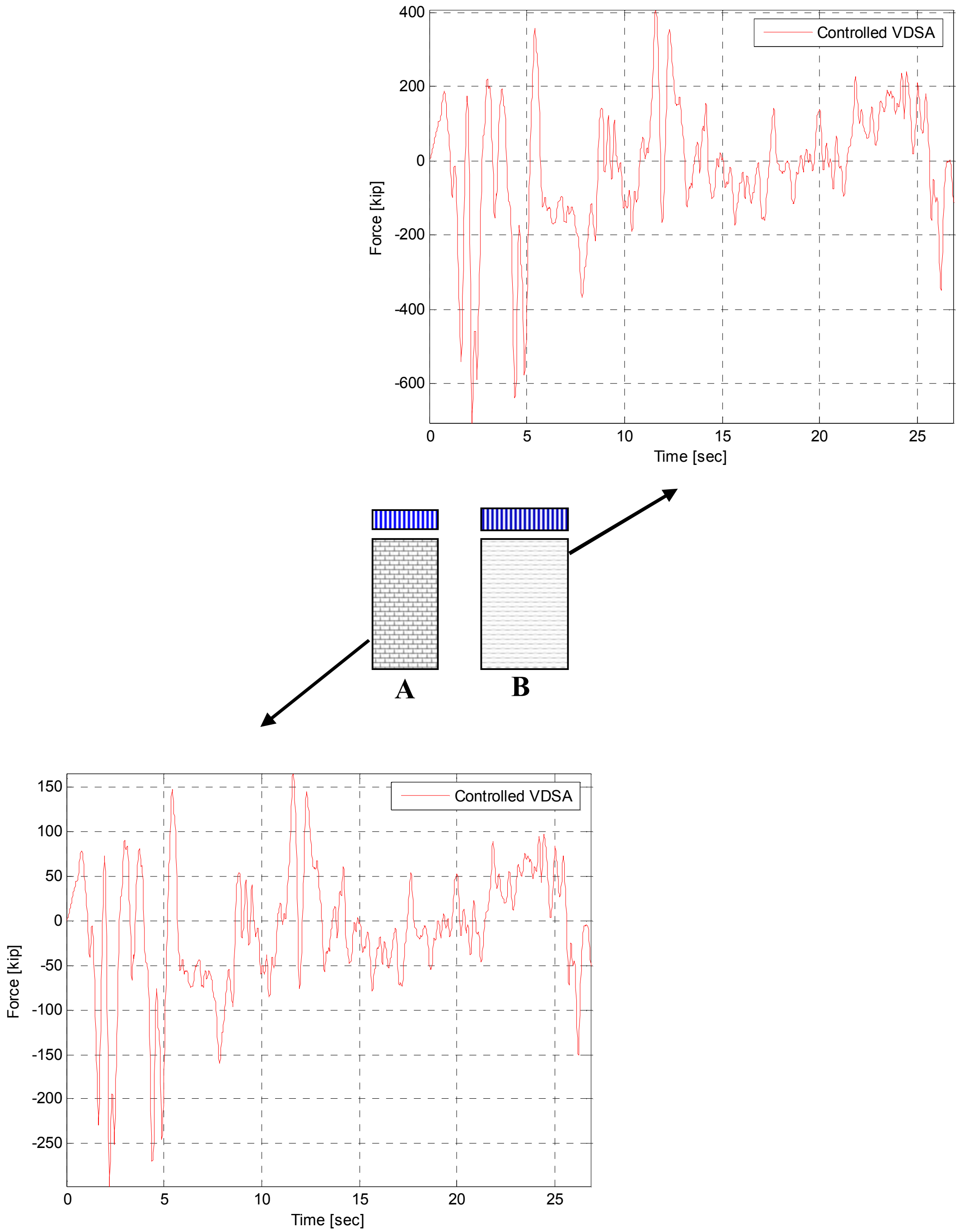


Figure 7.4: Base shear in the coupled sdof systems A and B for the El Centro record controlled with VDSA (*Closed-loop control*)

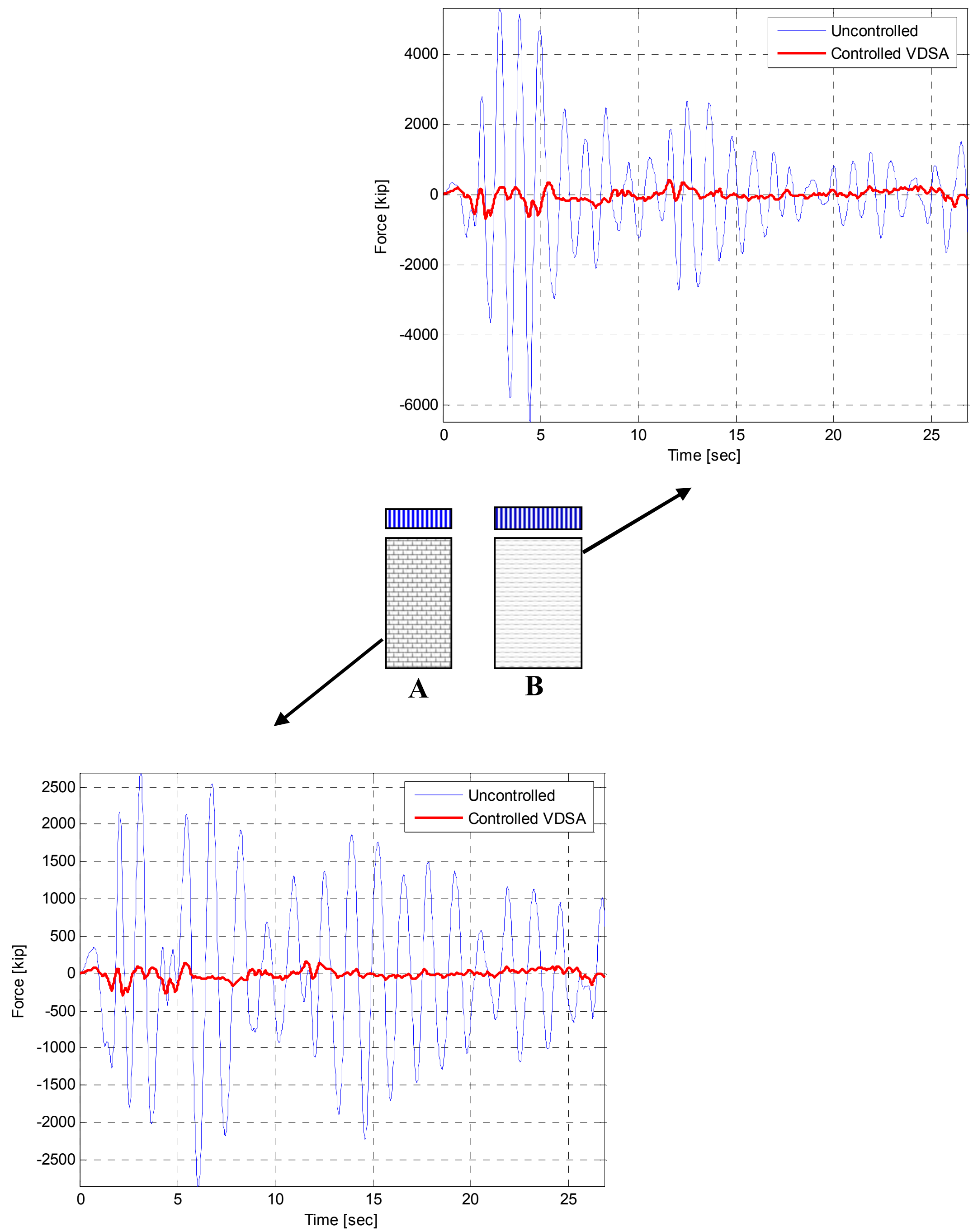


Figure 7.5: Base shear in the coupled sdof systems A and B for the El Centro record - Uncontrolled vs. VDSA (*Closed-loop control*)

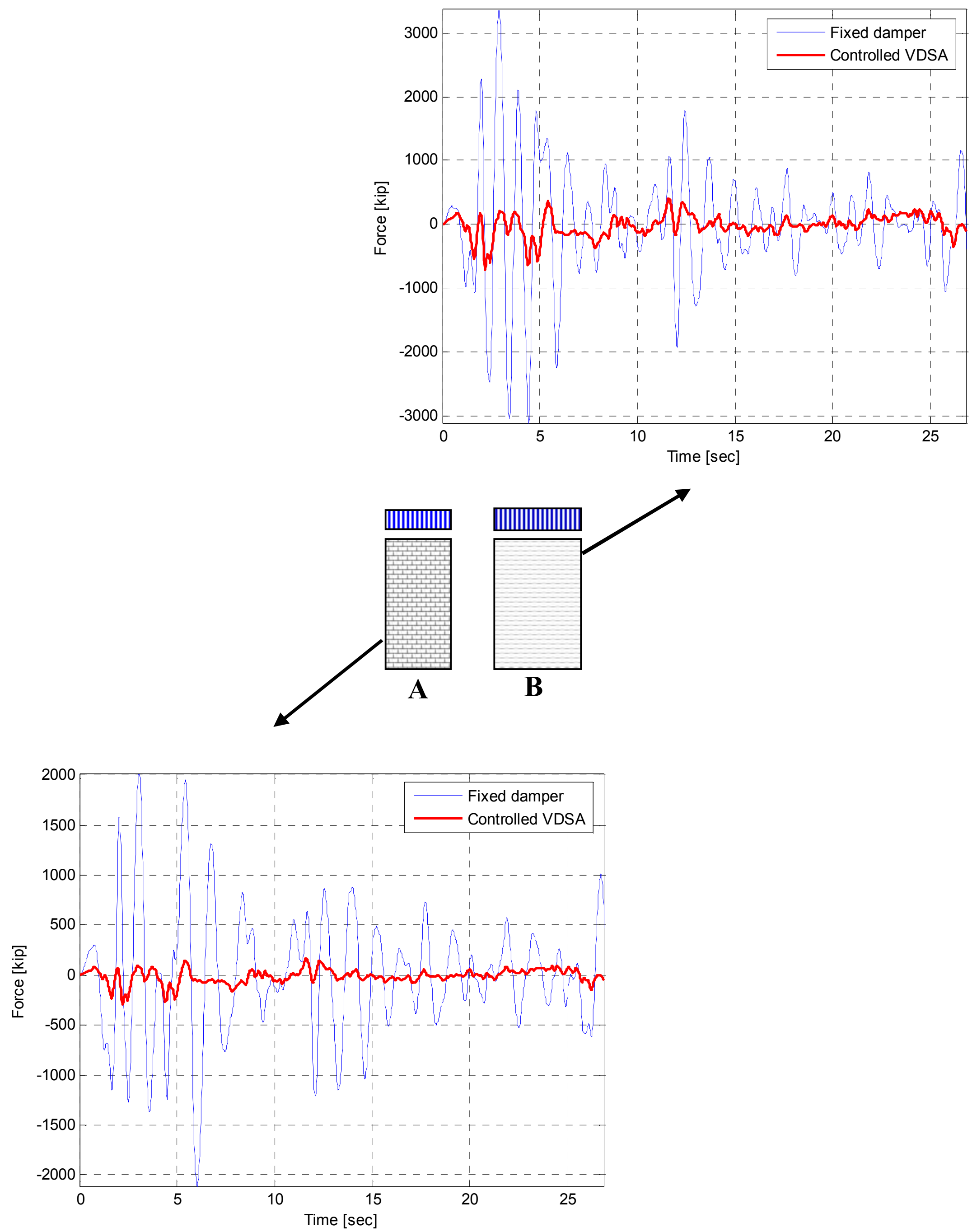


Figure 7.6: Base shear in the coupled sdof systems A and B for the El Centro record - Fixed damper vs. VDSA (*Closed-loop control*)

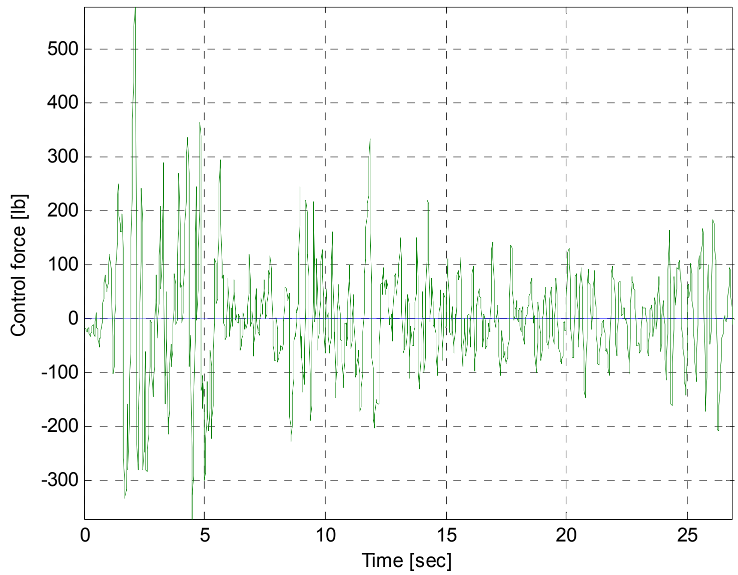


Figure 7.7: Control force in the coupled sdof systems A and B for the El Centro record (*Closed-loop control*)

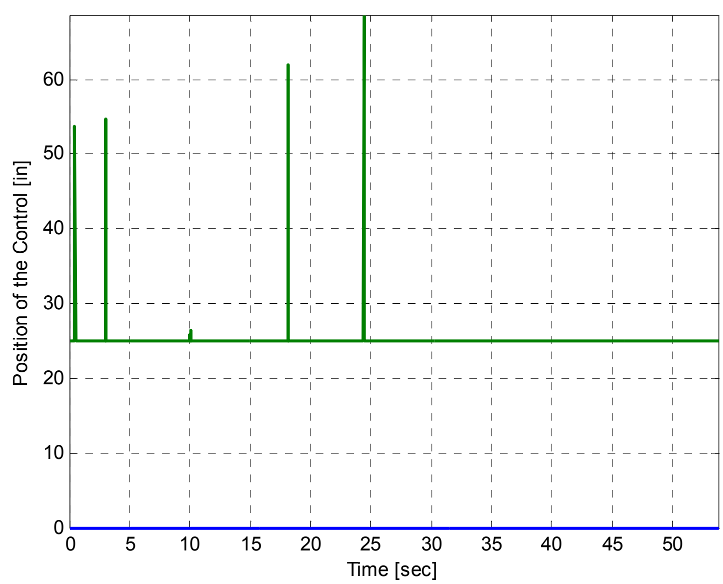


Figure 7.8: Variation of the position of the VDSA device in the coupled sdof systems A and B for the El Centro record (*Closed-loop control*)

7.2.1.b RESPONSE TO THE SAN FERNANDO RECORD

The relative displacement response in the two structures **A** and **B** controlled with the **VDSA** device and using the San Fernando record as input load are shown in Figures 7.9. Note that the time variation of the two displacements is about the same but will different magnitudes. Figures 7.10 and 7.11 show the relative displacements of the structures **A** and **B** with the **VDSA** along with the response of the uncontrolled structure and with fixed dampers, respectively. Figure 7.12 shows the time variation of the total base shear of the structures with the **VDSA** device. In Figures 7.13 and 7.14 are compared the total base shear in the original structure, the buildings with fixed dampers, and the structures controlled with the semiactive device. Figure 7.15 and 7.16 show the time variation of the control force and the position of the **VDSA** device, respectively. The maximum force required of the control device was 1.785 kip. The minimum height of the position of the **VDSA** device system was 25 in. Top better grasp the magnitude of the response reduction, it is interest to compare the peak value of the total base shear force as a fraction of the total weight of the two structures (28,800 kip). When the two structures are uncoupled without any protective system, this value reaches 67% whereas when the **VDSA** device is activated, the total base shear is only 14.8% of the total weight.

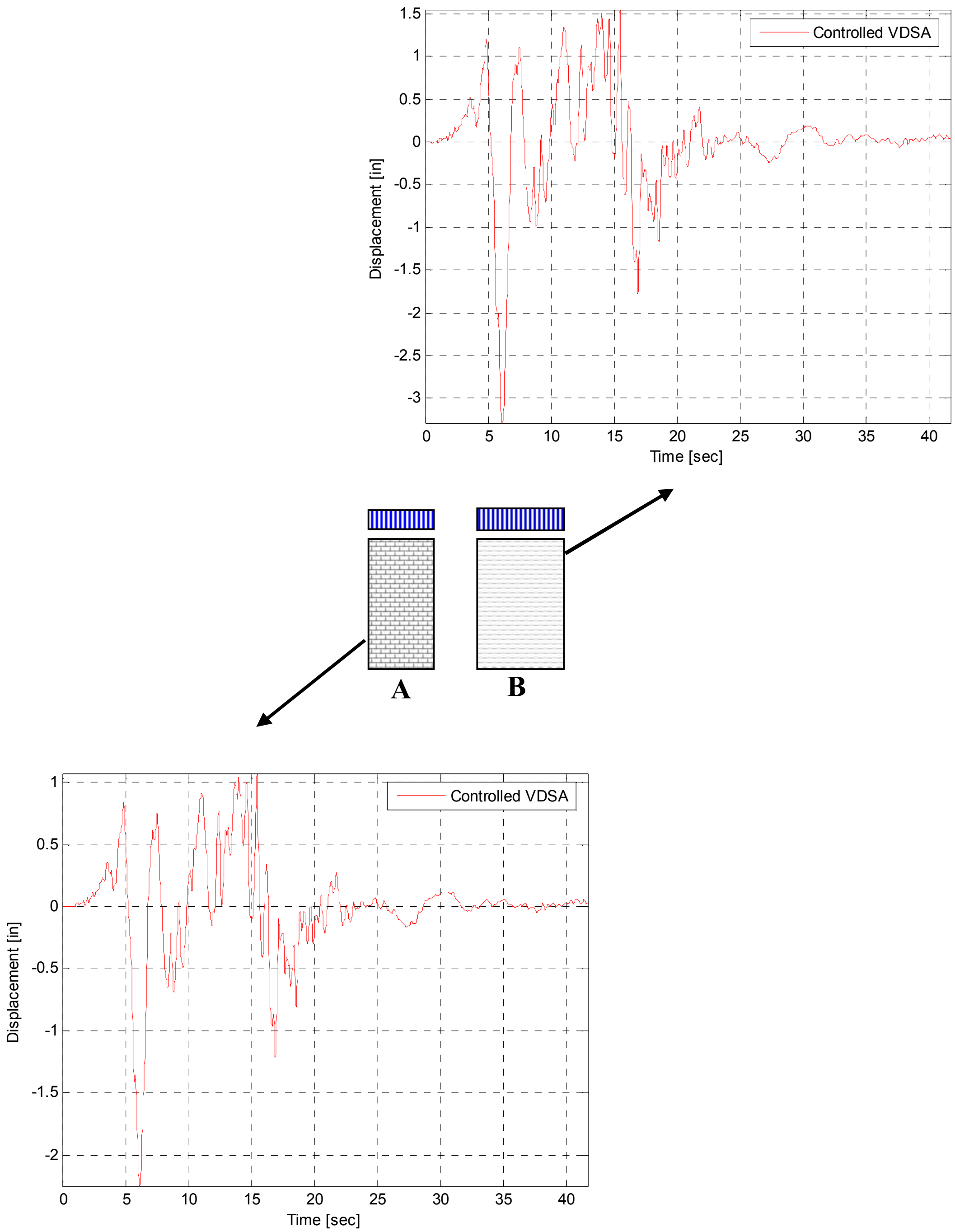


Figure 7.9: Relative displacement of the coupled sdof systems A and B for the San Fernando record controlled with VDSA (*Closed-loop control*)

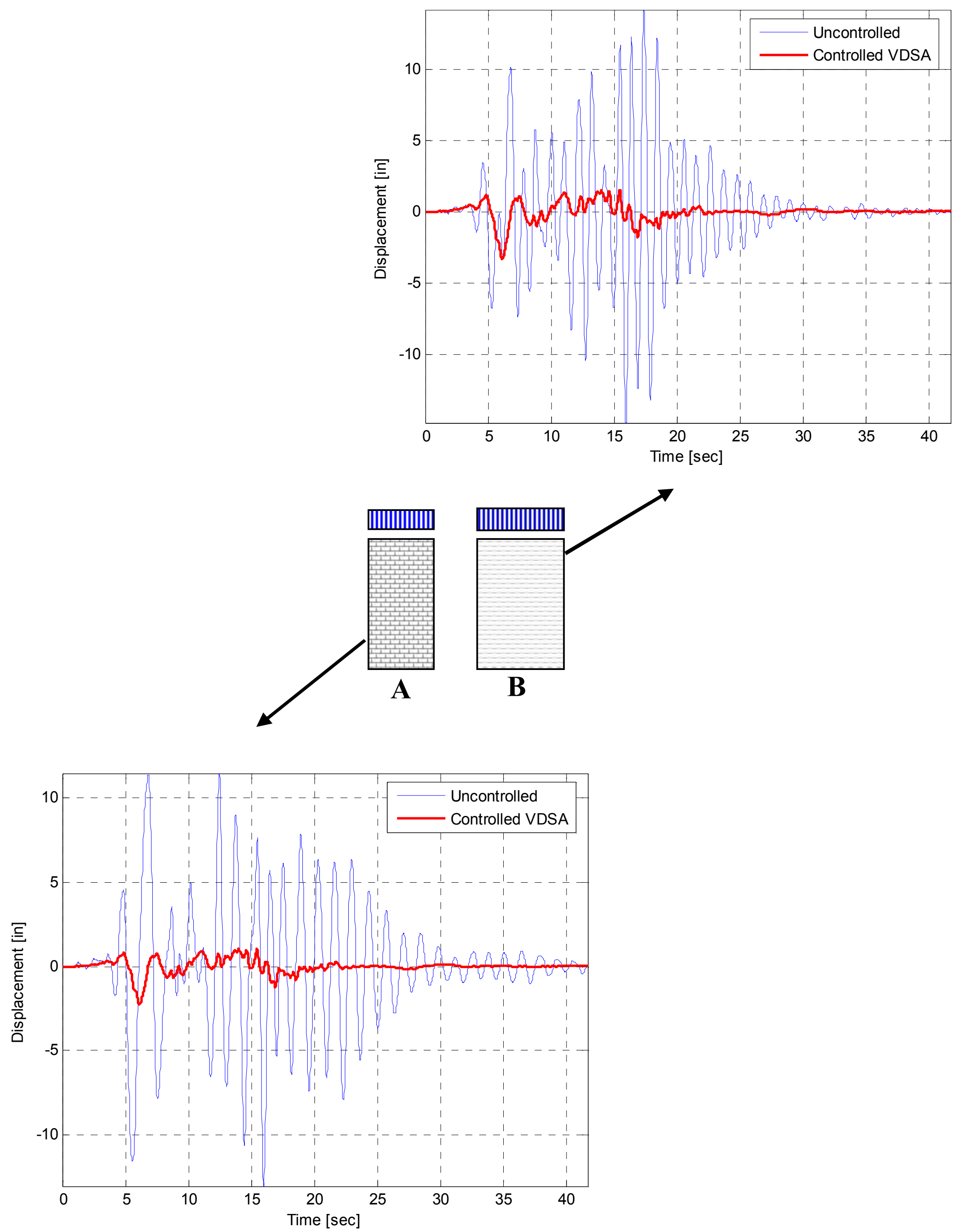


Figure 7.10: Relative displacement of the coupled sdof systems A and B for the San Fernando record - Uncontrolled vs. VDSA (*Closed-loop control*)

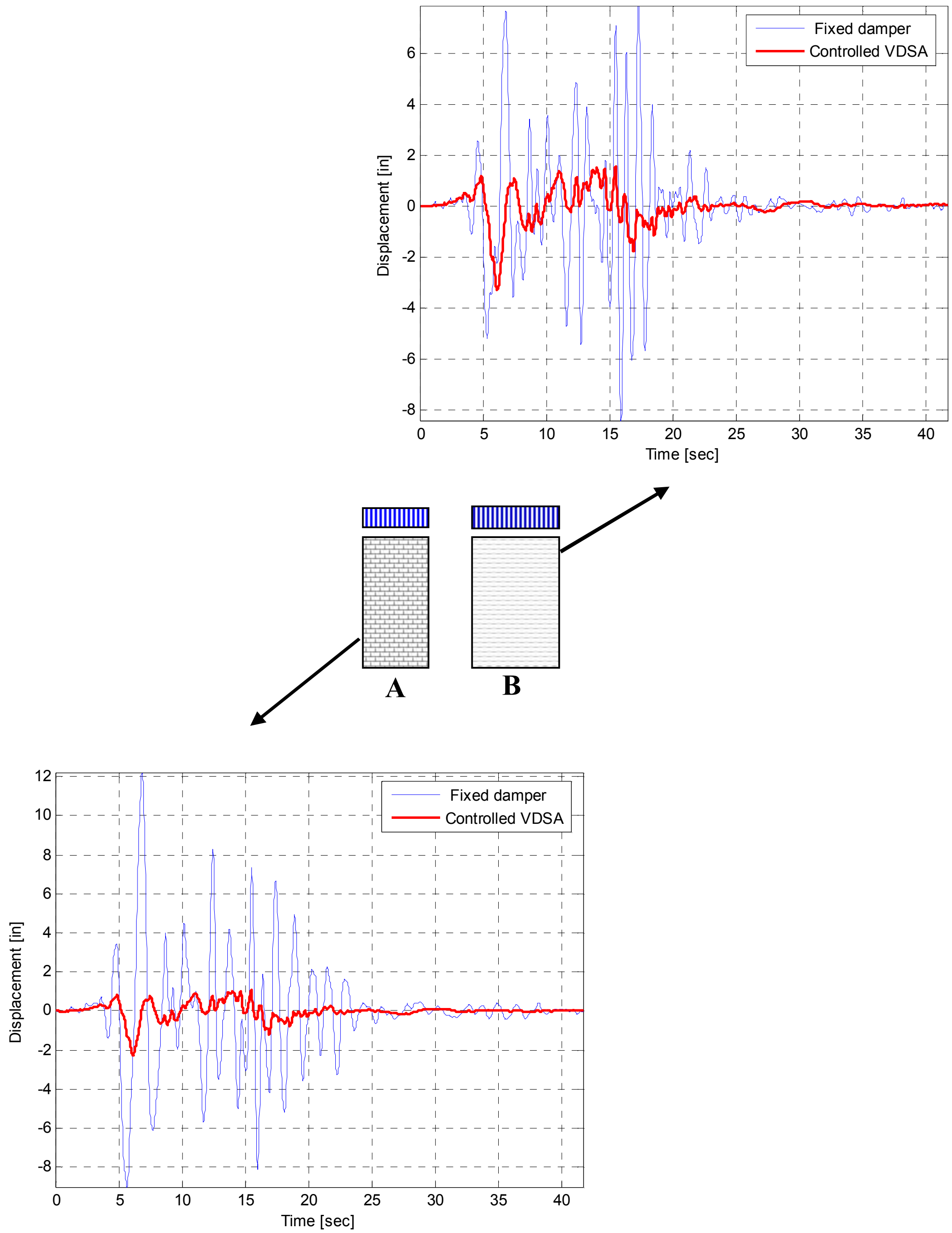


Figure 7.11: Relative displacement of the coupled sdof systems A and B for the San Fernando record - Fixed damper vs. VDSA (*Closed-loop control*)

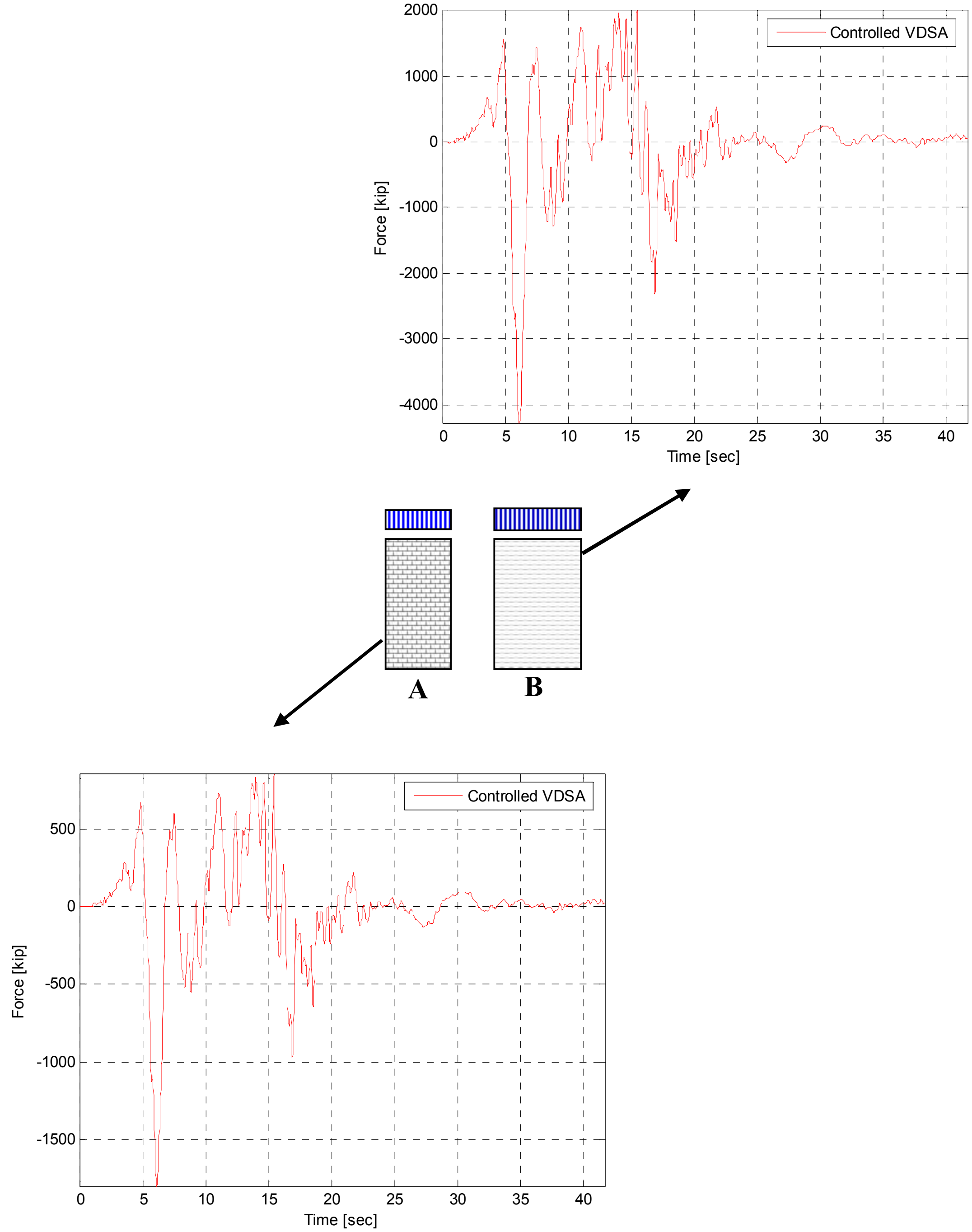


Figure 7.12: Base shear in the coupled sdof systems A and B for the San Fernando record controlled with VDSA (*Closed-loop control*)

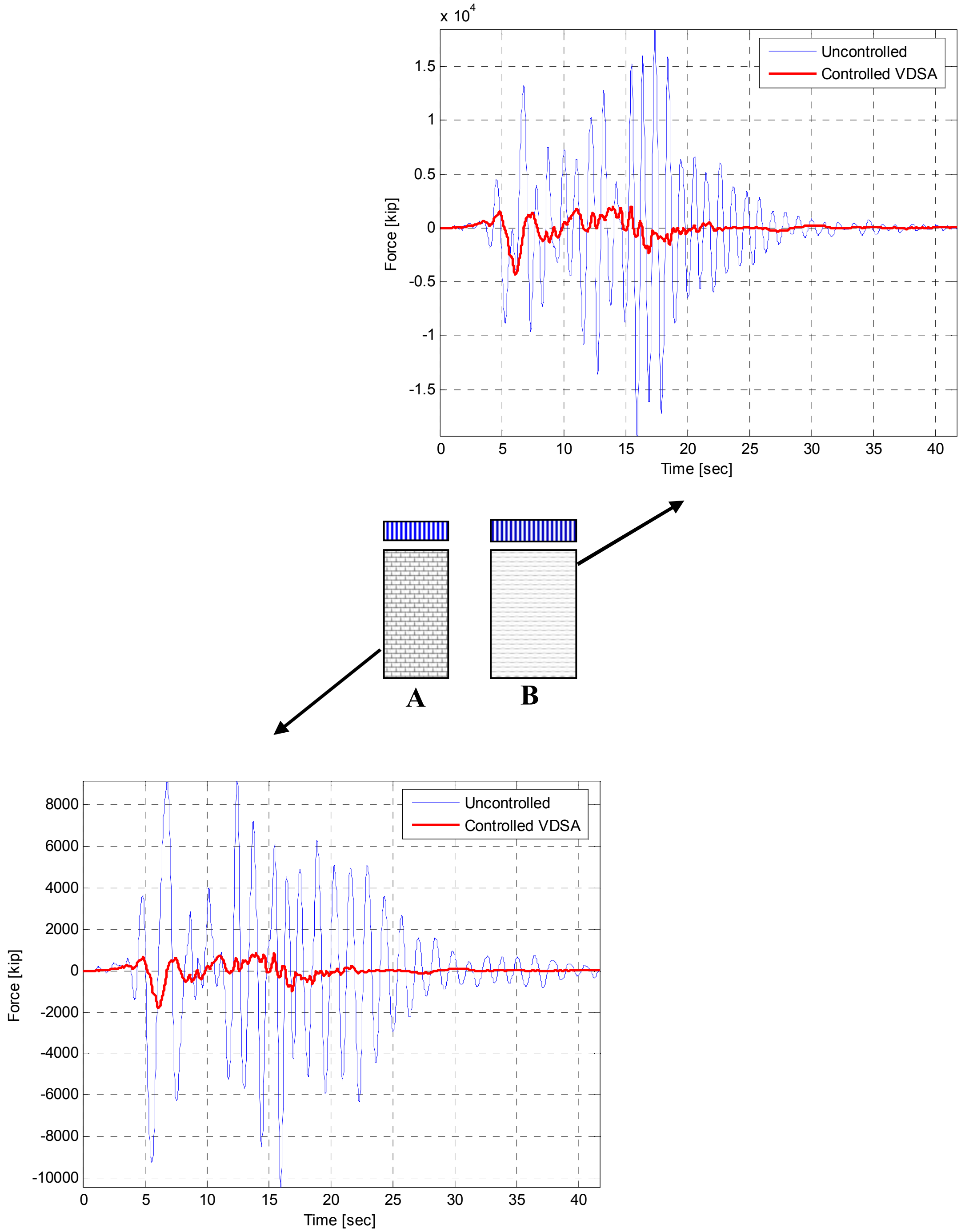


Figure 7.13: Base shear in the coupled sdof systems A and B for the San Fernando record - Uncontrolled vs. VDSA (*Closed-loop control*)

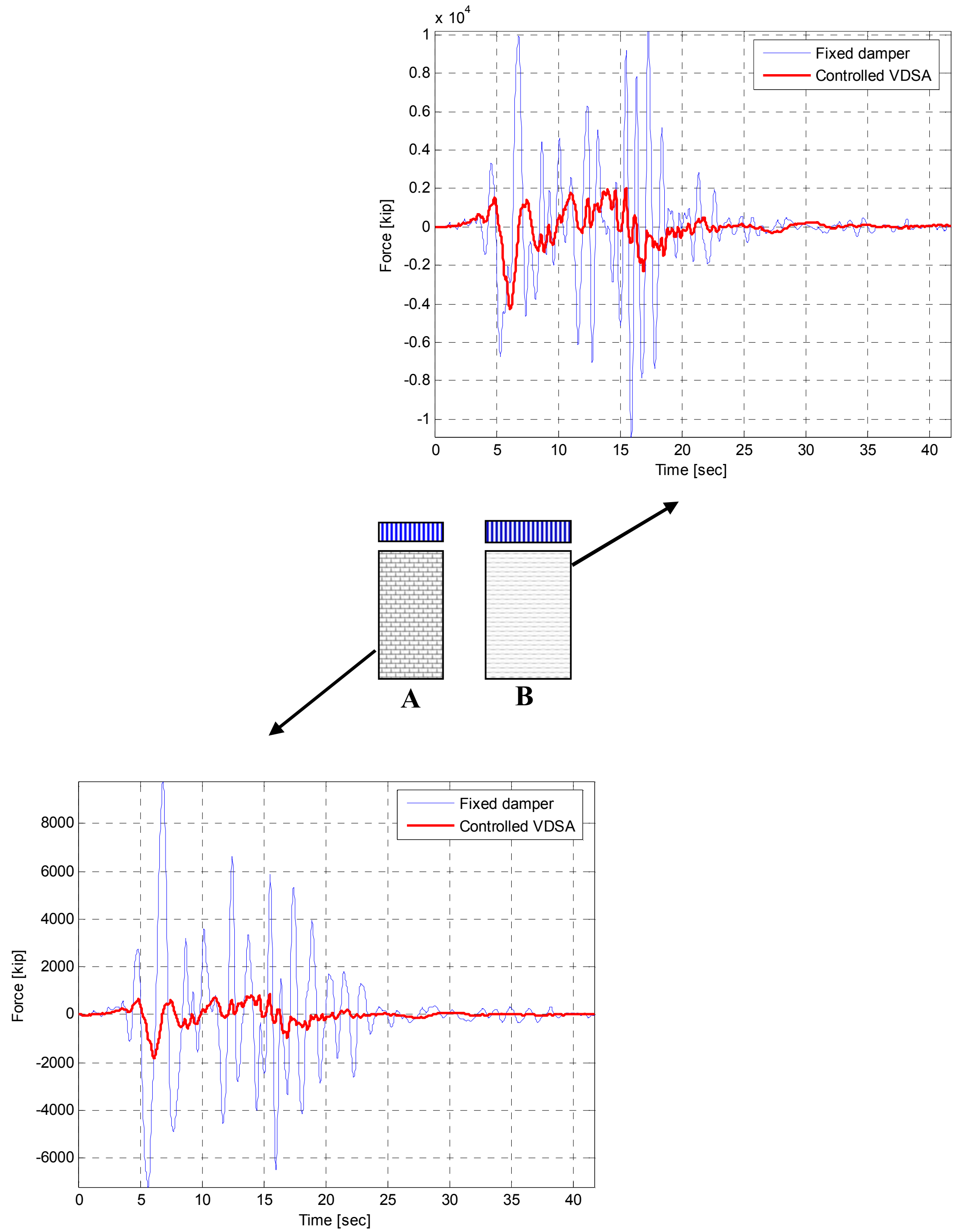


Figure 7.14: Base shear coupled sdof systems A and B for the San Fernando record - Fixed damper vs. VDSA (*Closed-loop control*)

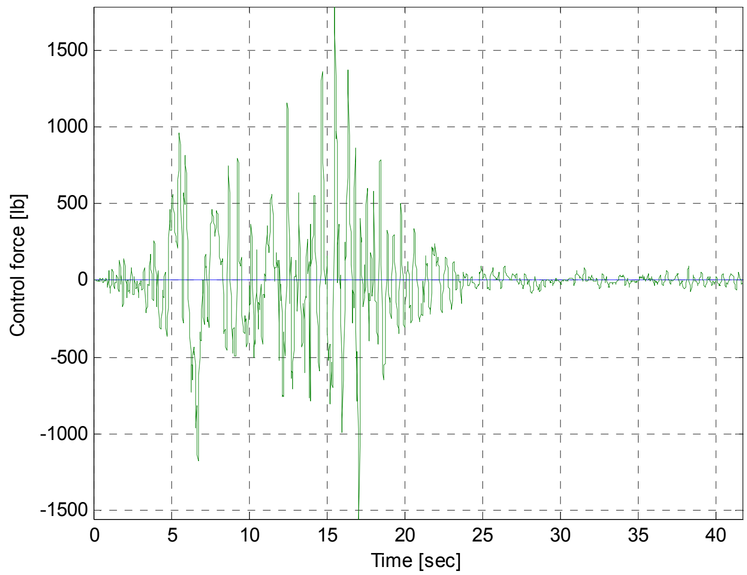


Figure 7.15: Control force in the coupled sdof systems A and B for the San Fernando record (*Closed-loop control*)

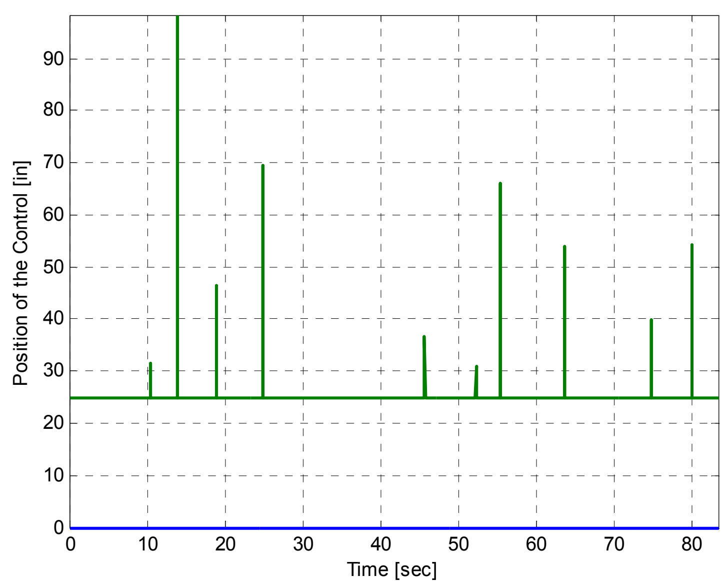


Figure 7.16: Variation of the position of the VDSA device in the coupled sdof systems A and B for the San Fernando record (*Closed-loop control*)

7.2.1.c RESPONSE TO THE FRIULI RECORD

The last sets of results are the relative displacements and total base shears for the structures **A** and **B** using the *Closed-loop control* algorithm to control the **VDSA** device and the 1976 Friuli earthquake record as input.

The response in the structures **A** and **B** in terms of the relative displacement and total base shear are presented and compared in Figures 7.17 to 7.22 for the three cases (uncontrolled, with fixed dampers and with the **VDSA** device). Finally, Figures 7.23 and 7.24 shows the time variation of the control force and the position of the lower end of the VDSA device, respectively. The peak value of the force in the control device and the minimum position used in the simulations are 0.781 kip and 25 in, respectively. When the structures are coupled and without any protective system, the maximum total base shear is 46% of the combined weight of the two structures (28,800 kip). When the VDSA device is on, this peak shear reduces 4% of the total weight.

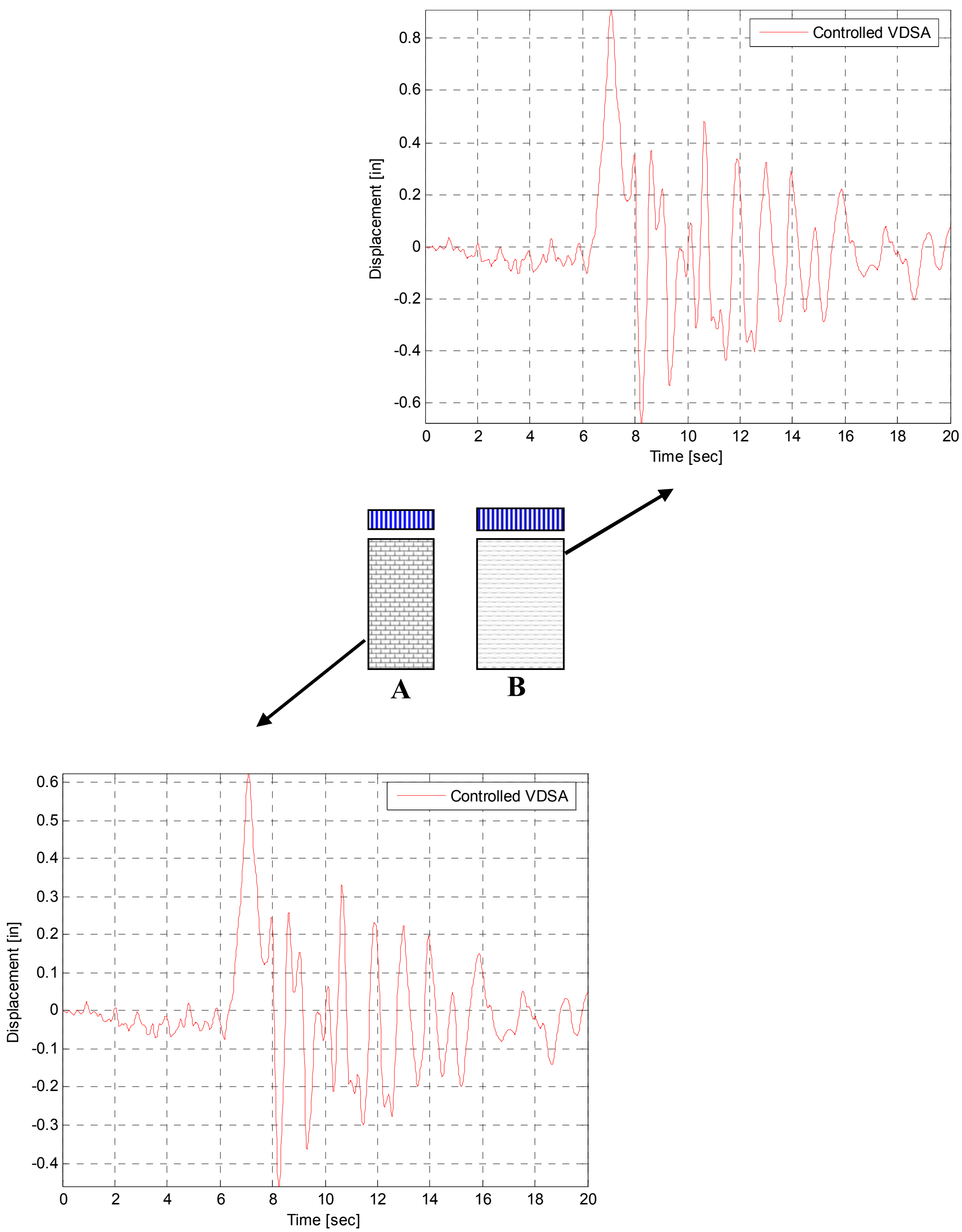


Figure 7.17: Relative displacement of the coupled sdof systems A and B for the Friuli record controlled with VDSA (*Closed-loop control*)

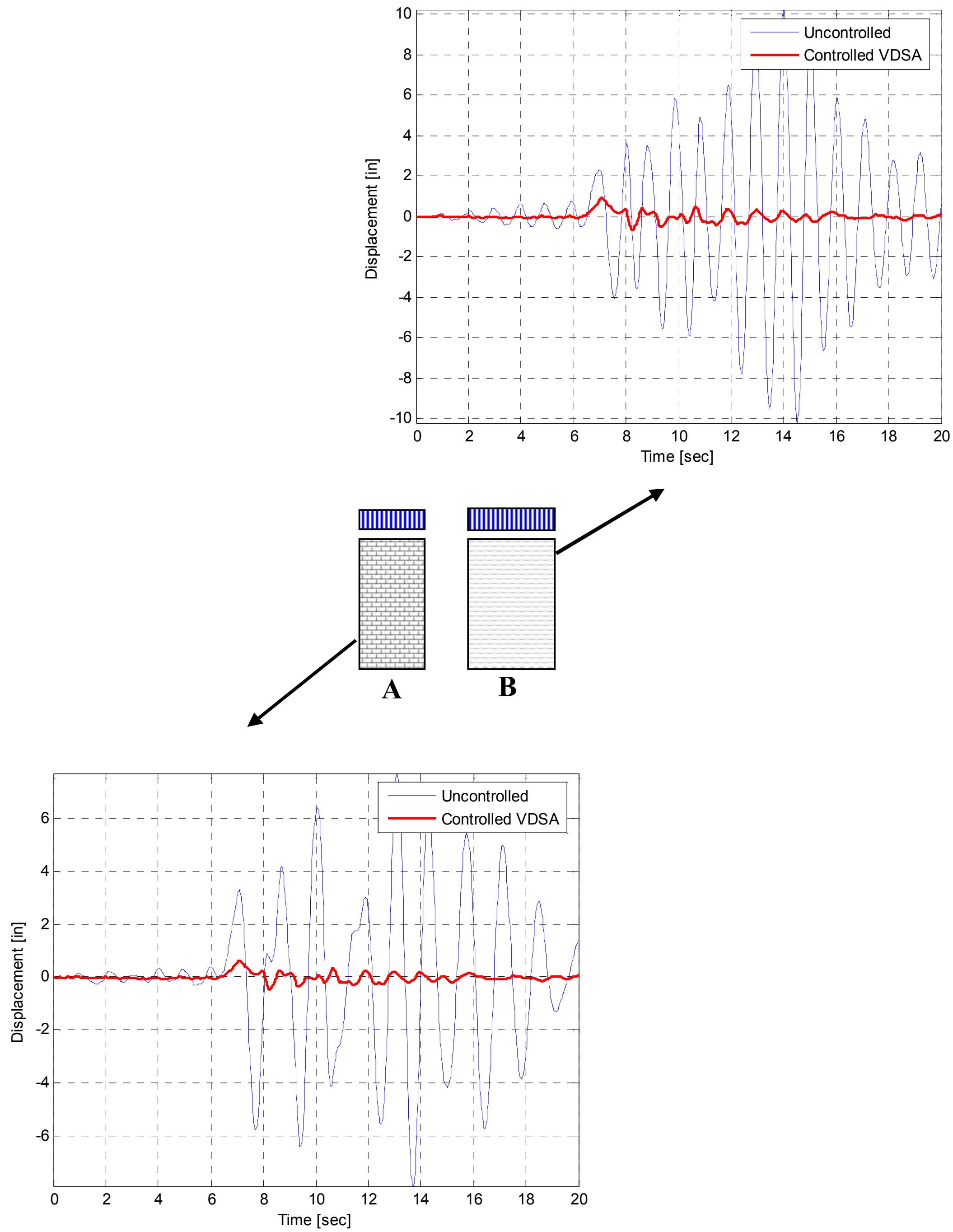


Figure 7.18: Relative displacement of the coupled sdof systems A and B for the Friuli record - Uncontrolled vs. VDSA: (*Closed-loop control*)

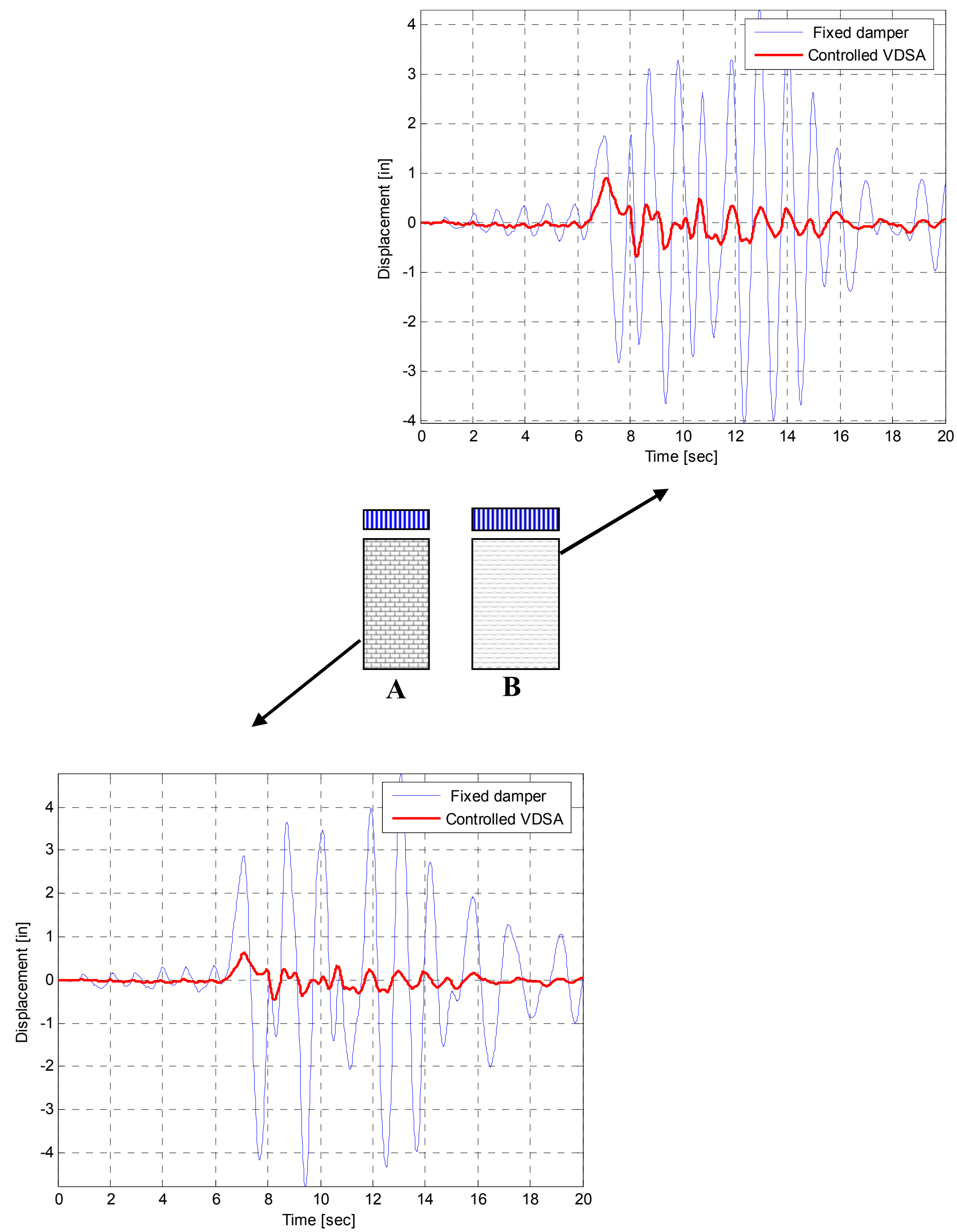


Figure 7.19: Relative displacement of the coupled sdof systems A and B for the Friuli record - Fixed damper vs. VDSA (*Closed-loop control*)

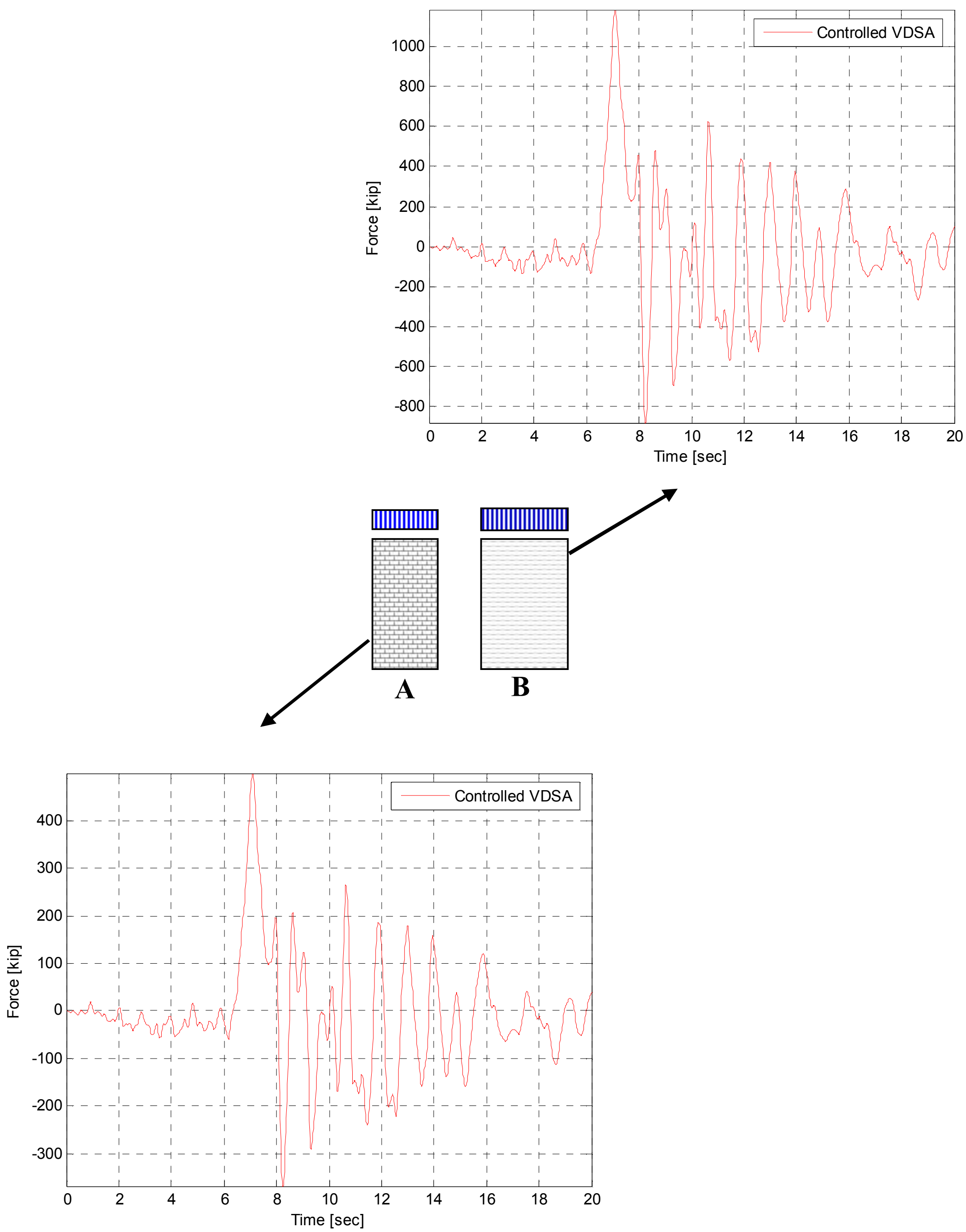


Figure 7.20: Base shear in the coupled sdof systems A and B for the Friuli record controlled with VDSA (*Closed-loop control*)

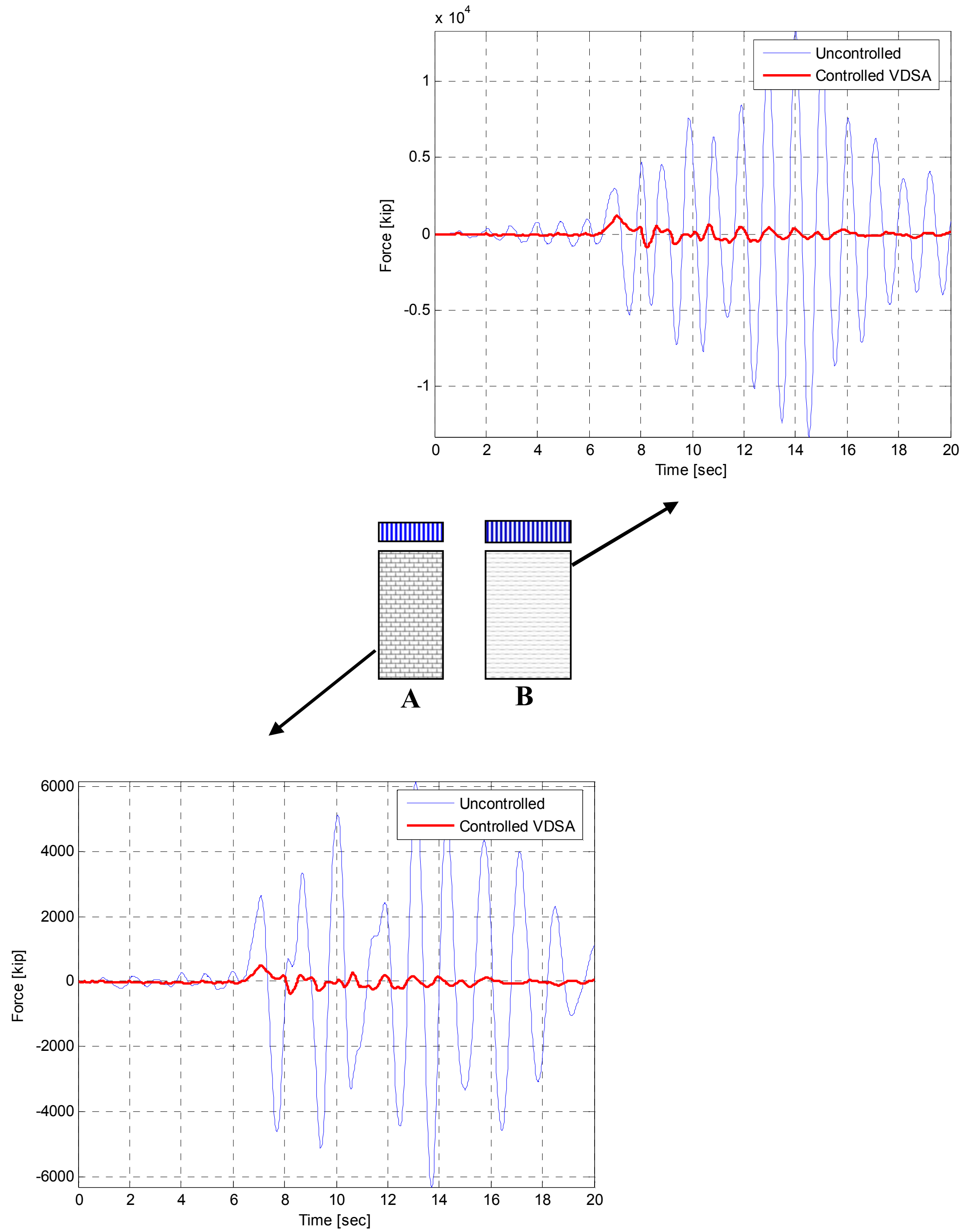


Figure 7.21: Base shear in the coupled sdof systems A and B for the Friuli record Uncontrolled vs. VDSA: (*Closed-loop control*)

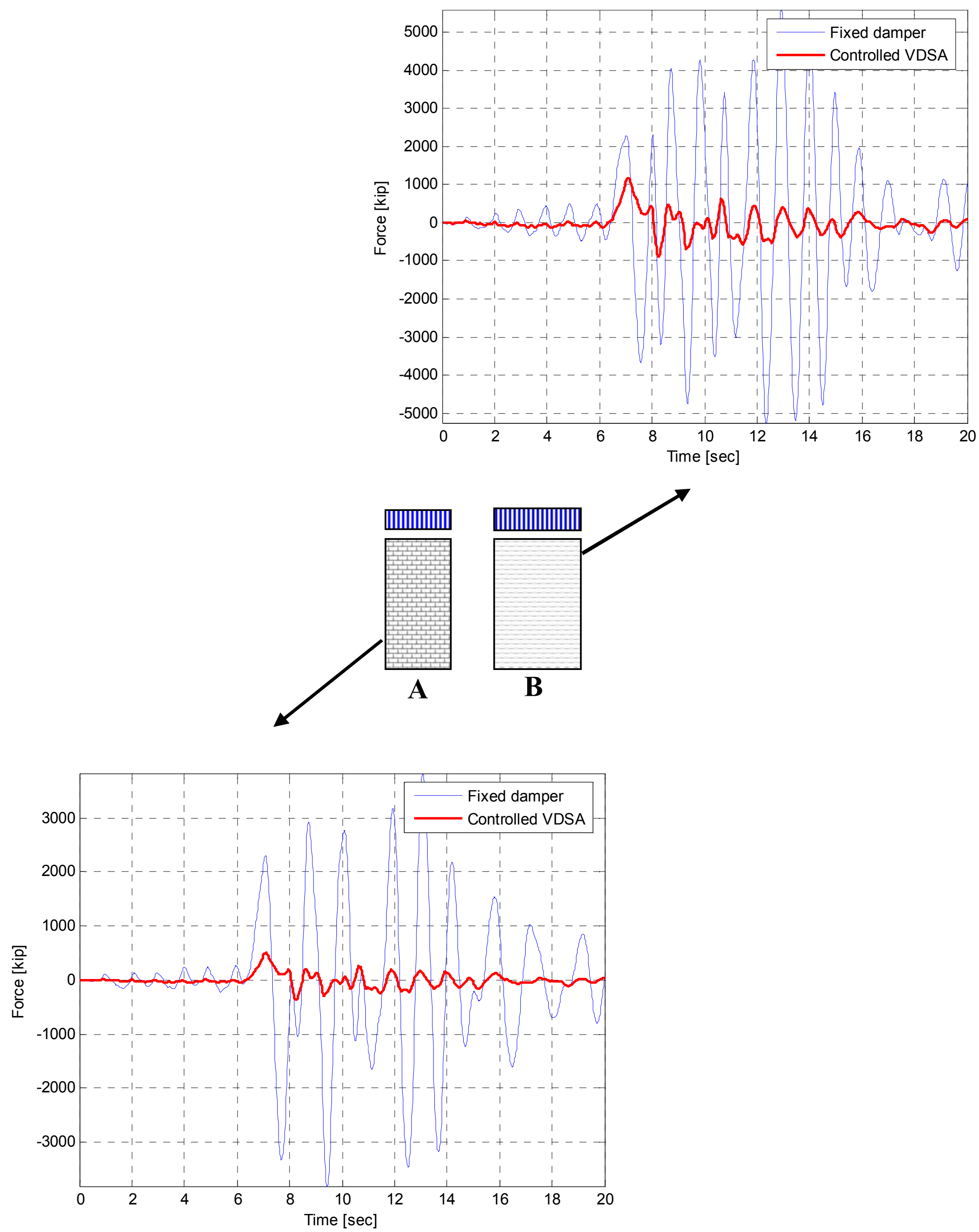


Figure 7.22: Base shear in the coupled sdof systems A and B for the Friuli record Fixed damper vs. VDSA (*Closed-loop control*)

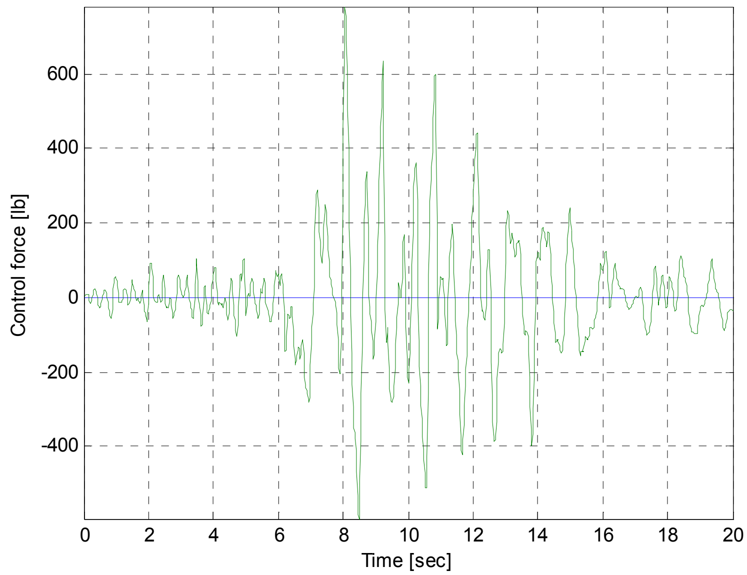


Figure 7.23: Control force in the coupled sdof systems A and B for the Friuli record (*Closed-loop control*)

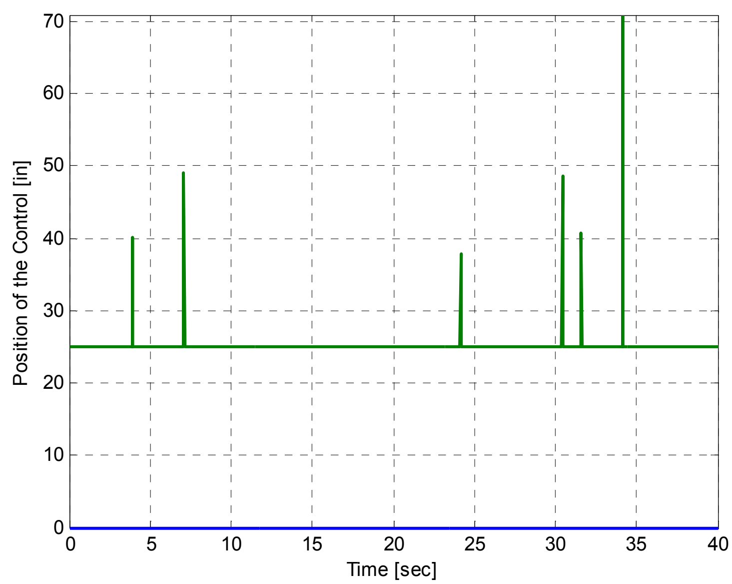


Figure 7.24: Variation of the position of the VDSA device in the coupled sdof systems A and B for the Friuli record (Closed-loop control)

Table 7.2 shows a summary of the responses displayed in Figures 7.1 to 7.6, for the El Centro earthquake Figures 7.9 to 7.14 for the San Fernando ground motion and Figures 7.17 to 7.22 for the Friulli earthquake in the coupled one degree of freedom structures. The modified *Closed-loop control* algorithm ***Qv*** is applied to control the **VDSA** device when it is installed between the two structures. Table 7.2 clearly demonstrates the advantages of the **VDSA** device along with control algorithm to reduce the seismic response of coupled structures that can be modeled as SDOF system.

Table 7.2: Maximum response of the SDOF coupled structures A and B without control and with a passive and semiactive system (*Closed-loop control*)

Earthquake	Displacement			Total base shear		
	Uncont. [in*]	Fixed damper [in*]	VDSA [in*]	Uncont. [kip⁺]	Fixed damper [kip⁺]	VDSA [kip⁺]
Structure A						
El Centro	3.5606	2.6375	0.3722	2848.5	2110.0	297.8
San Fernando	13.0256	12.2307	2.2513	10420.0	9784.5	1801.0
Friuli	7.8865	4.7842	0.6243	6309.2	3827.3	499.4
Structure B						
El Centro	4.9659	2.5856	0.5430	6455.6	3361.3	705.9
San Fernando	14.8279	8.3977	3.2865	19276.0	10917.0	4272.4
Friuli	10.2202	4.2993	0.9109	13286.0	5589.1	1184.2

*1 in = 2.54 cm, $^{+}$1 kip = 4.448 kN.

7.2.2 APPLICATION OF THE MODIFIED *CLOSED-OPEN-LOOP CONTROL* ALGORITHM *Qv*

The weighting matrix **Q** and **Qv** and the scalar **R** selected were identical to those used in the modified *Closed-loop control* algorithm ***Qv***. That is, **Q** = [***I***]x10^5, **Qv** =

[***I***]x10^2 and **R** = 10^{-3}.

In this example, as it occurred in those of the previous chapters, the maximum displacement response and total base shear obtained by modified *Closed-open-loop control* algorithm ***Qv*** were similar to the results obtained with modified *Close-loop control* algorithm ***Qv*** presented in Section 7.2.1.a to 7.21.c. The only observed differences occurred in the form in that varies the position of the VDSA device for each of the earthquakes. The variation in the height of the lower end of the semiactive device during the application of the El Centro, San Fernando and Friuli records are shown in Figures 7.25 to 7.27, respectively. These differences are not important and thus there is no real advantages in using one algorithm over the other.

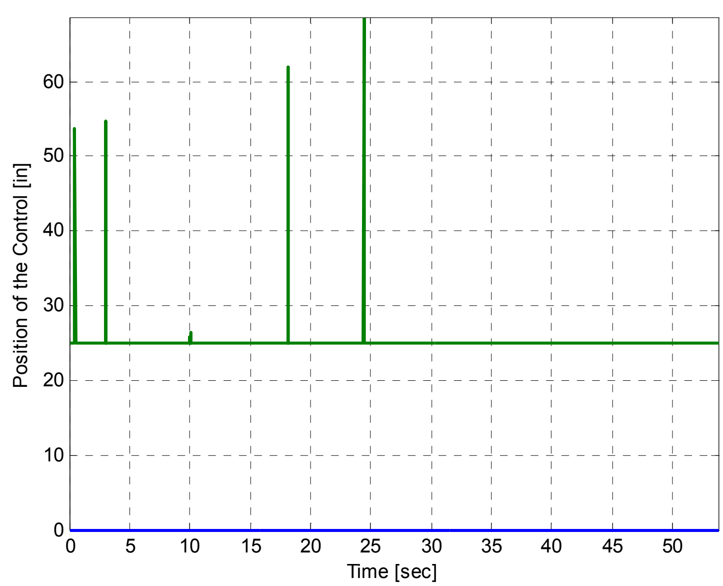


Figure 7.25: Variation of the position of the VDSA device in the coupled sdof systems A and B for the El Centro record (*Closed-open-loop control*)

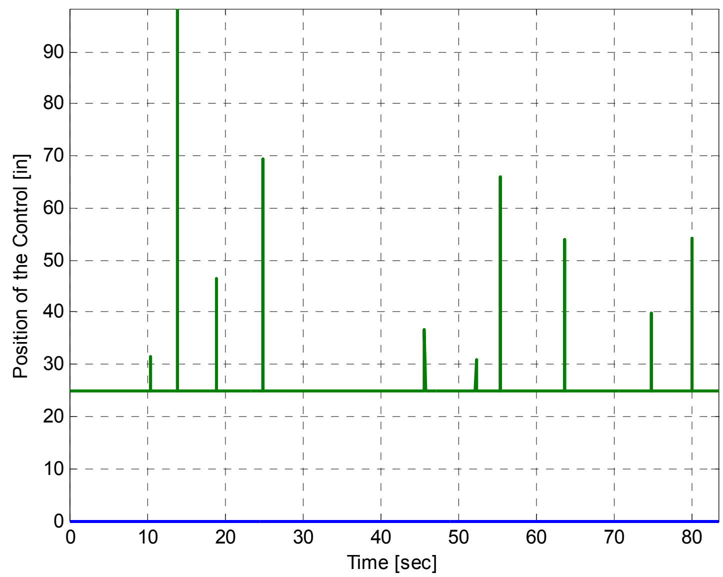
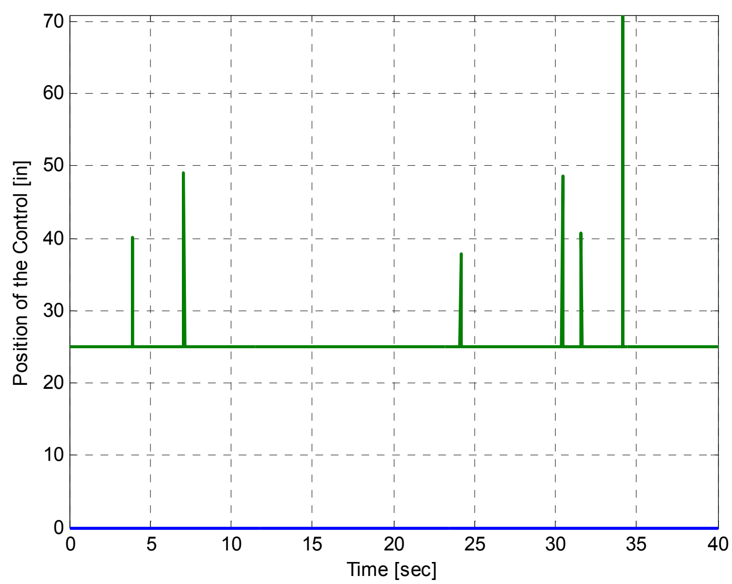


Figure 7.26: Variation of the position of the VDSA device in the coupled sdof systems A and B for the San Fernando record (Closed-open-loop control)



7.27: Variation of the position of the VDSA device in the coupled sdof systems A and B for the Friuli record (*Closed-open-loop control*)

7.2.3 DISCUSSION OF RESULTS

Table 7.4 presents a summary of the results (maximum control force, maximum relative displacement and maximum total base shear) obtained for the case of the coupled one-story structures **A** and **B** analyzed in this chapter. The results on which this table is based upon were shown in Figures 7.1 through 7.27. It is evident from the table that the **VDSA** device is capable of significantly reducing the response of the two coupled structures represented as single degrees of freedom systems. Table 7.4 display the results obtained with the modified *Closed-open-loop* and *Closed-loop control* algorithms ***Qv***. As it was mentioned before, in this example the force control, relative displacement and total base shear were reduced by the same amount with both algorithms.

Table 7.4: Maximum response of the SDOF coupled structures A and B (*Closed-loop control* and *Closed-Open-loop control*)

Earthquake	Control force		Displacement		Total base shear	
	Closed-loop [kip+]	***Closed-Open-loop*** [kip+]	***Closed-loop*** [in*]	***Closed-Open-loop*** [in*]	***Closed-loop*** [kip+]	***Closed-Open-loop*** [kip+]
Structure A						
El Centro	0.579	0.579	0.3722	0.3722	297.8	297.8
S. Fernando	1.785	1.785	2.2513	2.2513	1801.0	1801.0
Friuli	0.781	0.781	0.6243	0.6243	499.4	499.4
Structure B						
El Centro	0.579	0.579	0.5430	0.5430	705.9	705.9
S. Fernando	1.785	1.785	3.2865	3.2865	4272.4	4272.4
Friuli	0.781	0.781	0.9109	0.9109	1184.2	1184.2

*1 in = 2.54 cm, +1 kip = 4.448 kN.

7.3 TIME DELAY EFFECT

Table 7.13 shows a summary of the time delay effect in the maximum relative displacements of the uncontrolled (uncoupled) and controlled (coupled) structures, using the modified *Closed-loop control* algorithm ***Qv*** for El Centro, San Fernando and Friuli ground motions. Table 7.14 contains similar information but for the modified *Closed-open-loop control* algorithm ***Qv***.

The results for the controlled structures depend on the time delays (0.04 s, 0.06 s, 0.08 s, and 0.1 s). The column for 0.02 s is the "ideal case", the case when time delays are disregarded. As it was explained in previous chapter, the minimum time delay is equal to the time sampling rate (0.02 s in this example).

For the *Closed-loop control* the average in the maximum response is 0.53% for structures **A** and 0.33% for the structure **B**, considering the three seismic inputs. Thus, for this case the time delay effects are not important, at least when the delay is five times the sampling rate.

The result for the *Closed-open-loop control* are quite similar, there are only minors differences. Therefore the sensitivity of the algorithms to the time delay is the same.

Table 7.13: Summary of the time delay effect in the maximum response of SDOF structures controlled with the VDSA device (*Closed-loop control*)

Earthquake	Displacement					
	Uncontrolled [in*]	0.02 s [in*]	0.04 s [in*]	0.06 s [in*]	0.08 s [in*]	0.1 s [in*]
Structure A						
El Centro	3.5606	0.3722	0.3722	0.3731	0.3742	0.3753
S. Fernando	13.0256	2.2513	2.2514	2.2528	2.2540	2.2551
Friuli	7.8865	0.6243	0.6243	0.6257	0.6265	0.6278
Structure B						
El Centro	4.9659	0.5430	0.5431	0.5438	0.5449	0.5461
S. Fernando	14.8279	3.2865	3.2866	3.2873	3.2882	3.2890
Friuli	10.2202	0.9109	0.9109	0.9117	0.9124	0.9135

*1 in = 2.54 cm.

Table 7.14: Summary of the time delay effect in the maximum response of SDOF structures controlled with the VDSA device (*Closed-open-loop control*)

Earthquake	Displacement					
	Uncontrolled [in*]	0.02 s [in*]	0.04 s [in*]	0.06 s [in*]	0.08 s [in*]	0.1 s [in*]
Structure A						
El Centro	3.5606	0.3722	0.3722	0.3733	0.3740	0.3754
S. Fernando	13.0256	2.2513	2.2514	2.2529	2.2541	2.2551
Friuli	7.8865	0.6243	0.6243	0.6256	0.6266	0.6279
Structure B						
El Centro	4.9659	0.5430	0.5431	0.5439	0.5450	0.5463
S. Fernando	14.8279	3.2865	3.2866	3.2874	3.2882	3.2891
Friuli	10.2202	0.9109	0.9109	0.9117	0.9125	0.9137

*1 in = 2.54 cm.

CHAPTER VIII:

APPLICATION OF THE VDSA DEVICE AND MODIFIED ALGORITHMS Q_V TO MDOF COUPLED STRUCTURES

8.1 INTRODUCTION

This chapter present an example of the application of the **VDSA** device to coupled and reduce the seismic response of two multiple degree of freedom structures. One of coupled structures is an eight-story building and the other is the structure used in Chapter VI. As it was done in all previous examples the response compared are the relative displacement and total base shear for the structures controlled with the **VDSA** device and the same quantities for the structures uncontrolled and with fixed dampers. Again, the structures were subjected to the horizontal components of the three earthquakes used in Chapter V, VI and VII. The optimal variation of the position of the **VDSA** device is obtained by applying the modified *Closed-loop* and *Closed-open-loop* *control* algorithms ***Qv***.

8.2 EXAMPLE: MDOF STRUCTURES

The two structures to be coupled will be modeled as shear buildings with one degree of freedom per floor. The eight-story will be called structure **A** and the six-story building (described in Chapter VI) termed structure **B**. The total stiffness coefficients of the columns and the floor weights for the structure **A** are 5315 kip/in and 2205 kip, respectively. The damping ratio of the uncontrolled structures is assumed to be 5% for all modes. The damping ratio of the structures joined by passive dampers is assumed to be 30%. The damping constant C_o to achieve this modal damping is calculated with equation 3.5. The relevant modal properties and the other parameters for structure **A** is presented in Table 8.1.

Table 8.1: Properties and damping constants for the passive damper in structure A

Floor	$Cos^2\theta_1$	ϕ_1	ϕ_{r1}	ξ_{eff}	C_o
[#]				**[%]**	**[kip$^+$.sec/in*]**
8	0.9686	0.0103	0.0004	30	6
7	0.9686	0.0099	0.0007	30	6
6	0.9686	0.0092	0.0010	30	6
5	0.9686	0.0082	0.0012	30	6
4	0.9686	0.0070	0.0016	30	6
3	0.9686	0.0054	0.0017	30	6
2	0.9686	0.0037	0.0018	30	6
1	0.9686	0.0019	0.0019	30	6

*1 in = 2.54 cm, $^+$1 kip = 4.448 kN.

Three passive dampers configurations were considered for the structure **A**: a single damper at the lowest floor, four dampers in the four lowest floors and dampers in all eight floors. The three passive damper configuration in structure **A** are listed in Table 8.2. The **VDSA** device was installed in the fourth floor.

Table 8.2: Passive dampers location in structure A

Case	**I**	II	III
8th floor			X
7th floor			X
6th floor			X
5th floor			X
4th floor		X	X
3rd floor		X	X
2nd floor		X	X
1st floor	X	X	X

The natural periods of structure **A** are 0.6035, 0.2338, 0.1436, 0.1062, 0.0866, 0.0753, 0.0686 and 0.0651 sec. The natural periods of structure **B** are 0.5309, 0.1805, 0.1126, 0.0855, 0.0723 and 0.0659 sec.

The constant coefficients of the dampers that form the **VDSA** device are 15 kip.sec/in and 10 kip.sec/in for the dampers **A** and **B,** respectively.

8.2.1 APPLICATION OF THE MODIFIED *CLOSED- LOOP CONTROL* ALGORITHM *QV*

The weighting matrices **Q**, **Qv** and the scalar **R** are selected as:

$$Q = \left[\begin{array}{c:c} 10^9 * [I]_{16x16} & [0] \\ \hdashline [0] & 10^7 * [I]_{12x12} \end{array}\right]_{28x28} \quad Q_V = \left[\begin{array}{c:c} 10^4 * [I]_{8x8} & [0] \\ \hdashline [0] & 10 * [I]_{6x6} \end{array}\right]_{14x14} \quad R = 10^{-6}$$

8.2.1.a RESPONSE TO THE EL CENTRO RECORD

The first set of results for the two multiple degree of freedom structures is the response to the 1940 El Centro earthquake. Figures 8.1 to 8.6 show a comparison of the relative displacement time histories for the firsts up to the sixth the floors of structures **A** and **B** for the uncontrolled structure and controlled with the **VDSA** device. Figure 8.7 display the relative displacements of the uppermost two floors of structures **A**. Only the first twenty seven seconds of the response is shown. Figure 8.8 present the maximum relative displacement in each floor of the two structures for the five cases of analysis (i.e., uncontrolled structures, with the **VDSA** device, and with fixed dampers and configurations I, II and III). Figures 8.9 and 8.10 display the force control time history and the variation of the position of the **VDSA** device's lower end control, respectively. The maximum force generated by the control was 0.413 kip. The parameter $w_{\min}$ (minimum vertical position of the **VDSA** device) was set is equal to 30 in.

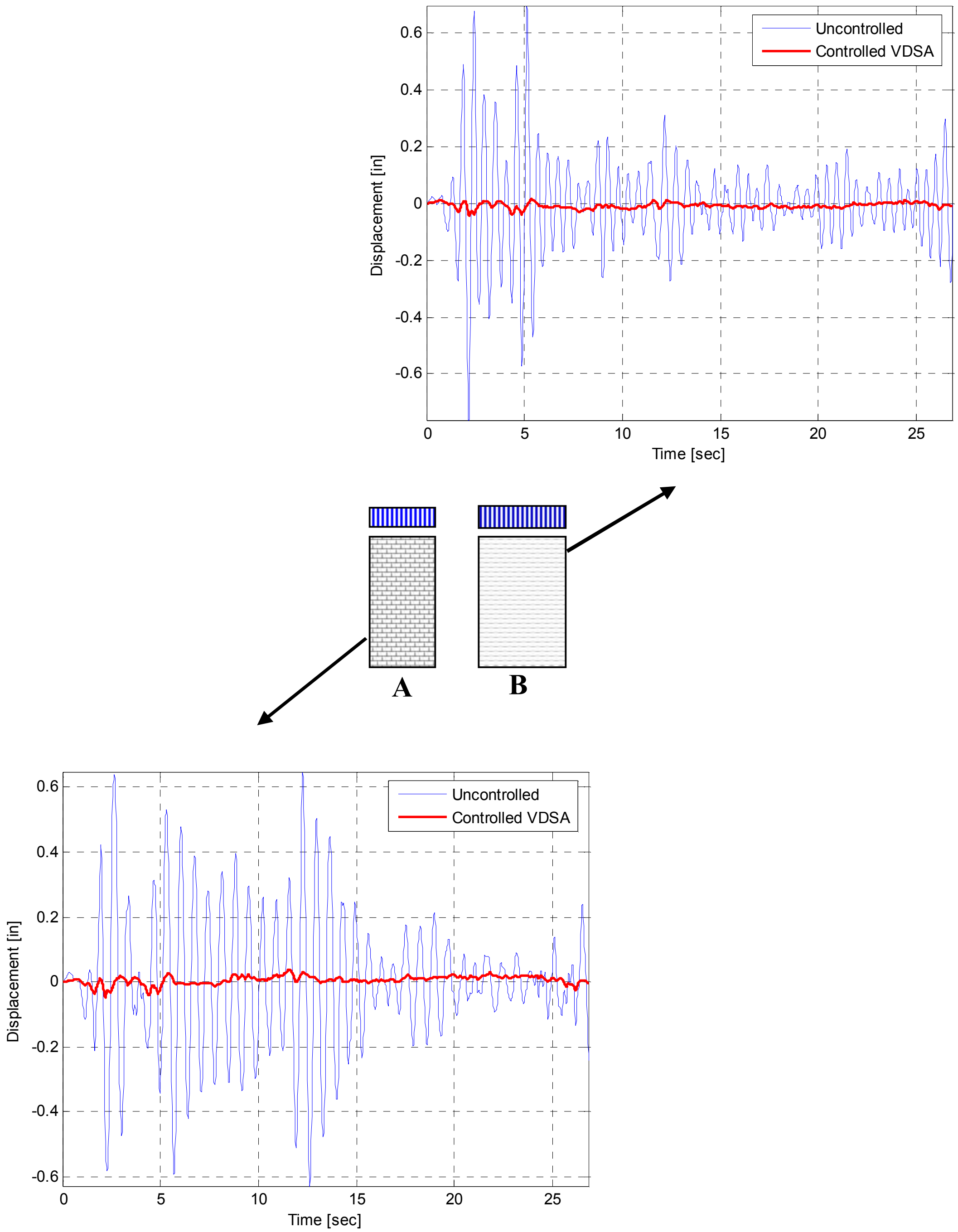


Figure 8.1: Relative displacement of the first floor of structures A and B for the El Centro record - Uncontrolled vs. VDSA (*Closed-loop control*)

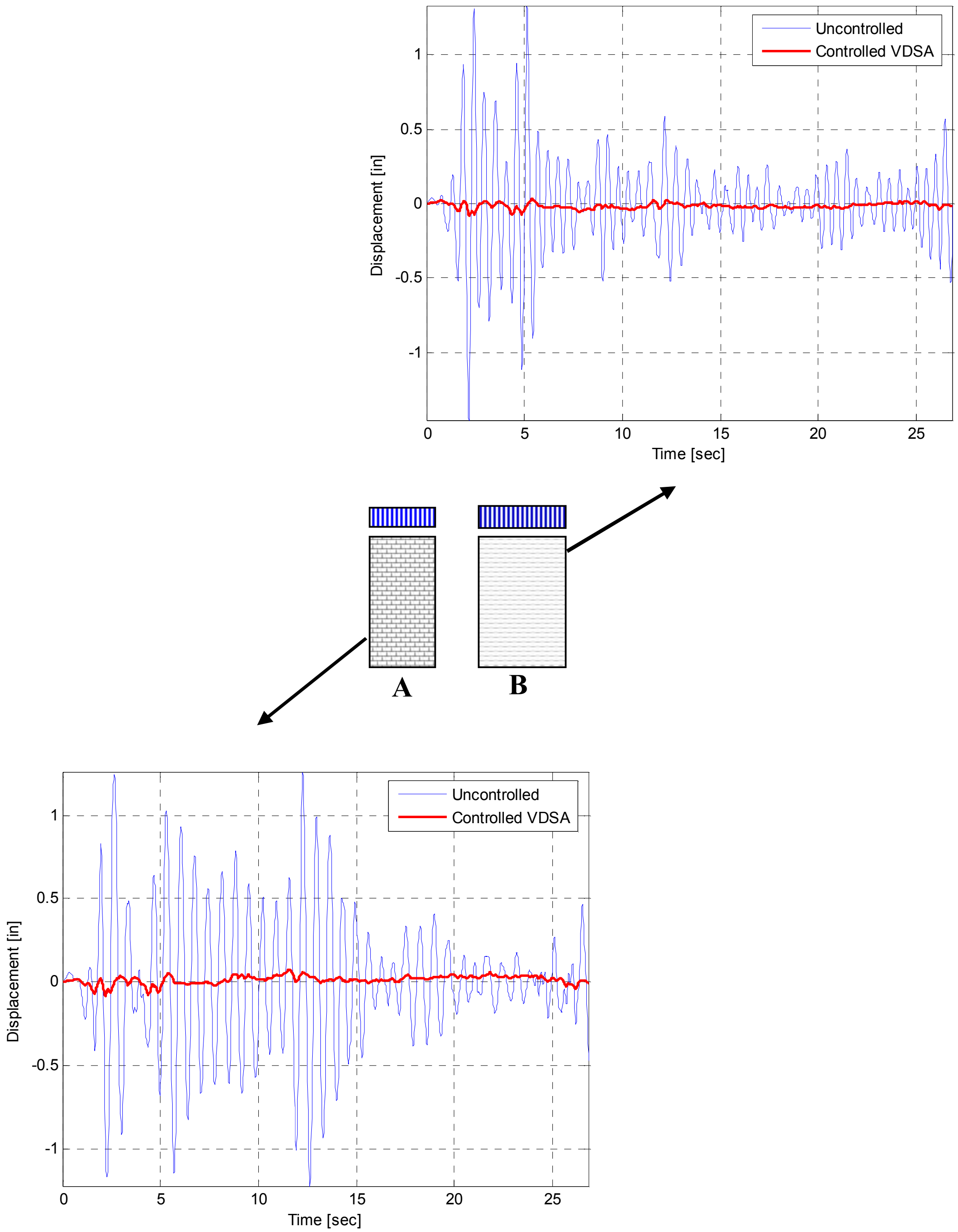


Figure 8.2: Relative displacement of the second floor of structures A and B for the El Centro record - Uncontrolled vs. VDSA (*Closed-loop control*)

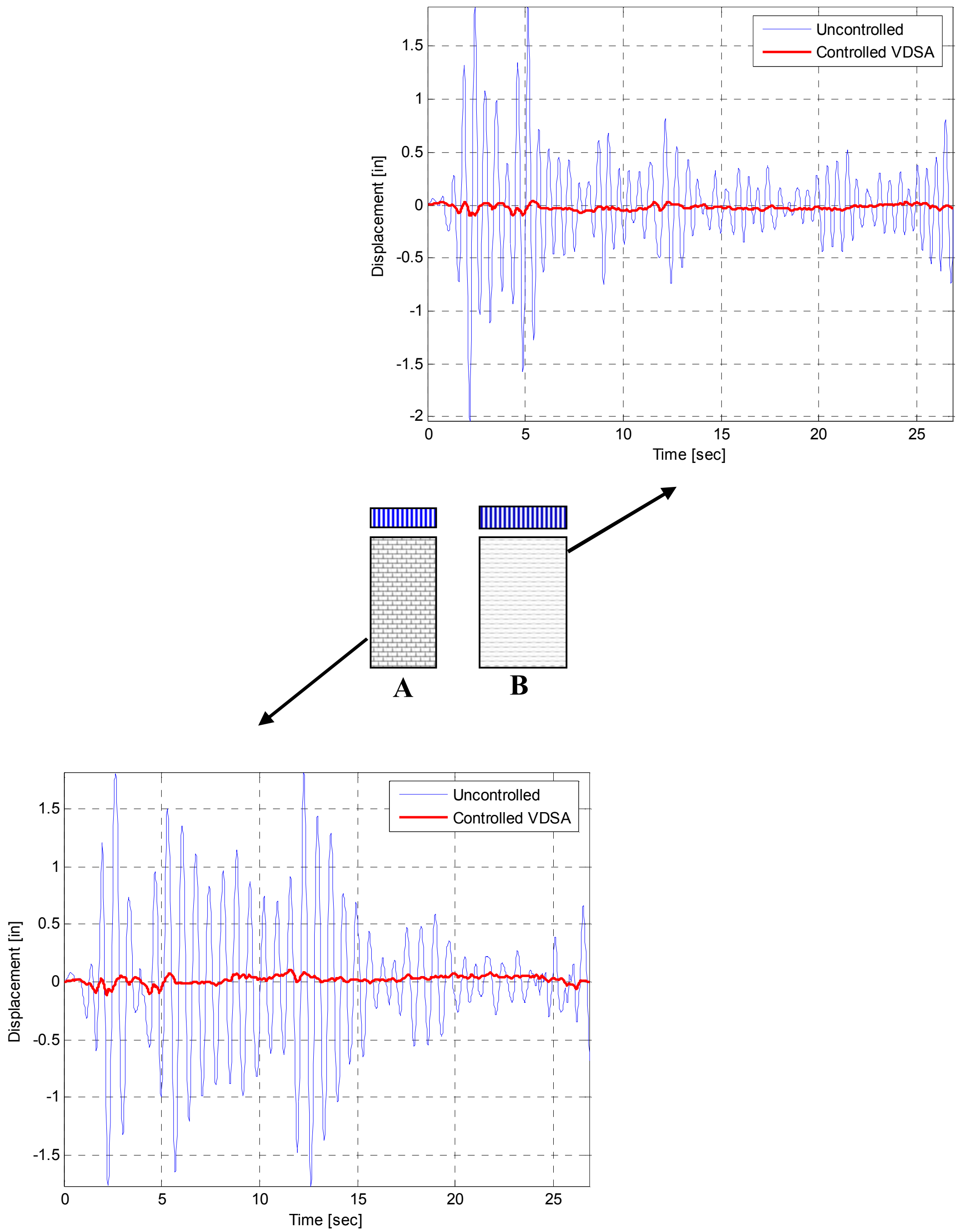


Figure 8.3: Relative displacement of the third floor of structures A and B for the El Centro record - Uncontrolled vs. VDSA (*Closed-loop control*)

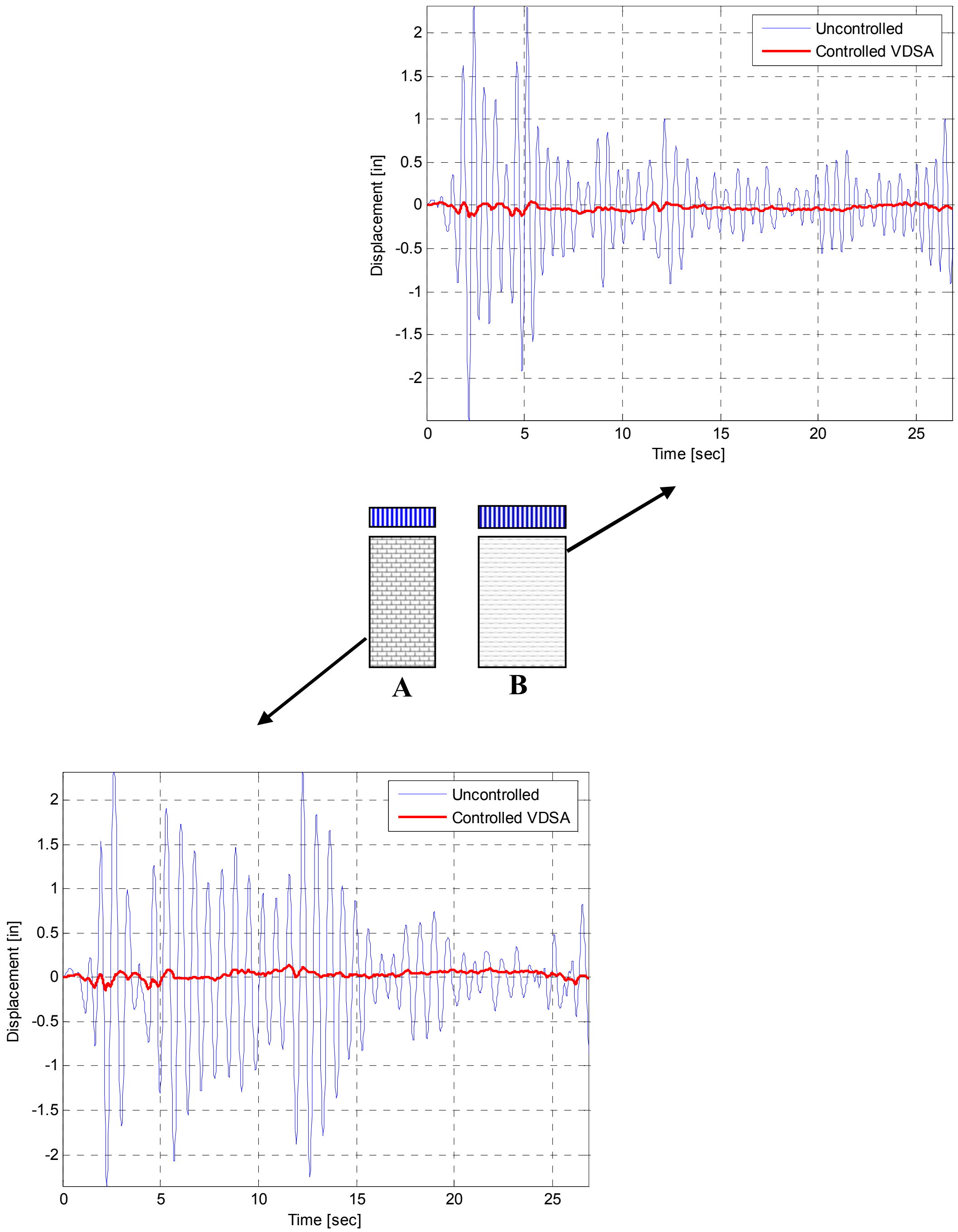


Figure 8.4: Relative displacement of the fourth floor of structures A and B for the El Centro record - Uncontrolled vs. VDSA (*Closed-loop control*)

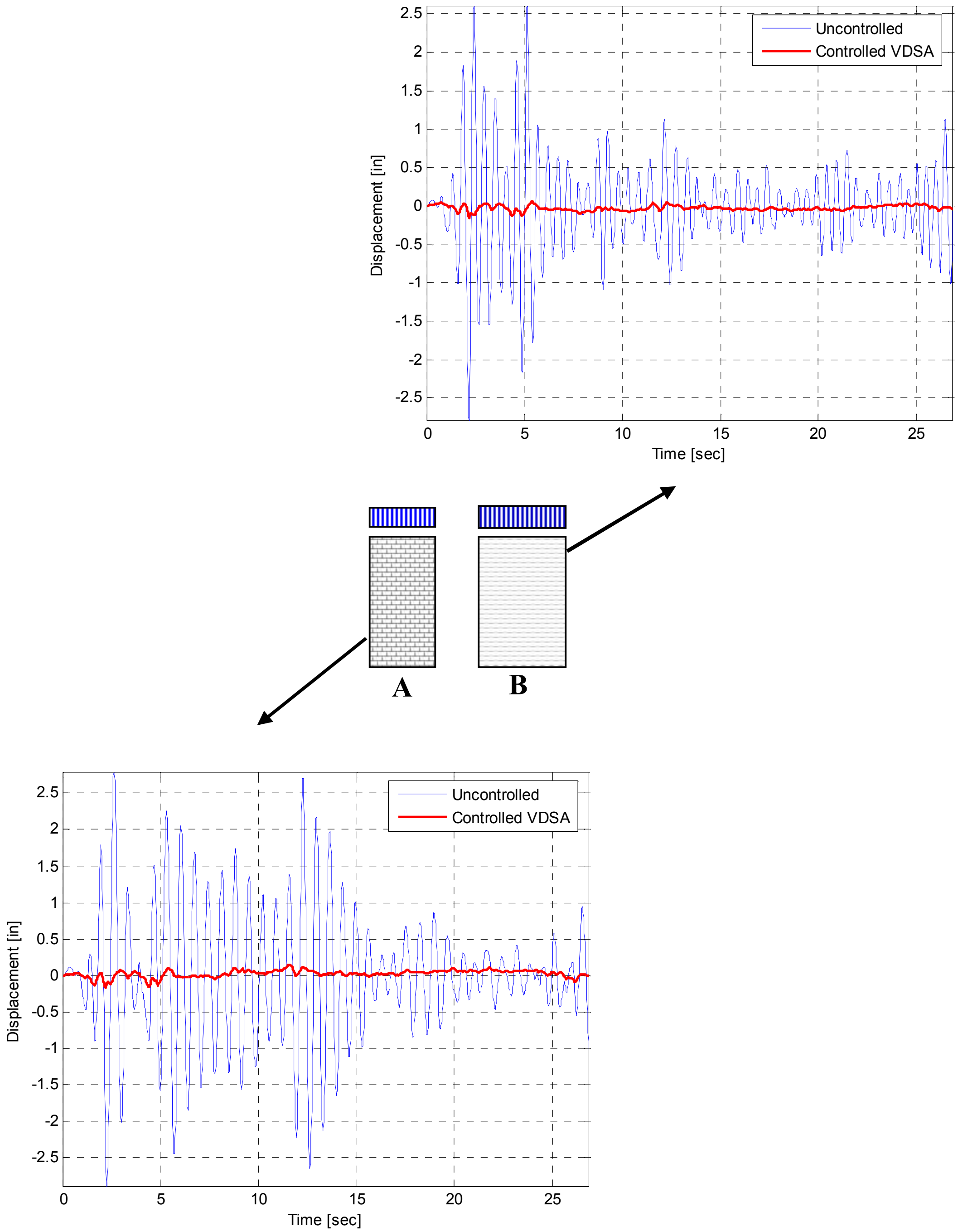


Figure 8.5: Relative displacement of the fifth floor of structures A and B for the El Centro record - Uncontrolled vs. VDSA (*Closed-loop control*)

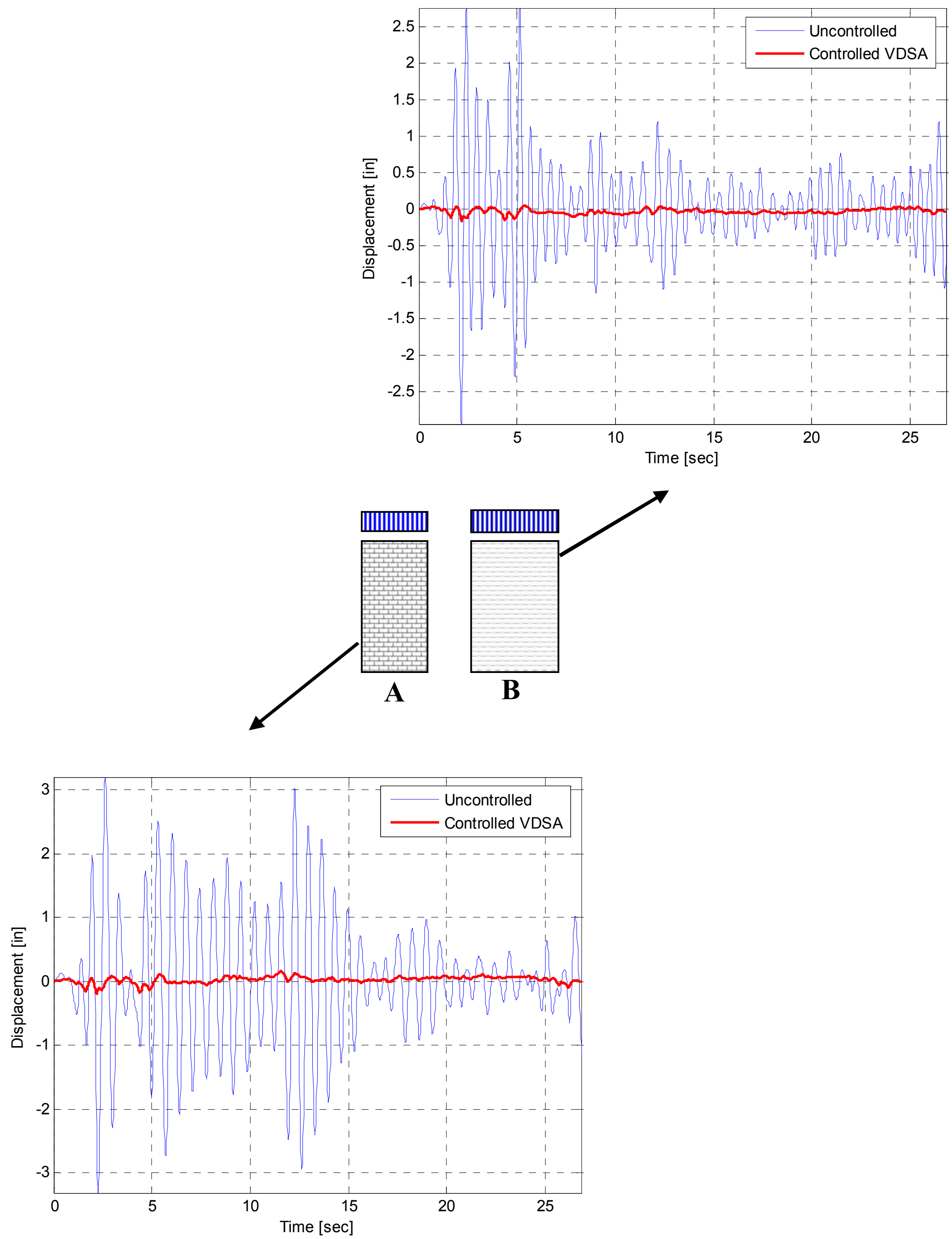


Figure 8.6: Relative displacement of the sixth floor of structures A and B for the El Centro record - Uncontrolled vs. VDSA (*Closed-loop control*)

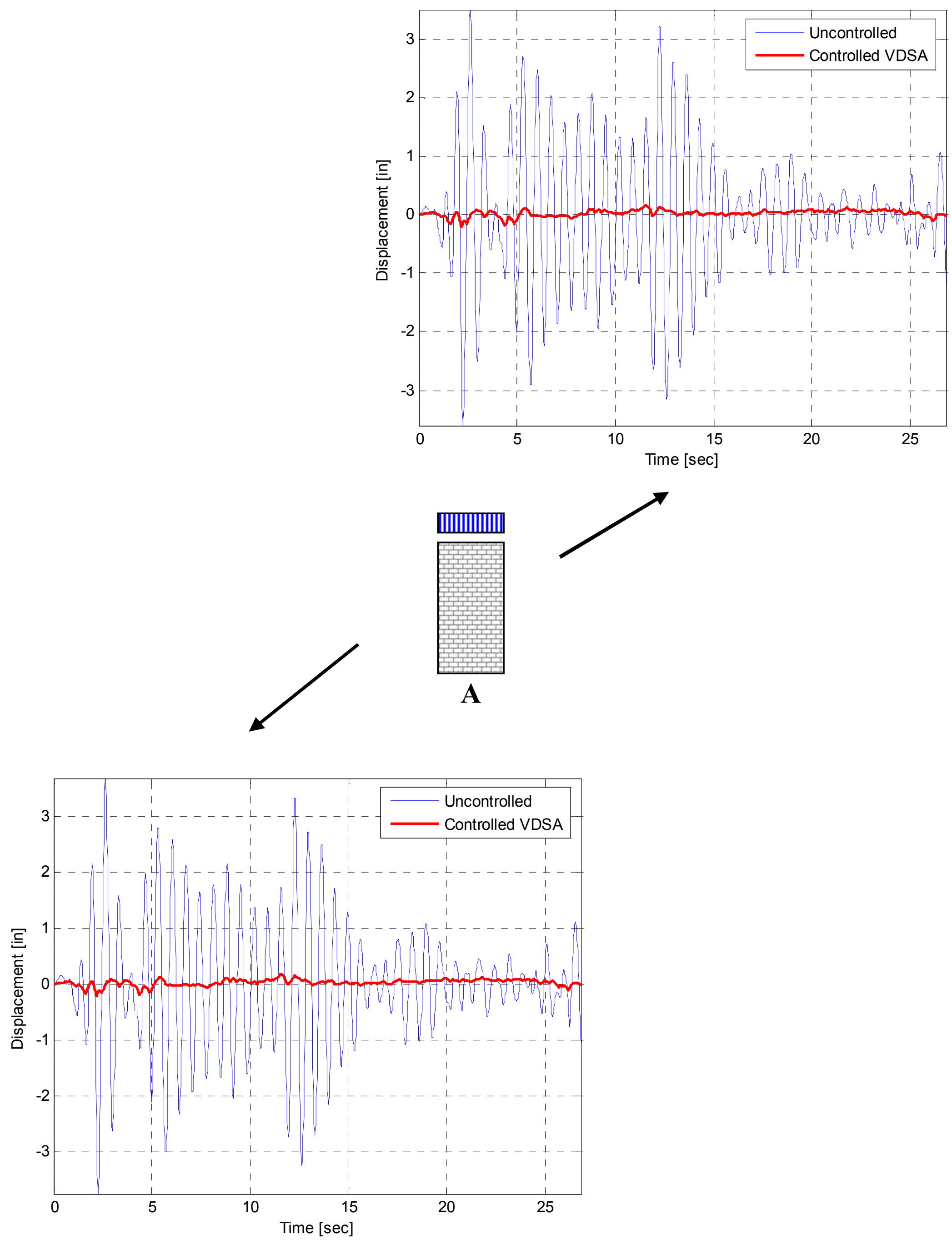


Figure 8.7: Relative displacement of the seventh and eighth floors of structure A for the El Centro record - Uncontrolled vs. VDSA (*Closed-loop control*)

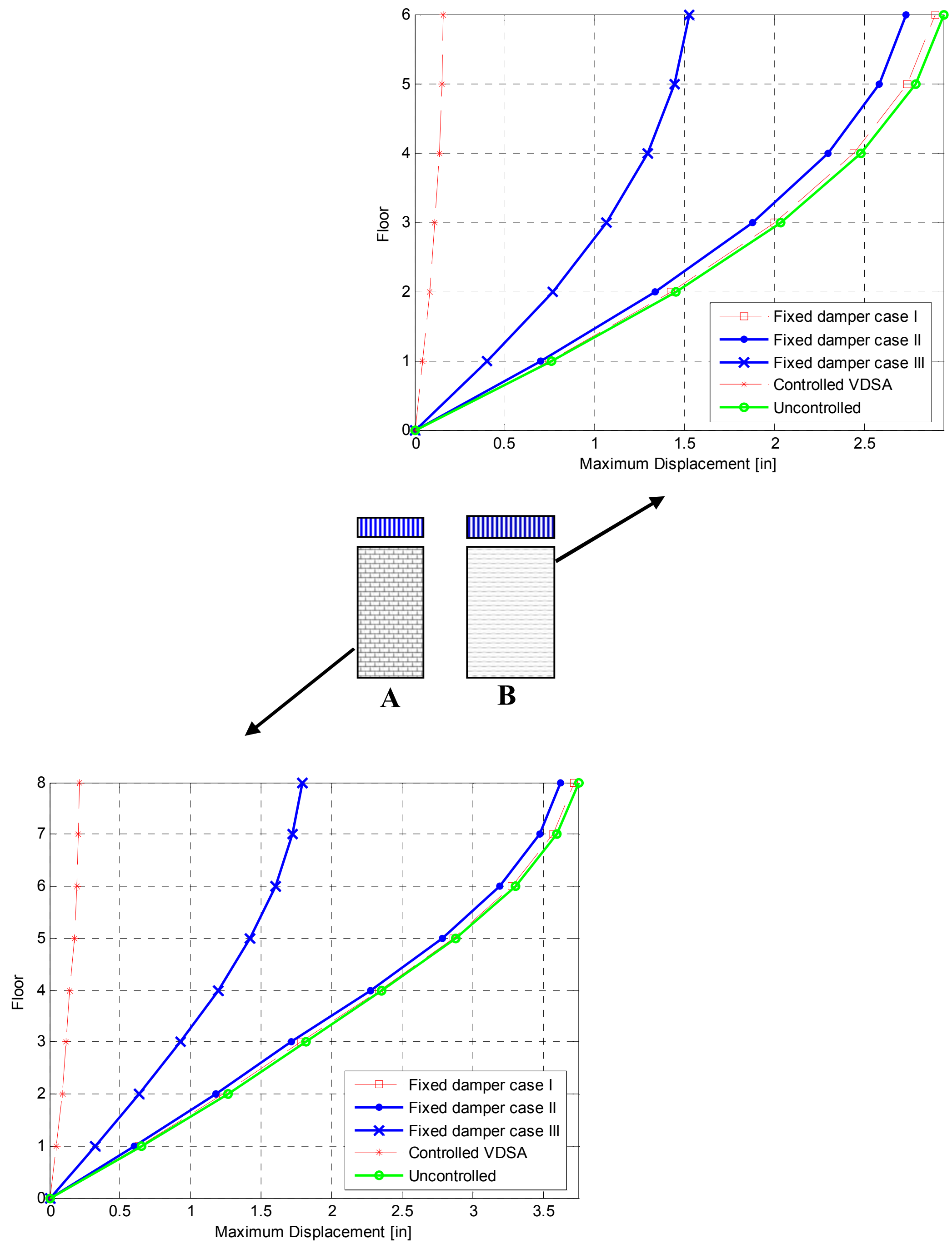


Figure 8.8: Maximum floor displacements of structures A and B for the El Centro record (*Closed-loop control*)

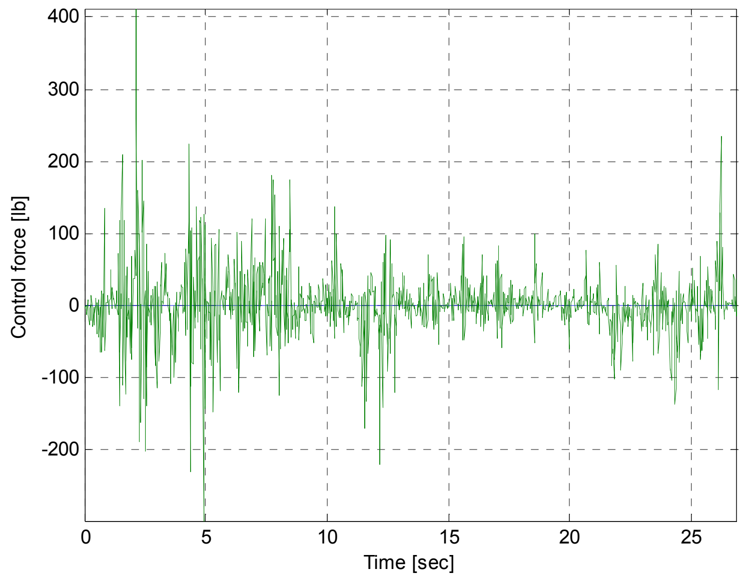


Figure 8.9: Control force in the coupled structures A and B for the El Centro record (*Closed-loop control*)

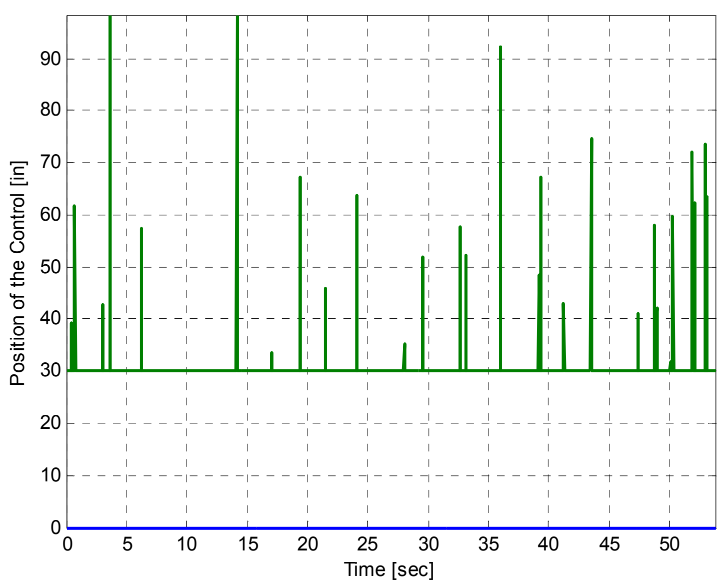


Figure 8.10: Variation of the position of the VDSA device in the coupled structures A and B for the El Centro record (*Closed-loop control*)

Table 8.3 shows a summary of the maximum relative displacements of structures **A** and **B** displayed in Figure 8.8, were the modified *Closed-loop control* algorithm ***Qv*** was used to control the **VDSA** device.

Table 8.3: Maximum displacements of the MDOF structures A and B without control, with passive and with the semiactive system for the El Centro record (*Closed-loop control*)

Floor	Displacement				
	Uncont.	Fixed damper Case I	Fixed damper Case II	Fixed damper Case III	VDSA
	[in*]	[in*]	[in*]	[in*]	[in*]
Structure A					
8th	3.7497	3.7220	3.6213	1.7939	0.2126
7th	3.6002	3.5735	3.4764	1.7254	0.2060
6th	3.3063	3.2816	3.1924	1.6017	0.1928
5th	2.8822	2.8612	2.7842	1.4245	0.1729
4th	2.3555	2.3401	2.2794	1.1931	0.1464
3rd	1.8228	1.7824	1.7188	0.9285	0.1199
2nd	1.2639	1.2326	1.1771	0.6365	0.0867
1st	0.6483	0.6319	0.6047	0.3233	0.0467
Structure B					
6th	2.9392	2.8943	2.7316	1.5288	0.1589
5th	2.7857	2.7423	2.5863	1.4481	0.1522
4th	2.4832	2.4427	2.3005	1.2961	0.1388
3rd	2.0359	2.0004	1.8807	1.0689	0.1142
2nd	1.4544	1.4273	1.3400	0.7692	0.0829
1st	0.7622	0.7468	0.7001	0.4054	0.0448

*1 in = 2.54 cm.

Table 8.4 presents a summary of the maximum total base shear for the all cases studied.

Table 8.4: Maximum base shear of the MDOF structures A and B without control, with passive dampers and with the semiactive system for the El Centro record (*Closed-loop control*)

Earthquake	Total base shear				
	Uncont.	Fixed damper Case I	Fixed damper Case II	Fixed damper Case III	VDSA
	[kip+]	[kip+]	[kip+]	[kip+]	[kip+]
Structure A					
El Centro	6891.1	6717.5	6427.6	3436.7	496.5
Structure B					
El Centro	8102.4	7938.5	7441.8	4309.9	476.5

+1 kip = 4.448 kN.

8.2.1.b RESPONSE TO THE SAN FERNANDO RECORD

The next set of figures corresponds to the results obtained with the San Fernando record. The relative displacement computed for the uncontrolled structures is compared with the structures controlled with the **VDSA** device in Figures 8.11 to 8.17. These comparisons were made for all the floors. Figure 8.18 shows the maximum relative displacement in each one of the floors obtained in the five cases studied. Note that among the three fixed dampers scheme, the configuration III (dampers in all floors) achieve the best results, as it could be expected. Even when compared with this most favorable case, the proposed semiactive system is able to further reduce the displacements. Figure 8.19 shows the control force time history whose maximum value was 2.512 kip. Figure 8.20 shows the height variation of the lower end of the **VDSA** device. The minimum height ($w_{\min}$) for the control was set equal to 30 in.

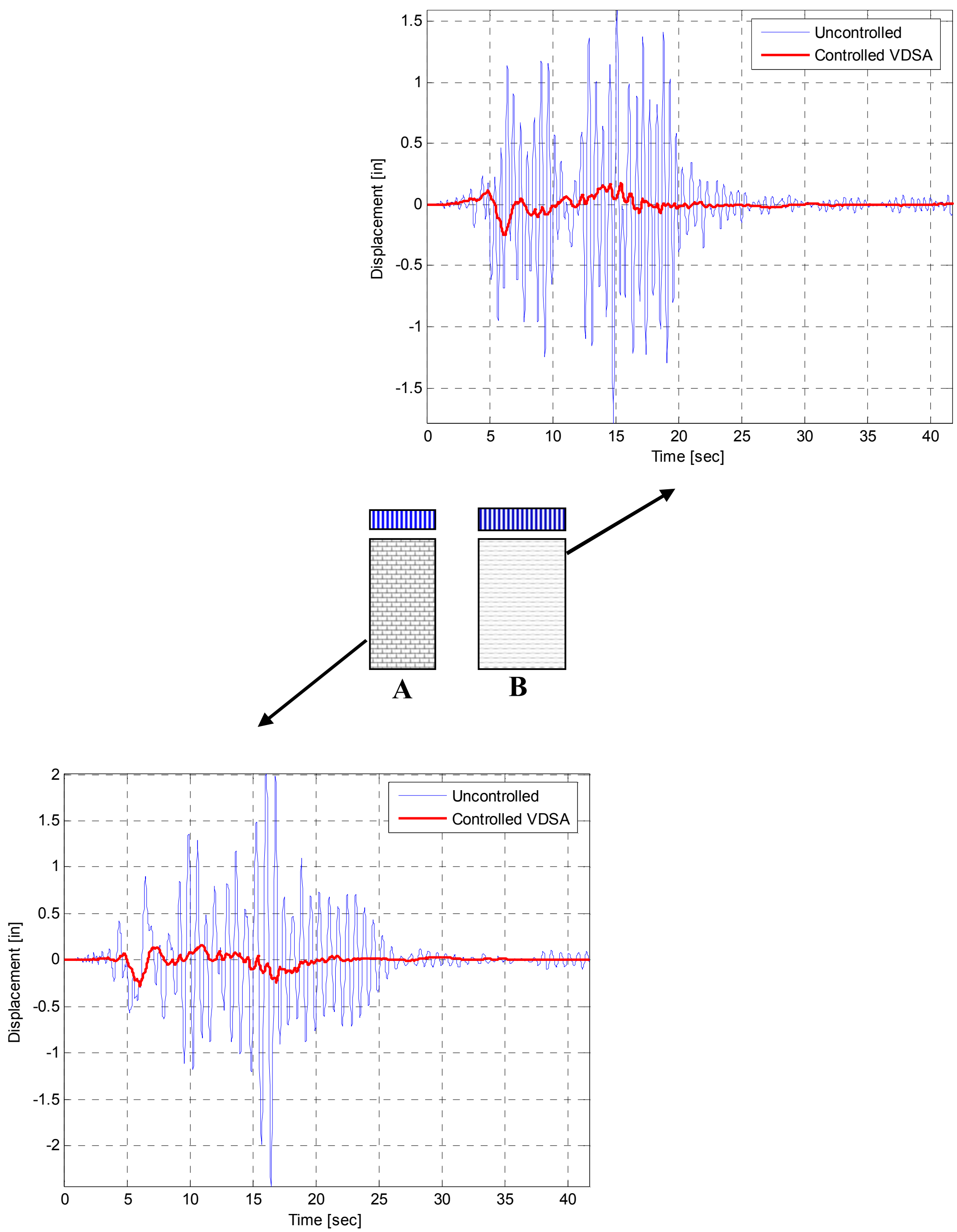


Figure 8.11: Relative displacement of the first floor of structures A and B for the San Fernando record - Uncontrolled vs. VDSA (*Closed-loop control*)

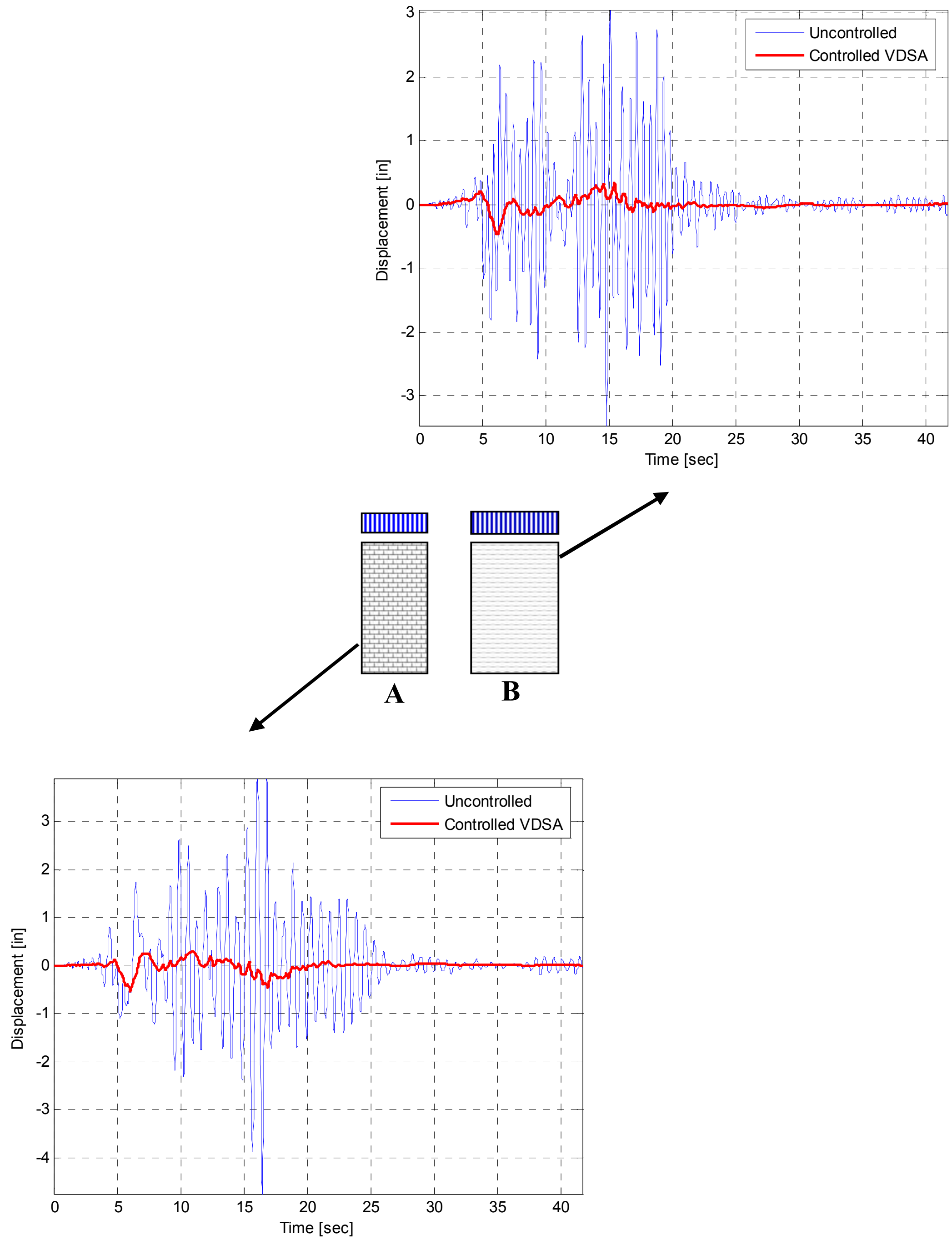


Figure 8.12: Relative displacement of the second floor of structures A and B for the San Fernando record - Uncontrolled vs. VDSA (*Closed-loop control*)

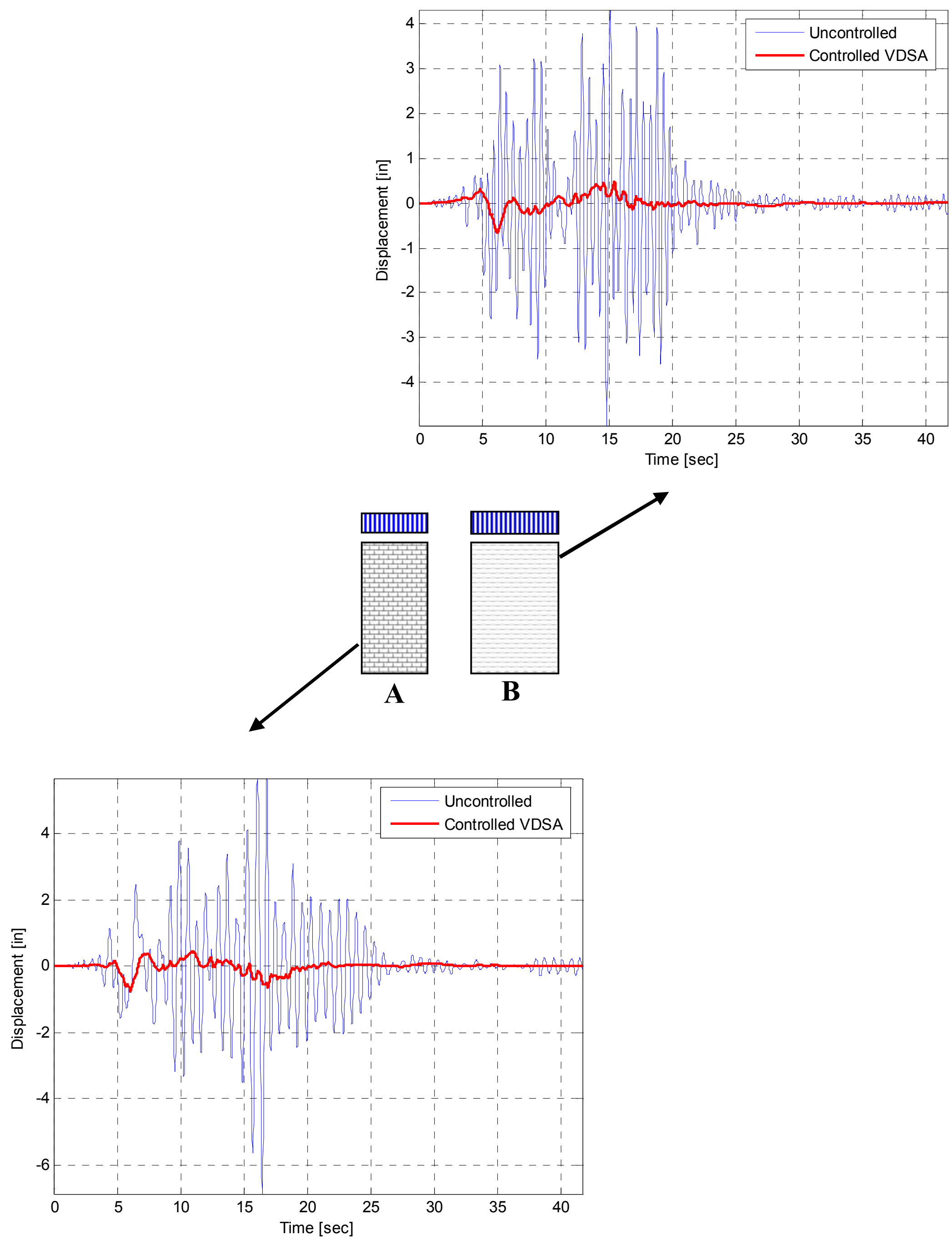


Figure 8.13: Relative displacement of the third floor of structures A and B for the San Fernando record - Uncontrolled vs. VDSA (*Closed-loop control*)

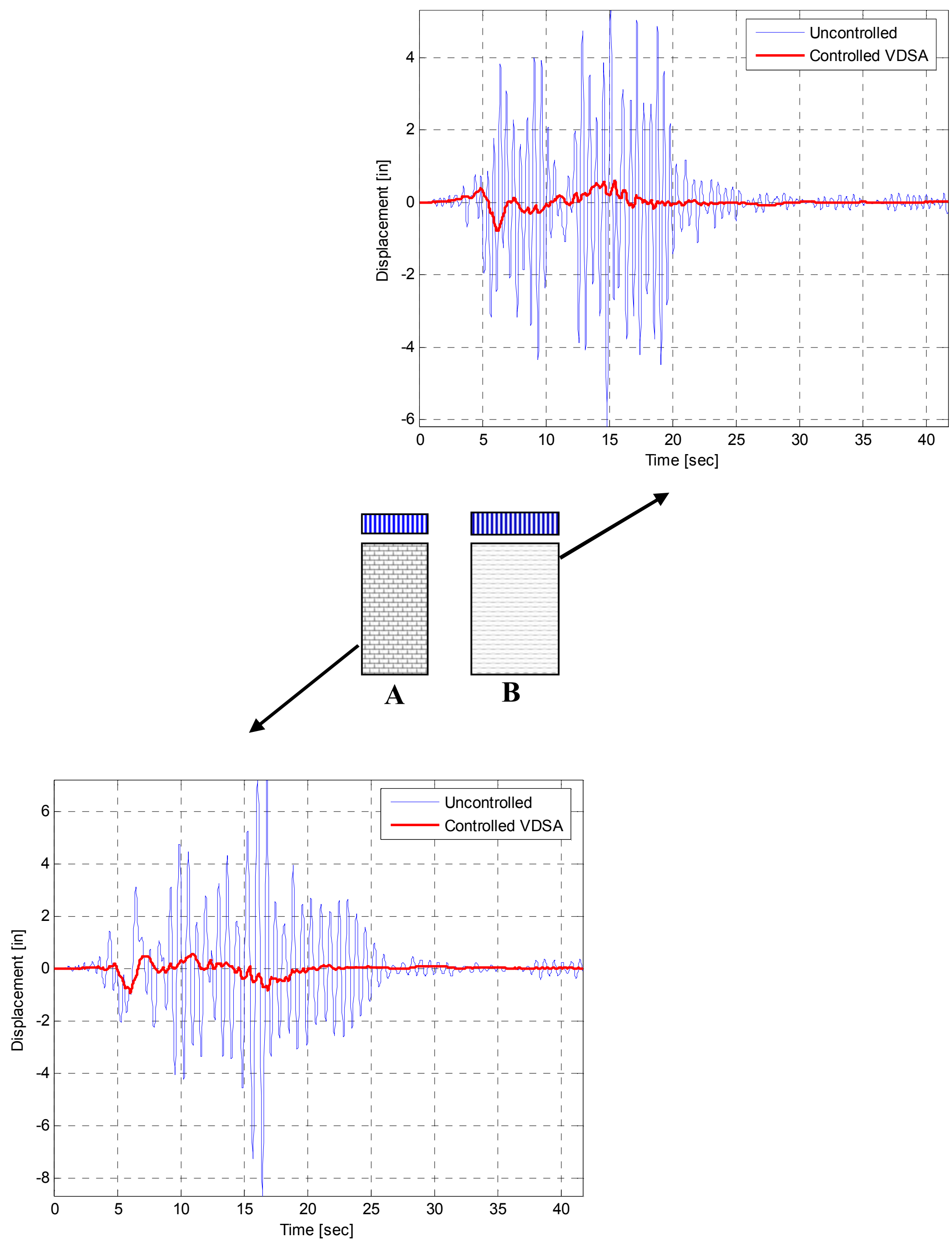


Figure 8.14: Relative displacement of the fourth floor of structures A and B for the San Fernando record - Uncontrolled vs. VDSA (*Closed-loop control*)

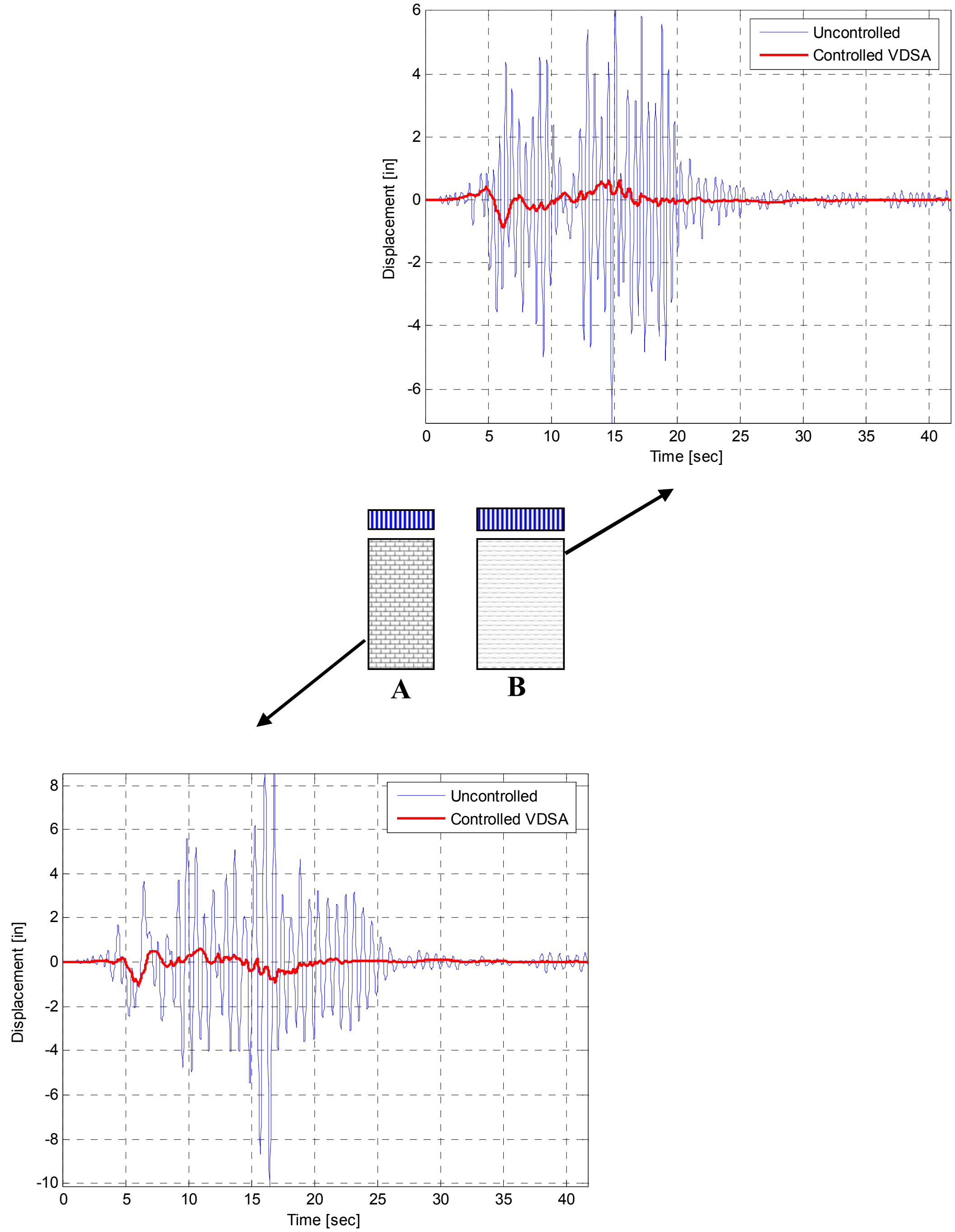


Figure 8.15: Relative displacement of the fifth floor of structures A and B for the San Fernando record - Uncontrolled vs. VDSA (*Closed-loop control*)

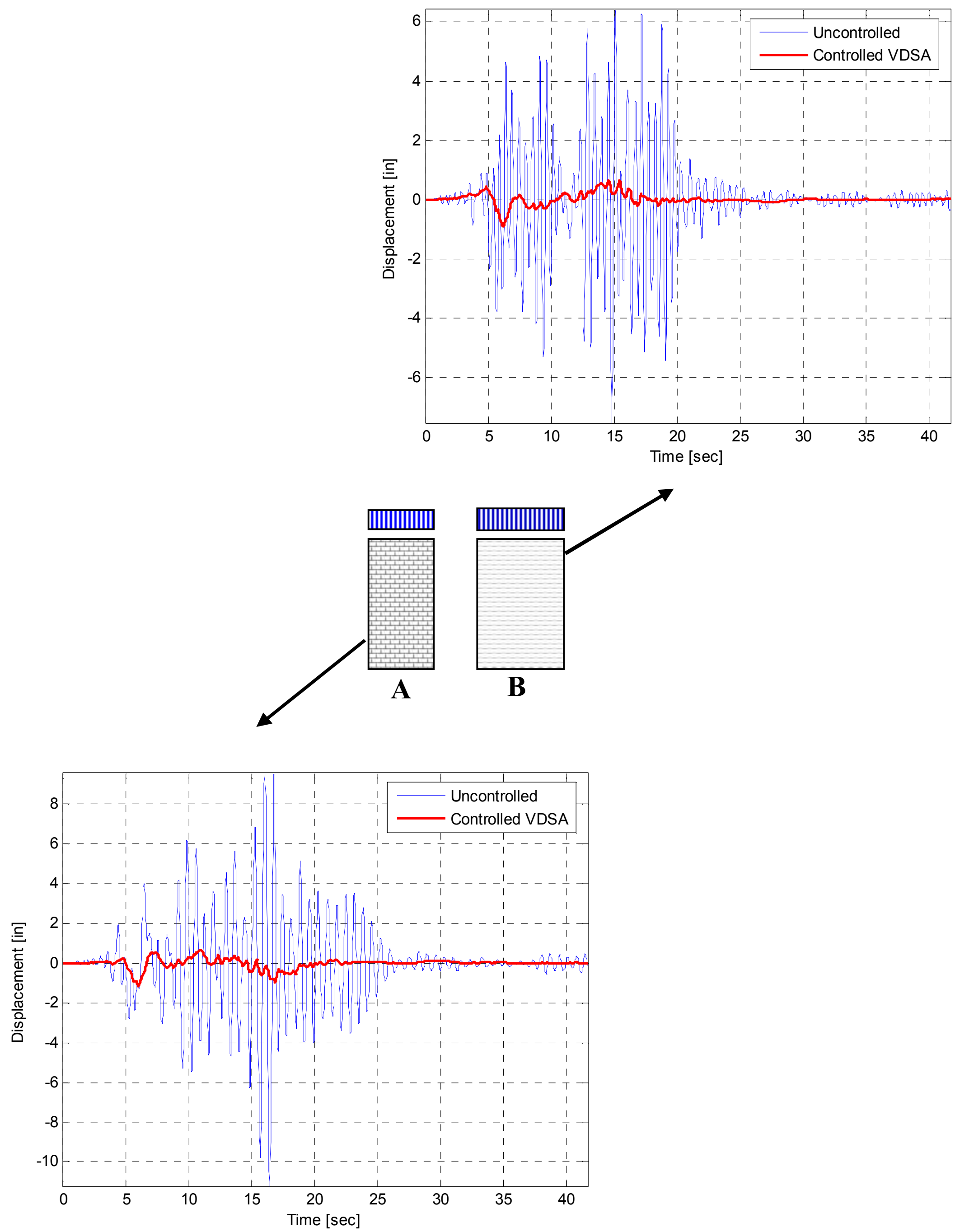


Figure 8.16: Relative displacement of the sixth floor of structures A and B for the San Fernando record - Uncontrolled vs. VDSA (*Closed-loop control*)

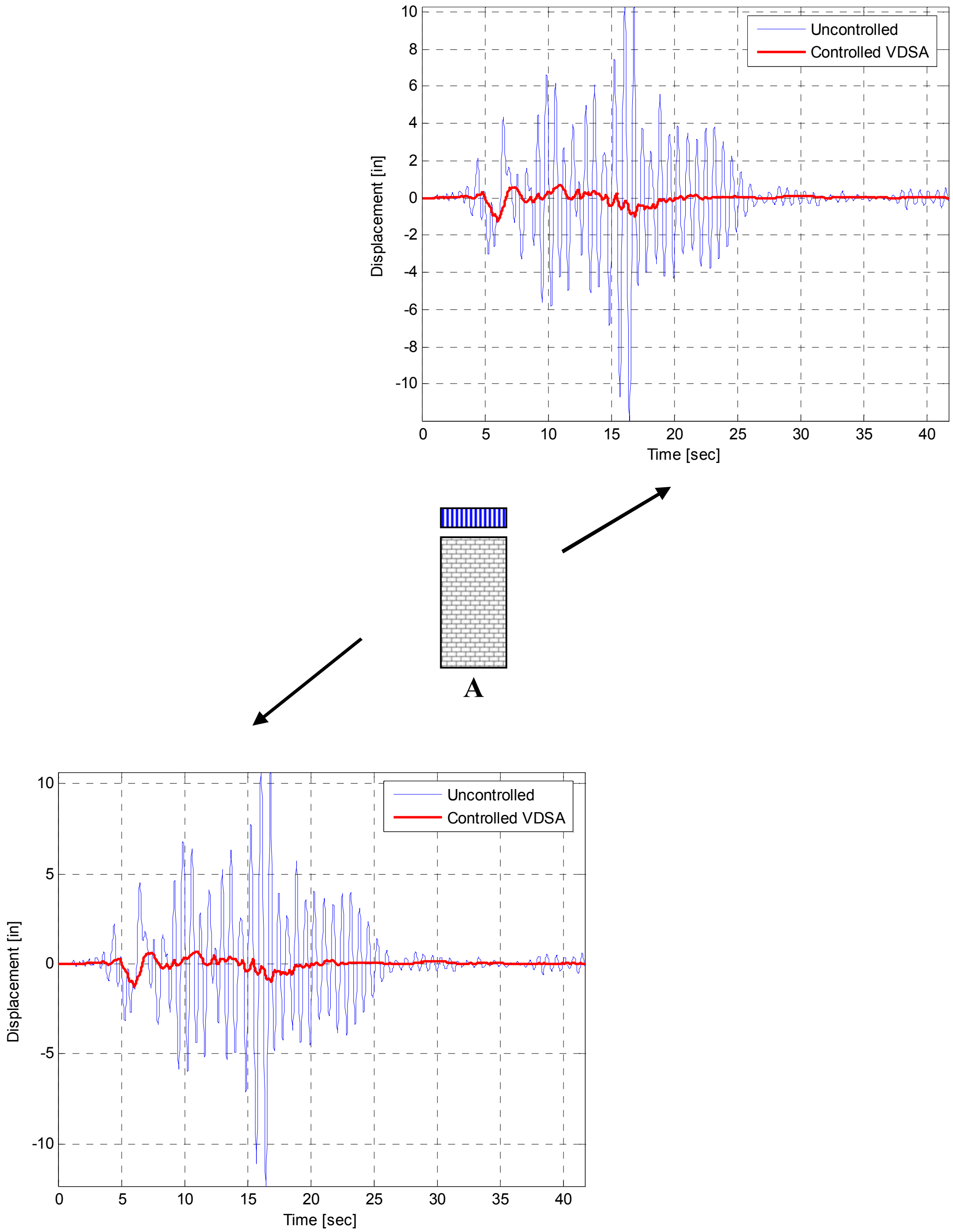


Figure 8.17: Relative displacement of the seventh and eighth floor of structure A for the San Fernando record - Uncontrolled vs. VDSA (*Closed-loop control*)

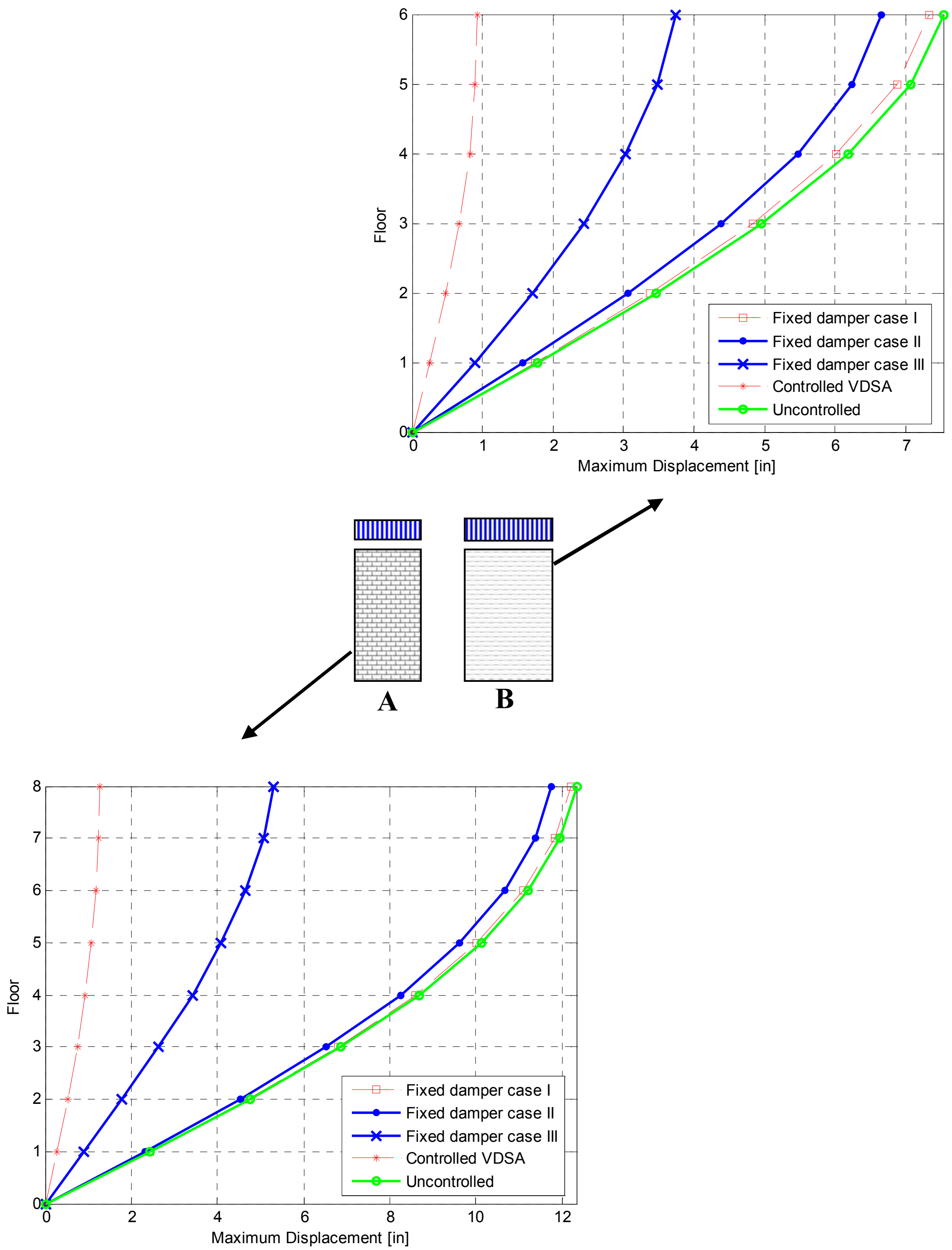


Figure 8.18: Maximum floor displacements of structures A and B for the San Fernando record (*Closed-loop control*)

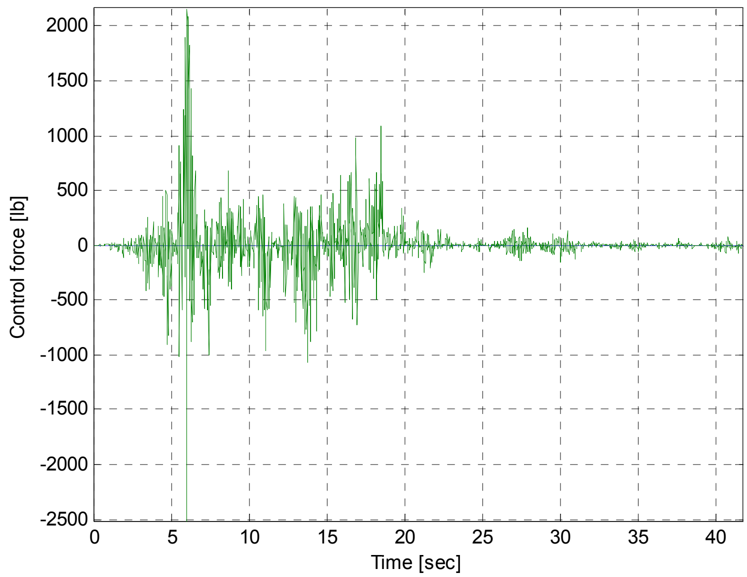


Figure 8.19: Control force in the coupled structures A and B for the San Fernando record (*Closed-loop control*)

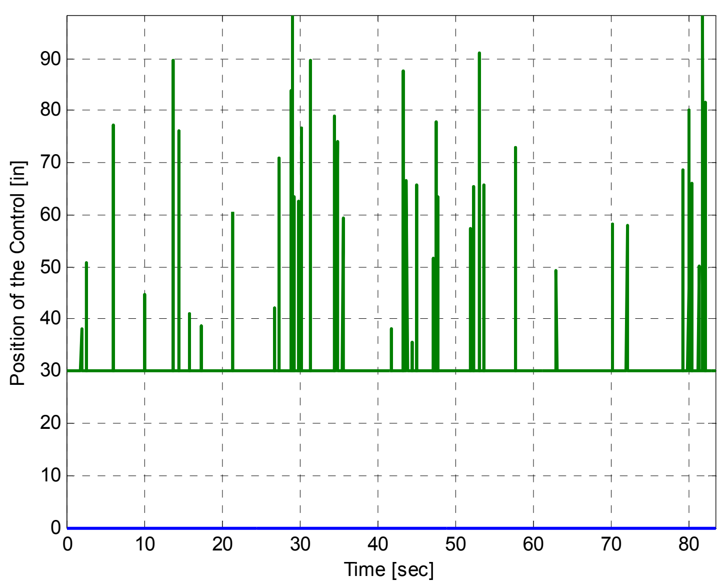


Figure 8.20: Variation of the position of the VDSA device in the coupled structures A and B for the San Fernando record (*Closed-loop control*)

Table 8.5 summarizes the maximum relative displacement displayed in Figure 8.18 for the two multi degree of freedom structures due to the San Fernando record. Again, the results in Table 8.5 demonstrates the benefits obtained by using the **VDSA** device with the modified *Closed-loop control* algorithm ***Qv*** in the reduction of the seismic response. Notice that the displacements are reduced by practically one order of magnitude thanks to the implementation of the semiactive device. For the top floor of building **A** the peak displacement decreased by 89.6% and for the sixth floor of building **B** the reduction was 87.7%.

Table 8.5: Maximum displacements of the MDOF structures A and B without control, with passive dampers and with the semiactive system for the San Fernando record (*Closed-loop control*)

Floor	Displacement				
	Uncont.	Fixed damper Case I	Fixed damper Case II	Fixed damper Case III	VDSA
	[in*]	[in*]	[in*]	[in*]	[in*]
Structure A					
8th	12.3300	12.2023	11.7388	5.2957	1.2771
7th	11.9537	11.8307	11.3824	5.0695	1.2415
6th	11.2169	11.1016	10.6718	4.6531	1.1701
5th	10.1276	10.0223	9.6270	4.0899	1.0631
4th	8.6762	8.5856	8.2460	3.4111	0.9205
3rd	6.8691	6.7995	6.5291	2.6346	0.7445
2nd	4.7590	4.7137	4.5240	1.7837	0.5320
1st	2.4368	2.4160	2.3183	0.9036	0.2839
Structure B					
6th	7.5278	7.3209	6.6504	3.7308	0.9238
5th	7.0656	6.8728	6.2438	3.4850	0.8882
4th	6.1855	6.0177	5.4672	3.0347	0.8169
3rd	4.9567	4.8232	4.3803	2.4325	0.6660
2nd	3.4634	3.3711	3.0570	1.7149	0.4797
1st	1.7855	1.7377	1.5753	0.8971	0.2577

*1 in = 2.54 cm.

Table 8.6 presents a summary of the maximum total base shear for the all cases. The reduction achieved by the **VDSA** device is about the same for both structures: 89.4% and 84.1% for building **A** and **B**, respectively.

Table 8.6: Maximum base shear of the MDOF structures A and B without control, with passive dampers and with the semiactive system for the San Fernando record (*Closed-loop control*)

Earthquake	Total base shear				
	Uncont.	Fixed damper Case I	Fixed damper Case II	Fixed damper Case III	VDSA
	[kip+]	[kip+]	[kip+]	[kip+]	[kip+]
Structure A					
San Fernando	25903.0	25682.0	24643.0	9605.2	2739.3
Structure B					
San Fernando	18980.0	18471.0	16746.0	9535.6	3017.7

+1 kip = 4.448 kN.

8.2.1.c RESPONSE TO THE FRIULI RECORD

The final set of results obtained by controlling the **VDSA** device with the *Closed-loop control* scheme is the response to the 1976 Friuli accelerogram. Again, the responses compared are the relative displacements of the masses of all floors for the structures in uncontrolled mode and controlled with the **VDSA** device. The results are presented in Figures 8.21 to 8.27 for the first twenty seconds of the ground motion. Figure 8.28 shows the variation with height of the maximum relative displacement for the five cases considered. Figures 8.29 and 8.30 show, respectively, the control force time history and the variation of the position of the control during the earthquake. The maximum force required from the mechanism was 0.576 kip. The minimum position for the control was 30 in.

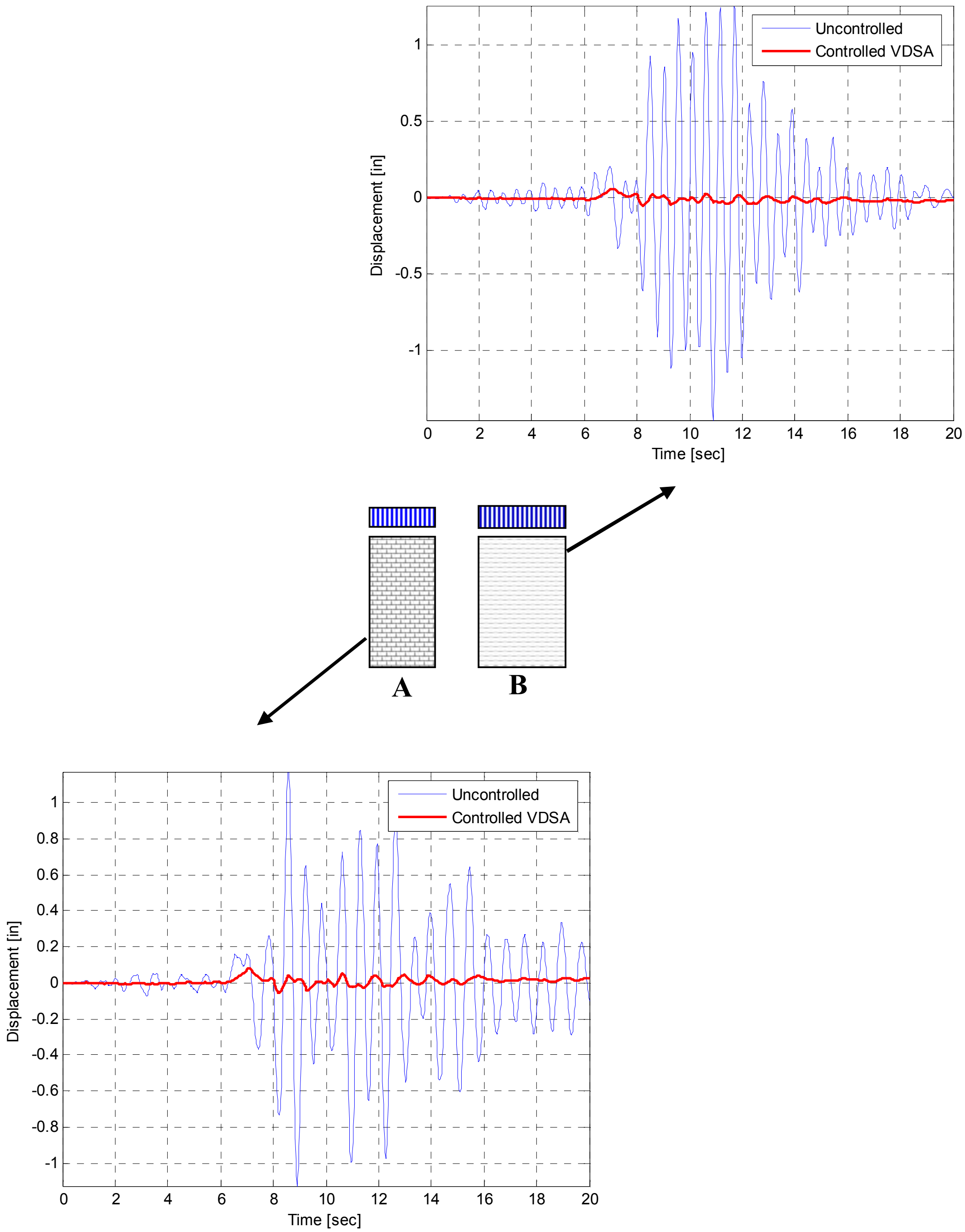


Figure 8.21: Relative displacement of the first floor of structures A and B for the Friuli record - Uncontrolled vs. VDSA (*Closed-loop control*)

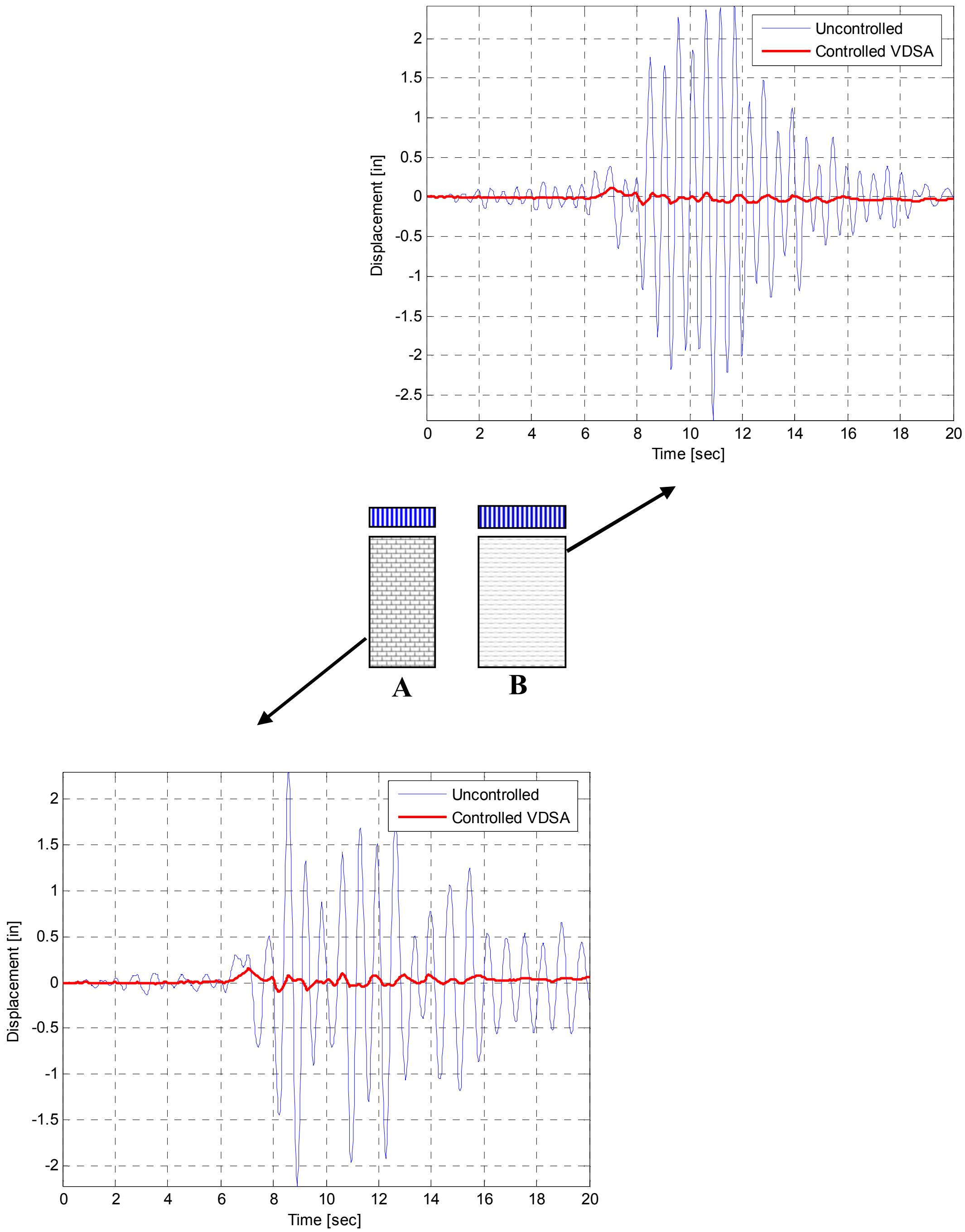


Figure 8.22: Relative displacement of the second floor of structures A and B for the Friuli record - Uncontrolled vs. VDSA (*Closed-loop control*)

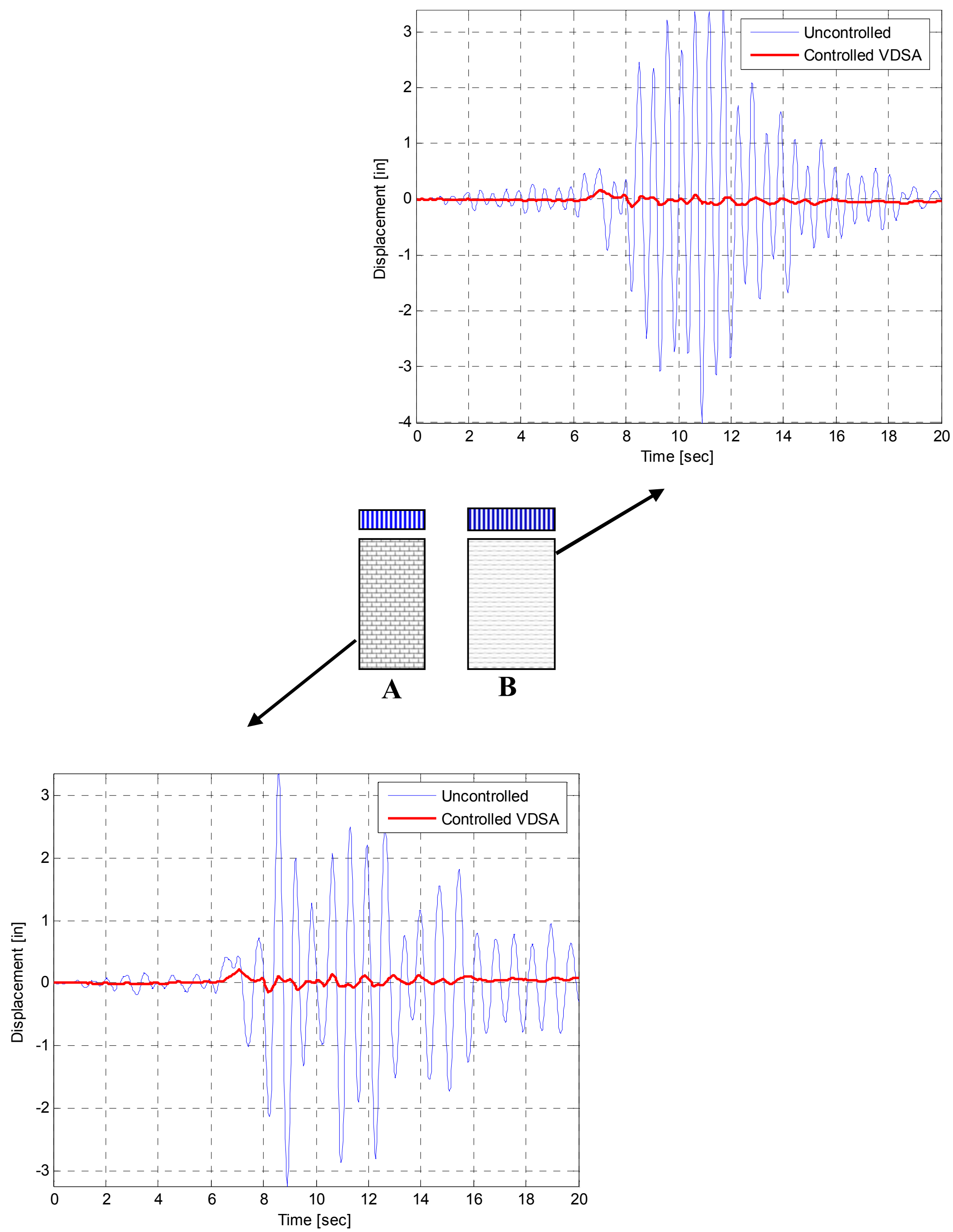


Figure 8.23: Relative displacement of the third floor of structures A and B for the Friuli record - Uncontrolled vs. VDSA (*Closed-loop control*)

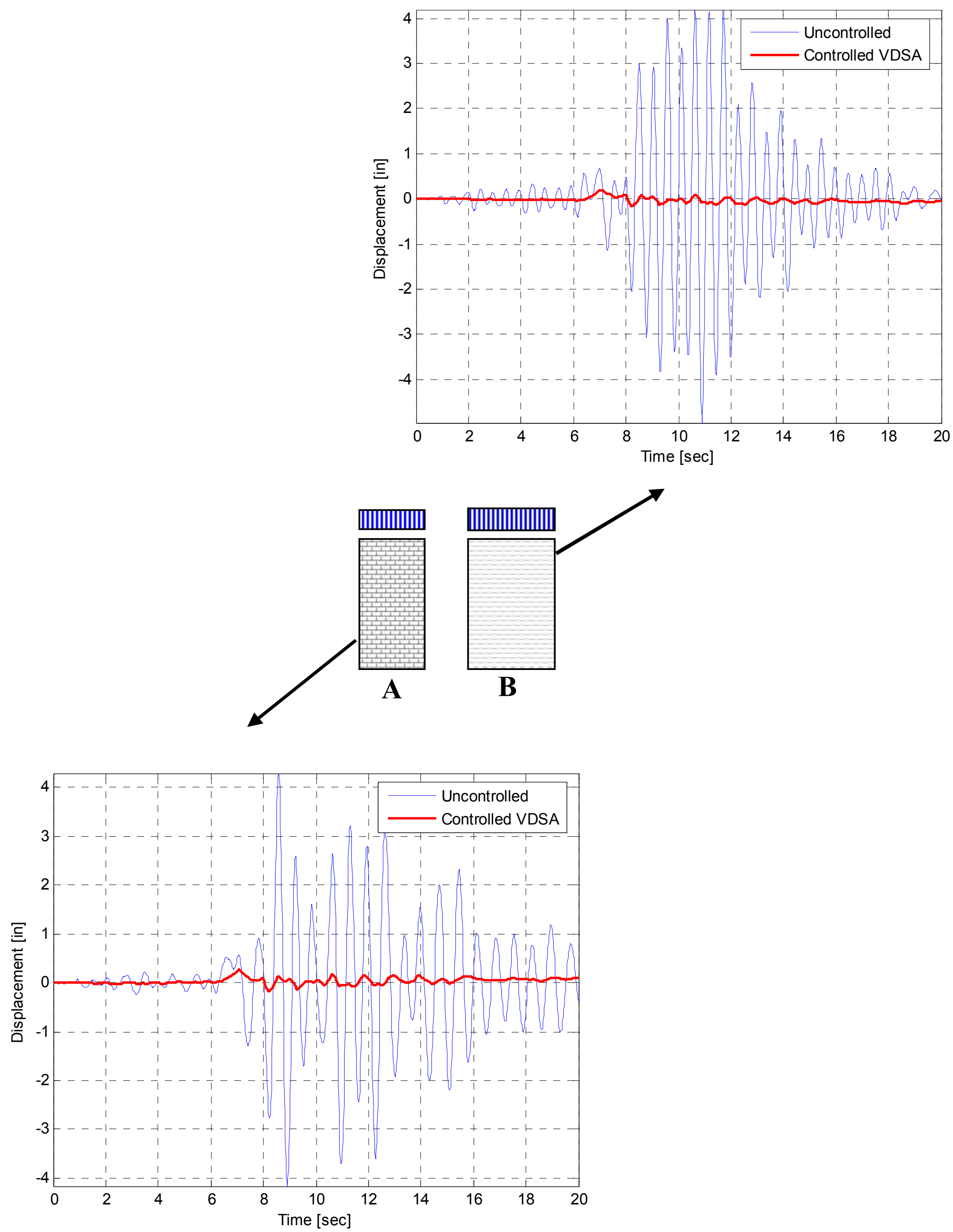


Figure 8.24: Relative displacement of the fourth floor of structures A and B for the Friuli record - Uncontrolled vs. VDSA (*Closed-loop control*)

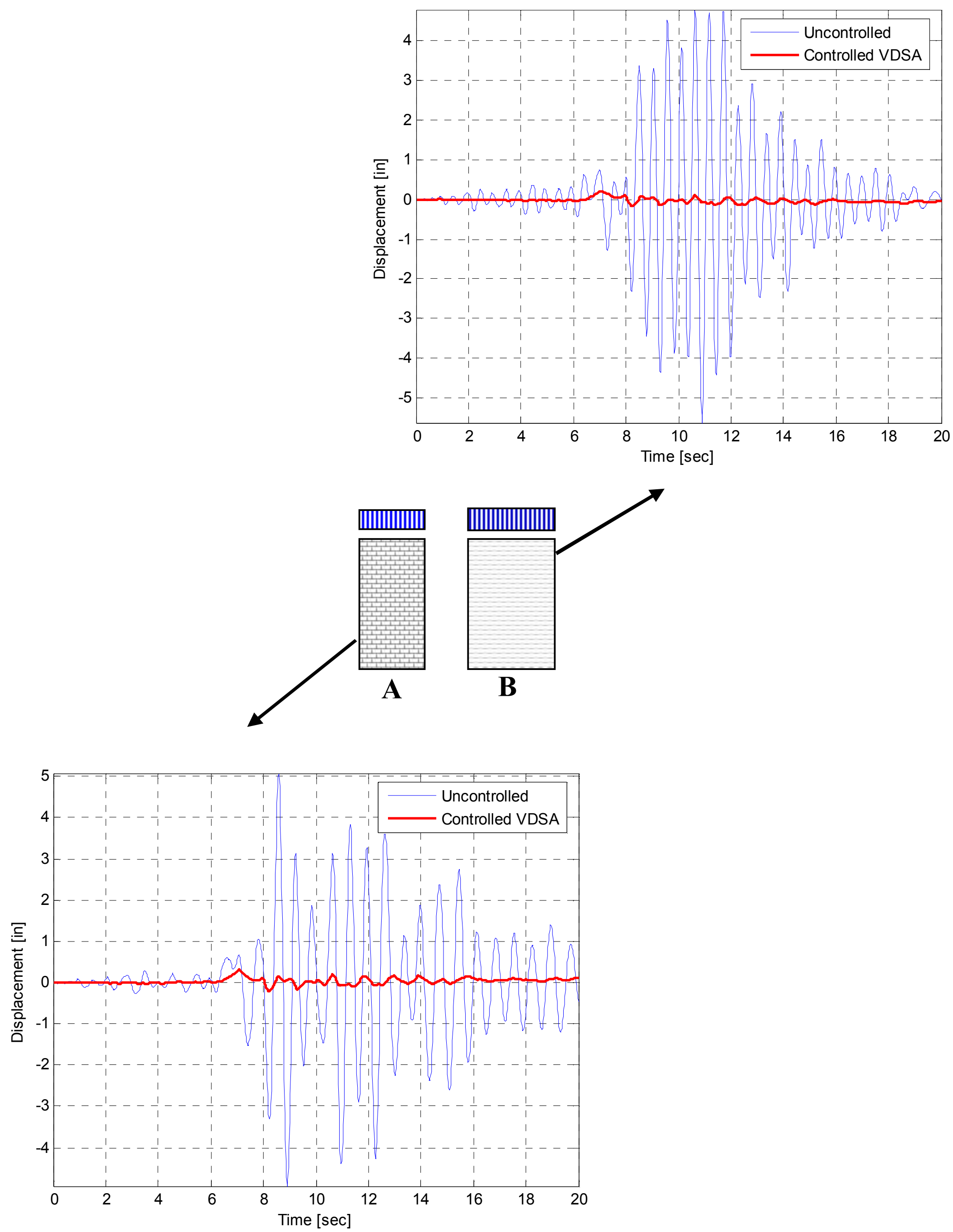


Figure 8.25: Relative displacement of the fifth floor of structures A and B for the Friuli record - Uncontrolled vs. VDSA (*Closed-loop control*)

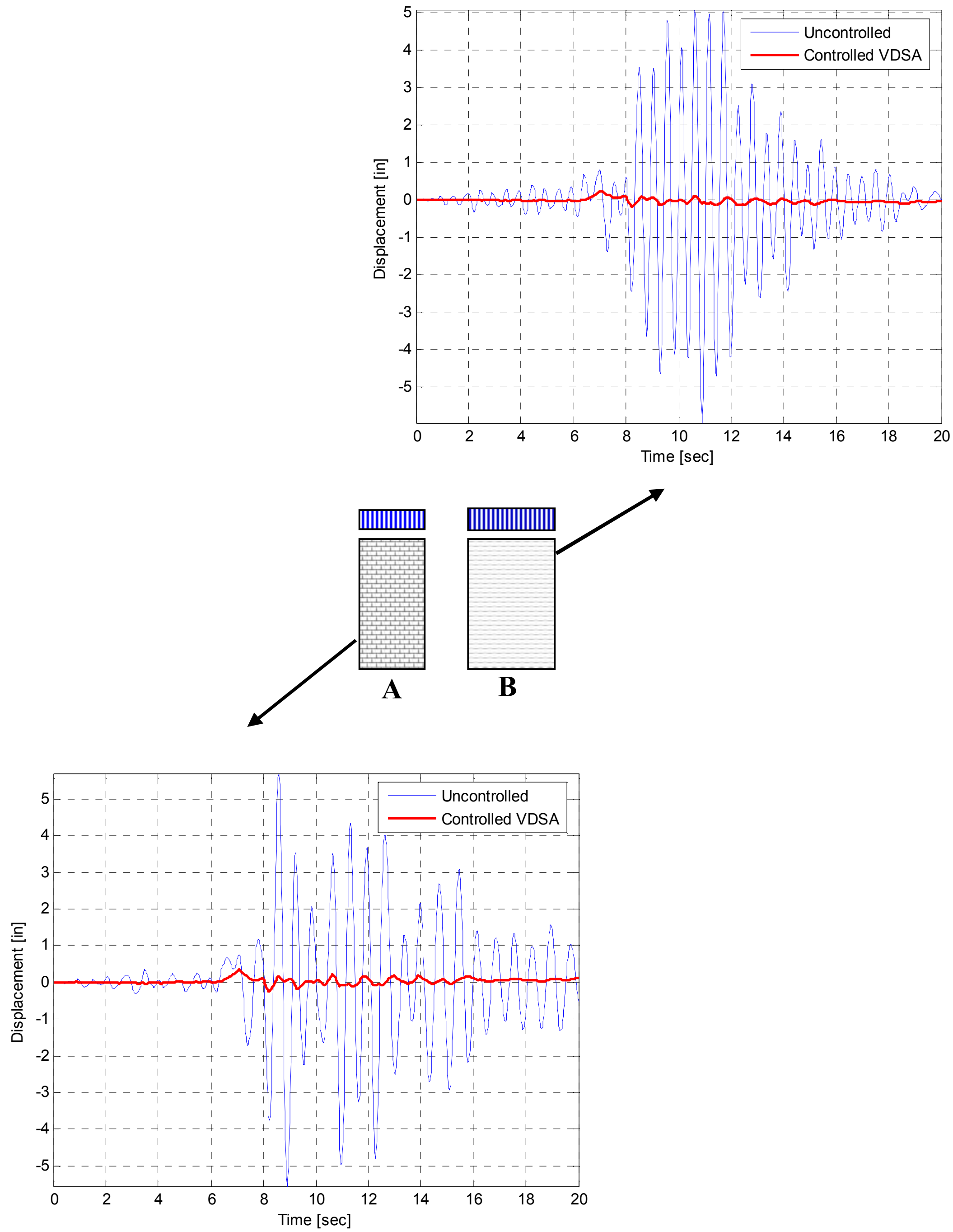


Figure 8.26: Relative displacement of the sixth floor of structures A and B for the Friuli record - Uncontrolled vs. VDSA (*Closed-loop control*)

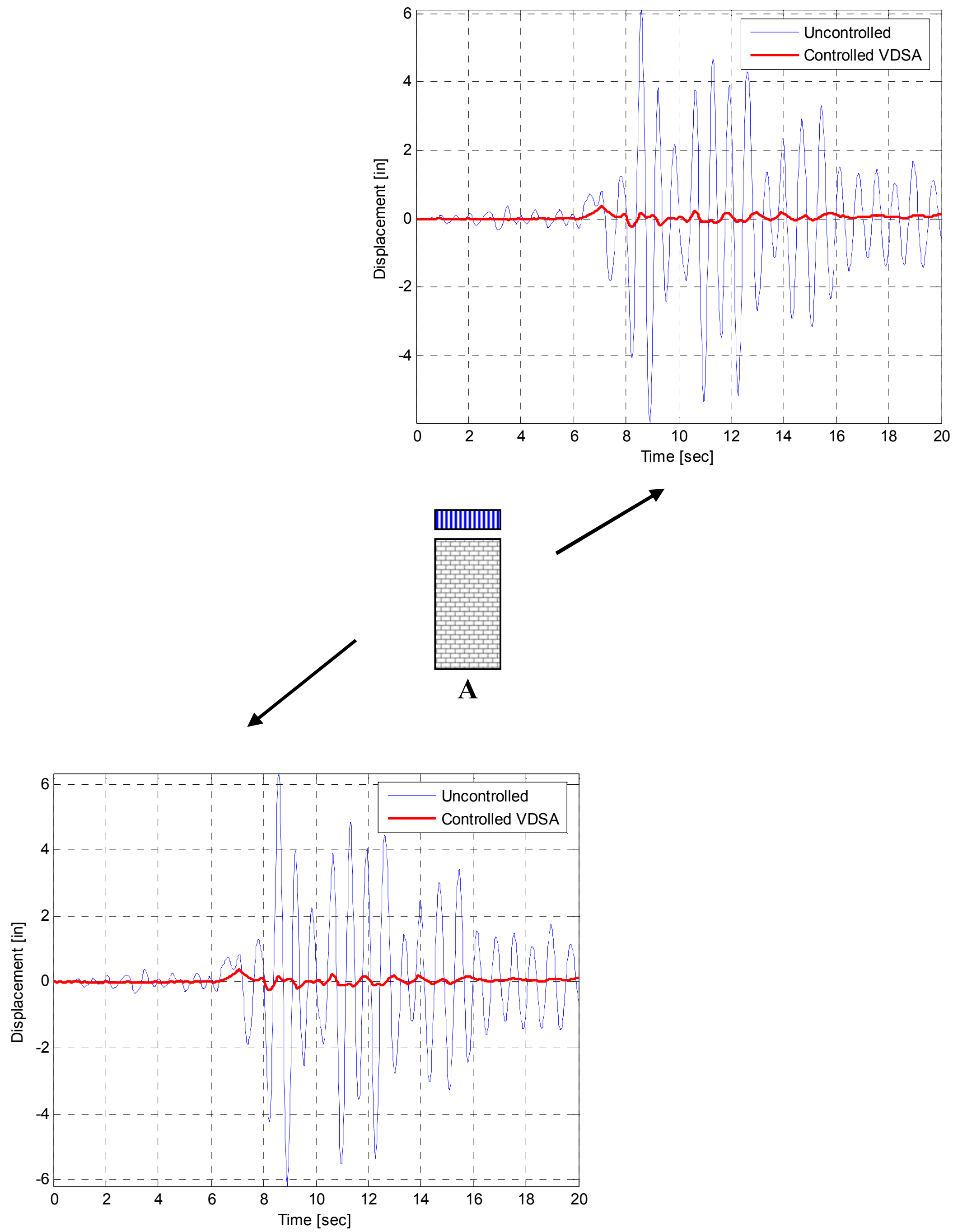


Figure 8.27: Relative displacement of the seventh and eighth floor of structure A for the Friuli record - Uncontrolled vs. VDSA (*Closed-loop control*)

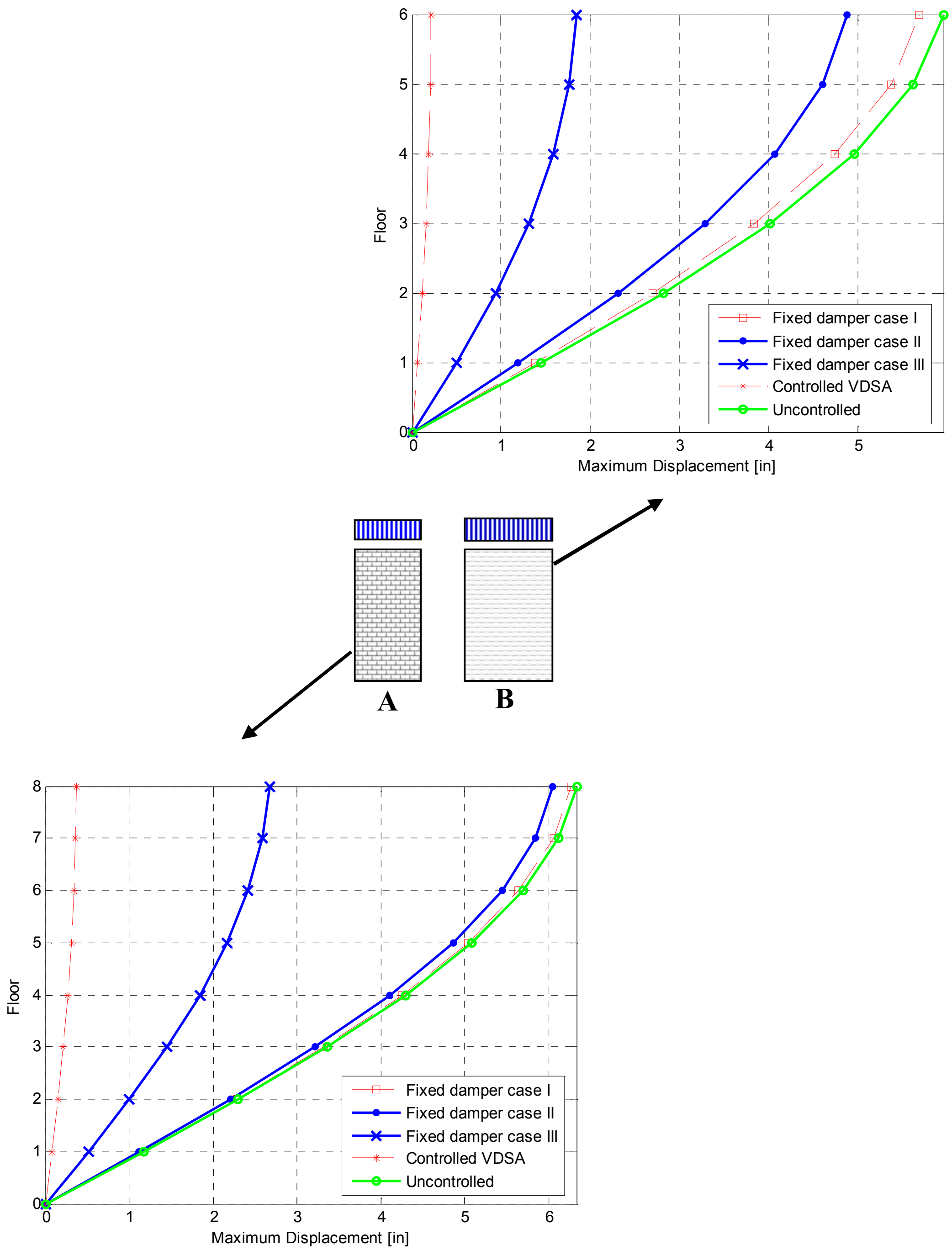


Figure 8.28: Maximum floor displacements of structures A and B for the Friuli record (*Closed-loop control*)

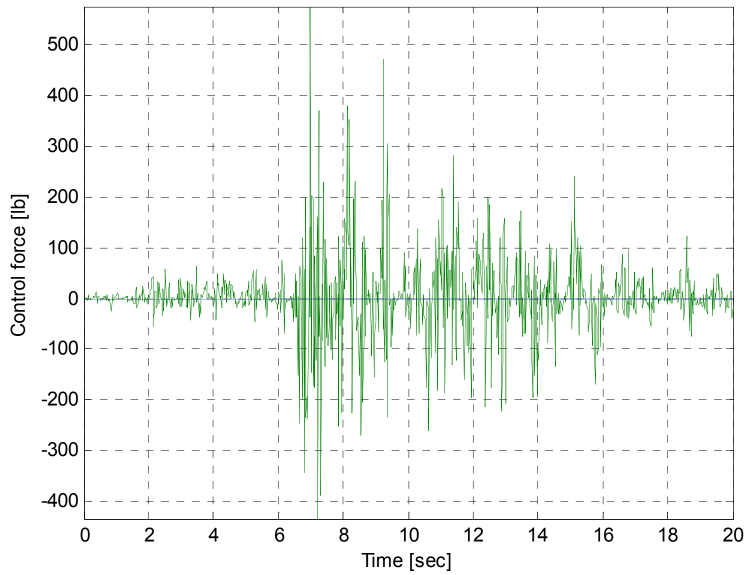


Figure 8.29: Control force in the coupled structures A and B for the Friuli record (*Closed-loop control*)

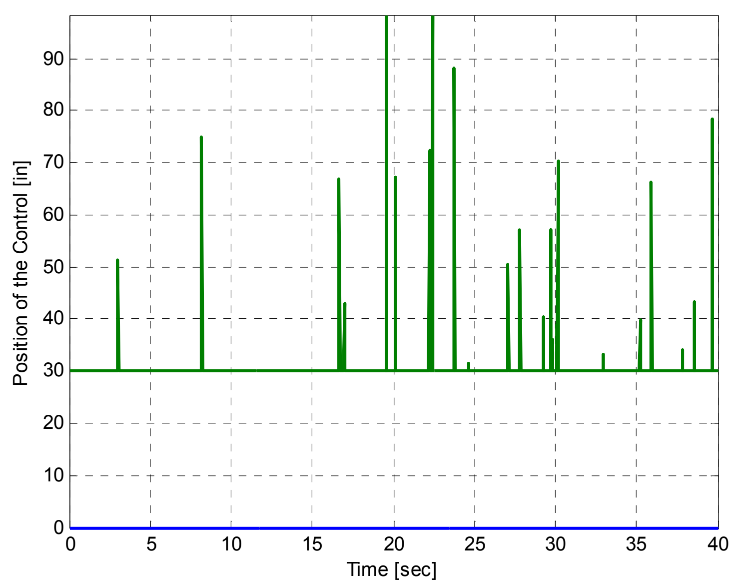


Figure 8.30: Variation of the position of the VDSA device in the coupled structures A and B for the Friuli record (*Closed-loop control*)

A summary of the results for the five cases analyzed and for the Friuli earthquake is presented in Table 8.7. These results correspond to those displayed in Figure 8.28. The peak displacement in the top floor of structure **A** decreased by 94.2% due to the application of the **VDSA** device. The reduction in the top floor of structure **B** was 96.4%.

Table 8.7: Maximum displacement of the MDOF structures A and B without control, with passive dampers and with the semiactive system for the Friuli record (*Closed-loop control*)

Floor	Displacement				
	Uncont.	Fixed damper Case I	Fixed damper Case II	Fixed damper Case III	VDSA
	[in*]	[in*]	[in*]	[in*]	[in*]
Structure A					
8th	6.3319	6.2717	6.0518	2.6765	0.3701
7th	6.1178	6.0603	5.8488	2.5881	0.3599
6th	5.6963	5.6438	5.4482	2.4162	0.3397
5th	5.0809	5.0350	4.8619	2.1710	0.3092
4th	4.2919	4.2537	4.1085	1.8483	0.2686
3rd	3.3557	3.3262	3.2132	1.4561	0.2169
2nd	2.3040	2.2841	2.2067	1.0062	0.1548
1st	1.1726	1.1630	1.1233	0.5196	0.0825
Structure B					
6th	5.9511	5.6891	4.8770	1.8504	0.2162
5th	5.6169	5.3690	4.6024	1.7586	0.2061
4th	4.9605	4.7409	4.0633	1.5787	0.1859
3rd	4.0097	3.8317	3.2834	1.3069	0.1546
2nd	2.8128	2.6882	2.3041	0.9457	0.1131
1st	1.4521	1.3829	1.1844	0.5051	0.0616

*1 in = 2.54 cm.

A summary of the maximum total base shear for the five cases is displayed in Table 8.8 when the proposed semiactive is working, the peak base shear in structure **A** was diminished by 93% compared to the force in the original structure. For structure **B** the peak base shear decreased by 95.8%

Table 8.8: Maximum base shear of the 2-MDOF structure A and B without control and with passives and semiactive system for the Friuli record (*Closed-loop control*)

Earthquake	Total base shear				
	Uncont.	Fixed damper Case I	Fixed damper Case II	Fixed damper Case III	VDSA
	[kip+]	[kip+]	[kip+]	[kip+]	[kip+]
Structure A					
Friuli	12465.0	12363.0	11941.0	5522.8	876.8
Structure B					
Friuli	15436.0	14700.0	12590.0	5368.7	655.1

+1 kip = 4.448 kN.

8.2.2 APPLICATION OF THE MODIFIED *CLOSED-OPEN-LOOP CONTROL* ALGORITHM *Qv*

The maximum displacements in all the floors and the maximum total base shear obtained for the structures **A** and **B** obtained by using the modified *Closed-loop control* algorithm ***Qv*** were the same than those obtained with the modified *Closed-open-loop control* algorithm ***Qv***. Thus, they will not be presented.

8.2.3 DISCUSSION OF RESULTS

The response of the uncontrolled and controlled coupled multi-story buildings was compared in Table 8.3 to 8.8, in which the maximum relative displacements for all floor and the base shear force due to the El Centro, San Fernando and Friuli ground motion, are presented. Again, it is shown that the **VDSA** device and the two control algorithms proposed are effective in reducing both responses in coupled structures. To obtain the results presented in this chapter the **VDSA** device was installed in the fourth floor. By trial and error, it was found that this position permits to obtain the best results. When fixed dampers were used, the best results were obtained with the configuration III (dampers in all floors of the two buildings). The reduction in the response of the buildings coupled with the **VDSA** device compared with the configuration III was between 70 and 95%. As it was done in the examples of Chapter V, VII and VII, the constant damping coefficient (C_o) assigned to the fixed dampers was the maximum practical value recommended in the literature.

8.3 TIME DELAY EFFECT

The effect of the time delay on the performance of the proposed VDSA device is examined here by means of a brief parametric study. Only, the effect on the peak displacements is presented.

Tables 8.9, 8.10 and 8.11 show a summary for of the maximum relative displacements obtained by accounting for the time delay. Each table corresponds to the response to El Centro, San Fernando and Friuli ground motions. The modified *Closed-loop control* algorithm ***Qv*** is used to obtain the optimal instantaneous position of the semiactive device.

The time delay considered are multiples of the time step (0.02 sec) used to sample the accelerogram. The response for Δt=0.02 sec correspond to the case when the time delay is neglected.

Observing the tables one can conclude that the attenuation in the maximum response is not significantly affected by the time delay.

For instance, the peak displacement at the top floor of structure A only increased by 1.55%, 0.21% and 0.68% when the structure is subjected to the El Centro, San Fernando and, Friuli records and the time delay is 0.1 sec. Of course, if the time delay keeps increasing eventually the effect will be more significant.

Table 8.9: Summary of the time delay effect in the maximum response of the two coupled MDOF structures controlled with the VDSA device for the El Centro record (*Closed-loop control*)

Floor	Displacement					
	Uncontrolled	**0.02 s**	**0.04 s**	**0.06 s**	**0.08 s**	**0.1 s**
	[in*]	**[in*]**	**[in*]**	**[in*]**	**[in*]**	**[in*]**
Structure A						
8th	3.7497	0.2126	0.2127	0.2135	0.2148	0.2159
7th	3.6002	0.2060	0.2060	0.2068	0.2075	0.2083
6th	3.3063	0.1928	0.1928	0.1934	0.1941	0.1952
5th	2.8822	0.1729	0.1729	0.1737	0.1746	0.1757
4th	2.3555	0.1464	0.1465	0.1473	0.1481	0.1489
3rd	1.8228	0.1199	0.1199	0.1208	0.1219	0.1228
2nd	1.2639	0.0867	0.0868	0.0874	0.0880	0.0889
1st	0.6483	0.0467	0.0467	0.0474	0.0483	0.0491
Structure B						
6th	2.9392	0.1589	0.1590	0.1599	0.1607	0.1619
5th	2.7857	0.1522	0.1522	0.1531	0.1538	0.1547
4th	2.4832	0.1388	0.1389	0.1398	0.1407	0.1419
3rd	2.0359	0.1142	0.1144	0.1152	0.1159	0.1168
2nd	1.4544	0.0829	0.0829	0.0838	0.0847	0.0858
1st	0.7622	0.0448	0.0448	0.0459	0.0467	0.0478

*1 in = 2.54 cm.

Table 8.10: Summary of the time delay effect in the maximum response of the two coupled MDOF structures controlled with the VDSA device for the San Fernando record (*Closed-loop control*)

Floor	Displacement					
	Uncontrolled	**0.02 s**	**0.04 s**	**0.06 s**	**0.08 s**	**0.1 s**
	[in*]	**[in*]**	**[in*]**	**[in*]**	**[in*]**	**[in*]**
Structure A						
8th	12.3300	1.2771	1.2771	1.2779	1.2787	1.2798
7th	11.9537	1.2415	1.2415	1.2424	1.2437	1.2442
6th	11.2169	1.1701	1.1702	1.1709	1.1718	1.1729
5th	10.1276	1.0631	1.0631	1.0640	1.0649	1.0658
4th	8.6762	0.9205	0.9205	0.9216	0.9224	0.9235
3rd	6.8691	0.7445	0.7445	0.7452	0.7461	0.7472
2nd	4.7590	0.5320	0.5320	0.5328	0.5337	0.5345
1st	2.4368	0.2839	0.2840	0.2848	0.2856	0.2865
Structure B						
6th	7.5278	0.9238	0.9238	0.9244	0.9257	0.9267
5th	7.0656	0.8882	0.8883	0.8891	0.8899	0.8909
4th	6.1855	0.8169	0.8170	0.8178	0.8185	0.8193
3rd	4.9567	0.6660	0.6660	0.6669	0.6678	0.6688
2nd	3.4634	0.4797	0.4798	0.4805	0.4816	0.4824
1st	1.7855	0.2577	0.2578	0.2585	0.2591	0.2599

*1 in = 2.54 cm.

Table 8.11: Summary of the time delay effect in the maximum response of the two coupled MDOF structures controlled with the VDSA device for the Friuli record (*Closed-loop control*)

Floor	Displacement					
	Uncontrolled	0.02 s	0.04 s	0.06 s	0.08 s	0.1 s
	[in*]	[in*]	[in*]	[in*]	[in*]	[in*]
Structure A						
8th	6.3319	0.3701	0.3701	0.3709	0.3718	0.3726
7th	6.1178	0.3599	0.3600	0.3609	0.3617	0.3625
6th	5.6963	0.3397	0.3398	0.3405	0.3419	0.3428
5th	5.0809	0.3092	0.3092	0.3101	0.3109	0.3118
4th	4.2919	0.2686	0.2686	0.2693	0.2703	0.2716
3rd	3.3557	0.2169	0.2170	0.2178	0.2187	0.2193
2nd	2.3040	0.1548	0.1548	0.1557	0.1562	0.1571
1st	1.1726	0.0825	0.0827	0.0833	0.0841	0.0849
Structure B						
6th	5.9511	0.2162	0.2163	0.2169	0.2179	0.2187
5th	5.6169	0.2061	0.2061	0.2072	0.2081	0.2090
4th	4.9605	0.1859	0.1861	0.1870	0.1878	0.1888
3rd	4.0097	0.1546	0.1546	0.1553	0.1561	0.1574
2nd	2.8128	0.1131	0.1133	0.1142	0.1153	0.1166
1st	1.4521	0.0616	0.0618	0.0627	0.0638	0.0647

*1 in = 2.54 cm.

CHAPTER IX:

CONCLUSIONS

9.1 SUMMARY

This dissertation proposed and validated by means of numerical simulations a new protective device for the reduction of the seismic response of civil engineering structures. The system was termed **V**ariable **D**amping **S**emi **A**ctive device (**VDSA**) and, as its name suggests, it consists of viscous fluid dampers that provide variable damping by varying the angle of the dampers. Because the only external force required to operate the device is that to move up or down the mechanism, the protective system is referred to as semiactive. The device is composed of two dampers in a configuration with the shape of a **V**. The top ends of the **V** are attached to the structure and the lower end can move vertically along a fixed rod. By changing the position of the lower end, the angle of the dampers changes and thus the amount of damping that they provide to the structure.

To operate the device to achieve a predefined objective in an optimal way, it is necessary to use a control law. In our case, it is desired to procure the most favorable amount of damping at every instant of time to drive the structure to rest. Two control algorithms based on modifications of the instantaneous optimal control scheme were introduced. The two versions of the algorithm are referred to as the modified *Closed-Loop* algorithm $\boldsymbol{Q_v}$ and the modified *Closed-Open Loop* algorithm $\boldsymbol{Q_v}$. The analytical derivation of these algorithms is presented in Chapter II.

The implementation and performance of the proposed **VDSA** device is investigated via numerical examples. The protective device is applied to two classes of structures. It is first applied to single, stand-alone structures and then to two closes structures that are coupled by the device. Before the numerical simulations were undertaken, it was necessary to develop the equations of motion that govern the behavior of the single and coupled structures with the **VDSA** device. The derivation of these equations presented in Chapters III and IV.

Chapters V to VIII contain the numerical simulations. Chapter V considers two single degree of freedom structures with very different dynamic characteristics. Chapter VI is dedicated to simulate the behavior of a multistory building fitted with the proposed device. The following two chapters deal with the application of the semiactive system to couple and control the seismic response of single and multi degree of freedom structures.

In all the numerical examples a set of three real accelerograms with different characteristics was used to demonstrate the effectiveness of the device. The accelerograms were not scaled or filtered, i.e. they were applied with the original acceleration values. The effect of the time delay between the measurement of the response and the motion of the device was studied by means of numerical experiments.

The response of the original structures was compared with that obtained when the **VDSA** system is in operation. The response quantities compared are the relative displacements with respect to the ground and the total shear force at the base. In addition, the response of the structure fitted with fixed dampers was examined side by side with the results obtained with the semiactive system. The following sections contain a summary and analysis of the results obtained from the numerical simulations.

9.2 STRUCTURES CONTROL OF STAND-ALONE

The results presented in Tables 5.2 and 5.3 for the SDOF stiff structure indicate that the maximum relative displacements due to the El Centro, San Fernando and Friuli accelerograms were reduced by 75.2, 74.4 and 79.0%, respectively, using the modified *Closed-loop control* algorithm ***Qv***. When the modified *Closed-open-loop control* algorithm ***Qv*** was used to define the position of the **VDSA** device, the reduction was

75.7, 74.8 and 79.6%. The displacements in the same SDOF system fitted with fixed dampers were reduced by 19.2, 2.0 and 8.9%.

The Example 2 presented in Chapter V was also a SDOF structure. However, in this case the structure was more flexible than that used in the previous example. The results in terms of relative displacements using as input the El Centro, San Fernando and Friuli records were presented in Tables 5.6 and 5.7 for the modified *Closed-loop control* and *Closed-open-loop control* algorithms, respectively. In this case a more significant reduction in the maximum displacement and total base shear was observed. A reduction in the displacements of up to 99% was achieved for the three ground motion records with both control algorithms. The reduction in the peak displacements for the structure with fixed damper was of 25.9% for the El Centro record, 6.1% for the San Fernando record and 39.3% for the Friuli record.

Chapter VI presented the numerical results for a MDOF structure. A six story building modeled as a shear building one DOF per floor was selected for the example. The relative displacement response of all floors for the three seismic records El Centro, San Fernando and Friuli earthquakes were presented in Table 6.3, 6.5 and 6.7, respectively. The **VDSA** device was installed in the fourth floor in all the cases. This was found to be the best position in terms of the response reduction. A reduction in the top floor displacement of 88.0% (El Centro), 84.4% (San Fernando) and 92.8% (Friuli) was obtained with the **VDSA** device controlled with both algorithms. These results were the same with both algorithms. To compare the effectiveness of the **VDSA** device with the viscous dampers in a fixed position, three configurations were for the examined for the latter case. The best results were observed when the structure has passive dampers installed in all floors (a configuration identified as Case III). In this case the reduction in the peak displacement at the top floor was 48, 50.4 and 68.9% for the three earthquakes in the usual order. The reduction in the displacements of the lower floors is not as dramatic as in the top floor. However, the proposed device was capable of achieving a notable decrease even for the lowest floor. For example, the reduction in peak displacements at the first floor obtained with the **VDSA** device was 71.3, 70.9 and 87.2% for El Centro, San Fernando and Friuli record, respectively. These percentages should be compared with the 46.8, 49.8 and 65.2% reduction obtained in the fixed dampers for case III.

9.3 CONTROL OF COUPLED STRUCTURES

Two cases of coupled structures were examined to assess the performance of the proposed **VDSA** device. In Chapter VII the case of two one-story structures modeled as SDOF systems was considered. The SDOF system identified as the **A** and **B** structure have different dynamic properties. Chapter VIII dealt with the case of two multi-story building modeled as MDOF shear building systems. One of the structure, identified as **A**, has eight stories and the other, denoted **B**, represents a six-story building.

A procedure similar to that used in the previous section was also used to analyze the results obtained for the coupled structures.

Table 7.2 presented the results for the two SDOF structures coupled via the proposed semiactive dampers. The maximum relative displacements due to the El Centro, San Fernando and Friuli accelerograms were reduced by 89.5, 82.7 and 92.1% in structure **A** and 89.1, 74.2 and 91.1% in structure **B**, respectively, when the structures were controlled with the **VDSA** device. When passive dampers were installed in all floors of the structures, the reduction in displacement was 25.9, 61 and 39.3% in structure **A**, whereas in structure **B** the reduction was 47.9, 43.4 and 57.9% for the El Centro, San Fernando and Friuli earthquake. The response reductions obtained with the VDSA device were the same irregardless of which of the two control algorithms was implemented.

Chapter VIII presented and example involving two MDOF structures coupled by the **VDSA** device. The results obtained of the relative displacement in each structure were presented in Table 8.3, 8.5 and 8.7 for all floors of the structure **A** and **B**, corresponding to El Centro, San Fernando and Friuli records, respectively. The reduction in the top floor displacement obtained for the three earthquakes (in the usual order) in the structures controlled with the **VDSA** device were: 94.3, 89.6 and 94.2% for structure **A** and 94.6, 87.7 and 96.4% for structure **B**. The reduction in the original response of the first floor were 92.8, 88.3 and 92.3% for structure **A** and 94.1, 85.6 and 95.8% for structure **B**. When the structures were furnished with fixed dampers in all floors (Case III) the displacements in the top floor of structure **A** were reduced by 52.2, 57.1 and 57.7%. For structure **B** the reduction were 47.9, 50.4 and 68.9%. The corresponding reductions in the first floor displacements of the structure **A** were 50.1, 62.9 and 55.7%, and for structure **B** 46.8, 49.8 and 65.2%.

From the reported results, it is evident that the **VDSA** device is capable of significantly attenuating the seismic response of single and multiple degree of freedom system. The results obtained in SDOF and MDOF structures (single and coupled) indicate that the proposed Variable Damping Semi-Active device (**VDSA**) is able to significantly reduce the relative displacements and base shear of structures subjected to earthquake ground motions. Two optimal control methodologies proposed, the modified *Closed-loop control* algorithms ***Qv*** and the modified *Closed-open-loop control* algorithms ***Qv***, were applied to select the displacement of the movable end of the **VDSA** device. This displacement also controls the effective damping coefficient of the dampers.

9.4 FUTURE WORKS

The aim of this dissertation was to put forth the new semiactive device and to validate it with numerical simulations in several types of structural systems. The performance of the proposed device was shown to be very promising, but before it is implemented in real structures, it is very important to perform experimental tests in scaled or full size models. Although this experimental verification was outside the scope of this study, it is realized that it is crucial for the final validation.

Another area for future research is the application of the **VDSA** device to other types of structures. For instance, it can be applied to bridge structures, towers, etc. In addition, its use to complement a seismic base isolation system can be another area of application.

Although this dissertation was concerned with the reduction of the response of buildings to strong earthquake ground motions, the proposed device can also be applied to mitigate the vibrations induced in tall, flexible structures by wind loads. Indeed, one of the first applications of conventional viscous fluid dampers was precisely to reduce the wind-induced oscillations.

Other more detailed models of the structures should also be considered to study special situations. For instance, three dimensional effects such as the coupling between the translational and torsional motion should be studied.

Finally, the models studied in this work had only one **VDSA** device. This proved to be enough to significantly reduce the seismic response. However, it is interesting to examine if there are any advantages in using more than one device. For instance, this could be beneficial of one of the devices is disabled.

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APPENDIX A:

THE STATE EQUATION

We have shown in Chapter II that the equations of motion of a viscously damped ***n***-degree of freedom system can be written in the compact form

$$\left[M\right]\{\ddot{x}(t)\}+\left[C\right]\{\dot{x}(t)\}+\left[K\right]\{x(t)\}=-\left[M\right]\{E\}\ddot{x}_g(t) \qquad \text{(A-1)}$$

where [*M*], [*C*] and [*K*] are ***nxn*** symmetric mass, damping and stiffness matrices, respectively, and $\{\ddot{x}(t)\}$, $\{\dot{x}(t)\}$ and $\{x(t)\}$ are the *n*-dimensional acceleration, velocity and displacement $\{x(t)\}$ vector, respectively and $\ddot{x}_g(t)$ is the base acceleration vector.

In the general case of viscous damping, the modal matrix does not diagonalize the damping matrix, so that no analytical solution of the equations of motion in the configuration space is possible. However, an analytical solution is possible in the ***state space***. To this end, we introduce an obvious identity, use equation A-1 and write

$$\begin{aligned}
\{\dot{x}(t)\} &= \{\dot{x}(t)\} \\
\{\ddot{x}(t)\} &= -[M]^{-1}[C]\{\dot{x}(t)\} - [M]^{-1}[K]\{x(t)\} - \{E\}\ddot{x}_g(t)
\end{aligned} \qquad \text{(A-2)}$$

Next, we define the ***state vector*** as the 2***n***-dimensional vector $z(t) = \left[\{x(t)\}^T \quad \{\dot{x}(t)\}^T\right]^T$ and write equations A-2 in the customary state form

$$\{\dot{z}(t)\} = [A]\{z(t)\} + [H]\{\ddot{x}_g(t)\} \qquad \text{(A-3)}$$

where

$$[A] = \left[\begin{array}{c|c} 0 & I \\ \hline -M^{-1}K & -M^{-1}C \end{array}\right] \qquad [H] = \left[\begin{array}{c} 0 \\ \hline -E \end{array}\right] \qquad \text{(A-4)}$$

Where $[A]$ and $[H]$ are constant coefficient matrices of dimensions $2nx2n$ and $2nx1$, respectively.

Equation A-3 represents the dynamics of a time-invariant linear system in the ***state-space*** form.

APPENDIX B:

PROGRAM: *CLOSED-LOOP CONTROL* AND VDSA DEVICE (SDOF) STRUCTURE

```
clc; clear all; close all

g = 386.4;                              % acceler. of gravity: in/s^2
% g = 9.8;                              % acceler. of gravity: m/s^2

n = 1;                                  % number of stories
% Wp = [14.4]*1e6;                      % weights of floor: lb
% k =  [1.2  ]*1e6;                     % total stiffness coefficients: lb/in
Wp = [14.4]*1e6;                        % weights of floor: lb Este
k =  [0.8  ]*1e6;                       % total stiffness coefficients: lb/in
% Wp = [9.4*g]*1e3;                     % weights of floor: lb
% k =  [(2*pi/1.5)^2*9.4  ]*1e3;        % total stiffness coefficients: lb/in

% Wp = [91.3250]*1e0;                   % weights of floor: lb
% k = [3.6054 ]*1e3;                    % total stiffness coefficients: lb/in
% Wp = [250]*1e3;                       % weights of floor: lb
% k = [20 ]*1e5;                        % total stiffness coefficients: lb/in
% Wp = [16.69*g]*1e0;                   % weights of floor: lb
% k =  [7934  ]*1e0;                    % total stiffness coefficients: lb/in
% Wp = [158*g]*1e3;                     % weights of floor: kg
% k = [25 ]*1e6;                        % total stiffness coefficients: kN/m
% Wp = [35.274*g]*1e3;                  % weights of floor: kg Este
% k = [(2*pi/0.2)^2*35.274 ]*1e3;       % total stiffness coefficients: kN/m
% Wp = [13.6298736]*1e6;                % weights of floor: kg
% k = [34.8140426]*1e6;                 % total stiffness coefficients: kN/m
% Wp = [13.630]*1e6;                    % weights of floor: kg
% k = [34.814]*1e6;                     % total stiffness coefficients: kN/m
% Wp = [16*g]*1e3;                      % weights of floor: kg
```

Appendix B:

Program: *Closed-Loop Control* and VDSA Device (SDOF) Structure

```
% k = [(2*pi/0.2)^2*16 ]*1e3;          % total stiffness coefficients: kN/m
% Wp = [3.14*g]*1e2;                   % weights of floor: kg
% k = [3.404]*1e5;                     % total stiffness coefficients: kN/m
%CoA= [25 ]*1e3;                       % total damping coefficients: lb.s/in
%CoB= [10 ]*1e3;                       % total damping coefficients: lb.s/in
dt = 0.02;                             % time step of accelerogram: sec
zita = [2]*1e-2;                       % modal damping ratios of building
U(1) =0;
Up(1)=0;
Ac{1}=0;
Act(1)=0;
h=120;
a=120;
w=60;
D=[1];
E=[1];
M  = diag( Wp/g ) ;
K  = diag( k );
Rm=0.18;
Wmax=h-a*Rm;

Wmin=h-a+25;
cte=k(1)/(Wp);

Q=[1,0;0,1]*10^2;
R=[1]*10^-4*1;
Uc(1)=0;
IR=inv(R);

%------------ Read and plot the ground acceleration time history ----------

nom = 'El Centro';                     % name of file with earthquake
                                       % addpath c:/MatlabFiles/Earthquakes;
                                       % directory with accelerograms
acc = load ([nom,'.txt']);             % read earthquake data file
[nr,nc] = size(acc);                   % columns and rows of data file
Xg(1:nr*nc) = acc';                    % copy accelerogram in a vector
xm = max( abs(Xg) );                   % original peak acceleration
Xg =  Xg*g;                            % scaled accelerogram
tf = (nr*nc-1)*dt;                     % final time of accelerogram
t  = 0: dt: tf ;                       % column vector with time steps
nt = length(t);                        % number of discrete time points
np = round( nt);                       % number of time points for plotting
z0 = zeros(1,np);                      % auxiliary vector for plotting

figure; plot(t,zeros(1,nt), t,Xg/g ); grid on; axis tight;
title(['Normalized earthquake ground acceleration: ',nom,' record']);
```

```
xlabel('Time [sec]'); ylabel('Acceleration [g]');
Xpg=cumsum(Xg)*dt;
figure; plot(t,zeros(1,nt), t,Xpg ); grid on; axis tight;
title(['Ground velocity: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/seg]');

Xppg=cumsum(Xpg)*dt;

figure; plot(t,zeros(1,nt), t,Xppg ); grid on; axis tight;
title(['Ground displacement: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

nn=nr*nc;
dampi(1)=0;
for i=2:nn
   z=[U(i-1);Up(i-1)/1];
   M  = diag( M ) ;
   K  = diag( k );
   [Phi,lam] =  eig(K,M);
   Phi = Phi * diag(1 ./ sqrt( diag(Phi'*M*Phi) ));
   [ws,id] = sort( sqrt(diag(lam)) );
   Phi = Phi(:,id);
   Ts  = 2*pi ./ ws ;
   Mi = inv(M) ;
   Cs=M* Phi* diag(2*(0.02).*ws)* Phi'* M;
   C=Cs+(CoA+CoB)*(a^2/(a^2+(h-w)^2));
   A=[0,1;-Mi*K,-Mi*C];
   Cpr=(1/2)*Mi*D*(CoA-CoB)*(a*(h-w)/(a^2+(h-w)^2));
   B=[0;Cpr];
   H=[0;-E];
   I=[1,0;0,1];

   %% New matrices

   Av=[0,1];
   Sv=[1];
   Qv=[1]*1e1;
   A2=Q+Av'*Qv*Av;
   A3=Av'*Qv*Sv;

   [T,lam1] =  eig(A);
   [ws1,id] = sort( sqrt(diag(lam1)) );
   lam2=diag(ws1);
   lam11=inv(T)*A*T;
   d=(exp(lam11*dt))*inv(T)*(z+dt/2*(B*Uc(i-1)+H*Xg(i-1)));
   Zt=(inv(I+(dt^2)/4*B*inv(R)*B'*A2)*(T*d-dt^2/4*B*inv(R)*B'*A3*Xpg(i)+dt/2*H*Xg(i)));
   Uc(i)=-(dt/2*inv(R)*B')*(A2*Zt+A3*Xpg(i));
```

```
    U(i)=(Zt(1));
    Up(i)=(Zt(2));
    Zc=0.43+0.37*sign(U(i)*Up(i));
    %Ac{i}=-Mi*C*Up(i)-Mi*K*U(i)-E*Xg(i)+Cpr;
    %Act(i)=Ac{i};
    w=h-a*(abs((CoA+CoB)/abs(Uc(i)/Up(i))-1))^(1/2);
    dampi(i)=C*Up(i);
    if w>Wmax
       w=Wmax;
    end

    if w<Wmin
       w=Wmin;
    end

    rr(i-1)=w;

end

rr(nn)=w;
bb(nn)=0;

figure; plot(t,zeros(1,nt),t,U ); grid on; axis tight;
title(['Relative displacement of top floor - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

figure; plot(t,zeros(1,nt),t,Up ); grid on; axis tight;
title(['Relative velocity of top floor - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

figure; plot(t,zeros(1,nt),t,Act ); grid on; axis tight;
title(['Relative acceleration of top floor - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Acceleration [in/sec^2]');

figure; plot(t,zeros(1,nt),t,Uc ); grid on; axis tight;
title(['Control Force - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Control Force [lb]');

figure; plot(t,zeros(1,nt),t,rr ); grid on; axis tight;
title(['Variation of Control - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Higth Control [in]');

figure; plot(t,zeros(1,nt),t,bb ); grid on; axis tight;
title(['Uc/Up - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Uc/Up');

%%%%  Uncontrolled
```

```
%--------- Form the state-space matrices and calculate the response -------
Mi = inv(M) ;
Ct=M* Phi* diag(2*(0.02+0.03).*ws)* Phi'* M;
A  = [-Mi*Ct, -Mi*K; eye(n), zeros(n) ] ;
B  = [-ones(n,1) ; zeros(n,1)] ;
C  = [-Mi*Ct, -Mi*K] ;
D  = zeros(n,1) ;

disp('*** Equivalent modal properties of the damped structure:')

damp(A)
[Y1,X1] = lsim(A,B,C,D,Xg,t);
V1  = K(1)*X1(:,n+1)/1e3;
um = max( abs(X1(:,n+1:2*n)) );
Vm = max( abs(V1) );

%------ Calculate the maximum response & plot the building response -------

V  = K(1)*X1(:,n+1)/1e3;
um = max( abs(X1(:,n+1:2*n)) );
Vm = max( abs(V1) );
V2 = K(1)*U/1e3;
damnc=Ct.*X1(:,n);

figure; plot(t(1:round(np/2)),X1(1:round(np/2)),t(1:round(np/2)),U(1:round(np/2)) ); grid on; xlabel('Time
[sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled',1); title(['Relative displacement of top floor -
Earthquake:',nom,' record']);

figure; plot( t(1:np),V1(1:np),t(1:np),V2(1:np) ); grid on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Controlled',1); title('Total base shear');

eladamnc=damnc+V1*1e3;

%------Damper Fixed Case I

eladam=dampi+V2*1e3;
w=Wmax;
Zeff=0.3;
Cd1=Zeff*pi*M*Phi^2/(2*Ts*(a^2/(a^2+(h-w)^2))*Phi^2);

Cd1= [0 2*Cd1];
for i=1:2

   Cop=Cd1(i);
   Ct1=2*zita*M*ws+2*Cop*(a^2/(a^2+(h-w)^2));
```

```
    A  = [-Mi*Ct1, -Mi*K; eye(n), zeros(n) ] ;
    B  = [-ones(n,1) ; zeros(n,1)] ;
    C  = [-Mi*Ct1, -Mi*K] ;
    D  = zeros(n,1) ;
    cos1=a^2/(a^2+(h-w)^2);

    disp('*** Equivalent modal properties of the damped structure:')

    damp(A);
    if i==1
        [Y3,X3] = lsim(A,B,C,D,Xg,t);
        V3  = K(1)*X3(:,n+1)/1e3;
        um = max( abs(X3(:,n+1:2*n)) );
        Vm = max( abs(V3) );
    else
        [Y4,X4] = lsim(A,B,C,D,Xg,t);
        V4  = K(1)*X4(:,n+1)/1e3;
        um = max( abs(X4(:,n+1:2*n)) );
        Vm = max( abs(V4) );
    end
end

%------Damper Fixed Case II

eladam=dampi+V2*1e3;
w=0;
Zeff=0.28;
Cd2=Zeff*pi*M*Phi^2/(Ts*(a^2/(a^2+(h-w)^2))*Phi^2);

Cd2= [0 2*Cd2];
for i=1:2

    Cos=Cd2(i);
    Ct2=2*zita*M*ws+Cos*(a^2/(a^2+(h-w)^2));
    A2 = [-Mi*Ct2, -Mi*K; eye(n), zeros(n) ] ;
    B  = [-ones(n,1) ; zeros(n,1)] ;
    C  = [-Mi*Ct2, -Mi*K] ;
    D  = zeros(n,1) ;
    cos2=a^2/(a^2+(h-w)^2);

    disp('*** Equivalent modal properties of the damped structure:')

    damp(A2);
    if i==1
        [Y5,X5] = lsim(A2,B,C,D,Xg,t);
        V5  = K(1)*X5(:,n+1)/1e3;
        um = max( abs(X5(:,n+1:2*n)) );
```

```
        Vm = max( abs(V5) );
    else
        [Y6,X6] = lsim(A2,B,C,D,Xg,t);
        V6  = K(1)*X6(:,n+1)/1e3;
        um = max( abs(X6(:,n+1:2*n)) );
        Vm = max( abs(V6) );
    end
end

figure; plot(t,zeros(1,nt),t,X1(:,n) ); grid on; axis tight;
title(['Relative velocity of top floor Uncontrolled - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

figure; plot(t(1:round(np/2)),X1(1:round(np/2),2*n),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),X4(1:round(np/2),2*n),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),U(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' fixed Damper','Controlled VDSA',1); title(['Relative
displacement of top floor - Earthquake:',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1(1:round(np/2),2*n),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),X4(1:round(np/2),2*n),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' fixed Damper',1); title(['Relative displacement of top
floor - Earthquake:',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1(1:round(np/2),2*n),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),U(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of top
floor - Earthquake:',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X4(1:round(np/2),2*n),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),U(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend(' fixed Damper','Controlled VDSA',1); title(['Relative displacement of
top floor - Earthquake:',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),V1(1:round(np/2)),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),V4(1:round(np/2)),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V2(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','fixed Damper','Controlled VDSA',1); title(['Total base shear -
Earthquake:',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1(1:round(np/2)),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V4(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','fixed Damper',1); title(['Total base shear - Earthquake:',nom,'
record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1(1:round(np/2)),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V2(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
```

```
ylabel('Force [kip]'),legend('Uncontrolled','Controlled VDSA',1); title(['Total base shear -
Earthquake:',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V4(1:round(np/2)),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V2(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('fixed Damper','Controlled VDSA',1); title(['Total base shear -
Earthquake:',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),U(1:round(np/2)),'-r','LineWidth',1),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Controlled VDSA',1); title(['Relative displacement of top floor -
Earthquake:',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),V2(1:round(np/2)),'-r','LineWidth',1),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Controlled VDSA',1); title(['Total base shear - Earthquake:',nom,' record']);
grid on;axis tight;


figure; plot(X1(1:np,2*n),V1(1:np),X4(1:np,2*n),V4(1:np),U(1:np),V2(1:np));grid on;
xlabel('Displacement [in]');
ylabel('Force Elastic [kip]'),legend('Uncontrolled',' fixed Damper','Controlled VDSA',1); title('Elastic');

figure; plot( U(1:np),dampi(1:np) ); grid on; xlabel('Displacement [in]');
ylabel('Damping [lb]'),legend('Controlled VDSA',2); title('Damping');

figure; plot( U(1:np),V2(1:np) ); grid on; xlabel('Displacement [in]');
ylabel('Elastic [lb]'),legend('Controlled VDSA',2); title('Elastic');

figure; plot( U(1:np),eladam(1:np) ); grid on; xlabel('Displacement [in]');
ylabel('Elastic + Damping [lb]'),legend('Controlled VDSA',2); title('Elastic + Damping');

figure; plot( X1(1:np,2*n),damnc(1:np) ); grid on; xlabel('Displacement [in]');
ylabel('Damping [lb]'),legend('Uncontrolled',2); title('Damping');

figure; plot( X1(1:np,2*n),V1(1:np) ); grid on; xlabel('Displacement [in]');
ylabel('Elastic [lb]'),legend('Uncontrolled',2); title('Elastic');

figure; plot( X1(1:np,2*n),eladamnc(1:np) ); grid on; xlabel('Displacement [in]');
ylabel('Elastic + Damping [lb]'),legend('Uncontrolled',2); title('Elastic + Damping');

En1=cumsum(abs(V1(1:np)))*dt;
En2=cumsum(abs(V2(1:np)))*dt;
En3=cumsum(abs(dampi(1:np)))*dt;
En4=cumsum(abs(damnc(1:np)))*dt;
En6=En1+En4;
En5=En2+En3;

figure; plot(t,En1,t,En4,t,zeros(1,np) ); grid on; axis tight; xlabel('Time [sec]');
```

```
ylabel('Energy'),legend('Elastic','Damping','Elastic + Damping',4); title(['Energy Force Uncontrolled -
Earthquake:',nom,' record']);

figure; plot(t,En2,t,En3,t,zeros(1,np) ); grid on; axis tight; xlabel('Time [sec]');
ylabel('Energy'),legend('Elastic','Damping','Elastic + Damping',4); title(['Energy Force Controlled -
Earthquake:',nom,' record']);
```

APPENDIX C:

PROGRAM: *CLOSED-OPEN-LOOP CONTROL* AND VDSA DEVICE (SDOF) STRUCTURE

```
clc; clear all; close all

g = 386.4;                          % acceler. of gravity: in/s^2
% g = 9.8;                          % acceler. of gravity: m/s^2

n = 1;                              % number of stories
% Wp = [14.4]*1e6;                  % weights of floor: lb
% k =  [1.2  ]*1e6;                 % total stiffness coefficients: lb/in
Wp = [14.4]*1e6;                    % weights of floor: lb
k =  [0.8  ]*1e6;                   % total stiffness coefficients: lb/in
% Wp = [9.4*g]*1e3;                 % weights of floor: lb
% k =  [(2*pi/1.5)^2*9.4  ]*1e3;    % total stiffness coefficients: lb/in

% Wp = [91.3250]*1e0;               % weights of floor: lb
% k = [3.6054 ]*1e3;                % total stiffness coefficients: lb/in
% Wp = [250]*1e3;                   % weights of floor: lb
% k = [20 ]*1e5;                    % total stiffness coefficients: lb/in
% Wp = [16.69*g]*1e0;               % weights of floor: lb
% k =  [7934  ]*1e0;                % total stiffness coefficients: lb/in
% Wp = [158*g]*1e3;                 % weights of floor: kg
% k = [25 ]*1e6;                    % total stiffness coefficients: kN/m
% Wp = [35.274*g]*1e3;              % weights of floor: kg
% k = [(2*pi/0.2)^2*35.274 ]*1e3;   % total stiffness coefficients: kN/m
% Wp = [13.6298736]*1e6;            % weights of floor: kg
% k = [34.8140426]*1e6;             % total stiffness coefficients: kN/m
% Wp = [13.630]*1e6;                % weights of floor: kg
% k = [34.814]*1e6;                 % total stiffness coefficients: kN/m
% Wp = [16*g]*1e3;                  % weights of floor: kg
```

```
% k = [(2*pi/0.2)^2*16 ]*1e3;                 % total stiffness coefficients: kN/m

CoA= [25 ]*1e3;                               % total damping coefficients: lb.s/in
CoB= [10 ]*1e3;                               % total damping coefficients: lb.s/in

dt = 0.02;                                    % time step of accelerogram: sec
zita = [2]*1e-2;                              % modal damping ratios of building
U(1) =0;
Up(1)=0;
Ac{1}=0;
Act(1)=0;
h=120;
a=120;
w=60;
D=[1];
E=[1];
M  = diag( Wp/g ) ;
K  = diag( k );
Rm=0.18;
Wmax=h-a*Rm;

Wmin=h-a+20;
cte=k(1)/(Wp);

Q=[1,0;0,1]*10^2;
R=[1]*10^-4*1;
Uc(1)=0;
IR=inv(R);

%------------ Read and plot the ground acceleration time history ----------

nom = 'San Fernando';                         % name of file with earthquake
                                              % addpath c:/MatlabFiles/Earthquakes;
                                              % directory with accelerograms
acc = load ([nom,'.txt']);                    % read earthquake data file
[nr,nc] = size(acc);                          % columns and rows of data file
Xg(1:nr*nc) = acc';                           % copy accelerogram in a vector
xm = max( abs(Xg) );                          % original peak acceleration
Xg =  Xg*g;                                   % scaled accelerogram
tf = (nr*nc-1)*dt;                            % final time of accelerogram
t  = 0: dt: tf ;                              % column vector with time steps
nt = length(t);                               % number of discrete time points
np = round( nt);                              % number of time points for plotting
z0 = zeros(1,np);                             % auxiliary vector for plotting

figure; plot(t,zeros(1,nt), t,Xg/g ); grid on; axis tight;
title(['Normalized earthquake ground acceleration: ',nom,' record']);
```

```
xlabel('Time [sec]'); ylabel('Acceleration [g]');

Xpg=cumsum(Xg)*dt;

figure; plot(t,zeros(1,nt), t,Xpg ); grid on; axis tight;
title(['Ground velocity: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/seg]');

Xppg=cumsum(Xpg)*dt;

figure; plot(t,zeros(1,nt), t,Xppg ); grid on; axis tight;
title(['Ground displacement: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');
%---- Calculate the matrices & modal properties of the undamped bldg. -----
nn=nr*nc;
dampi(1)=0;
for i=2:nn
   z=[U(i-1);Up(i-1)/1];
   M  = diag( M ) ;
   K  = diag( k );
   [Phi,lam] =  eig(K,M);
   Phi = Phi * diag(1 ./ sqrt( diag(Phi'*M*Phi) ));
   [ws,id] = sort( sqrt(diag(lam)) );
   Phi = Phi(:,id);
   Ts  = 2*pi ./ ws ;
   Mi = inv(M) ;
   Cs=2*zita*ws*M;
   C=Cs+(CoA+CoB)*(a^2/(a^2+(h-w)^2));
   A=[0,1;-Mi*K,-Mi*C];
   Cpr=(1/2)*Mi*D*(CoA-CoB)*(a*(h-w)/(a^2+(h-w)^2));
   B=[0;Cpr];
   H=[0;-E];
   I=[1,0;0,1];

   %% New Matrices

   Av=[0,1];
   Sv=[1];
   Qv=[1]*1e1;

   [T,lam1] =  eig(A);
   [ws1,id] = sort( sqrt(diag(lam1)) );
   lam2=diag(ws1);
   lam11=inv(T)*A*T;
   d=(exp(lam11*dt))*inv(T)*(z+dt/2*(B*Uc(i-1)+H*Xg(i-1)));

   P1=(-1)*inv(I+((dt^2)/8)*Q*B*IR*B')*(Q+2*Av'*Qv*Av);
```

```
   P2=(-1)*inv((dt^2)/8*Q*B*inv(R)*B'+I)*(Q*(T*d+dt/2*H*Xg(i))+2*Av'*Qv*Sv*Xpg(i));

    Zt=(inv(I-(dt^2)/8*B*inv(R)*B'*P1)*(T*d+(dt^2)/8*B*inv(R)*B'*P2+dt/2*H*Xg(i)));
   LP=(P1*Zt+P2);
   Uc(i)=(dt/4*inv(R)*B'*LP);

   U(i)=(Zt(1));
   Up(i)=(Zt(2));
   Zc=0.43+0.37*sign(U(i)*Up(i));
    bb(i-1)=a3;
   %Ac{i}=-Mi*C*Up(i)-Mi*K*U(i)-E*Xg(i)+Cpr;
   %Act(i)=Ac{i};
   w=h-a*(abs((CoA+CoB)/abs(Uc(i)/Up(i))-1))^(1/2);
   %rr(i-1)=w;
   dampi(i)=C*Up(i);
   if w>Wmax
      w=Wmax;
   end

   if w<Wmin
      w=Wmin;
   end

   rr(i-1)=w;

end

rr(nn)=w;
bb(nn)=0;

figure; plot(t,zeros(1,nt),t,U ); grid on; axis tight;
title(['Relative displacement of top floor - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

figure; plot(t,zeros(1,nt),t,Up ); grid on; axis tight;
title(['Relative velocity of top floor - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

figure; plot(t,zeros(1,nt),t,Act ); grid on; axis tight;
title(['Relative acceleration of top floor - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Acceleration [in/sec^2]');

figure; plot(t,zeros(1,nt),t,Uc ); grid on; axis tight;
title(['Control Force - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Control Force [lb]');

figure; plot(t,zeros(1,nt),t,rr ); grid on; axis tight;
```

```
title(['Variation of Control - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Higth Control [in]');
figure; plot(t,zeros(1,nt),t,bb ); grid on; axis tight;
title(['Uc/Up - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Uc/Up');

%%%% Uncontrolled

%--------- Form the state-space matrices and calculate the response -------

Mi = inv(M) ;
Ct=2*(0.02+0.03)*M*ws;
A  = [-Mi*Ct, -Mi*K; eye(n), zeros(n) ] ;
B  = [-ones(n,1) ; zeros(n,1)] ;
C  = [-Mi*Ct, -Mi*K] ;
D  = zeros(n,1) ;

disp('*** Equivalent modal properties of the damped structure:')

damp(A)
[Y1,X1] = lsim(A,B,C,D,Xg,t);
V1  = K(1)*X1(:,n+1)/1e3;
um = max( abs(X1(:,n+1:2*n)) );
Vm = max( abs(V1) );

%------ Calculate the maximum response & plot the building response -------

V  = K(1)*X1(:,n+1)/1e3;
um = max( abs(X1(:,n+1:2*n)) );
Vm = max( abs(V1) );
V2 = K(1)*U/1e3;
damnc=Ct.*X1(:,n);

figure; plot(t(1:np),X1(1:np,2*n),t(1:np),U(1:np) ); grid on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled',1); title(['Relative displacement of top floor -
Earthquake:',nom,' record']);

figure; plot( t(1:np),V1(1:np),t(1:np),V2(1:np) ); grid on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Controlled',1); title('Total base shear');

eladamnc=damnc+V1*1e3;

%------Damper Fixed Case I

eladam=dampi+V2*1e3;
w=Wmax;
Zeff=0.3;
```

```
Cd1=Zeff*pi*M*Phi^2/(2*Ts*(a^2/(a^2+(h-w)^2))*Phi^2);

Cd1= [0 Cd1];
for i=1:2

   Cop=Cd1(i);
   Ct1=2*zita*M*ws+2*Cop*(a^2/(a^2+(h-w)^2));
   A  = [-Mi*Ct1, -Mi*K; eye(n), zeros(n) ] ;
   B  = [-ones(n,1) ; zeros(n,1)] ;
   C  = [-Mi*Ct1, -Mi*K] ;
   D  = zeros(n,1) ;
   cos1=a^2/(a^2+(h-w)^2);

   disp('*** Equivalent modal properties of the damped structure:')

   damp(A);
   if i==1
      [Y3,X3] = lsim(A,B,C,D,Xg,t);
      V3  = K(1)*X3(:,n+1)/1e3;
      um = max( abs(X3(:,n+1:2*n)) );
      Vm = max( abs(V3) );
   else
      [Y4,X4] = lsim(A,B,C,D,Xg,t);
      V4  = K(1)*X4(:,n+1)/1e3;
      um = max( abs(X4(:,n+1:2*n)) );
      Vm = max( abs(V4) );
   end

end

%------Damper Fixed Case II

eladam=dampi+V2*1e3;
w=0;
Zeff=0.3;
Cd2=Zeff*pi*M*Phi^2/(Ts*(a^2/(a^2+(h-w)^2))*Phi^2);

Cd2= [0 Cd2];
for i=1:2

   Cos=Cd2(i);
   Ct2=2*zita*M*ws+Cos*(a^2/(a^2+(h-w)^2));
   A2 = [-Mi*Ct2, -Mi*K; eye(n), zeros(n) ] ;
   B  = [-ones(n,1) ; zeros(n,1)] ;
   C  = [-Mi*Ct2, -Mi*K] ;
   D  = zeros(n,1) ;
   cos2=a^2/(a^2+(h-w)^2);
```

```
    disp('*** Equivalent modal properties of the damped structure:')
    damp(A2);
    if i==1
       [Y5,X5] = lsim(A2,B,C,D,Xg,t);
       V5  = K(1)*X5(:,n+1)/1e3;
       um = max( abs(X5(:,n+1:2*n)) );
       Vm = max( abs(V5) );
    else
       [Y6,X6] = lsim(A2,B,C,D,Xg,t);
       V6  = K(1)*X6(:,n+1)/1e3;
       um = max( abs(X6(:,n+1:2*n)) );
       Vm = max( abs(V6) );
    end

end

figure; plot(t,zeros(1,nt),t,X1(:,n) ); grid on; axis tight;
title(['Relative velocity of top floor Uncontrolled - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');


figure; plot(t(1:round(np/2)),X1(1:round(np/2),2*n),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),X4(1:round(np/2),2*n),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),U(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' fixed Damper','Controlled VDSA',1); title(['Relative
displacement of top floor - Earthquake:',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1(1:round(np/2),2*n),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),X4(1:round(np/2),2*n),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' fixed Damper',1); title(['Relative displacement of top
floor - Earthquake:',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1(1:round(np/2),2*n),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),U(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of top
floor - Earthquake:',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X4(1:round(np/2),2*n),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),U(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend(' fixed Damper','Controlled VDSA',1); title(['Relative displacement of
top floor - Earthquake:',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),V1(1:round(np/2)),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),V4(1:round(np/2)),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V2(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','fixed Damper','Controlled VDSA',1); title(['Total base shear -
```

```
Earthquake:',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1(1:round(np/2)),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V4(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','fixed Damper',1); title(['Total base shear - Earthquake:',nom,'
record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1(1:round(np/2)),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V2(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Controlled VDSA',1); title(['Total base shear -
Earthquake:',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V4(1:round(np/2)),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V2(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('fixed Damper','Controlled VDSA',1); title(['Total base shear -
Earthquake:',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),U(1:round(np/2)),'-r','LineWidth',1),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Controlled VDSA',1); title(['Relative displacement of top floor -
Earthquake:',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),V2(1:round(np/2)),'-r','LineWidth',1),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Controlled VDSA',1); title(['Total base shear - Earthquake:',nom,' record']);
grid on;axis tight;

figure; plot(X1(1:np,2*n),V1(1:np),X4(1:np,2*n),V4(1:np),U(1:np),V2(1:np) ); grid on;
xlabel('Displacement [in]');
ylabel('Force Elastic [kip]'),legend('Uncontrolled',' fixed Damper','Controlled VDSA',1); title('Elastic');

figure; plot( U(1:np),dampi(1:np) ); grid on; xlabel('Displacement [in]');
ylabel('Damping [lb]'),legend('Controlled VDSA',2); title('Damping');

figure; plot( U(1:np),V2(1:np) ); grid on; xlabel('Displacement [in]');
ylabel('Elastic [lb]'),legend('Controlled VDSA',2); title('Elastic');

figure; plot( U(1:np),eladam(1:np) ); grid on; xlabel('Displacement [in]');
ylabel('Elastic + Damping [lb]'),legend('Controlled VDSA',2); title('Elastic + Damping');

figure; plot( X1(1:np,2*n),damnc(1:np) ); grid on; xlabel('Displacement [in]');
ylabel('Damping [lb]'),legend('Uncontrolled',2); title('Damping');

figure; plot( X1(1:np,2*n),V1(1:np) ); grid on; xlabel('Displacement [in]');
ylabel('Elastic [lb]'),legend('Uncontrolled',2); title('Elastic');

figure; plot( X1(1:np,2*n),eladamnc(1:np) ); grid on; xlabel('Displacement [in]');
ylabel('Elastic + Damping [lb]'),legend('Uncontrolled',2); title('Elastic + Damping');
```

```
En1=cumsum(abs(V1(1:np)))*dt;
En2=cumsum(abs(V2(1:np)))*dt;
En3=cumsum(abs(dampi(1:np)))*dt;
En4=cumsum(abs(damnc(1:np)))*dt;
En6=En1+En4;
En5=En2+En3;

figure; plot(t,En1,t,En4,t,zeros(1,np) ); grid on; axis tight; xlabel('Time [sec]');
ylabel('Energy'),legend('Elastic','Damping','Elastic + Damping',4); title(['Energy Force Uncontrolled -
Earthquake:',nom,' record']);

figure; plot(t,En2,t,En3,t,zeros(1,np) ); grid on; axis tight; xlabel('Time [sec]');
ylabel('Energy'),legend('Elastic','Damping','Elastic + Damping',4); title(['Energy Force Controlled -
Earthquake:',nom,' record']);
```

APPENDIX D:

PROGRAM: *CLOSED-LOOP CONTROL* AND VDSA DEVICE (MDOF) STRUCTURE

```
clc; clear all; close all

g = 386.4;                                    % acceler. of gravity: in/s^2
% g = 9.8;                                    % acceler. of gravity: m/s^2

n = 6;                                        % number of stories

Wp = [345.6; 345.6; 345.6; 345.6; 345.6;345.6;345.6;345.6];           % weights of floor: lb
k =  [3.404; 3.404; 3.404; 3.404; 3.404;3.404;3.404;3.404]*1e5*0.225/0.0254;   % total stiffness
coefficients: lb/in

k = [3000; 3000; 3000; 3000; 3000; 3000]*1e3*0.225/0.0254*0.2;        % weights of floor: lb
Wp =  [1; 1; 1; 1; 1; 1]*1e3*2.205;            % total stiffness coefficients: lb/in

CoA= [25 ]*1e3;                               % total damping coefficients: lb.s/in
CoB= [10 ]*1e3;                               % total damping coefficients: lb.s/in

RP=1;                                         % Floor
nco=1;                                        % Control numbers
pco=4;                                        % location of the control
Co1=zeros(n,nco);
Co1(pco,1)=CoA+CoB;
D=diag(zeros(1,n));
D(pco,pco)=1;
D2=zeros(n,nco);
D2(pco,1)=1;
Co2=Co1(pco,1);
```

Appendix D:
Program: *Closed-Loop Control* and VDSA Device (MDOF) Structure

```
Q=diag(ones(1,2*n))*10^(4);
K  = diag(k) + diag( [k(2:n) ; 0] ) - diag( k(2:n),1 ) - diag( k(2:n),-1 );
dt = 0.02;                                   % time step of accelerogram: sec
zita = [2]*1e-2;                             % modal damping ratios of building
h=120;
a=120;
E=ones(n,1);
Rm=0.18;
Wmax=h-a*Rm;

Wmin=h-a+30;
w=Wmax;

R=diag(ones(1,nco).*10^(-1));
IR=inv(R);

Ur(1)=0;
r=zeros(n,1);
d1=zeros(2*n,1);
Uc{1}=zeros(nco,1);
U{1}=r;
Up{1}=r;
Ud(1)=0;
Upd(1)=0;
Ucd(1)=0;
Acd(1)=0;

Urr2=zeros(n,1);
Urr22=zeros(n,1);
Ac{1}=0;
Ucv(1)=0;
J(1)=0;

%------------ Read and plot the ground acceleration time history ----------

nom = 'El Centro';                     % name of file with earthquake
                                       % addpath c:/MatlabFiles/Earthquakes;
                                       % directory with accelerograms
acc = load ([nom,'.txt']);             % read earthquake data file
[nr,nc] = size(acc);                   % columns and rows of data file
Xg(1:nr*nc) = acc';                    % copy accelerogram in a vector
xm = max( abs(Xg) );                   % original peak acceleration
Xg =  Xg*g;                            % scaled accelerogram
tf = (nr*nc-1)*dt;                     % final time of accelerogram
t  = 0: dt: tf ;                       % column vector with time steps
nt = length(t);                        % number of discrete time points
np = round( nt);                       % number of time points for plotting
```

```
z0 = zeros(1,np);                          % auxiliary vector for plotting

figure; plot(t,zeros(1,nt), t,Xg/g ); grid on; axis tight;
title(['Normalized earthquake ground acceleration: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Acceleration [g]');

Xpg=cumsum(Xg)*dt;

figure; plot(t,zeros(1,nt), t,Xpg ); grid on; axis tight;
title(['Ground velocity: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/seg]');

Xppg=cumsum(Xpg)*dt;

figure; plot(t,zeros(1,nt), t,Xppg ); grid on; axis tight;
title(['Ground displacement: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

%---- Calculate the matrices & modal properties  -----

I=diag(ones(1,n));
I2=diag(ones(1,2*n));
nn=nr*nc;
dampi(1)=0;

M  = diag( Wp ) ;
K  = diag(k) + diag( [k(2:n) ; 0] ) - diag( k(2:n),1 ) - diag( k(2:n),-1 );
[Phi,lam] =  eig(K,M);
Phi = Phi * diag(1 ./ sqrt( diag(Phi'*M*Phi) ));
[ws,id] = sort( sqrt(diag(lam)) );
Phi = Phi(:,id);
Ts  = 2*pi ./ ws ;
Mi = inv(M) ;
Cs= M* Phi*diag(2*(zita).*ws)* Phi'* M;

%% New Matrices

Av=[zeros(n),diag(ones(1,n))];
Sv=ones(n,1);
Qv=diag(ones(n,1)).*1e2;
A2=Q+Av'*Qv*Av;
A3=Av'*Qv*Sv;

matr=zeros(nn,n);
for i=2:nn
   z=[U{i-1};Up{i-1}];
   c=D*Co1.*(a^2/(a^2+(h-w)^2));
```

```
   Cd=diag(c);
   Cd(pco,pco-1)=-Cd(pco,pco);
   Cd(pco-1,pco)=-Cd(pco,pco);
   C=Cs+Cd;
   A=[zeros(n),I;-Mi*K,-Mi*C];
   Cpr=(1/2)*Mi*D2*(CoA-CoB)*(a*(h-w)/(a^2+(h-w)^2));
   B=[zeros(n,nco);Cpr];
   H=[zeros(n,1);-E];

   [T,lam1] =  eig(A);
   [ws1,id] = sort( sqrt(diag(lam1)) );
   lam2=diag(ws1);
   lam11=inv(T)*A*T;
   paso=(exp(((lam11.*dt))));
   d =diag(diag(paso))*inv(T)*(z+dt/2*(B*Uc{i-1}+H*Xg(i-1)));
   Zt=(inv(diag(ones(1,2*n))+(dt^2)/4*B*inv(R)*B'*A2)*(T*d-
(dt^2)/4*B*inv(R)*B'*A3*Xpg(i)+dt/2*H*Xg(i)));

   Uc{i}=-(dt/2*inv(R)*B')*(A2*Zt+A3*Xpg(i));

   U{i}=(Zt(1:n,:));

   Up{i}=(Zt(n+1:2*n,:));

   %Ac{i}=-Mi*C*Up{i}-Mi*K*U{i}-E*Xg(i)+Cpr;
   %Urr4=Ac{i};
   %Acd(i)=Urr4(RP);
   Urr1=U{i};
   Ud1(i)=Urr1(RP);
   Udn(i)=Urr1(n);
   Urr2=Up{i};
   Upd(i)=Urr2(1);
   Updc(i)=Urr2(pco);
   Updn(i)=Urr2(n);
   Updd(i)=Urr2(RP);
   Urr3=Uc{i};
   Ucd(i)=Urr3(1);
   a3=Ucd(i)/Upd(i);
   bb(i-1)=a3;

   w=h-a*(abs(Co1(pco,1)/abs(Ucd(i)/Updc(i))-1))^(1/2);

    if w>Wmax
      w=Wmax;
   end

   if w<Wmin
```

```
      w=Wmin;
   end
      rr(i-1)=w;

   for j=1:6
      matr(i,j)=Urr1(j);
   end

end

rr(nn)=w;
bb(nn)=0;

figure; plot(t,zeros(1,nt),t,Udn ); grid on; axis tight;
title(['Relative displacement of top floor - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

figure; plot(t,zeros(1,nt),t,Updn ); grid on; axis tight;
title(['Relative velocity of top floor - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

figure; plot(t,zeros(1,nt),t,Ud1 ); grid on; axis tight;
title(['Relative displacement of first floor - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

figure; plot(t,zeros(1,nt),t,Upd ); grid on; axis tight;
title(['Relative velocity of first floor - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

% figure; plot(t,zeros(1,nt),t,Acd ); grid on; axis tight;
% title(['Relative acceleration of top floor - Earthquake:',nom,' record']);
% xlabel('Time [sec]'); ylabel('Acceleration [in/sec^2]');

figure; plot(t,zeros(1,nt),t,Ucd ); grid on; axis tight;
title(['Control Force - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Control Force [lb]');

figure; plot(t,zeros(1,nt),t,rr ); grid on; axis tight;
title(['Variation of Control - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Higth Control [in]');

figure; plot(t,zeros(1,nt),t,bb ); grid on; axis tight;
title(['Uc/Up - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Uc/Up');

% %%%% Uncontrolled
```

```
%--------- Form the state-space matrices and calculate the response -------
Mi = inv(M) ;
Ct=M* Phi* diag(2*(0.02+0.03).*ws)* Phi'* M;
A  = [-Mi*Ct, -Mi*K; eye(n), zeros(n) ] ;
B  = [-ones(n,1) ; zeros(n,1)] ;
C  = [-Mi*Ct, -Mi*K] ;
D  = zeros(n,1) ;

disp('*** Equivalent modal properties of the damped structure:')

damp(A)
[Y1,X1] = lsim(A,B,C,D,Xg,t);
V1  = K(1)*X1(:,n+1)/1e3;
um1 = max( abs(X1(:,n+1:2*n)) );
Vm1 = max( abs(V1) );

%------ Calculate the maximum response & plot the building response -------

% V  = K(1)*X1(:,n+1)/1e3;
% um = max( abs(X1(:,n+1:2*n)) );
% Vm = max( abs(V1) );
V2 = K(1)*Ud1/1e3;
umc = max( abs(matr(:,1:n)) );
Vm1 = max( abs(V1) );
Vm2 = max( abs(V2) );
% damnc=Ct.*X1(:,n);

figure; plot(t(1:np),X1(1:np,2*n) ); grid on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',1); title(['Relative displacement of top floor -
Earthquake:',nom,' record']);

figure; plot( t(1:np),V1(1:np) ); grid on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled',1); title('Total base shear');


% figure; plot(t(1:np),X1(1:np,2*n),t(1:np),Udn(1:np) ); grid on; xlabel('Time [sec]');
% ylabel('Displacement [in]'),legend('Uncontrolled','Controlled',1); title(['Relative displacement of top floor
- Earthquake:',nom,' record']);
%
% figure; plot( t(1:np),V1(1:np),t(1:np),V2(1:np) ); grid on; xlabel('Time [sec]');
% ylabel('Force [kip]'),legend('Uncontrolled','Controlled',1); title('Total base shear');

%------Damper Fixed Case I

w=Wmax;
Zeff=0.3;
Phi1=zeros(n,1);
```

```
for j=1:n-1
   Phi1(j,1)=Phi(j+1,1);
end
Phi1r=Phi(:,1)-Phi1(:,1);
sum1=0;
sum2=0;
for j=1:n
   sum1=sum1+Zeff*pi*M(j,j)*Phi(j,1)^2;
   sum2=sum2+2*Ts(1,1)*(a^2/(a^2+(h-w)^2))*Phi1r(j,1)^2;
end
Cd1=sum1/sum2;

Cd1= [0 Cd1];
for i=1:2
   D3=diag(zeros(1,n));
   D3(1,1)=1;
   Cop=Cd1(i);
   Ct1=M* Phi* diag(2*(0.02+0.03).*ws)* Phi'* M+2*Cop*(a^2/(a^2+(h-w)^2)).*D3;
   A  = [-Mi*Ct1, -Mi*K; eye(n), zeros(n) ] ;
   B  = [-ones(n,1) ; zeros(n,1)] ;
   C  = [-Mi*Ct1, -Mi*K] ;
   D  = zeros(n,1) ;
   cos1=a^2/(a^2+(h-w)^2);

   disp('*** Equivalent modal properties of the damped structure:')

   damp(A);
   if i==1
      [Y3,X3] = lsim(A,B,C,D,Xg,t);
      V3  = K(1)*X3(:,n+1)/1e3;
      um = max( abs(X3(:,n+1:2*n)) );
      Vm = max( abs(V3) );
   else
      [Y4,X4] = lsim(A,B,C,D,Xg,t);
      V4  = K(1)*X4(:,n+1)/1e3;
      um4 = max( abs(X4(:,n+1:2*n)) );
      Vm4 = max( abs(V4) );
   end

end

%------Damper Fixed Case II

for i=1:2
   D3=diag(zeros(1,n));
   D3(1,1)=1;
   D3(2,2)=1;
```

```
    D3(1,1)=1;
    Cop=Cd1(i);
    Ct1=M* Phi* diag(2*(0.02+0.03).*ws)* Phi'* M+2*Cop*(a^2/(a^2+(h-w)^2)).*D3;
    A  = [-Mi*Ct1, -Mi*K; eye(n), zeros(n) ] ;
    B  = [-ones(n,1) ; zeros(n,1)] ;
    C  = [-Mi*Ct1, -Mi*K] ;
    D  = zeros(n,1) ;
    cos1=a^2/(a^2+(h-w)^2);

    disp('*** Equivalent modal properties of the damped structure:')

    damp(A);
    if i==1
        [Y3,X3] = lsim(A,B,C,D,Xg,t);
        V3  = K(1)*X3(:,n+1)/1e3;
        um = max( abs(X3(:,n+1:2*n)) );
        Vm = max( abs(V3) );
    else
        [Y5,X5] = lsim(A,B,C,D,Xg,t);
        V5  = K(1)*X5(:,n+1)/1e3;
        um5 = max( abs(X5(:,n+1:2*n)) );
        Vm5 = max( abs(V5) );
    end

end

%------Damper Fixed Case III

for i=1:2
    D3=diag(ones(1,n));
    Cop=Cd1(i);
    Ct1=M* Phi* diag(2*(0.02+0.03).*ws)* Phi'* M+2*Cop*(a^2/(a^2+(h-w)^2)).*D3;
    A  = [-Mi*Ct1, -Mi*K; eye(n), zeros(n) ] ;
    B  = [-ones(n,1) ; zeros(n,1)] ;
    C  = [-Mi*Ct1, -Mi*K] ;
    D  = zeros(n,1) ;
    cos1=a^2/(a^2+(h-w)^2);

    disp('*** Equivalent modal properties of the damped structure:')

    damp(A);
    if i==1
        [Y3,X3] = lsim(A,B,C,D,Xg,t);
        V3  = K(1)*X3(:,n+1)/1e3;
        um = max( abs(X3(:,n+1:2*n)) );
        Vm = max( abs(V3) );
    else
```

```
      [Y6,X6] = lsim(A,B,C,D,Xg,t);
      V6  = K(1)*X6(:,n+1)/1e3;
      um6 = max( abs(X6(:,n+1:2*n)) );
      Vm6 = max( abs(V6) );
   end
end

figure; plot(t,zeros(1,nt),t,X1(:,n) ); grid on; axis tight;
title(['Relative velocity of top floor Uncontrolled - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

% Relative displacement of top floor

figure; plot(t(1:round(np/2)),X1(1:round(np/2),2*n),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),X6(1:round(np/2),2*n),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Udn(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III','Controlled VDSA',1);
title(['Relative displacement of the top floor - Earthquake:',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1(1:round(np/2),2*n),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),X6(1:round(np/2),2*n),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III',1); title(['Relative displacement
of the top floor - Earthquake:',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1(1:round(np/2),2*n),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Udn(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of the
top floor - Earthquake:',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X6(1:round(np/2),2*n),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Udn(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend(' Fixed Damper Case III','Controlled VDSA',1); title(['Relative
displacement of the top floor - Earthquake:',nom,' record']); grid on ;axis tight;

% Relative displacement of first floor

figure; plot(t(1:round(np/2)),X1(1:round(np/2),n+1),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),X6(1:round(np/2),n+1),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Ud1(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III','Controlled VDSA',1);
title(['Relative displacement of the first floor - Earthquake:',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1(1:round(np/2),n+1),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),X6(1:round(np/2),n+1),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III',1); title(['Relative displacement
of the first floor - Earthquake:',nom,' record']); grid on ;axis tight;
```

```
figure; plot(t(1:round(np/2)),X1(1:round(np/2),n+1),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Ud1(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of the
first floor - Earthquake:',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X6(1:round(np/2),n+1),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Ud1(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend(' Fixed Damper Case III','Controlled VDSA',1); title(['Relative
displacement of the first floor - Earthquake:',nom,' record']); grid on ;axis tight;
% Relative displacement of all floor

for j=1:n

   figure; plot(t(1:round(np/2)),X1(1:round(np/2),j+n),'-b','LineWidth',0.3),hold on
   plot(t(1:round(np/2)),matr(1:round(np/2),j),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
   ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of
the  ', int2str(j), '  floor - Earthquake:',nom,' record']); grid on ;axis tight;
end

% Total base Shear

figure; plot(t(1:round(np/2)),V1(1:round(np/2)),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),V6(1:round(np/2)),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V2(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Fixed Damper Case III','Controlled VDSA',1); title(['Total base
shear - Earthquake:',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1(1:round(np/2)),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V6(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Fixed Damper Case III',1); title(['Total base shear -
Earthquake:',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1(1:round(np/2)),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V2(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Controlled VDSA',1); title(['Total base shear -
Earthquake:',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V6(1:round(np/2)),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V2(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Fixed Damper Case III','Controlled VDSA',1); title(['Total base shear -
Earthquake:',nom,' record']); grid on;axis tight;

% Displacement for different case

piso=[0:1:n];
case1=[0,um4];
case2=[0,um5];
```

```
case3=[0,um6];
contro=[0,umc];
nocon=[0,um1];

figure; plot(case1,piso,'--rs','MarkerSize',5,'LineWidth',1),hold on
        plot(case2,piso,'.-b','MarkerSize',15,'LineWidth',1.5),hold on
        plot(case3,piso,'-xb','MarkerSize',10,'LineWidth',1.5),hold on
        plot(contro,piso,'*--r','MarkerSize',5,'LineWidth',1),hold on
        plot(nocon,piso,'o-g','MarkerSize',5,'LineWidth',1.6),hold on; xlabel('Maximum Displacement [in]');
ylabel('Floor'),legend('fixed Damper Case I','fixed Damper Case II','fixed Damper Case III','Controlled
VDSA','Uncontrolled',4); title(['Maximum Displacement.: ',nom,' record']); grid on;axis tight;

figure; plot(X1(1:np,2*n),V1(1:np),X4(1:np,2*n),V4(1:np),Udn(1:np),V2(1:np) ); grid on;
xlabel('Displacement [in]');
ylabel('Force Elastic [kip]'),legend('Uncontrolled',' fixed Damper','Controlled VDSA',1); title('Elastic');

% figure; plot( Udn(1:np),dampi(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Damping [lb]'),legend('Controlled VDSA',2); title('Damping');

figure; plot( Udn(1:np),V2(1:np) ); grid on; xlabel('Displacement [in]');
ylabel('Elastic [lb]'),legend('Controlled VDSA',2); title('Elastic');

% figure; plot( U(1:np),eladam(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Elastic + Damping [lb]'),legend('Controlled VDSA',2); title('Elastic + Damping');

% figure; plot( X1(1:np,2*n),damnc(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Damping [lb]'),legend('Uncontrolled',2); title('Damping');

figure; plot( X1(1:np,2*n),V1(1:np) ); grid on; xlabel('Displacement [in]');
ylabel('Elastic [lb]'),legend('Uncontrolled',2); title('Elastic');

% figure; plot( X1(1:np,2*n),eladamnc(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Elastic + Damping [lb]'),legend('Uncontrolled',2); title('Elastic + Damping');

% figure; plot( t,X1(1:np,n),t,Up ); grid on; xlabel('Time [sec]');
% ylabel('Velocity [in/sec]'),legend('Uncontrolled','Controlled',1); title(['Relative velocity of top floor -
Earthquake:',nom,' record']);

% En1=cumsum(abs(V1(1:np)))*dt;
% En2=cumsum(abs(V2(1:np)))*dt;
% En3=cumsum(abs(dampi(1:np)))*dt;
% En4=cumsum(abs(damnc(1:np)))*dt;
% En6=En1+En4;
% En5=En2+En3;
% En5=cumsum(abs(eladam(1:np)))*dt;
% En6=cumsum(abs(eladamnc(1:np)))*dt;
% figure; plot(t,En1,t,En4,t,zeros(1,np) ); grid on; axis tight; xlabel('Time [sec]');
```

```
% ylabel('Energy'),legend('Elastic','Damping','Elastic + Damping',4); title(['Energy Force Uncontrolled - Earthquake:',nom,' record']);
%
% figure; plot(t,En2,t,En3,t,zeros(1,np) ); grid on; axis tight; xlabel('Time [sec]');
% ylabel('Energy'),legend('Elastic','Damping','Elastic + Damping',4); title(['Energy Force Controlled - Earthquake:',nom,' record']);
```

APPENDIX E:

PROGRAM: *CLOSED-OPEN-LOOP CONTROL* AND VDSA DEVICE (MDOF) STRUCTURE

```
g = 386.4;                                   % acceler. of gravity: in/s^2
% g = 9.8;                                   % acceler. of gravity: m/s^2

n = 6;                                       % number of stories

Wp = [345.6; 345.6; 345.6; 345.6; 345.6;345.6;345.6;345.6];          % weights of floor: lb
k =  [3.404; 3.404; 3.404; 3.404; 3.404;3.404;3.404;3.404]*1e5*0.225/0.0254;      % total stiffness
coefficients: lb/in

k = [3000; 3000; 3000; 3000; 3000; 3000]*1e3*0.225/0.0254*0.2;     % weights of floor: lb
Wp =  [1; 1; 1; 1; 1; 1]*1e3*2.205;                    % total stiffness coefficients: lb/in

CoA= [25 ]*1e3;                              % total damping coefficients: lb.s/in
CoB= [10 ]*1e3;                              % total damping coefficients: lb.s/in

RP=1;                                        % Floor
nco=1;                                       % Control numbers
pco=4;                                       % Location of the control
Co1=zeros(n,nco);
Co1(pco,1)=CoA+CoB;
D=diag(zeros(1,n));
D(pco,pco)=1;
D2=zeros(n,nco);
D2(pco,1)=1;
Co2=Co1(pco,1);

Q=diag(ones(1,2*n))*10^(4);
K  = diag(k) + diag( [k(2:n) ; 0] ) - diag( k(2:n),1 ) - diag( k(2:n),-1 );
```

Appendix E: Program: *Closed-Open-Loop Control* and VDSA Device (MDOF) Structure

```
dt = 0.02;                          % time step of accelerogram: sec
zita = [2]*1e-2;                    % modal damping ratios of building
h=120;
a=120;
E=ones(n,1);
Rm=0.18;
Wmax=h-a*Rm;

Wmin=h-a+30;
w=Wmax;

R=diag(ones(1,nco).*10^(-3));
IR=inv(R);

Ur(1)=0;
r=zeros(n,1);
d1=zeros(2*n,1);
Uc{1}=zeros(nco,1);
U{1}=r;
Up{1}=r;
Ud(1)=0;
Upd(1)=0;
Ucd(1)=0;
Acd(1)=0;

Urr2=zeros(n,1);
Urr22=zeros(n,1);
Ac{1}=0;
Ucv(1)=0;
J(1)=0;


%------------ Read and plot the ground acceleration time history ----------

nom = 'Friuli';                     % name of file with earthquake
                                    % addpath c:/MatlabFiles/Earthquakes;
                                    % directory with accelerograms
acc = load ([nom,'.txt']);          % read earthquake data file
[nr,nc] = size(acc);                % columns and rows of data file
Xg(1:nr*nc) = acc';                 % copy accelerogram in a vector
xm = max( abs(Xg) );                % original peak acceleration
Xg =  Xg*g;                         % scaled accelerogram
tf = (nr*nc-1)*dt;                  % final time of accelerogram
t  = 0: dt: tf ;                    % column vector with time steps
nt = length(t);                     % number of discrete time points
np = round( nt);                    % number of time points for plotting
```

```
z0 = zeros(1,np);                                % auxiliary vector for plotting

figure; plot(t,zeros(1,nt), t,Xg/g ); grid on; axis tight;
title(['Normalized earthquake ground acceleration: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Acceleration [g]');

Xpg=cumsum(Xg)*dt;

figure; plot(t,zeros(1,nt), t,Xpg ); grid on; axis tight;
title(['Ground velocity: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/seg]');

Xppg=cumsum(Xpg)*dt;

figure; plot(t,zeros(1,nt), t,Xppg ); grid on; axis tight;
title(['Ground displacement: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');
%---- Calculate the matrices & modal properties  -----
I=diag(ones(1,n));
I2=diag(ones(1,2*n));
nn=nr*nc;
dampi(1)=0;

M  = diag( Wp ) ;
K  = diag(k) + diag( [k(2:n) ; 0] ) - diag( k(2:n),1 ) - diag( k(2:n),-1 );
[Phi,lam] =  eig(K,M);
Phi = Phi * diag(1 ./ sqrt( diag(Phi'*M*Phi) ));
[ws,id] = sort( sqrt(diag(lam)) );
Phi = Phi(:,id);
Ts  = 2*pi ./ ws ;
Mi = inv(M) ;
Cs= M* Phi*diag(2*(zita).*ws)* Phi'* M;

%% New Matrices

Av=[zeros(n),diag(ones(1,n))];
Sv=ones(n,1);
Qv=diag(ones(n,1)).*1e2;
A2=Q+Av'*Qv*Av;
A3=Av'*Qv*Sv;

matr=zeros(nn,n);
for i=2:nn
   z=[U{i-1};Up{i-1}];
   c=D*Co1.*(a^2/(a^2+(h-w)^2));
   Cd=diag(c);
   Cd(pco,pco-1)=-Cd(pco,pco);
```

```
Cd(pco-1,pco)=-Cd(pco,pco);
C=Cs+Cd;
A=[zeros(n),I;-Mi*K,-Mi*C];
Cpr=(1/2)*Mi*D2*(CoA-CoB)*(a*(h-w)/(a^2+(h-w)^2));
B=[zeros(n,nco);Cpr];
H=[zeros(n,1);-E];

[T,lam1] =  eig(A);
[ws1,id] = sort( sqrt(diag(lam1)) );
lam2=diag(ws1);
lam11=inv(T)*A*T;
paso=(exp(((lam11.*dt))));
d =diag(diag(paso))*inv(T)*(z+dt/2*(B*Uc{i-1}+H*Xg(i-1)));

P1=(-1)*inv(diag(ones(1,2*n))+((dt^2)/8)*Q*B*inv(R)*B')*(Q+2*Av'*Qv*Av);
P2=(-
1)*inv((dt^2)/8*Q*B*inv(R)*B'+diag(ones(1,2*n)))*(Q*(T*d+dt/2*H*Xg(i))+2*Av'*Qv*Sv*Xpg(i));


Zt=(inv(diag(ones(1,2*n))-(dt^2)/8*B*inv(R)*B'*P1)*(T*d+(dt^2)/8*B*inv(R)*B'*P2+dt/2*H*Xg(i)));
LP=(P1*Zt+P2);
Uc{i}=(dt/4*inv(R)*B'*LP);

U{i}=(Zt(1:n,:));

Up{i}=(Zt(n+1:2*n,:));

%Ac{i}=-Mi*C*Up{i}-Mi*K*U{i}-E*Xg(i)+Cpr;
%Urr4=Ac{i};
Acd(i)=Urr4(RP);
Urr1=U{i};
Ud1(i)=Urr1(RP);
Udn(i)=Urr1(n);
Urr2=Up{i};
Upd(i)=Urr2(1);
Updc(i)=Urr2(pco);
Updn(i)=Urr2(n);
Updd(i)=Urr2(RP);
Urr3=Uc{i};
Ucd(i)=Urr3(1);
 bb(i-1)=a3;

w=h-a*(abs(Co1(pco,1)/abs(Ucd(i)/Updc(i))-1))^(1/2);

 if w>Wmax
   w=Wmax;
end
```

```
    if w<Wmin
       w=Wmin;
    end

    rr(i-1)=w;

    for j=1:6
       matr(i,j)=Urr1(j);
    end

end

rr(nn)=w;
bb(nn)=0;

figure; plot(t,zeros(1,nt),t,Udn ); grid on; axis tight;
title(['Relative displacement of top floor - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

figure; plot(t,zeros(1,nt),t,Updn ); grid on; axis tight;
title(['Relative velocity of top floor - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

figure; plot(t,zeros(1,nt),t,Ud1 ); grid on; axis tight;
title(['Relative displacement of first floor - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

figure; plot(t,zeros(1,nt),t,Upd ); grid on; axis tight;
title(['Relative velocity of first floor - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

% figure; plot(t,zeros(1,nt),t,Acd ); grid on; axis tight;
% title(['Relative acceleration of top floor - Earthquake:',nom,' record']);
% xlabel('Time [sec]'); ylabel('Acceleration [in/sec^2]');

figure; plot(t,zeros(1,nt),t,Ucd ); grid on; axis tight;
title(['Control Force - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Control Force [lb]');

figure; plot(t,zeros(1,nt),t,rr ); grid on; axis tight;
title(['Variation of Control - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Higth Control [in]');

figure; plot(t,zeros(1,nt),t,bb ); grid on; axis tight;
title(['Uc/Up - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Uc/Up');
```

Program: *Closed-Open-Loop Control* and VDSA Device (MDOF) Structure

```
% %%%% Uncontrolled
%--------- Form the state-space matrices and calculate the response -------

Mi = inv(M) ;
Ct=M* Phi* diag(2*(0.02+0.03).*ws)* Phi'* M;
A  = [-Mi*Ct, -Mi*K; eye(n), zeros(n) ] ;
B  = [-ones(n,1) ; zeros(n,1)] ;
C  = [-Mi*Ct, -Mi*K] ;
D  = zeros(n,1) ;

disp('*** Equivalent modal properties of the damped structure:')

damp(A)
[Y1,X1] = lsim(A,B,C,D,Xg,t);
V1  = K(1)*X1(:,n+1)/1e3;
um1 = max( abs(X1(:,n+1:2*n)) );
Vm1 = max( abs(V1) );

%------ Calculate the maximum response & plot the building response -------

% V  = K(1)*X1(:,n+1)/1e3;
% um = max( abs(X1(:,n+1:2*n)) );
% Vm = max( abs(V1) );
V2 = K(1)*Ud1/1e3;
umc = max( abs(matr(:,1:n)) );
Vm1 = max( abs(V1) );
% damnc=Ct.*X1(:,n);

figure; plot(t(1:np),X1(1:np,2*n) ); grid on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',1); title(['Relative displacement of top floor -
Earthquake:',nom,' record']);

figure; plot( t(1:np),V1(1:np) ); grid on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled',1); title('Total base shear');

% figure; plot(t(1:np),X1(1:np,2*n),t(1:np),Udn(1:np) ); grid on; xlabel('Time [sec]');
% ylabel('Displacement [in]'),legend('Uncontrolled','Controlled',1); title(['Relative displacement of top floor
- Earthquake:',nom,' record']);

% figure; plot( t(1:np),V1(1:np),t(1:np),V2(1:np) ); grid on; xlabel('Time [sec]');
% ylabel('Force [kip]'),legend('Uncontrolled','Controlled',1); title('Total base shear');

%------Damper Fixed Case I

w=Wmax;
Zeff=0.3;
```

```
Phi1=zeros(n,1);
for j=1:n-1
   Phi1(j,1)=Phi(j+1,1);
end
Phi1r=Phi(:,1)-Phi1(:,1);
sum1=0;
sum2=0;
for j=1:n
   sum1=sum1+Zeff*pi*M(j,j)*Phi(j,1)^2;
   sum2=sum2+2*Ts(1,1)*(a^2/(a^2+(h-w)^2))*Phi1r(j,1)^2;
end
Cd1=sum1/sum2;

Cd1= [0 Cd1];
for i=1:2
   D3=diag(zeros(1,n));
   D3(1,1)=1;
   Cop=Cd1(i);
   Ct1=M* Phi* diag(2*(0.02+0.03).*ws)* Phi'* M+2*Cop*(a^2/(a^2+(h-w)^2)).*D3;
   A  = [-Mi*Ct1, -Mi*K; eye(n), zeros(n) ] ;
   B  = [-ones(n,1) ; zeros(n,1)] ;
   C  = [-Mi*Ct1, -Mi*K] ;
   D  = zeros(n,1) ;
   cos1=a^2/(a^2+(h-w)^2);

   disp('*** Equivalent modal properties of the damped structure:')

   damp(A);
   if i==1
      [Y3,X3] = lsim(A,B,C,D,Xg,t);
      V3  = K(1)*X3(:,n+1)/1e3;
      um = max( abs(X3(:,n+1:2*n)) );
      Vm = max( abs(V3) );
   else
      [Y4,X4] = lsim(A,B,C,D,Xg,t);
      V4  = K(1)*X4(:,n+1)/1e3;
      um4 = max( abs(X4(:,n+1:2*n)) );
      Vm4 = max( abs(V4) );
   end

end

%------Damper Fixed Case II

for i=1:2
   D3=diag(zeros(1,n));
   D3(1,1)=1;
```

```
   D3(2,2)=1;
   D3(1,1)=1;
   Cop=Cd1(i);
   Ct1=M* Phi* diag(2*(0.02+0.03).*ws)* Phi'* M+2*Cop*(a^2/(a^2+(h-w)^2)).*D3;
   A  = [-Mi*Ct1, -Mi*K; eye(n), zeros(n) ] ;
   B  = [-ones(n,1) ; zeros(n,1)] ;
   C  = [-Mi*Ct1, -Mi*K] ;
   D  = zeros(n,1) ;
   cos1=a^2/(a^2+(h-w)^2);

   disp('*** Equivalent modal properties of the damped structure:')

   damp(A);
   if i==1
      [Y3,X3] = lsim(A,B,C,D,Xg,t);
      V3  = K(1)*X3(:,n+1)/1e3;
      um = max( abs(X3(:,n+1:2*n)) );
      Vm = max( abs(V3) );
   else
      [Y5,X5] = lsim(A,B,C,D,Xg,t);
      V5  = K(1)*X5(:,n+1)/1e3;
      um5 = max( abs(X5(:,n+1:2*n)) );
      Vm5 = max( abs(V5) );
   end

end

%------Damper Fixed Case III

for i=1:2
   D3=diag(ones(1,n));
   Cop=Cd1(i);
   Ct1=M* Phi* diag(2*(0.02+0.03).*ws)* Phi'* M+2*Cop*(a^2/(a^2+(h-w)^2)).*D3;
   A  = [-Mi*Ct1, -Mi*K; eye(n), zeros(n) ] ;
   B  = [-ones(n,1) ; zeros(n,1)] ;
   C  = [-Mi*Ct1, -Mi*K] ;
   D  = zeros(n,1) ;
   cos1=a^2/(a^2+(h-w)^2);

   disp('*** Equivalent modal properties of the damped structure:')

   damp(A);
   if i==1
      [Y3,X3] = lsim(A,B,C,D,Xg,t);
      V3  = K(1)*X3(:,n+1)/1e3;
      um = max( abs(X3(:,n+1:2*n)) );
      Vm = max( abs(V3) );
```

```
   else
      [Y6,X6] = lsim(A,B,C,D,Xg,t);
      V6  = K(1)*X6(:,n+1)/1e3;
      um6 = max( abs(X6(:,n+1:2*n)) );
      Vm6 = max( abs(V6) );
   end
end
figure; plot(t,zeros(1,nt),t,X1(:,n) ); grid on; axis tight;
title(['Relative velocity of top floor Uncontrolled - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

% Relative displacement of top floor

figure; plot(t(1:round(np/2)),X1(1:round(np/2),2*n),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),X6(1:round(np/2),2*n),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Udn(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III','Controlled VDSA',1);
title(['Relative displacement of the top floor - Earthquake:',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1(1:round(np/2),2*n),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),X6(1:round(np/2),2*n),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III',1); title(['Relative displacement
of the top floor - Earthquake:',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1(1:round(np/2),2*n),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Udn(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of the
top floor - Earthquake:',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X6(1:round(np/2),2*n),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Udn(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend(' Fixed Damper Case III','Controlled VDSA',1); title(['Relative
displacement of the top floor - Earthquake:',nom,' record']); grid on ;axis tight;

% Relative displacement of first floor

figure; plot(t(1:round(np/2)),X1(1:round(np/2),n+1),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),X6(1:round(np/2),n+1),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Ud1(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III','Controlled VDSA',1);
title(['Relative displacement of the first floor - Earthquake:',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1(1:round(np/2),n+1),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),X6(1:round(np/2),n+1),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III',1); title(['Relative displacement
of the first floor - Earthquake:',nom,' record']); grid on ;axis tight;
```

```
figure; plot(t(1:round(np/2)),X1(1:round(np/2),n+1),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Ud1(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of the
first floor - Earthquake:',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X6(1:round(np/2),n+1),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Ud1(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend(' Fixed Damper Case III','Controlled VDSA',1); title(['Relative
displacement of the first floor - Earthquake:',nom,' record']); grid on ;axis tight;

% Relative displacement of all floor

for j=1:n
   figure; plot(t(1:round(np/2)),X1(1:round(np/2),j+n),'-b','LineWidth',0.3),hold on
   plot(t(1:round(np/2)),matr(1:round(np/2),j),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
   ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of
the  ', int2str(j), '  floor - Earthquake:',nom,' record']); grid on ;axis tight;
end

% Total base Shear

figure; plot(t(1:round(np/2)),V1(1:round(np/2)),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),V6(1:round(np/2)),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V2(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Fixed Damper Case III','Controlled VDSA',1); title(['Total base
shear - Earthquake:',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1(1:round(np/2)),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V6(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Fixed Damper Case III',1); title(['Total base shear -
Earthquake:',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1(1:round(np/2)),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V2(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Controlled VDSA',1); title(['Total base shear -
Earthquake:',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V6(1:round(np/2)),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V2(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Fixed Damper Case III','Controlled VDSA',1); title(['Total base shear -
Earthquake:',nom,' record']); grid on;axis tight;

% Displacement for different case

piso=[0:1:n];
case1=[0,um4];
case2=[0,um5];
```

```
case3=[0,um6];
contro=[0,umc];
nocon=[0,um1];

figure; plot(case1,piso,'--rs','MarkerSize',5,'LineWidth',1),hold on
        plot(case2,piso,'.-b','MarkerSize',15,'LineWidth',1.5),hold on
        plot(case3,piso,'-xb','MarkerSize',10,'LineWidth',1.5),hold on
        plot(contro,piso,'*--r','MarkerSize',5,'LineWidth',1),hold on
        plot(nocon,piso,'o-g','MarkerSize',5,'LineWidth',1.6),hold on; xlabel('Maximum Displacement [in]');
ylabel('Floor'),legend('fixed Damper Case I','fixed Damper Case II','fixed Damper Case III','Controlled
VDSA','Uncontrolled',4); title(['Maximum Displacement: ',nom,' record']); grid on;axis tight;

figure; plot(X1(1:np,2*n),V1(1:np),X4(1:np,2*n),V6(1:np),Udn(1:np),V2(1:np) ); grid on;
xlabel('Displacement [in]');
ylabel('Force Elastic [kip]'),legend('Uncontrolled',' Fixed Damper Case III','Controlled VDSA',1);
title('Elastic');

% figure; plot( Udn(1:np),dampi(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Damping [lb]'),legend('Controlled VDSA',2); title('Damping');

figure; plot( Udn(1:np),V2(1:np) ); grid on; xlabel('Displacement [in]');
ylabel('Elastic [lb]'),legend('Controlled VDSA',2); title('Elastic');

% figure; plot( U(1:np),eladam(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Elastic + Damping [lb]'),legend('Controlled VDSA',2); title('Elastic + Damping');

% figure; plot( X1(1:np,2*n),damnc(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Damping [lb]'),legend('Uncontrolled',2); title('Damping');

figure; plot( X1(1:np,2*n),V1(1:np) ); grid on; xlabel('Displacement [in]');
ylabel('Elastic [lb]'),legend('Uncontrolled',2); title('Elastic');

% figure; plot( X1(1:np,2*n),eladamnc(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Elastic + Damping [lb]'),legend('Uncontrolled',2); title('Elastic + Damping');

% figure; plot( t,X1(1:np,n),t,Up ); grid on; xlabel('Time [sec]');
% ylabel('Velocity [in/sec]'),legend('Uncontrolled','Controlled',1); title(['Relative velocity of top floor -
Earthquake:',nom,' record']);

% En1=cumsum(abs(V1(1:np)))*dt;
% En2=cumsum(abs(V2(1:np)))*dt;
% En3=cumsum(abs(dampi(1:np)))*dt;
% En4=cumsum(abs(damnc(1:np)))*dt;
% En6=En1+En4;
% En5=En2+En3;
% En5=cumsum(abs(eladam(1:np)))*dt;
% En6=cumsum(abs(eladamnc(1:np)))*dt;
```

```
% figure; plot(t,En1,t,En4,t,zeros(1,np) ); grid on; axis tight; xlabel('Time [sec]');
% ylabel('Energy'),legend('Elastic','Damping','Elastic + Damping',4); title(['Energy Force Uncontrolled - 
Earthquake:',nom,' record']);
%
% figure; plot(t,En2,t,En3,t,zeros(1,np) ); grid on; axis tight; xlabel('Time [sec]');
% ylabel('Energy'),legend('Elastic','Damping','Elastic + Damping',4); title(['Energy Force Controlled - 
Earthquake:',nom,' record']);
```

APPENDIX F:

PROGRAM: *CLOSED-LOOP CONTROL* AND VDSA DEVICE (SDOF) COUPLED STRUCTURES

```
clc; clear all; close all

g = 386.4;                                   % acceler. of gravity: in/s^2
% g = 9.8;                                   % acceler. of gravity: m/s^2

nA = 1;                                      % number of stories A
nB = 1;                                      % number of stories B
% Wp = [14.4]*1e6;                           % weights of floor: lb
% k =  [1.2  ]*1e6;                          % total stiffness coefficients: lb/in
WpA = [14.4]*1e6;                            % weights of floor A: lb
WpB = [14.4]*1e6;                            % weights of floor B: lb
kA =  [0.8  ]*1e6;                           % total stiffness coefficients A: lb/in
kB =  [1.3  ]*1e6;                           % total stiffness coefficients B: lb/in
% Wp = [9.4*g]*1e3;                          % weights of floor: lb
% k =  [(2*pi/1.5)^2*9.4  ]*1e3;             % total stiffness coefficients: lb/in
% Wp = [91.3250]*1e0;                        % weights of floor: lb
% k = [3.6054 ]*1e3;                         % total stiffness coefficients: lb/in
% Wp = [250]*1e3;                            % weights of floor: lb
% k = [20 ]*1e5;                             % total stiffness coefficients: lb/in
% Wp = [16.69*g]*1e0;                        % weights of floor: lb
% k =  [7934  ]*1e0;                         % total stiffness coefficients: lb/in
% Wp = [158*g]*1e3;                          % weights of floor: kg
% k = [25 ]*1e6;                             % total stiffness coefficients: kN/m
% Wp = [35.274*g]*1e3;                       % weights of floor: kg Este
% k = [(2*pi/0.2)^2*35.274 ]*1e3;            % total stiffness coefficients: kN/m
% Wp = [13.6298736]*1e6;                     % weights of floor: kg
% k = [34.8140426]*1e6;                      % total stiffness coefficients: kN/m
% Wp = [13.630]*1e6;                         % weights of floor: kg
```

```
% k = [34.814]*1e6;                       % total stiffness coefficients: kN/m
% Wp = [16*g]*1e3;                        % weights of floor: kg
% k = [(2*pi/0.2)^2*16 ]*1e3;             % total stiffness coefficients: kN/m
% Wp = [3.14*g]*1e2;                      % weights of floor: kg
% k = [3.404]*1e5;                        % total stiffness coefficients: kN/m

CoA= [15 ]*1e3;                           % total damping coefficients: lb.s/in
CoB= [10 ]*1e3;                           % total damping coefficients: lb.s/in

dt = 0.02;                                % time step of accelerogram: sec
zita = [2]*1e-2;                          % modal damping ratios of building

UA(1) =0;
UB(1) =0;
UpA(1)=0;
UpB(1)=0;
Ac{1}=0;
Act(1)=0;
h=120;
a=120;
w=60;
D=[1];
E=diag(ones(nA+nB));
MA  = diag( WpA/g ) ;
MB  = diag( WpB/g ) ;
KA  = diag( kA );
KB  = diag( kB );
Rm=0.18;
Wmax=h-a*Rm;

Wmin=h-a+25;

Q=[diag(diag(ones(2*(nA+nB))))]*10^5;
R=[1]*10^-3*1;
Uc(1)=0;
IR=inv(R);

%------------ Read and plot the ground acceleration time history ----------

nom = 'El Centro';                        % name of file with earthquake
                                          % addpath c:/MatlabFiles/Earthquakes;
                                          % directory with accelerograms
acc = load ([nom,'.txt']);                % read earthquake data file
[nr,nc] = size(acc);                      % columns and rows of data file
Xg(1:nr*nc) = acc';                       % copy accelerogram in a vector
xm = max( abs(Xg) );                      % original peak acceleration
Xg =  Xg*g;                               % scaled accelerogram
```

```
tf = (nr*nc-1)*dt;                        % final time of accelerogram
t  = 0: dt: tf ;                          % column vector with time steps
nt = length(t);                           % number of discrete time points
np = round( nt);                          % number of time points for plotting
z0 = zeros(1,np);                         % auxiliary vector for plotting

figure; plot(t,zeros(1,nt), t,Xg/g ); grid on; axis tight;
title(['Normalized earthquake ground acceleration: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Acceleration [g]');

Xpg=cumsum(Xg)*dt;

figure; plot(t,zeros(1,nt), t,Xpg ); grid on; axis tight;
title(['Ground velocity: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/seg]');

Xppg=cumsum(Xpg)*dt;

figure; plot(t,zeros(1,nt), t,Xppg ); grid on; axis tight;
title(['Ground displacement: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

%---- Calculate the matrices & modal properties of the undamped bldg. -----

nn=nr*nc;
dampi(1)=0;
for i=2:nn
   z=[UA(i-1);UB(i-1);UpA(i-1);UpB(i-1)];
   MA  = diag( MA ) ;
   KA  = diag( kA );
   MB  = diag( MB ) ;
   KB  = diag( kB );

   % Modal Properties structure A

   [PhiA,lamA] =  eig(KA,MA);
   PhiA = PhiA * diag(1 ./ sqrt( diag(PhiA'*MA*PhiA) ));
   [wsA,id] = sort( sqrt(diag(lamA)) );
   PhiA = PhiA(:,id);
   TsA  = 2*pi ./ wsA ;
   MiA = inv(MA) ;
   CsA=MA* PhiA* diag(2*(0.02).*wsA)* PhiA'* MA;

   % Modal Properties structure B

   [PhiB,lamB] =  eig(KB,MB);
   PhiB = PhiB * diag(1 ./ sqrt( diag(PhiB'*MB*PhiB) ));
```

```
[wsB,id] = sort( sqrt(diag(lamB)) );
PhiB = PhiB(:,id);
TsB  = 2*pi ./ wsB ;
MiB = inv(MB) ;
CsB=MB* PhiB* diag(2*(0.02).*wsB)* PhiB'* MB;

CA=CsA+CoA*(a^2/(a^2+(h-w)^2));
CB=CsB+CoB*(a^2/(a^2+(h-w)^2));

Bc=[MiA*KA,0;0,MiB*KB];
Ac=[MiA*CsA,0;0,MiB*CsB];

A=[zeros(nA+nB),diag(diag(ones(nA+nB)));-Bc,-Ac];

CprA=(1/2)*MiA*D*CoA*(a*(h-w)/(a^2+(h-w)^2));
CprB=-(1/2)*MiB*D*CoB*(a*(h-w)/(a^2+(h-w)^2));

B=[diag(zeros(nA+nB));CprA;CprB];
H=[diag(zeros(nA+nB));-E];
I=[1,0;0,1];

%% New Matrices

Av=[diag(diag(zeros((nA+nB)))),diag(diag(ones((nA+nB))))];
Sv=[diag(ones((nA+nB)))];
Qv=[diag(diag(ones((nA+nB))))]*1e2;
A2=Q+Av'*Qv*Av;
A3=Av'*Qv*Sv;

[T,lam1] =  eig(A);
[ws1,id] = sort( sqrt(diag(lam1)) );
lam2=diag(ws1);
lam11=inv(T)*A*T;
paso=(exp(((lam11.*dt))));
d=paso*inv(T)*(z+dt/2*(B*Uc(i-1)+H*Xg(i-1)));
Zt=(inv(diag(diag(ones(2*(nA+nB))))+(dt^2)/4*B*inv(R)*B'*A2)*(T*d-
dt^2/4*B*inv(R)*B'*A3*Xpg(i)+dt/2*H*Xg(i)));
Uc(i)=-(dt/2*inv(R)*B')*(A2*Zt+A3*Xpg(i));
UA(i)=(Zt(1));
UB(i)=(Zt(2));
UpA(i)=(Zt(3));
UpB(i)=(Zt(4));
%AcA{i}=-MiA*CA*UpA(i)-MiA*KA*UA(i)-E(1,1)*Xg(i)+CprA;
%ActA(i)=AcA{i};
%AcB{i}=-MiB*CB*UpB(i)-MiB*KB*UB(i)-E(2,1)*Xg(i)+CprB;
%ActB(i)=AcB{i};
w=h-a*(abs(CoA/abs(Uc(i)/UpA(i))-1))^(1/2);
```

```
    %dampi(i)=C*Up(i);
    %dampiB(i)=C*UpB(i);
    if w>Wmax
       w=Wmax;
    end

    if w<Wmin
       w=Wmin;
    end

    rr(i-1)=w;

end

rr(nn)=w;
bb(nn)=0;

figure; plot(t,zeros(1,nt),t,UA ); grid on; axis tight;
title(['Relative displacement of top floor structure A :',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

figure; plot(t,zeros(1,nt),t,UB ); grid on; axis tight;
title(['Relative displacement of top floor structure B :',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

figure; plot(t,zeros(1,nt),t,UpA ); grid on; axis tight;
title(['Relative velocity of top floor structure A :',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

figure; plot(t,zeros(1,nt),t,UpB ); grid on; axis tight;
title(['Relative velocity of top floor structure B :',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

figure; plot(t,zeros(1,nt),t,ActA ); grid on; axis tight;
title(['Relative acceleration of top floor structure A:',nom,' record']);
xlabel('Time [sec]'); ylabel('Acceleration [in/sec^2]');

figure; plot(t,zeros(1,nt),t,ActB ); grid on; axis tight;
title(['Relative acceleration of top floor structure B:',nom,' record']);
xlabel('Time [sec]'); ylabel('Acceleration [in/sec^2]');

figure; plot(t,zeros(1,nt),t,Uc ); grid on; axis tight;
title(['Control Force - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Control Force [lb]');

figure; plot(t,zeros(1,nt),t,rr ); grid on; axis tight;
title(['Variation of Control - Earthquake:',nom,' record']);
```

```
xlabel('Time [sec]'); ylabel('Higth Control [in]');
figure; plot(t,zeros(1,nt),t,bb ); grid on; axis tight;
title(['Uc/Up - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Uc/Up');

%%%% Uncontrolled

%--------- Form the state-space matrices and calculate the response structure A-------

MiA = inv(MA) ;
CtA=MA* PhiA* diag(2*(0.02+0.03).*wsA)* PhiA'* MA;
A  = [-MiA*CtA, -MiA*KA; eye(nA), zeros(nA) ] ;
B  = [-ones(nA,1) ; zeros(nA,1)] ;
C  = [-MiA*CtA, -MiA*KA] ;
D  = zeros(nA,1) ;

disp('*** Equivalent modal properties of the damped structure:')

damp(A)
[Y1A,X1A] = lsim(A,B,C,D,Xg,t);
V1A  = KA(1)*X1A(:,nA+1)/1e3;
um1A = max( abs(X1A(:,nA+1:2*nA)) );
Vm1A = max( abs(V1A) );

%------ Calculate the maximum response & plot the building response -------

% VA  = KA(1)*X1A(:,nA+1)/1e3;
% um1A = max( abs(X1A(:,nA+1:2*nA)) );
% Vm1A = max( abs(V1A) );
V2A = KA(1)*UA/1e3;
% damnc=CtA.*X1A(:,n);

figure; plot(t(1:round(np/2)),X1A(1:round(np/2)),t(1:round(np/2)),UA(1:round(np/2)) ); grid on;
xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled',1); title(['Relative displacement of top floor
structure A :',nom,' record']);

figure; plot( t(1:np),V1A(1:np),t(1:np),V2A(1:np) ); grid on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Controlled',1); title('Total base shear structure A');

% eladamnc=damnc+V1A*1e3;

%--------- Form the state-space matrices and calculate the response structure B-------

MiB = inv(MB) ;
CtB=MB* PhiB* diag(2*(0.02+0.03).*wsB)* PhiB'* MB;
A  = [-MiB*CtB, -MiB*KB; eye(nB), zeros(nB) ] ;
```

```
B  = [-ones(nB,1) ; zeros(nB,1)] ;
C  = [-MiB*CtB, -MiB*KB] ;
D  = zeros(nB,1) ;

disp('*** Equivalent modal properties of the damped structure:')

damp(A)
[Y1B,X1B] = lsim(A,B,C,D,Xg,t);
V1B  = KB(1)*X1B(:,nB+1)/1e3;
um1B = max( abs(X1B(:,nB+1:2*nB)) );
Vm1B = max( abs(V1B) );

%------ Calculate the maximum response & plot the building response -------

% VB  = KB(1)*X1B(:,nB+1)/1e3;
% umB = max( abs(X1B(:,nB+1:2*nB)) );
% VmB = max( abs(V1B) );
V2B = KB(1)*UB/1e3;
% damnc=CtB.*X1B(:,nB);

figure; plot(t(1:round(np/2)),X1B(1:round(np/2)),t(1:round(np/2)),UB(1:round(np/2)) ); grid on;
xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled',1); title(['Relative displacement of top floor
structure A :',nom,' record']);

figure; plot( t(1:np),V1B(1:np),t(1:np),V2B(1:np) ); grid on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Controlled',1); title('Total base shear structure A');

% eladamnc=damnc+V1B*1e3;

%------Damper Fixed Case I structure A

% eladam=dampi+V2*1e3;
w=Wmax;
Zeff=0.3;
Cd1A=Zeff*pi*MA*PhiA^2/(2*TsA*(a^2/(a^2+(h-w)^2))*PhiA^2);

Cd1A= [0 2*Cd1A];
for i=1:2

   CopA=Cd1A(i);
   Ct1A=2*zita*MA*wsA+2*CopA*(a^2/(a^2+(h-w)^2));
   A  = [-MiA*Ct1A, -MiA*KA; eye(nA), zeros(nA) ] ;
   B  = [-ones(nA,1) ; zeros(nA,1)] ;
   C  = [-MiA*Ct1A, -MiA*KA] ;
   D  = zeros(nA,1) ;
   cos1=a^2/(a^2+(h-w)^2);
```

```
    disp('*** Equivalent modal properties of the damped structure:')
    damp(A);
    if i==1
        [Y3A,X3A] = lsim(A,B,C,D,Xg,t);
        V3A = KA(1)*X3A(:,nA+1)/1e3;
        um3A = max( abs(X3A(:,nA+1:2*nA)) );
        Vm3A = max( abs(V3A) );
    else
        [Y4A,X4A] = lsim(A,B,C,D,Xg,t);
        V4A = KA(1)*X4A(:,nA+1)/1e3;
        um4A = max( abs(X4A(:,nA+1:2*nA)) );
        Vm4A = max( abs(V4A) );
    end

end

%------Damper Fixed Case I structure B

% eladam=dampi+V2*1e3;
w=Wmax;
Zeff=0.3;
Cd1B=Zeff*pi*MB*PhiB^2/(2*TsB*(a^2/(a^2+(h-w)^2))*PhiB^2);

Cd1B= [0 2*Cd1B];
for i=1:2

    CopB=Cd1B(i);
    Ct1B=2*zita*MB*wsB+2*CopB*(a^2/(a^2+(h-w)^2));
    A  = [-MiB*Ct1B, -MiB*KB; eye(nB), zeros(nB) ] ;
    B  = [-ones(nB,1) ; zeros(nB,1)] ;
    C  = [-MiB*Ct1B, -MiB*KB] ;
    D  = zeros(nB,1) ;
    cos1=a^2/(a^2+(h-w)^2);

    disp('*** Equivalent modal properties of the damped structure:')

    damp(A);
    if i==1
        [Y3B,X3B] = lsim(A,B,C,D,Xg,t);
        V3B  = KB(1)*X3B(:,nB+1)/1e3;
        um3B = max( abs(X3B(:,nB+1:2*nB)) );
        Vm3B = max( abs(V3B) );
    else
        [Y4B,X4B] = lsim(A,B,C,D,Xg,t);
        V4B = KB(1)*X4B(:,nB+1)/1e3;
        um4B = max( abs(X4B(:,nB+1:2*nB)) );
        Vm4B = max( abs(V4B) );
```

```
  end
end
%------Damper Fixed Case II

% eladam=dampi+V2*1e3;
% w=0;
% Zeff=0.28;
% Cd2=Zeff*pi*M*Phi^2/(Ts*(a^2/(a^2+(h-w)^2))*Phi^2);
%
% Cd2= [0 2*Cd2];
% for i=1:2
%
%     Cos=Cd2(i);
%     Ct2=2*zita*M*ws+Cos*(a^2/(a^2+(h-w)^2));
%     A2 = [-Mi*Ct2, -Mi*K; eye(n), zeros(n) ] ;
%     B  = [-ones(n,1) ; zeros(n,1)] ;
%     C  = [-Mi*Ct2, -Mi*K] ;
%     D  = zeros(n,1) ;
%     cos2=a^2/(a^2+(h-w)^2);

%     disp('*** Equivalent modal properties of the damped structure:')

%     damp(A2);
%     if i==1
%        [Y5,X5] = lsim(A2,B,C,D,Xg,t);
%        V5  = K(1)*X5(:,n+1)/1e3;
%        um = max( abs(X5(:,n+1:2*n)) );
%        Vm = max( abs(V5) );
%     else
%        [Y6,X6] = lsim(A2,B,C,D,Xg,t);
%        V6  = K(1)*X6(:,n+1)/1e3;
%        um = max( abs(X6(:,n+1:2*n)) );
%        Vm = max( abs(V6) );
%     end
%
% end

figure; plot(t,zeros(1,nt),t,X1A(:,nA) ); grid on; axis tight;
title(['Relative velocity of top floor structure A Uncontrolled - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

figure; plot(t,zeros(1,nt),t,X1B(:,nB) ); grid on; axis tight;
title(['Relative velocity of top floor structure B Uncontrolled - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

% Structure A (Displacement)
```

```
figure; plot(t(1:round(np/2)),X1A(1:round(np/2),2*nA),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),X4A(1:round(np/2),2*nA),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),UA(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' fixed Damper','Controlled VDSA',1); title(['Relative
displacement of top floor structure A :',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1A(1:round(np/2),2*nA),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),X4A(1:round(np/2),2*nA),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' fixed Damper',1); title(['Relative displacement of top
floor structure A :',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1A(1:round(np/2),2*nA),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),UA(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of top
floor structure A :',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X4A(1:round(np/2),2*nA),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),UA(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend(' fixed Damper','Controlled VDSA',1); title(['Relative displacement of
top floor structure A :',nom,' record']); grid on ;axis tight;

% Structure B (Displacement)

figure; plot(t(1:round(np/2)),X1B(1:round(np/2),2*nB),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),X4B(1:round(np/2),2*nB),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),UB(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' fixed Damper','Controlled VDSA',1); title(['Relative
displacement of top floor structure B :',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1B(1:round(np/2),2*nB),'-b','LineWidth',0.3),hold on
      plot(t(1:round(np/2)),X4B(1:round(np/2),2*nB),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' fixed Damper',1); title(['Relative displacement of top
floor structure B :',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1B(1:round(np/2),2*nB),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),UB(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of top
floor structure B :',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X4B(1:round(np/2),2*nB),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),UB(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend(' fixed Damper','Controlled VDSA',1); title(['Relative displacement of
top floor structure B :',nom,' record']); grid on ;axis tight;

% Structure A (Total base shear)

figure; plot(t(1:round(np/2)),V1A(1:round(np/2)),'--b','LineWidth',1),hold on
```

```
plot(t(1:round(np/2)),V4A(1:round(np/2)),'.-r','LineWidth',0.3),hold on
plot(t(1:round(np/2)),V2A(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','fixed Damper','Controlled VDSA',1); title(['Total base shear
structure A :',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1A(1:round(np/2)),'-b','LineWidth',0.3),hold on
plot(t(1:round(np/2)),V4A(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','fixed Damper',1); title(['Total base shear structure A :',nom,'
record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1A(1:round(np/2)),'-b','LineWidth',0.3),hold on
plot(t(1:round(np/2)),V2A(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Controlled VDSA',1); title(['Total base shear structure A :',nom,'
record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V4A(1:round(np/2)),'-b','LineWidth',0.3),hold on
plot(t(1:round(np/2)),V2A(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('fixed Damper','Controlled VDSA',1); title(['Total base shear structure A
:',nom,' record']); grid on;axis tight;

% Structure B (Total base shear)

figure; plot(t(1:round(np/2)),V1B(1:round(np/2)),'--b','LineWidth',1),hold on
plot(t(1:round(np/2)),V4B(1:round(np/2)),'.-r','LineWidth',0.3),hold on
plot(t(1:round(np/2)),V2B(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','fixed Damper','Controlled VDSA',1); title(['Total base shear
structure B :',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1B(1:round(np/2)),'-b','LineWidth',0.3),hold on
plot(t(1:round(np/2)),V4B(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','fixed Damper',1); title(['Total base shear structure B :',nom,'
record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1B(1:round(np/2)),'-b','LineWidth',0.3),hold on
plot(t(1:round(np/2)),V2B(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Controlled VDSA',1); title(['Total base shear structure B :',nom,'
record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V4B(1:round(np/2)),'-b','LineWidth',0.3),hold on
plot(t(1:round(np/2)),V2B(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('fixed Damper','Controlled VDSA',1); title(['Total base shear structure B
:',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),UA(1:round(np/2)),'-r','LineWidth',1),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Controlled VDSA',1); title(['Relative displacement of top floor structure
A :',nom,' record']); grid on ;axis tight;
```

```
figure; plot(t(1:round(np/2)),UB(1:round(np/2)),'-r','LineWidth',1),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Controlled VDSA',1); title(['Relative displacement of top floor structure
B :',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),V2A(1:round(np/2)),'-r','LineWidth',1),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Controlled VDSA',1); title(['Total base shear structure A :',nom,' record']); grid
on;axis tight;

figure; plot(t(1:round(np/2)),V2B(1:round(np/2)),'-r','LineWidth',1),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Controlled VDSA',1); title(['Total base shear structure B :',nom,' record']); grid
on;axis tight;

% figure; plot(X1(1:np,2*n),V1(1:np),X4(1:np,2*n),V4(1:np),U(1:np),V2(1:np) ); grid on;
xlabel('Displacement [in]');
% ylabel('Force Elastic [kip]'),legend('Uncontrolled',' fixed Damper','Controlled VDSA',1); title('Elastic');
%
% figure; plot( U(1:np),dampi(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Damping [lb]'),legend('Controlled VDSA',2); title('Damping');
%
% figure; plot( U(1:np),V2(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Elastic [lb]'),legend('Controlled VDSA',2); title('Elastic');
%
% figure; plot( U(1:np),eladam(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Elastic + Damping [lb]'),legend('Controlled VDSA',2); title('Elastic + Damping');
%
% figure; plot( X1(1:np,2*n),damnc(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Damping [lb]'),legend('Uncontrolled',2); title('Damping');
%
% figure; plot( X1(1:np,2*n),V1(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Elastic [lb]'),legend('Uncontrolled',2); title('Elastic');
%
% figure; plot( X1(1:np,2*n),eladamnc(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Elastic + Damping [lb]'),legend('Uncontrolled',2); title('Elastic + Damping');
%
% En1=cumsum(abs(V1(1:np)))*dt;
% En2=cumsum(abs(V2(1:np)))*dt;
% En3=cumsum(abs(dampi(1:np)))*dt;
% En4=cumsum(abs(damnc(1:np)))*dt;
% En6=En1+En4;
% En5=En2+En3;
% figure; plot(t,En1,t,En4,t,zeros(1,np) ); grid on; axis tight; xlabel('Time [sec]');
% ylabel('Energy'),legend('Elastic','Damping','Elastic + Damping',4); title(['Energy Force Uncontrolled -
Earthquake:',nom,' record']);
%
% figure; plot(t,En2,t,En3,t,zeros(1,np) ); grid on; axis tight; xlabel('Time [sec]');
% ylabel('Energy'),legend('Elastic','Damping','Elastic + Damping',4); title(['Energy Force Controlled -
Earthquake:',nom,' record']);
```

APPENDIX G:

PROGRAM: *CLOSED-OPEN-LOOP CONTROL* AND VDSA DEVICE (SDOF) COUPLED STRUCTURES

```
clc; clear all; close all

g = 386.4;                          % acceler. of gravity: in/s^2
% g = 9.8;                          % acceler. of gravity: m/s^2
nA = 1;                             % number of stories A
nB = 1;                             % number of stories B
% Wp = [14.4]*1e6;                  % weights of floor: lb
% k =  [1.2  ]*1e6;                 % total stiffness coefficients: lb/in
WpA = [14.4]*1e6;                   % weights of floor A: lb
WpB = [14.4]*1e6;                   % weights of floor B: lb
kA =  [0.8  ]*1e6;                  % total stiffness coefficients A: lb/in
kB =  [1.3  ]*1e6;                  % total stiffness coefficients B: lb/in
% Wp = [9.4*g]*1e3;                 % weights of floor: lb
% k =  [(2*pi/1.5)^2*9.4  ]*1e3;    % total stiffness coefficients: lb/in

% Wp = [91.3250]*1e0;               % weights of floor: lb
% k = [3.6054 ]*1e3;                % total stiffness coefficients: lb/in
% Wp = [250]*1e3;                   % weights of floor: lb
% k = [20 ]*1e5;                    % total stiffness coefficients: lb/in
% Wp = [16.69*g]*1e0;               % weights of floor: lb
% k =  [7934  ]*1e0;                % total stiffness coefficients: lb/in
% Wp = [158*g]*1e3;                 % weights of floor: kg
% k = [25 ]*1e6;                    % total stiffness coefficients: kN/m
% Wp = [35.274*g]*1e3;              % weights of floor: kg
% k = [(2*pi/0.2)^2*35.274 ]*1e3;   % total stiffness coefficients: kN/m
% Wp = [13.6298736]*1e6;            % weights of floor: kg
% k = [34.8140426]*1e6;             % total stiffness coefficients: kN/m
% Wp = [13.630]*1e6;                % weights of floor: kg
```

Appendix G: Program: *Closed-Open-Loop Control* and VDSA Device (SDOF) Coupled Structures

```
% k = [34.814]*1e6;                      % total stiffness coefficients: kN/m
% Wp = [16*g]*1e3;                       % weights of floor: kg
% k = [(2*pi/0.2)^2*16 ]*1e3;            % total stiffness coefficients: kN/m
% Wp = [3.14*g]*1e2;                     % weights of floor: kg
% k = [3.404]*1e5;                       % total stiffness coefficients: kN/m

CoA= [15 ]*1e3;                          % total damping coefficients: lb.s/in
CoB= [10 ]*1e3;                          % total damping coefficients: lb.s/in

dt = 0.02;                               % time step of accelerogram: sec
zita = [2]*1e-2;                         % modal damping ratios of building

UA(1) =0;
UB(1) =0;
UpA(1)=0;
UpB(1)=0;
Ac{1}=0;
Act(1)=0;
h=120;
a=120;
w=60;
D=[1];
E=diag(ones(nA+nB));
MA  = diag( WpA/g ) ;
MB  = diag( WpB/g ) ;
KA  = diag( kA );
KB  = diag( kB );
Rm=0.18;
Wmax=h-a*Rm;

Wmin=h-a+25;
%cte=k(1)/(Wp);

Q=[diag(diag(ones(2*(nA+nB))))]*10^5;
R=[1]*10^-3*1;
Uc(1)=0;
IR=inv(R);

%------------ Read and plot the ground acceleration time history ----------

nom = 'Friuli';                          % name of file with earthquake
                                         % addpath c:/MatlabFiles/Earthquakes;
                                         % directory with accelerograms
acc = load ([nom,'.txt']);               % read earthquake data file
[nr,nc] = size(acc);                     % columns and rows of data file
Xg(1:nr*nc) = acc';                      % copy accelerogram in a vector
xm = max( abs(Xg) );                     % original peak acceleration
```

```
Xg =  Xg*g;                               % scaled accelerogram
tf = (nr*nc-1)*dt;                        % final time of accelerogram
t  = 0: dt: tf ;                          % column vector with time steps
nt = length(t);                           % number of discrete time points
np = round( nt);                          % number of time points for plotting
z0 = zeros(1,np);                         % auxiliary vector for plotting

figure; plot(t,zeros(1,nt), t,Xg/g ); grid on; axis tight;
title(['Normalized earthquake ground acceleration: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Acceleration [g]');

Xpg=cumsum(Xg)*dt;

figure; plot(t,zeros(1,nt), t,Xpg ); grid on; axis tight;
title(['Ground velocity: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/seg]');

Xppg=cumsum(Xpg)*dt;

figure; plot(t,zeros(1,nt), t,Xppg ); grid on; axis tight;
title(['Ground displacement: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

%---- Calculate the matrices & modal properties of the undamped bldg. -----

nn=nr*nc;
dampi(1)=0;
for i=2:nn
   z=[UA(i-1);UB(i-1);UpA(i-1);UpB(i-1)];
   MA  = diag( MA ) ;
   KA  = diag( kA );
   MB  = diag( MB ) ;
   KB  = diag( kB );

   % Modal Properties structure A

   [PhiA,lamA] =  eig(KA,MA);
   PhiA = PhiA * diag(1 ./ sqrt( diag(PhiA'*MA*PhiA) ));
   [wsA,id] = sort( sqrt(diag(lamA)) );
   PhiA = PhiA(:,id);
   TsA  = 2*pi ./ wsA ;
   MiA = inv(MA) ;
   CsA=MA* PhiA* diag(2*(0.02).*wsA)* PhiA'* MA;

   % Modal Properties structure B

   [PhiB,lamB] =  eig(KB,MB);
```

```
PhiB = PhiB * diag(1 ./ sqrt( diag(PhiB'*MB*PhiB) ));
[wsB,id] = sort( sqrt(diag(lamB)) );
PhiB = PhiB(:,id);
TsB  = 2*pi ./ wsB ;
MiB = inv(MB) ;
CsB=MB* PhiB* diag(2*(0.02).*wsB)* PhiB'* MB;

CA=CsA+CoA*(a^2/(a^2+(h-w)^2));
CB=CsB+CoB*(a^2/(a^2+(h-w)^2));

Bc=[MiA*KA,0;0,MiB*KB];
Ac=[MiA*CsA,0;0,MiB*CsB];

A=[zeros(nA+nB),diag(diag(ones(nA+nB)));-Bc,-Ac];

CprA=(1/2)*MiA*D*CoA*(a*(h-w)/(a^2+(h-w)^2));
CprB=-(1/2)*MiB*D*CoB*(a*(h-w)/(a^2+(h-w)^2));

B=[diag(zeros(nA+nB));CprA;CprB];
H=[diag(zeros(nA+nB));-E];
I=[1,0;0,1];

%% New Matrices

Av=[diag(diag(zeros((nA+nB)))),diag(diag(ones((nA+nB))))];
Sv=[diag(ones((nA+nB)))];
Qv=[diag(diag(ones((nA+nB))))]*1e2;
A2=Q+Av'*Qv*Av;
A3=Av'*Qv*Sv;

[T,lam1] =  eig(A);
[ws1,id] = sort( sqrt(diag(lam1)) );
lam2=diag(ws1);
lam11=inv(T)*A*T;
paso=(exp(((lam11.*dt))));
d=paso*inv(T)*(z+dt/2*(B*Uc(i-1)+H*Xg(i-1)));

P1=(-1)*inv(diag(diag(ones(2*(nA+nB))))+((dt^2)/8)*Q*B*IR*B')*(Q+2*Av'*Qv*Av);
P2=(-
1)*inv((dt^2)/8*Q*B*inv(R)*B'+diag(diag(ones(2*(nA+nB)))))*(Q*(T*d+dt/2*H*Xg(i))+2*Av'*Qv*Sv*
Xpg(i));


Zt=(inv(diag(diag(ones(2*(nA+nB))))-
(dt^2)/8*B*inv(R)*B'*P1)*(T*d+(dt^2)/8*B*inv(R)*B'*P2+dt/2*H*Xg(i)));
LP=(P1*Zt+P2);
Uc(i)=(dt/4*inv(R)*B'*LP);
```

```
    UA(i)=(Zt(1));
    UB(i)=(Zt(2));
    UpA(i)=(Zt(3));
    UpB(i)=(Zt(4));
    %AcA{i}=-MiA*CA*UpA(i)-MiA*KA*UA(i)-E(1,1)*Xg(i)+CprA;
    %ActA(i)=AcA{i};
    %AcB{i}=-MiB*CB*UpB(i)-MiB*KB*UB(i)-E(2,1)*Xg(i)+CprB;
    %ActB(i)=AcB{i};
    w=h-a*(abs(CoA/abs(Uc(i)/UpA(i))-1))^(1/2);
    %dampiA(i)=C*UpA(i);
    %dampiB(i)=C*UpB(i);
    if w>Wmax
       w=Wmax;
    end

    if w<Wmin
       w=Wmin;
    end

    rr(i-1)=w;

end

rr(nn)=w;
bb(nn)=0;

figure; plot(t,zeros(1,nt),t,UA ); grid on; axis tight;
title(['Relative displacement of top floor structure A :',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

figure; plot(t,zeros(1,nt),t,UB ); grid on; axis tight;
title(['Relative displacement of top floor structure B :',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

figure; plot(t,zeros(1,nt),t,UpA ); grid on; axis tight;
title(['Relative velocity of top floor structure A :',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

figure; plot(t,zeros(1,nt),t,UpB ); grid on; axis tight;
title(['Relative velocity of top floor structure B :',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

figure; plot(t,zeros(1,nt),t,ActA ); grid on; axis tight;
title(['Relative acceleration of top floor structure A:',nom,' record']);
xlabel('Time [sec]'); ylabel('Acceleration [in/sec^2]');
```

```
figure; plot(t,zeros(1,nt),t,ActB ); grid on; axis tight;
title(['Relative acceleration of top floor structure B:',nom,' record']);
xlabel('Time [sec]'); ylabel('Acceleration [in/sec^2]');

figure; plot(t,zeros(1,nt),t,Uc ); grid on; axis tight;
title(['Control Force - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Control Force [lb]');

figure; plot(t,zeros(1,nt),t,rr ); grid on; axis tight;
title(['Variation of Control - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Higth Control [in]');

figure; plot(t,zeros(1,nt),t,bb ); grid on; axis tight;
title(['Uc/Up - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Uc/Up');

%%%% Uncontrolled

%--------- Form the state-space matrices and calculate the response structure A-------

MiA = inv(MA) ;
CtA=MA* PhiA* diag(2*(0.02+0.03).*wsA)* PhiA'* MA;
A  = [-MiA*CtA, -MiA*KA; eye(nA), zeros(nA) ] ;
B  = [-ones(nA,1) ; zeros(nA,1)] ;
C  = [-MiA*CtA, -MiA*KA] ;
D  = zeros(nA,1) ;

disp('*** Equivalent modal properties of the damped structure:')

damp(A)
[Y1A,X1A] = lsim(A,B,C,D,Xg,t);
V1A  = KA(1)*X1A(:,nA+1)/1e3;
umA = max( abs(X1A(:,nA+1:2*nA)) );
VmA = max( abs(V1A) );

%------ Calculate the maximum response & plot the building response -------

% VA  = KA(1)*X1A(:,nA+1)/1e3;
% umA = max( abs(X1A(:,nA+1:2*nA)) );
% VmA = max( abs(V1A) );
V2A = KA(1)*UA/1e3;
% damnc=CtA.*X1A(:,n);

figure; plot(t(1:round(np/2)),X1A(1:round(np/2)),t(1:round(np/2)),UA(1:round(np/2)) ); grid on;
xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled',1); title(['Relative displacement of top floor
structure A :',nom,' record']);
```

```
figure; plot( t(1:np),V1A(1:np),t(1:np),V2A(1:np) ); grid on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Controlled',1); title('Total base shear structure A');

% eladamnc=damnc+V1A*1e3;

%--------- Form the state-space matrices and calculate the response structure B-------

MiB = inv(MB) ;
CtB=MB* PhiB* diag(2*(0.02+0.03).*wsB)* PhiB'* MB;
A  = [-MiB*CtB, -MiB*KB; eye(nB), zeros(nB) ] ;
B  = [-ones(nB,1) ; zeros(nB,1)] ;
C  = [-MiB*CtB, -MiB*KB] ;
D  = zeros(nB,1) ;

disp('*** Equivalent modal properties of the damped structure:')

damp(A)
[Y1B,X1B] = lsim(A,B,C,D,Xg,t);
V1B  = KB(1)*X1B(:,nB+1)/1e3;
umB = max( abs(X1B(:,nB+1:2*nB)) );
VmB = max( abs(V1B) );

%------ Calculate the maximum response & plot the building response -------

% VB  = KB(1)*X1B(:,nB+1)/1e3;
% umB = max( abs(X1B(:,nB+1:2*nB)) );
% VmB = max( abs(V1B) );
V2B = KB(1)*UB/1e3;
% damnc=CtB.*X1B(:,nB);

figure; plot(t(1:round(np/2)),X1B(1:round(np/2)),t(1:round(np/2)),UB(1:round(np/2)) ); grid on;
xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled',1); title(['Relative displacement of top floor
structure A :',nom,' record']);

figure; plot( t(1:np),V1B(1:np),t(1:np),V2B(1:np) ); grid on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Controlled',1); title('Total base shear structure A');

% eladamnc=damnc+V1B*1e3;

%------Damper Fixed Case I structure A

% eladam=dampi+V2*1e3;
w=Wmax;
Zeff=0.3;
Cd1A=Zeff*pi*MA*PhiA^2/(2*TsA*(a^2/(a^2+(h-w)^2))*PhiA^2);
```

```
Cd1A= [0 2*Cd1A];
for i=1:2

   CopA=Cd1A(i);
   Ct1A=2*zita*MA*wsA+2*CopA*(a^2/(a^2+(h-w)^2));
   A  = [-MiA*Ct1A, -MiA*KA; eye(nA), zeros(nA) ] ;
   B  = [-ones(nA,1) ; zeros(nA,1)] ;
   C  = [-MiA*Ct1A, -MiA*KA] ;
   D  = zeros(nA,1) ;
   cos1=a^2/(a^2+(h-w)^2);

   disp('*** Equivalent modal properties of the damped structure:')

   damp(A);
   if i==1
      [Y3A,X3A] = lsim(A,B,C,D,Xg,t);
      V3A = KA(1)*X3A(:,nA+1)/1e3;
      um3A = max( abs(X3A(:,nA+1:2*nA)) );
      Vm3A = max( abs(V3A) );
   else
      [Y4A,X4A] = lsim(A,B,C,D,Xg,t);
      V4A = KA(1)*X4A(:,nA+1)/1e3;
      um4A = max( abs(X4A(:,nA+1:2*nA)) );
      Vm4A = max( abs(V4A) );
   end

end

%------Damper Fixed Case I structure B

% eladam=dampi+V2*1e3;
w=Wmax;
Zeff=0.3;
Cd1B=Zeff*pi*MB*PhiB^2/(2*TsB*(a^2/(a^2+(h-w)^2))*PhiB^2);

Cd1B= [0 2*Cd1B];
for i=1:2

   CopB=Cd1B(i);
   Ct1B=2*zita*MB*wsB+2*CopB*(a^2/(a^2+(h-w)^2));
   A  = [-MiB*Ct1B, -MiB*KB; eye(nB), zeros(nB) ] ;
   B  = [-ones(nB,1) ; zeros(nB,1)] ;
   C  = [-MiB*Ct1B, -MiB*KB] ;
   D  = zeros(nB,1) ;
   cos1=a^2/(a^2+(h-w)^2);
```

```
   disp('*** Equivalent modal properties of the damped structure:')
   damp(A);
   if i==1
      [Y3B,X3B] = lsim(A,B,C,D,Xg,t);
      V3B  = KB(1)*X3B(:,nB+1)/1e3;
      um3B = max( abs(X3B(:,nB+1:2*nB)) );
      Vm3B = max( abs(V3B) );
   else
      [Y4B,X4B] = lsim(A,B,C,D,Xg,t);
      V4B = KB(1)*X4B(:,nB+1)/1e3;
      um4B = max( abs(X4B(:,nB+1:2*nB)) );
      Vm4B = max( abs(V4B) );
   end

end

%------Damper Fixed Case II

% eladam=dampi+V2*1e3;
% w=0;
% Zeff=0.28;
% Cd2=Zeff*pi*M*Phi^2/(Ts*(a^2/(a^2+(h-w)^2))*Phi^2);
%
% Cd2= [0 2*Cd2];
% for i=1:2
%
%    Cos=Cd2(i);
%    Ct2=2*zita*M*ws+Cos*(a^2/(a^2+(h-w)^2));
%    A2 = [-Mi*Ct2, -Mi*K; eye(n), zeros(n) ] ;
%    B  = [-ones(n,1) ; zeros(n,1)] ;
%    C  = [-Mi*Ct2, -Mi*K] ;
%    D  = zeros(n,1) ;
%    cos2=a^2/(a^2+(h-w)^2);

%    disp('*** Equivalent modal properties of the damped structure:')

%    damp(A2);
%    if i==1
%       [Y5,X5] = lsim(A2,B,C,D,Xg,t);
%       V5  = K(1)*X5(:,n+1)/1e3;
%       um = max( abs(X5(:,n+1:2*n)) );
%       Vm = max( abs(V5) );
%    else
%       [Y6,X6] = lsim(A2,B,C,D,Xg,t);
%       V6  = K(1)*X6(:,n+1)/1e3;
%       um = max( abs(X6(:,n+1:2*n)) );
%       Vm = max( abs(V6) );
```

```
%     end
%
% end

figure; plot(t,zeros(1,nt),t,X1A(:,nA) ); grid on; axis tight;
title(['Relative velocity of top floor structure A Uncontrolled - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

figure; plot(t,zeros(1,nt),t,X1B(:,nB) ); grid on; axis tight;
title(['Relative velocity of top floor structure B Uncontrolled - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

% Structure A (Displacement)

figure; plot(t(1:round(np/2)),X1A(1:round(np/2),2*nA),'--b','LineWidth',1),hold on
         plot(t(1:round(np/2)),X4A(1:round(np/2),2*nA),'.-r','LineWidth',0.3),hold on
         plot(t(1:round(np/2)),UA(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' fixed Damper','Controlled VDSA',1); title(['Relative
displacement of top floor structure A :',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1A(1:round(np/2),2*nA),'-b','LineWidth',0.3),hold on
         plot(t(1:round(np/2)),X4A(1:round(np/2),2*nA),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' fixed Damper',1); title(['Relative displacement of top
floor structure A :',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1A(1:round(np/2),2*nA),'-b','LineWidth',0.3),hold on
         plot(t(1:round(np/2)),UA(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of top
floor structure A :',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X4A(1:round(np/2),2*nA),'-b','LineWidth',0.3),hold on
         plot(t(1:round(np/2)),UA(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend(' fixed Damper','Controlled VDSA',1); title(['Relative displacement of
top floor structure A :',nom,' record']); grid on ;axis tight;

% Structure B (Displacement)

figure; plot(t(1:round(np/2)),X1B(1:round(np/2),2*nB),'--b','LineWidth',1),hold on
         plot(t(1:round(np/2)),X4B(1:round(np/2),2*nB),'.-r','LineWidth',0.3),hold on
         plot(t(1:round(np/2)),UB(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' fixed Damper','Controlled VDSA',1); title(['Relative
displacement of top floor structure B :',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1B(1:round(np/2),2*nB),'-b','LineWidth',0.3),hold on
         plot(t(1:round(np/2)),X4B(1:round(np/2),2*nB),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' fixed Damper',1); title(['Relative displacement of top
floor structure B :',nom,' record']); grid on ;axis tight;
```

```
figure; plot(t(1:round(np/2)),X1B(1:round(np/2),2*nB),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),UB(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of top
floor structure B :',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X4B(1:round(np/2),2*nB),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),UB(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend(' fixed Damper','Controlled VDSA',1); title(['Relative displacement of
top floor structure B :',nom,' record']); grid on ;axis tight;

% Structure A (Total base shear)

figure; plot(t(1:round(np/2)),V1A(1:round(np/2)),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),V4A(1:round(np/2)),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V2A(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','fixed Damper','Controlled VDSA',1); title(['Total base shear
structure A :',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1A(1:round(np/2)),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V4A(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','fixed Damper',1); title(['Total base shear structure A :',nom,'
record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1A(1:round(np/2)),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V2A(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Controlled VDSA',1); title(['Total base shear structure A :',nom,'
record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V4A(1:round(np/2)),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V2A(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('fixed Damper','Controlled VDSA',1); title(['Total base shear structure A
:',nom,' record']); grid on;axis tight;

% Structure B (Total base shear)

figure; plot(t(1:round(np/2)),V1B(1:round(np/2)),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),V4B(1:round(np/2)),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V2B(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','fixed Damper','Controlled VDSA',1); title(['Total base shear
structure B :',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1B(1:round(np/2)),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V4B(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','fixed Damper',1); title(['Total base shear structure B :',nom,'
record']); grid on;axis tight;
```

```
figure; plot(t(1:round(np/2)),V1B(1:round(np/2)),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V2B(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Controlled VDSA',1); title(['Total base shear structure B :',nom,'
record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V4B(1:round(np/2)),'-b','LineWidth',0.3),hold on
         plot(t(1:round(np/2)),V2B(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('fixed Damper','Controlled VDSA',1); title(['Total base shear structure B
:',nom,' record']); grid on;axis tight;



figure; plot(t(1:round(np/2)),UA(1:round(np/2)),'-r','LineWidth',1),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Controlled VDSA',1); title(['Relative displacement of top floor structure
A :',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),UB(1:round(np/2)),'-r','LineWidth',1),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Controlled VDSA',1); title(['Relative displacement of top floor structure
B :',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),V2A(1:round(np/2)),'-r','LineWidth',1),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Controlled VDSA',1); title(['Total base shear structure A :',nom,' record']); grid
on;axis tight;

figure; plot(t(1:round(np/2)),V2B(1:round(np/2)),'-r','LineWidth',1),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Controlled VDSA',1); title(['Total base shear structure B :',nom,' record']); grid
on;axis tight;

% figure; plot(X1(1:np,2*n),V1(1:np),X4(1:np,2*n),V4(1:np),U(1:np),V2(1:np) ); grid on;
xlabel('Displacement [in]');
% ylabel('Force Elastic [kip]'),legend('Uncontrolled',' fixed Damper','Controlled VDSA',1); title('Elastic');
%
% figure; plot( U(1:np),dampi(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Damping [lb]'),legend('Controlled VDSA',2); title('Damping');
%
% figure; plot( U(1:np),V2(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Elastic [lb]'),legend('Controlled VDSA',2); title('Elastic');
%
% figure; plot( U(1:np),eladam(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Elastic + Damping [lb]'),legend('Controlled VDSA',2); title('Elastic + Damping');
%
% figure; plot( X1(1:np,2*n),damnc(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Damping [lb]'),legend('Uncontrolled',2); title('Damping');
%
% figure; plot( X1(1:np,2*n),V1(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Elastic [lb]'),legend('Uncontrolled',2); title('Elastic');
%
```

```
% figure; plot( X1(1:np,2*n),eladamnc(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Elastic + Damping [lb]'),legend('Uncontrolled',2); title('Elastic + Damping');
%
% En1=cumsum(abs(V1(1:np)))*dt;
% En2=cumsum(abs(V2(1:np)))*dt;
% En3=cumsum(abs(dampi(1:np)))*dt;
% En4=cumsum(abs(damnc(1:np)))*dt;
% En6=En1+En4;
% En5=En2+En3;
%
% figure; plot(t,En1,t,En4,t,zeros(1,np) ); grid on; axis tight; xlabel('Time [sec]');
% ylabel('Energy'),legend('Elastic','Damping','Elastic + Damping',4); title(['Energy Force Uncontrolled -
Earthquake:',nom,' record']);
%
% figure; plot(t,En2,t,En3,t,zeros(1,np) ); grid on; axis tight; xlabel('Time [sec]');
% ylabel('Energy'),legend('Elastic','Damping','Elastic + Damping',4); title(['Energy Force Controlled -
Earthquake:',nom,' record']);
```

APPENDIX H:

PROGRAM: *CLOSED-LOOP CONTROL* AND VDSA DEVICE (MDOF) COUPLED STRUCTURES

```
clc; clear all; close all

g = 386.4;                          % acceler. of gravity: in/s^2
% g = 9.8;                          % acceler. of gravity: m/s^2

nA = 8;                             % number of stories
nB = 6;                             % number of stories

% WpA = [345.6; 345.6; 345.6; 345.6; 345.6;345.6;345.6;345.6]*1e3*2.205;     % weights of floor: lb
% kA =  [3.404; 3.404; 3.404; 3.404; 3.404;3.404;3.404;3.404]*1e6*0.225/0.0254;     % total stiffness
coefficients: lb/in

kA = [3000; 3000; 3000; 3000; 3000; 3000; 3000; 3000]*1e3*0.225/0.0254*0.2;   % weights of floor: lb
WpA =  [1; 1; 1; 1; 1; 1; 1; 1]*1e3*2.205;         % total stiffness coefficients: lb/in

kB = [3000; 3000; 3000; 3000; 3000; 3000]*1e3*0.225/0.0254*0.2;     % weights of floor: lb
WpB =  [1; 1; 1; 1; 1; 1]*1e3*2.205;               % total stiffness coefficients: lb/in

% kA = [3000; 3000; 3000; 3000; 3000; 3000]*1e3*0.225/0.0254*0.2;             % weights of floor: lb
% WpA =  [1; 1; 1; 1; 1; 1]*1e3*2.205;             % total stiffness coefficients: lb/in

CoA= [15 ]*1e3;                     % total damping coefficients: lb.s/in
CoB= [10 ]*1e3;                     % total damping coefficients: lb.s/in

RP=1;                               % floor
nco=1;                              % position of the control
pco=4;
Co1A=zeros(nA,nco);
```

```
Co1A(pco,1)=CoA;
Co1B=zeros(nB,nco);
Co1B(pco,1)=CoB;

DA=diag(zeros(1,nA));
DA(pco,pco)=1;
D2A=zeros(nA,nco);
D2A(pco,1)=1;
Co2A=Co1A(pco,1);

DB=diag(zeros(1,nB));
DB(pco,pco)=1;
D2B=zeros(nB,nco);
D2B(pco,1)=1;
Co2B=Co1B(pco,1);

Q=[diag(diag(ones(2*(nA))))*10^9,((zeros(2*nA,2*nB)));((zeros(2*nB,2*nA))),diag(diag(ones((2*nB))))*
10^7];
KA  = diag(kA) + diag( [kA(2:nA) ; 0] ) - diag( kA(2:nA),1 ) - diag( kA(2:nA),-1 );
KB  = diag(kB) + diag( [kB(2:nB) ; 0] ) - diag( kB(2:nB),1 ) - diag( kB(2:nB),-1 );

dt = 0.02;                          % time step of accelerogram: sec
zita = [2]*1e-2;                    % modal damping ratios of building

E=diag(ones(nA+nB));
MA  = diag( WpA ) ;
MB  = diag( WpB ) ;

h=120;
a=120;
E=diag(ones(nA+nB));
Rm=0.18;
Wmax=h-a*Rm;

Wmin=h-a+30;
w=Wmax;

R=diag(ones(1,nco).*10^(-6));
IR=inv(R);

Ur(1)=0;
r=zeros(nA+nB,1);
d1=zeros(2*(nA+nB),1);
Uc{1}=zeros(nco,1);
U{1}=r;
Up{1}=r;
Ud(1)=0;
```

```
Upd(1)=0;
Ucd(1)=0;
Acd(1)=0;
Urr2A=zeros(nA,1);
Urr22A=zeros(nA,1);
AcA{1}=0;

Urr2B=zeros(nB,1);
Urr22B=zeros(nB,1);
AcB{1}=0;

Ucv(1)=0;
J(1)=0;

%------------ Read and plot the ground acceleration time history ----------

nom = 'Friuli';                 % name of file with earthquake
                                % addpath c:/MatlabFiles/Earthquakes;
                                % directory with accelerograms
acc = load ([nom,'.txt']);      % read earthquake data file
[nr,nc] = size(acc);            % columns and rows of data file
Xg(1:nr*nc) = acc';             % copy accelerogram in a vector
xm = max( abs(Xg) );            % original peak acceleration
Xg =  Xg*g;                     % scaled accelerogram
tf = (nr*nc-1)*dt;              % final time of accelerogram
t  = 0: dt: tf ;                % column vector with time steps
nt = length(t);                 % number of discrete time points
np = round( nt);                % number of time points for plotting
z0 = zeros(1,np);               % auxiliary vector for plotting

figure; plot(t,zeros(1,nt), t,Xg/g ); grid on; axis tight;
title(['Normalized earthquake ground acceleration: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Acceleration [g]');

Xpg=cumsum(Xg)*dt;

figure; plot(t,zeros(1,nt), t,Xpg ); grid on; axis tight;
title(['Ground velocity: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/seg]');

Xppg=cumsum(Xpg)*dt;

figure; plot(t,zeros(1,nt), t,Xppg ); grid on; axis tight;
title(['Ground displacement: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

%---- Calculate the matrices & modal properties  -----
```

```
nn=nr*nc;
dampi(1)=0;
 % Modal Properties structure A

    [PhiA,lamA] =  eig(KA,MA);
    PhiA = PhiA * diag(1 ./ sqrt( diag(PhiA'*MA*PhiA) ));
    [wsA,id] = sort( sqrt(diag(lamA)) );
    PhiA = PhiA(:,id);
    TsA  = 2*pi ./ wsA ;
    MiA = inv(MA) ;
    CsA=MA* PhiA* diag(2*(0.02).*wsA)* PhiA'* MA;

 % Modal Properties structure B

    [PhiB,lamB] =  eig(KB,MB);
    PhiB = PhiB * diag(1 ./ sqrt( diag(PhiB'*MB*PhiB) ));
    [wsB,id] = sort( sqrt(diag(lamB)) );
    PhiB = PhiB(:,id);
    TsB  = 2*pi ./ wsB ;
    MiB = inv(MB) ;
    CsB=MB* PhiB* diag(2*(0.02).*wsB)* PhiB'* MB;

matrA=zeros(nn,nA);
matrB=zeros(nn,nB);

for i=2:nn
    z=[U{i-1};Up{i-1}];

    cA=DA*Co1A.*(a^2/(a^2+(h-w)^2));
    CdA=diag(cA);
    CdA(pco,pco-1)=-CdA(pco,pco);
    CdA(pco-1,pco)=-CdA(pco,pco);
    CA=CsA+CdA;

    cB=DB*Co1B.*(a^2/(a^2+(h-w)^2));
    CdB=diag(cB);
    CdB(pco,pco-1)=-CdB(pco,pco);
    CdB(pco-1,pco)=-CdB(pco,pco);
    CB=CsB+CdB;

    CprA=(1/2)*MiA*D2A*CoA*(a*(h-w)/(a^2+(h-w)^2));
    CprB=-(1/2)*MiB*D2B*CoB*(a*(h-w)/(a^2+(h-w)^2)

    Bc=[MiA*KA,((zeros(nA,nB)));((zeros(nB,nA))),MiB*KB];
    Ac=[MiA*CsA,((zeros(nA,nB)));((zeros(nB,nA))),MiB*CsB];

    %Ac(pco-1,pco)=MiA(pco,pco)*CdA(pco,pco);
```

```
%Ac(pco,pco-1)=MiA(pco,pco)*CdA(pco,pco);
 %Ac(pco+nA,pco)=MiB(pco,pco)*CdB(pco,pco);
 %Ac(pco,pco+nA)=MiB(pco,pco)*CdB(pco,pco);

 A=[zeros(nA+nB),diag(diag(ones(nA+nB)));-Bc,-Ac];

 B=[(zeros(nA+nB,nco));CprA;CprB];
 H=[diag(zeros(nA+nB));-E];

 %% New Matrices

 Av=[diag(diag(zeros((nA+nB)))),diag(diag(ones((nA+nB))))];
 Sv=[diag(ones((nA+nB)))];
 %Qv=[diag(diag(ones((nA+nB))))]*1e1;
 Qv=[diag(diag(ones((nA))))*10^4,((zeros(nA,nB)));((zeros(nB,nA))),diag(diag(ones((nB))))*10^1];
 A2=Q+Av'*Qv*Av;
 A3=Av'*Qv*Sv;

 [T,lam1] =  eig(A);
 [ws1,id] = sort( sqrt(diag(lam1)) );
 lam2=diag(ws1);
 lam11=inv(T)*A*T;
 paso=(exp(((lam11.*dt))));
 d =diag(diag(paso))*inv(T)*(z+dt/2*(B*Uc{i-1}+H*Xg(i-1)));
 Zt=(inv(diag(diag(ones(2*(nA+nB))))+(dt^2)/4*B*inv(R)*B'*A2)*(T*d-
(dt^2)/4*B*inv(R)*B'*A3*Xpg(i)+dt/2*H*Xg(i)));

 Uc{i}=-(dt/2*inv(R)*B')*(A2*Zt+A3*Xpg(i));

 UA{i}=(Zt(1:nA,:));
 UB{i}=(Zt(nA+1:nA+nB,:));

 UpA{i}=(Zt(nA+nB+1:2*nA+nB,:));
 UpB{i}=(Zt(2*nA+nB+1:2*(nA+nB),:));

 U{i}=[UA{i};UB{i}];
 Up{i}=[UpA{i};UpB{i}];

 %AcA{i}=-MiA*CA*UpA{i}-MiA*KA*UA{i}-E(1:nA,1)*Xg(i)+CprA;
 %AcB{i}=-MiB*CB*UpB{i}-MiB*KB*UB{i}-E(1:nB,1)*Xg(i)+CprB;

 %Urr4A=AcA{i};
 %AcdA(i)=Urr4A(RP);
 %Urr4B=AcB{i};
 %AcdB(i)=Urr4B(RP);

 Urr1A=UA{i};
```

```
    Ud1A(i)=Urr1A(RP);
    UdnA(i)=Urr1A(nA);
    Urr2A=UpA{i};
    UpdA(i)=Urr2A(1);
    UpdcA(i)=Urr2A(pco);
    UpdnA(i)=Urr2A(nA);
    UpddA(i)=Urr2A(RP);

    Urr1B=UB{i};
    Ud1B(i)=Urr1B(RP);
    UdnB(i)=Urr1B(nB);
    Urr2B=UpB{i};
    UpdB(i)=Urr2B(1);
    UpdcB(i)=Urr2B(pco);
    UpdnB(i)=Urr2B(nB);
    UpddB(i)=Urr2B(RP);

    Urr3=Uc{i};
    Ucd(i)=Urr3(1);

    w=h-a*(abs(Co1A(pco,1)/abs(Ucd(i)/UpdcA(i))-1))^(1/2);

    if w>Wmax
       w=Wmax;
    end

    if w<Wmin
       w=Wmin;
    end

    rr(i-1)=w;

    for j=1:nA
       matrA(i,j)=Urr1A(j);
    end

    for j=1:nB
       matrB(i,j)=Urr1B(j);
    end

end

rr(nn)=w;
bb(nn)=0;

% Results Structure A
```

```
figure; plot(t,zeros(1,nt),t,UdnA ); grid on; axis tight;
title(['Relative displacement of top floor structure A: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');
figure; plot(t,zeros(1,nt),t,UpdnA ); grid on; axis tight;
title(['Relative velocity of top floor structure A: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

figure; plot(t,zeros(1,nt),t,Ud1A ); grid on; axis tight;
title(['Relative displacement of first floor structure A: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

figure; plot(t,zeros(1,nt),t,UpdA ); grid on; axis tight;
title(['Relative velocity of first floor structure A: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

% Results Structure B

figure; plot(t,zeros(1,nt),t,UdnB ); grid on; axis tight;
title(['Relative displacement of top floor structure B: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

figure; plot(t,zeros(1,nt),t,UpdnB ); grid on; axis tight;
title(['Relative velocity of top floor structure B: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

figure; plot(t,zeros(1,nt),t,Ud1B ); grid on; axis tight;
title(['Relative displacement of first floor structure B: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

figure; plot(t,zeros(1,nt),t,UpdB ); grid on; axis tight;
title(['Relative velocity of first floor structure B: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

figure; plot(t,zeros(1,nt),t,Acd ); grid on; axis tight;
title(['Relative acceleration of top floor - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Acceleration [in/sec^2]');

figure; plot(t,zeros(1,nt),t,Ucd ); grid on; axis tight;
title(['Control Force - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Control Force [lb]');

figure; plot(t,zeros(1,nt),t,rr ); grid on; axis tight;
title(['Variation of Control - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Higth Control [in]');

figure; plot(t,zeros(1,nt),t,bb ); grid on; axis tight;
title(['Uc/Up - Earthquake:',nom,' record']);
```

```
xlabel('Time [sec]'); ylabel('Uc/Up');

%%%%%%  Uncontrolled
% Structure A

%--------- Form the state-space matrices and calculate the response -------

MiA = inv(MA) ;
CtA=MA* PhiA* diag(2*(0.02+0.03).*wsA)* PhiA'* MA;
A  = [-MiA*CtA, -MiA*KA; eye(nA), zeros(nA) ] ;
B  = [-ones(nA,1) ; zeros(nA,1)] ;
C  = [-MiA*CtA, -MiA*KA] ;
D  = zeros(nA,1) ;

disp('*** Equivalent modal properties of the damped structure:')

damp(A)
[Y1A,X1A] = lsim(A,B,C,D,Xg,t);
V1A  = KA(1)*X1A(:,nA+1)/1e3;
um1A = max( abs(X1A(:,nA+1:2*nA)) );
Vm1A = max( abs(V1A) );

%------ Calculate the maximum response & plot the building response -------

V2A = KA(1)*Ud1A/1e3;
umcA = max( abs(matrA(:,1:nA)) );
Vm1A = max( abs(V1A) );
Vm2A = max( abs(V2A) );

figure; plot(t(1:np),X1A(1:np,2*nA) ); grid on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',1); title(['Relative displacement of top floor structure A:
',nom,' record']);

figure; plot( t(1:np),V1A(1:np) ); grid on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled',1); title('Total base shear structure A');


% figure; plot(t(1:np),X1(1:np,2*n),t(1:np),Udn(1:np) ); grid on; xlabel('Time [sec]');
% ylabel('Displacement [in]'),legend('Uncontrolled','Controlled',1); title(['Relative displacement of top floor
- Earthquake:',nom,' record']);
%
% figure; plot( t(1:np),V1(1:np),t(1:np),V2(1:np) ); grid on; xlabel('Time [sec]');
% ylabel('Force [kip]'),legend('Uncontrolled','Controlled',1); title('Total base shear');

% Structure B

%--------- Form the state-space matrices and calculate the response -------
```

```
MiB = inv(MB) ;
CtB=MB* PhiB* diag(2*(0.02+0.03).*wsB)* PhiB'* MB;
A  = [-MiB*CtB, -MiB*KB; eye(nB), zeros(nB) ] ;
B  = [-ones(nB,1) ; zeros(nB,1)] ;
C  = [-MiB*CtB, -MiB*KB] ;
D  = zeros(nB,1) ;

disp('*** Equivalent modal properties of the damped structure:')

damp(A)
[Y1B,X1B] = lsim(A,B,C,D,Xg,t);
V1B  = KB(1)*X1B(:,nB+1)/1e3;
um1B = max( abs(X1B(:,nB+1:2*nB)) );
Vm1B = max( abs(V1B) );

%------ Calculate the maximum response & plot the building response -------

V2B = KB(1)*Ud1B/1e3;
umcB = max( abs(matrB(:,1:nB)) );
Vm1B = max( abs(V1B) );
Vm2B = max( abs(V2B) );
% damnc=Ct.*X1(:,n);

figure; plot(t(1:np),X1B(1:np,2*nB) ); grid on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',1); title(['Relative displacement of top floor structure B:
',nom,' record']);

figure; plot( t(1:np),V1B(1:np) ); grid on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled',1); title('Total base shear structure B');

%------Damper Fixed Case I

% Structure A

w=Wmax;
Zeff=0.3;
Phi1A=zeros(nA,1);
for j=1:nA-1
   Phi1A(j,1)=PhiA(j+1,1);
end
Phi1rA=PhiA(:,1)-Phi1A(:,1);
sum1=0;
sum2=0;
for j=1:nA
   sum1=sum1+Zeff*pi*MA(j,j)*PhiA(j,1)^2;
   sum2=sum2+2*TsA(1,1)*(a^2/(a^2+(h-w)^2))*Phi1rA(j,1)^2;
end
```

```
Cd1A=sum1/sum2;

Cd1A= [0 Cd1A];
for i=1:2
   D3A=diag(zeros(1,nA));
   D3A(1,1)=1;
   CopA=Cd1A(i);
   Ct1A=MA* PhiA* diag(2*(0.02+0.03).*wsA)* PhiA'* MA+2*CopA*(a^2/(a^2+(h-w)^2)).*D3A;
   A  = [-MiA*Ct1A, -MiA*KA; eye(nA), zeros(nA) ] ;
   B  = [-ones(nA,1) ; zeros(nA,1)] ;
   C  = [-MiA*Ct1A, -MiA*KA] ;
   D  = zeros(nA,1) ;
   cos1=a^2/(a^2+(h-w)^2);

   disp('*** Equivalent modal properties of the damped structure:')

   damp(A);
   if i==1
      [Y3A,X3A] = lsim(A,B,C,D,Xg,t);
      V3A  = KA(1)*X3A(:,nA+1)/1e3;
      umA = max( abs(X3A(:,nA+1:2*nA)) );
      VmA = max( abs(V3A) );
   else
      [Y4A,X4A] = lsim(A,B,C,D,Xg,t);
      V4A  = KA(1)*X4A(:,nA+1)/1e3;
      um4A = max( abs(X4A(:,nA+1:2*nA)) );
      Vm4A = max( abs(V4A) );
   end

end

% Structure B

w=Wmax;
Zeff=0.3;
Phi1B=zeros(nB,1);
for j=1:nB-1
   Phi1B(j,1)=PhiB(j+1,1);
end
Phi1rB=PhiB(:,1)-Phi1B(:,1);
sum1=0;
sum2=0;
for j=1:nB
   sum1=sum1+Zeff*pi*MB(j,j)*PhiB(j,1)^2;
   sum2=sum2+2*TsB(1,1)*(a^2/(a^2+(h-w)^2))*Phi1rB(j,1)^2;
end
Cd1B=sum1/sum2;
```

```
Cd1B= [0 Cd1B];
for i=1:2
   D3B=diag(zeros(1,nB));
   D3B(1,1)=1;
   CopB=Cd1B(i);
   Ct1B=MB* PhiB* diag(2*(0.02+0.03).*wsB)* PhiB'* MB+2*CopB*(a^2/(a^2+(h-w)^2)).*D3B;
   A  = [-MiB*Ct1B, -MiB*KB; eye(nB), zeros(nB) ] ;
   B  = [-ones(nB,1) ; zeros(nB,1)] ;
   C  = [-MiB*Ct1B, -MiB*KB] ;
   D  = zeros(nB,1) ;
   cos1=a^2/(a^2+(h-w)^2);

   disp('*** Equivalent modal properties of the damped structure:')

   damp(A);
   if i==1
      [Y3B,X3B] = lsim(A,B,C,D,Xg,t);
      V3B  = KB(1)*X3B(:,nB+1)/1e3;
      umB = max( abs(X3B(:,nB+1:2*nB)) );
      VmB = max( abs(V3B) );
   else
      [Y4B,X4B] = lsim(A,B,C,D,Xg,t);
      V4B  = KB(1)*X4B(:,nB+1)/1e3;
      um4B = max( abs(X4B(:,nB+1:2*nB)) );
      Vm4B = max( abs(V4B) );
   end

end

%------Damper Fixed Case II

% Structure A

for i=1:2
   D3A=diag(zeros(1,nA));
   D3A(1,1)=1;
   D3A(2,2)=1;
   D3A(1,1)=1;
   CopA=Cd1A(i);
   Ct1A=MA* PhiA* diag(2*(0.02+0.03).*wsA)* PhiA'* MA+2*CopA*(a^2/(a^2+(h-w)^2)).*D3A;
   A  = [-MiA*Ct1A, -MiA*KA; eye(nA), zeros(nA) ] ;
   B  = [-ones(nA,1) ; zeros(nA,1)] ;
   C  = [-MiA*Ct1A, -MiA*KA] ;
   D  = zeros(nA,1) ;
   cos1=a^2/(a^2+(h-w)^2);
```

```
    disp('*** Equivalent modal properties of the damped structure:')

    damp(A);
    if i==1
        [Y3A,X3A] = lsim(A,B,C,D,Xg,t);
        V3A  = KA(1)*X3A(:,nA+1)/1e3;
        umA = max( abs(X3A(:,nA+1:2*nA)) );
        VmA = max( abs(V3A) );
    else
        [Y5A,X5A] = lsim(A,B,C,D,Xg,t);
        V5A  = KA(1)*X5A(:,nA+1)/1e3;
        um5A = max( abs(X5A(:,nA+1:2*nA)) );
        Vm5A = max( abs(V5A) );
    end

end

% Structure B

for i=1:2
    D3B=diag(zeros(1,nB));
    D3B(1,1)=1;
    D3B(2,2)=1;
    D3B(1,1)=1;
    CopB=Cd1B(i);
    Ct1B=MB* PhiB* diag(2*(0.02+0.03).*wsB)* PhiB'* MB+2*CopB*(a^2/(a^2+(h-w)^2)).*D3B;
    A  = [-MiB*Ct1B, -MiB*KB; eye(nB), zeros(nB) ] ;
    B  = [-ones(nB,1) ; zeros(nB,1)] ;
    C  = [-MiB*Ct1B, -MiB*KB] ;
    D  = zeros(nB,1) ;
    cos1=a^2/(a^2+(h-w)^2);

    disp('*** Equivalent modal properties of the damped structure:')

    damp(A);
    if i==1
        [Y3B,X3B] = lsim(A,B,C,D,Xg,t);
        V3B  = KB(1)*X3B(:,nB+1)/1e3;
        umB = max( abs(X3B(:,nB+1:2*nB)) );
        VmB = max( abs(V3B) );
    else
        [Y5B,X5B] = lsim(A,B,C,D,Xg,t);
        V5B  = KB(1)*X5B(:,nB+1)/1e3;
        um5B = max( abs(X5B(:,nB+1:2*nB)) );
        Vm5B = max( abs(V5B) );
    end
```

```
end

%------Damper Fixed Case III

% Structure A

for i=1:2
   D3A=diag(ones(1,nA));
   CopA=Cd1A(i);
   Ct1A=MA* PhiA* diag(2*(0.02+0.03).*wsA)* PhiA'* MA+2*CopA*(a^2/(a^2+(h-w)^2)).*D3A;
   A  = [-MiA*Ct1A, -MiA*KA; eye(nA), zeros(nA) ] ;
   B  = [-ones(nA,1) ; zeros(nA,1)] ;
   C  = [-MiA*Ct1A, -MiA*KA] ;
   D  = zeros(nA,1) ;
   cos1=a^2/(a^2+(h-w)^2);

   disp('*** Equivalent modal properties of the damped structure:')

   damp(A);
   if i==1
      [Y3A,X3A] = lsim(A,B,C,D,Xg,t);
      V3A  = KA(1)*X3A(:,nA+1)/1e3;
      umA = max( abs(X3A(:,nA+1:2*nA)) );
      VmA = max( abs(V3A) );
   else
      [Y6A,X6A] = lsim(A,B,C,D,Xg,t);
      V6A  = KA(1)*X6A(:,nA+1)/1e3;
      um6A = max( abs(X6A(:,nA+1:2*nA)) );
      Vm6A = max( abs(V6A) );
   end

end

% Structure B

for i=1:2
   D3B=diag(ones(1,nB));
   CopB=Cd1B(i);
   Ct1B=MB* PhiB* diag(2*(0.02+0.03).*wsB)* PhiB'* MB+2*CopB*(a^2/(a^2+(h-w)^2)).*D3B;
   A  = [-MiB*Ct1B, -MiB*KB; eye(nB), zeros(nB) ] ;
   B  = [-ones(nB,1) ; zeros(nB,1)] ;
   C  = [-MiB*Ct1B, -MiB*KB] ;
   D  = zeros(nB,1) ;
   cos1=a^2/(a^2+(h-w)^2);

   disp('*** Equivalent modal properties of the damped structure:')
```

```
    damp(A);
    if i==1
       [Y3B,X3B] = lsim(A,B,C,D,Xg,t);
       V3B  = KB(1)*X3B(:,nB+1)/1e3;
       umB = max( abs(X3B(:,nB+1:2*nB)) );
       VmB = max( abs(V3B) );
    else
       [Y6B,X6B] = lsim(A,B,C,D,Xg,t);
       V6B  = KB(1)*X6B(:,nB+1)/1e3;
       um6B = max( abs(X6B(:,nB+1:2*nB)) );
       Vm6B = max( abs(V6B) );
    end

end

% figure; plot(t,zeros(1,nt),t,X1(:,n) ); grid on; axis tight;
% title(['Relative velocity of top floor Uncontrolled - Earthquake:',nom,' record']);
% xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

% Relative displacement of top floor structure A

figure; plot(t(1:round(np/2)),X1A(1:round(np/2),2*nA),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),X6A(1:round(np/2),2*nA),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),UdnA(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III','Controlled VDSA',1);
title(['Relative displacement of the top floor structure A: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1A(1:round(np/2),2*nA),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),X6A(1:round(np/2),2*nA),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III',1); title(['Relative displacement
of the top floor structure A: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1A(1:round(np/2),2*nA),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),UdnA(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of the
top floor structure A: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X6A(1:round(np/2),2*nA),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),UdnA(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend(' Fixed Damper Case III','Controlled VDSA',1); title(['Relative
displacement of the top floor structure A: ',nom,' record']); grid on ;axis tight;

% Relative displacement of top floor structure B

figure; plot(t(1:round(np/2)),X1B(1:round(np/2),2*nB),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),X6B(1:round(np/2),2*nB),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),UdnB(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
```

```
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III','Controlled VDSA',1);
title(['Relative displacement of the top floor structure B: ',nom,' record']); grid on ;axis tight;
figure; plot(t(1:round(np/2)),X1B(1:round(np/2),2*nB),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),X6B(1:round(np/2),2*nB),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III',1); title(['Relative displacement
of the top floor structure B: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1B(1:round(np/2),2*nB),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),UdnB(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of the
top floor structure B: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X6B(1:round(np/2),2*nB),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),UdnB(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend(' Fixed Damper Case III','Controlled VDSA',1); title(['Relative
displacement of the top floor structure B: ',nom,' record']); grid on ;axis tight;

% Relative displacement of first floor structure A

figure; plot(t(1:round(np/2)),X1A(1:round(np/2),nA+1),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),X6A(1:round(np/2),nA+1),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Ud1A(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III','Controlled VDSA',1);
title(['Relative displacement of the first floor structure A: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1A(1:round(np/2),nA+1),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),X6A(1:round(np/2),nA+1),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III',1); title(['Relative displacement
of the first floor structure A: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1A(1:round(np/2),nA+1),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Ud1A(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of the
first floor structure A: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X6A(1:round(np/2),nA+1),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Ud1A(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend(' Fixed Damper Case III','Controlled VDSA',1); title(['Relative
displacement of the first floor structure A: ',nom,' record']); grid on ;axis tight;

% Relative displacement of first floor structure B

figure; plot(t(1:round(np/2)),X1B(1:round(np/2),nB+1),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),X6B(1:round(np/2),nB+1),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Ud1B(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III','Controlled VDSA',1);
title(['Relative displacement of the first floor structure B: ',nom,' record']); grid on ;axis tight;
```

```
figure; plot(t(1:round(np/2)),X1B(1:round(np/2),nB+1),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),X6B(1:round(np/2),nB+1),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III',1); title(['Relative displacement
of the first floor structure B: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1B(1:round(np/2),nB+1),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Ud1B(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of the
first floor structure B: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X6B(1:round(np/2),nB+1),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Ud1B(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend(' Fixed Damper Case III','Controlled VDSA',1); title(['Relative
displacement of the first floor structure B: ',nom,' record']); grid on ;axis tight;

% Relative displacement of all floor structure A

for j=1:nA
   figure; plot(t(1:round(np/2)),X1A(1:round(np/2),j+nA),'-b','LineWidth',0.3),hold on
   plot(t(1:round(np/2)),matrA(1:round(np/2),j),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
   ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of
the  ', int2str(j), '  floor structure A: ',nom,' record']); grid on ;axis tight;
end

% Relative displacement of all floor structure B

for j=1:nB
   figure; plot(t(1:round(np/2)),X1B(1:round(np/2),j+nB),'-b','LineWidth',0.3),hold on
   plot(t(1:round(np/2)),matrB(1:round(np/2),j),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
   ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of
the  ', int2str(j), '  floor structure B: ',nom,' record']); grid on ;axis tight;
end

% Total base Shear structure A

figure; plot(t(1:round(np/2)),V1A(1:round(np/2)),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),V6A(1:round(np/2)),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V2A(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Fixed Damper Case III','Controlled VDSA',1); title(['Total base
shear structure A: ',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1A(1:round(np/2)),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V6A(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Fixed Damper Case III',1); title(['Total base shear structure A:
',nom,' record']); grid on;axis tight;
```

```
figure; plot(t(1:round(np/2)),V1A(1:round(np/2)),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V2A(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Controlled VDSA',1); title(['Total base shear structure A: ',nom,'
record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V6A(1:round(np/2)),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V2A(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Fixed Damper Case III','Controlled VDSA',1); title(['Total base shear structure
A: ',nom,' record']); grid on;axis tight;

% Total base Shear structure B
figure; plot(t(1:round(np/2)),V1B(1:round(np/2)),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),V6B(1:round(np/2)),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),V2B(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Fixed Damper Case III','Controlled VDSA',1); title(['Total base
shear structure B: ',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1B(1:round(np/2)),'-b','LineWidth',0.3),hold on
      plot(t(1:round(np/2)),V6B(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Fixed Damper Case III',1); title(['Total base shear structure B:
',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1B(1:round(np/2)),'-b','LineWidth',0.3),hold on
      plot(t(1:round(np/2)),V2B(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Controlled VDSA',1); title(['Total base shear structure B: ',nom,'
record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V6B(1:round(np/2)),'-b','LineWidth',0.3),hold on
      plot(t(1:round(np/2)),V2B(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Fixed Damper Case III','Controlled VDSA',1); title(['Total base shear structure
B: ',nom,' record']); grid on;axis tight;
% Displacement for different case structure A

pisoA=[0:1:nA];
case1A=[0,um4A];
case2A=[0,um5A];
case3A=[0,um6A];
controA=[0,umcA];
noconA=[0,um1A];

figure; plot(case1A,pisoA,'--rs','MarkerSize',5,'LineWidth',1),hold on
        plot(case2A,pisoA,'.-b','MarkerSize',15,'LineWidth',1.5),hold on
        plot(case3A,pisoA,'-xb','MarkerSize',10,'LineWidth',1.5),hold on
        plot(controA,pisoA,'*--r','MarkerSize',5,'LineWidth',1),hold on
        plot(noconA,pisoA,'o-g','MarkerSize',5,'LineWidth',1.6),hold on; xlabel('Maximum Displacement
[in]');
ylabel('Floor'),legend('fixed Damper Case I','fixed Damper Case II','fixed Damper Case III','Controlled
```

```
VDSA','Uncontrolled',4); title(['Maximum Displacement structure A: ',nom,' record']); grid on;axis tight;
% Displacement for different case structure B

pisoB=[0:1:nB];
case1B=[0,um4B];
case2B=[0,um5B];
case3B=[0,um6B];
controB=[0,umcB];
noconB=[0,um1B];

figure; plot(case1B,pisoB,'--rs','MarkerSize',5,'LineWidth',1),hold on
        plot(case2B,pisoB,'.-b','MarkerSize',15,'LineWidth',1.5),hold on
        plot(case3B,pisoB,'-xb','MarkerSize',10,'LineWidth',1.5),hold on
        plot(controB,pisoB,'*--r','MarkerSize',5,'LineWidth',1),hold on
        plot(noconB,pisoB,'o-g','MarkerSize',5,'LineWidth',1.6),hold on; xlabel('Maximum Displacement
[in]');
ylabel('Floor'),legend('fixed Damper Case I','fixed Damper Case II','fixed Damper Case III','Controlled
VDSA','Uncontrolled',4); title(['Maximum Displacement structure B: ',nom,' record']); grid on;axis tight;

% figure; plot(X1(1:np,2*n),V1(1:np),X4(1:np,2*n),V4(1:np),Udn(1:np),V2(1:np) ); grid on;
xlabel('Displacement [in]');
% ylabel('Force Elastic [kip]'),legend('Uncontrolled',' fixed Damper','Controlled VDSA',1); title('Elastic');

% figure; plot( Udn(1:np),dampi(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Damping [lb]'),legend('Controlled VDSA',2); title('Damping');

% figure; plot( Udn(1:np),V2(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Elastic [lb]'),legend('Controlled VDSA',2); title('Elastic');

% figure; plot( U(1:np),eladam(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Elastic + Damping [lb]'),legend('Controlled VDSA',2); title('Elastic + Damping');

% figure; plot( X1(1:np,2*n),damnc(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Damping [lb]'),legend('Uncontrolled',2); title('Damping');

% figure; plot( X1(1:np,2*n),V1(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Elastic [lb]'),legend('Uncontrolled',2); title('Elastic');

% figure; plot( X1(1:np,2*n),eladamnc(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Elastic + Damping [lb]'),legend('Uncontrolled',2); title('Elastic + Damping');

% figure; plot( t,X1(1:np,n),t,Up ); grid on; xlabel('Time [sec]');
% ylabel('Velocity [in/sec]'),legend('Uncontrolled','Controlled',1); title(['Relative velocity of top floor -
Earthquake:',nom,' record']);

% En1=cumsum(abs(V1(1:np)))*dt;
% En2=cumsum(abs(V2(1:np)))*dt;
```

```
% En3=cumsum(abs(dampi(1:np)))*dt;
% En4=cumsum(abs(damnc(1:np)))*dt;
% En6=En1+En4;
% En5=En2+En3;
% En5=cumsum(abs(eladam(1:np)))*dt;
% En6=cumsum(abs(eladamnc(1:np)))*dt;

% figure; plot(t,En1,t,En4,t,zeros(1,np) ); grid on; axis tight; xlabel('Time [sec]');
% ylabel('Energy'),legend('Elastic','Damping','Elastic + Damping',4); title(['Energy Force Uncontrolled -
Earthquake:',nom,' record']);
%
% figure; plot(t,En2,t,En3,t,zeros(1,np) ); grid on; axis tight; xlabel('Time [sec]');
% ylabel('Energy'),legend('Elastic','Damping','Elastic + Damping',4); title(['Energy Force Controlled -
Earthquake:',nom,' record']);
```

APPENDIX I:

PROGRAM: *CLOSED-OPEN-LOOP CONTROL* AND VDSA DEVICE (MDOF) COUPLED STRUCTURES

```
clc; clear all; close all

g = 386.4;                                  % acceler. of gravity: in/s^2
% g = 9.8;                                  % acceler. of gravity: m/s^2

nA = 8;                                     % number of stories
nB = 6;                                     % number of stories

% WpA = [345.6; 345.6; 345.6; 345.6; 345.6;345.6;345.6;345.6]*1e3*2.205;      % weights of floor: lb
% kA =  [3.404; 3.404; 3.404; 3.404; 3.404;3.404;3.404;3.404]*1e6*0.225/0.0254;          % total stiffness
coefficients: lb/in
kA = [3000; 3000; 3000; 3000; 3000; 3000; 3000; 3000]*1e3*0.225/0.0254*0.2;    % weights of floor: lb
WpA =  [1; 1; 1; 1; 1; 1; 1; 1]*1e3*2.205;         % total stiffness coefficients: lb/in
kB = [3000; 3000; 3000; 3000; 3000; 3000]*1e3*0.225/0.0254*0.2;             % weights of floor: lb
WpB =  [1; 1; 1; 1; 1; 1]*1e3*2.205;              % total stiffness coefficients: lb/in
% kA = [3000; 3000; 3000; 3000; 3000; 3000]*1e3*0.225/0.0254*0.2;          % weights of floor: lb
% WpA =  [1; 1; 1; 1; 1; 1]*1e3*2.205;            % total stiffness coefficients: lb/in

CoA= [15 ]*1e3;                   % total damping coefficients: lb.s/in
CoB= [10 ]*1e3;                   % total damping coefficients: lb.s/in

RP=1;                             % floor
nco=1;                            % position of control
pco=4;
Co1A=zeros(nA,nco);
Co1A(pco,1)=CoA;
Co1B=zeros(nB,nco);
Co1B(pco,1)=CoB;
```

```
DA=diag(zeros(1,nA));
DA(pco,pco)=1;
D2A=zeros(nA,nco);
D2A(pco,1)=1;
Co2A=Co1A(pco,1);

DB=diag(zeros(1,nB));
DB(pco,pco)=1;
D2B=zeros(nB,nco);
D2B(pco,1)=1;
Co2B=Co1B(pco,1);

Q=[diag(diag(ones(2*(nA+nB)))) ]*10^(7);
Q=[diag(diag(ones(2*(nA))))*10^9,((zeros(2*nA,2*nB)));((zeros(2*nB,2*nA))),diag(diag(ones((2*nB))))*
10^7];
KA  = diag(kA) + diag( [kA(2:nA) ; 0] ) - diag( kA(2:nA),1 ) - diag( kA(2:nA),-1 );
KB  = diag(kB) + diag( [kB(2:nB) ; 0] ) - diag( kB(2:nB),1 ) - diag( kB(2:nB),-1 );

dt = 0.02;                    % time step of accelerogram: sec
zita = [2]*1e-2;              % modal damping ratios of building

E=diag(ones(nA+nB));
MA  = diag( WpA ) ;
MB  = diag( WpB ) ;

h=120;
a=120;
E=diag(ones(nA+nB));
Rm=0.18;
Wmax=h-a*Rm;

Wmin=h-a+20;
w=Wmax;

R=diag(ones(1,nco).*10^(3));
IR=inv(R);

Ur(1)=0;
r=zeros(nA+nB,1);
d1=zeros(2*(nA+nB),1);
Uc{1}=zeros(nco,1);
U{1}=r;
Up{1}=r;
Ud(1)=0;
Upd(1)=0;
Ucd(1)=0;
Acd(1)=0;
```

```
Urr2A=zeros(nA,1);
Urr22A=zeros(nA,1);
AcA{1}=0;

Urr2B=zeros(nB,1);
Urr22B=zeros(nB,1);
AcB{1}=0;

Ucv(1)=0;
J(1)=0;

%------------ Read and plot the ground acceleration time history ----------

nom = 'Friuli';                    % name of file with earthquake
                                   % addpath c:/MatlabFiles/Earthquakes;
                                   % directory with accelerograms
acc = load ([nom,'.txt']);         % read earthquake data file
[nr,nc] = size(acc);               % columns and rows of data file
Xg(1:nr*nc) = acc';                % copy accelerogram in a vector
xm = max( abs(Xg) );               % original peak acceleration
Xg =  Xg*g;                        % scaled accelerogram
tf = (nr*nc-1)*dt;                 % final time of accelerogram
t  = 0: dt: tf ;                   % column vector with time steps
nt = length(t);                    % number of discrete time points
np = round( nt);                   % number of time points for plotting
z0 = zeros(1,np);                  % auxiliary vector for plotting

figure; plot(t,zeros(1,nt), t,Xg/g ); grid on; axis tight;
title(['Normalized earthquake ground acceleration: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Acceleration [g]');

Xpg=cumsum(Xg)*dt;

figure; plot(t,zeros(1,nt), t,Xpg ); grid on; axis tight;
title(['Ground velocity: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/seg]');

Xppg=cumsum(Xpg)*dt;

figure; plot(t,zeros(1,nt), t,Xppg ); grid on; axis tight;
title(['Ground displacement: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

%---- Calculate the matrices & modal properties  -----

nn=nr*nc;
```

```
dampi(1)=0;
 % Modal Properties structure A
   [PhiA,lamA] =  eig(KA,MA);
   PhiA = PhiA * diag(1 ./ sqrt( diag(PhiA'*MA*PhiA) ));
   [wsA,id] = sort( sqrt(diag(lamA)) );
   PhiA = PhiA(:,id);
   TsA  = 2*pi ./ wsA ;
   MiA = inv(MA) ;
   CsA=MA* PhiA* diag(2*(0.02).*wsA)* PhiA'* MA;

 % Modal Properties structure B

   [PhiB,lamB] =  eig(KB,MB);
   PhiB = PhiB * diag(1 ./ sqrt( diag(PhiB'*MB*PhiB) ));
   [wsB,id] = sort( sqrt(diag(lamB)) );
   PhiB = PhiB(:,id);
   TsB  = 2*pi ./ wsB ;
   MiB = inv(MB) ;
   CsB=MB* PhiB* diag(2*(0.02).*wsB)* PhiB'* MB;

matrA=zeros(nn,nA);
matrB=zeros(nn,nB);

for i=2:nn
   z=[U{i-1};Up{i-1}];

   cA=DA*Co1A.*(a^2/(a^2+(h-w)^2));
   CdA=diag(cA);
   CdA(pco,pco-1)=-CdA(pco,pco);
   CdA(pco-1,pco)=-CdA(pco,pco);
   CA=CsA+CdA;

   cB=DB*Co1B.*(a^2/(a^2+(h-w)^2));
   CdB=diag(cB);
   CdB(pco,pco-1)=-CdB(pco,pco);
   CdB(pco-1,pco)=-CdB(pco,pco);
   CB=CsB+CdB;

   CprA=(1/2)*MiA*D2A*CoA*(a*(h-w)/(a^2+(h-w)^2));
   CprB=-(1/2)*MiB*D2B*CoB*(a*(h-w)/(a^2+(h-w)^2));

   Bc=[MiA*KA,((zeros(nA,nB)));((zeros(nB,nA))),MiB*KB];
   Ac=[MiA*CsA,((zeros(nA,nB)));((zeros(nB,nA))),MiB*CsB];

   %Ac(pco-1,pco)=MiA(pco,pco)*CdA(pco,pco);
   %Ac(pco,pco-1)=MiA(pco,pco)*CdA(pco,pco);
   %Ac(pco+nA,pco)=MiB(pco,pco)*CdB(pco,pco);
```

```
%Ac(pco,pco+nA)=MiB(pco,pco)*CdB(pco,pco);

A=[zeros(nA+nB),diag(diag(ones(nA+nB)));-Bc,-Ac];
 B=[(zeros(nA+nB,nco));CprA;CprB];
H=[diag(zeros(nA+nB));-E];

%% New Matrices

Av=[diag(diag(zeros((nA+nB)))),diag(diag(ones((nA+nB))))];
Sv=[diag(ones((nA+nB)))];
%Qv=[diag(diag(ones((nA+nB))))]*1e1;
Qv=[diag(diag(ones((nA))))*10^4,((zeros(nA,nB)));((zeros(nB,nA))),diag(diag(ones((nB))))*10^1];
A2=Q+Av'*Qv*Av;
A3=Av'*Qv*Sv;

[T,lam1] =  eig(A);
[ws1,id] = sort( sqrt(diag(lam1)) );
lam2=diag(ws1);
lam11=inv(T)*A*T;
paso=(exp(((lam11.*dt))));
d =diag(diag(paso))*inv(T)*(z+dt/2*(B*Uc{i-1}+H*Xg(i-1)));

P1=(-1)*inv(diag(diag(ones(2*(nA+nB))))+((dt^2)/8)*Q*B*IR*B')*(Q+2*Av'*Qv*Av);
P2=(-
1)*inv((dt^2)/8*Q*B*inv(R)*B'+diag(diag(ones(2*(nA+nB)))))*(Q*(T*d+dt/2*H*Xg(i))+2*Av'*Qv*Sv*
Xpg(i));

  Zt=(inv(diag(diag(ones(2*(nA+nB))))-
(dt^2)/8*B*inv(R)*B'*P1)*(T*d+(dt^2)/8*B*inv(R)*B'*P2+dt/2*H*Xg(i)));
LP=(P1*Zt+P2);
Uc{i}=(dt/4*inv(R)*B'*LP);

UA{i}=(Zt(1:nA,:));
UB{i}=(Zt(nA+1:nA+nB,:));

UpA{i}=(Zt(nA+nB+1:2*nA+nB,:));
UpB{i}=(Zt(2*nA+nB+1:2*(nA+nB),:));

U{i}=[UA{i};UB{i}];
Up{i}=[UpA{i};UpB{i}];

%AcA{i}=-MiA*CA*UpA{i}-MiA*KA*UA{i}-E(1:nA,1)*Xg(i)+CprA;
%AcB{i}=-MiB*CB*UpB{i}-MiB*KB*UB{i}-E(1:nB,1)*Xg(i)+CprB;

%Urr4A=AcA{i};
%AcdA(i)=Urr4A(RP);
%Urr4B=AcB{i};
```

```
    %AcdB(i)=Urr4B(RP);

    Urr1A=UA{i};
    Ud1A(i)=Urr1A(RP);
    UdnA(i)=Urr1A(nA);
    Urr2A=UpA{i};
    UpdA(i)=Urr2A(1);
    UpdcA(i)=Urr2A(pco);
    UpdnA(i)=Urr2A(nA);
    UpddA(i)=Urr2A(RP);

    Urr1B=UB{i};
    Ud1B(i)=Urr1B(RP);
    UdnB(i)=Urr1B(nB);
    Urr2B=UpB{i};
    UpdB(i)=Urr2B(1);
    UpdcB(i)=Urr2B(pco);
    UpdnB(i)=Urr2B(nB);
    UpddB(i)=Urr2B(RP);

    Urr3=Uc{i};
    Ucd(i)=Urr3(1);

    w=h-a*(abs(Co1A(pco,1)/abs(Ucd(i)/UpdcA(i))-1))^(1/2);


    if w>Wmax
       w=Wmax;
    end

    if w<Wmin
       w=Wmin;
    end

    rr(i-1)=w;

    for j=1:nA
       matrA(i,j)=Urr1A(j);
    end

    for j=1:nB
       matrB(i,j)=Urr1B(j);
    end

end

rr(nn)=w;
```

```
bb(nn)=0;

% Results Structure A

figure; plot(t,zeros(1,nt),t,UdnA ); grid on; axis tight;
title(['Relative displacement of top floor structure A: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

figure; plot(t,zeros(1,nt),t,UpdnA ); grid on; axis tight;
title(['Relative velocity of top floor structure A: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

figure; plot(t,zeros(1,nt),t,Ud1A ); grid on; axis tight;
title(['Relative displacement of first floor structure A: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

figure; plot(t,zeros(1,nt),t,UpdA ); grid on; axis tight;
title(['Relative velocity of first floor structure A: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

% Results Structure B

figure; plot(t,zeros(1,nt),t,UdnB ); grid on; axis tight;
title(['Relative displacement of top floor structure B: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

figure; plot(t,zeros(1,nt),t,UpdnB ); grid on; axis tight;
title(['Relative velocity of top floor structure B: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

figure; plot(t,zeros(1,nt),t,Ud1B ); grid on; axis tight;
title(['Relative displacement of first floor structure B: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Displacement [in]');

figure; plot(t,zeros(1,nt),t,UpdB ); grid on; axis tight;
title(['Relative velocity of first floor structure B: ',nom,' record']);
xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

figure; plot(t,zeros(1,nt),t,Acd ); grid on; axis tight;
title(['Relative acceleration of top floor - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Acceleration [in/sec^2]');

figure; plot(t,zeros(1,nt),t,Ucd ); grid on; axis tight;
title(['Control Force - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Control Force [lb]');

figure; plot(t,zeros(1,nt),t,rr ); grid on; axis tight;
```

```
title(['Variation of Control - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Higth Control [in]');
figure; plot(t,zeros(1,nt),t,bb ); grid on; axis tight;
title(['Uc/Up - Earthquake:',nom,' record']);
xlabel('Time [sec]'); ylabel('Uc/Up');

%%%%%% Uncontrolled

% Structure A

%--------- Form the state-space matrices and calculate the response -------

MiA = inv(MA) ;
CtA=MA* PhiA* diag(2*(0.02+0.03).*wsA)* PhiA'* MA;
A  = [-MiA*CtA, -MiA*KA; eye(nA), zeros(nA) ] ;
B  = [-ones(nA,1) ; zeros(nA,1)] ;
C  = [-MiA*CtA, -MiA*KA] ;
D  = zeros(nA,1) ;

disp('*** Equivalent modal properties of the damped structure:')

damp(A)
[Y1A,X1A] = lsim(A,B,C,D,Xg,t);
V1A  = KA(1)*X1A(:,nA+1)/1e3;
um1A = max( abs(X1A(:,nA+1:2*nA)) );
Vm1A = max( abs(V1A) );

%------ Calculate the maximum response & plot the building response -------

V2A = KA(1)*Ud1A/1e3;
umcA = max( abs(matrA(:,1:nA)) );
Vm1A = max( abs(V1A) );
Vm2A = max( abs(V2A) );

figure; plot(t(1:np),X1A(1:np,2*nA) ); grid on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',1); title(['Relative displacement of top floor structure A:
',nom,' record']);

figure; plot( t(1:np),V1A(1:np) ); grid on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled',1); title('Total base shear structure A');


% figure; plot(t(1:np),X1(1:np,2*n),t(1:np),Udn(1:np) ); grid on; xlabel('Time [sec]');
% ylabel('Displacement [in]'),legend('Uncontrolled','Controlled',1); title(['Relative displacement of top floor
- Earthquake:',nom,' record']);
%
% figure; plot( t(1:np),V1(1:np),t(1:np),V2(1:np) ); grid on; xlabel('Time [sec]');
```

```
% ylabel('Force [kip]'),legend('Uncontrolled','Controlled',1); title('Total base shear');

% Structure B

%--------- Form the state-space matrices and calculate the response -------

MiB = inv(MB) ;
CtB=MB* PhiB* diag(2*(0.02+0.03).*wsB)* PhiB'* MB;
A  = [-MiB*CtB, -MiB*KB; eye(nB), zeros(nB) ] ;
B  = [-ones(nB,1) ; zeros(nB,1)] ;
C  = [-MiB*CtB, -MiB*KB] ;
D  = zeros(nB,1) ;

disp('*** Equivalent modal properties of the damped structure:')

damp(A)
[Y1B,X1B] = lsim(A,B,C,D,Xg,t);
V1B  = KB(1)*X1B(:,nB+1)/1e3;
um1B = max( abs(X1B(:,nB+1:2*nB)) );
Vm1B = max( abs(V1B) );

%------ Calculate the maximum response & plot the building response -------

V2B = KB(1)*Ud1B/1e3;
umcB = max( abs(matrB(:,1:nB)) );
Vm1B = max( abs(V1B) );
Vm2B = max( abs(V2B) );
% damnc=Ct.*X1(:,n);

figure; plot(t(1:np),X1B(1:np,2*nB) ); grid on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',1); title(['Relative displacement of top floor structure B: ',nom,' record']);

figure; plot( t(1:np),V1B(1:np) ); grid on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled',1); title('Total base shear structure B');

%------Damper Fixed Case I

% Structure A

w=Wmax;
Zeff=0.3;
Phi1A=zeros(nA,1);
for j=1:nA-1
   Phi1A(j,1)=PhiA(j+1,1);
end
Phi1rA=PhiA(:,1)-Phi1A(:,1);
```

```
sum1=0;
sum2=0;
for j=1:nA
   sum1=sum1+Zeff*pi*MA(j,j)*PhiA(j,1)^2;
   sum2=sum2+2*TsA(1,1)*(a^2/(a^2+(h-w)^2))*Phi1rA(j,1)^2;
end
Cd1A=sum1/sum2;

Cd1A= [0 Cd1A];
for i=1:2
   D3A=diag(zeros(1,nA));
   D3A(1,1)=1;
   CopA=Cd1A(i);
   Ct1A=MA* PhiA* diag(2*(0.02+0.03).*wsA)* PhiA'* MA+2*CopA*(a^2/(a^2+(h-w)^2)).*D3A;
   A  = [-MiA*Ct1A, -MiA*KA; eye(nA), zeros(nA) ] ;
   B  = [-ones(nA,1) ; zeros(nA,1)] ;
   C  = [-MiA*Ct1A, -MiA*KA] ;
   D  = zeros(nA,1) ;
   cos1=a^2/(a^2+(h-w)^2);

   disp('*** Equivalent modal properties of the damped structure:')

   damp(A);
   if i==1
      [Y3A,X3A] = lsim(A,B,C,D,Xg,t);
      V3A  = KA(1)*X3A(:,nA+1)/1e3;
      umA = max( abs(X3A(:,nA+1:2*nA)) );
      VmA = max( abs(V3A) );
   else
      [Y4A,X4A] = lsim(A,B,C,D,Xg,t);
      V4A  = KA(1)*X4A(:,nA+1)/1e3;
      um4A = max( abs(X4A(:,nA+1:2*nA)) );
      Vm4A = max( abs(V4A) );
   end

end

% Structure B

w=Wmax;
Zeff=0.3;
Phi1B=zeros(nB,1);
for j=1:nB-1
   Phi1B(j,1)=PhiB(j+1,1);
end
Phi1rB=PhiB(:,1)-Phi1B(:,1);
sum1=0;
```

```
sum2=0;
for j=1:nB
   sum1=sum1+Zeff*pi*MB(j,j)*PhiB(j,1)^2;
   sum2=sum2+2*TsB(1,1)*(a^2/(a^2+(h-w)^2))*Phi1rB(j,1)^2;
end
Cd1B=sum1/sum2;

Cd1B= [0 Cd1B];
for i=1:2
   D3B=diag(zeros(1,nB));
   D3B(1,1)=1;
   CopB=Cd1B(i);
   Ct1B=MB* PhiB* diag(2*(0.02+0.03).*wsB)* PhiB'* MB+2*CopB*(a^2/(a^2+(h-w)^2)).*D3B;
   A  = [-MiB*Ct1B, -MiB*KB; eye(nB), zeros(nB) ] ;
   B  = [-ones(nB,1) ; zeros(nB,1)] ;
   C  = [-MiB*Ct1B, -MiB*KB] ;
   D  = zeros(nB,1) ;
   cos1=a^2/(a^2+(h-w)^2);

   disp('*** Equivalent modal properties of the damped structure:')

   damp(A);
   if i==1
      [Y3B,X3B] = lsim(A,B,C,D,Xg,t);
      V3B  = KB(1)*X3B(:,nB+1)/1e3;
      umB = max( abs(X3B(:,nB+1:2*nB)) );
      VmB = max( abs(V3B) );
   else
      [Y4B,X4B] = lsim(A,B,C,D,Xg,t);
      V4B  = KB(1)*X4B(:,nB+1)/1e3;
      um4B = max( abs(X4B(:,nB+1:2*nB)) );
      Vm4B = max( abs(V4B) );
   end

end

%------Damper Fixed Case II

% Structure A

for i=1:2
   D3A=diag(zeros(1,nA));
   D3A(1,1)=1;
   D3A(2,2)=1;
   D3A(1,1)=1;
   CopA=Cd1A(i);
   Ct1A=MA* PhiA* diag(2*(0.02+0.03).*wsA)* PhiA'* MA+2*CopA*(a^2/(a^2+(h-w)^2)).*D3A;
```

```
    A  = [-MiA*Ct1A, -MiA*KA; eye(nA), zeros(nA) ] ;
    B  = [-ones(nA,1) ; zeros(nA,1)] ;
    C  = [-MiA*Ct1A, -MiA*KA] ;
    D  = zeros(nA,1) ;
    cos1=a^2/(a^2+(h-w)^2);

    disp('*** Equivalent modal properties of the damped structure:')

    damp(A);
    if i==1
       [Y3A,X3A] = lsim(A,B,C,D,Xg,t);
       V3A  = KA(1)*X3A(:,nA+1)/1e3;
       umA = max( abs(X3A(:,nA+1:2*nA)) );
       VmA = max( abs(V3A) );
    else
       [Y5A,X5A] = lsim(A,B,C,D,Xg,t);
       V5A  = KA(1)*X5A(:,nA+1)/1e3;
       um5A = max( abs(X5A(:,nA+1:2*nA)) );
       Vm5A = max( abs(V5A) );
    end

end

% Structure B

for i=1:2
    D3B=diag(zeros(1,nB));
    D3B(1,1)=1;
    D3B(2,2)=1;
    D3B(1,1)=1;
    CopB=Cd1B(i);
    Ct1B=MB* PhiB* diag(2*(0.02+0.03).*wsB)* PhiB'* MB+2*CopB*(a^2/(a^2+(h-w)^2)).*D3B;
    A  = [-MiB*Ct1B, -MiB*KB; eye(nB), zeros(nB) ] ;
    B  = [-ones(nB,1) ; zeros(nB,1)] ;
    C  = [-MiB*Ct1B, -MiB*KB] ;
    D  = zeros(nB,1) ;
    cos1=a^2/(a^2+(h-w)^2);

    disp('*** Equivalent modal properties of the damped structure:')

    damp(A);
    if i==1
       [Y3B,X3B] = lsim(A,B,C,D,Xg,t);
       V3B  = KB(1)*X3B(:,nB+1)/1e3;
       umB = max( abs(X3B(:,nB+1:2*nB)) );
       VmB = max( abs(V3B) );
    else
```

```
        [Y5B,X5B] = lsim(A,B,C,D,Xg,t);
        V5B  = KB(1)*X5B(:,nB+1)/1e3;
        um5B = max( abs(X5B(:,nB+1:2*nB)) );
        Vm5B = max( abs(V5B) );
    end

end

%------Damper Fixed Case III

% Structure A
for i=1:2
    D3A=diag(ones(1,nA));
    CopA=Cd1A(i);
    Ct1A=MA* PhiA* diag(2*(0.02+0.03).*wsA)* PhiA'* MA+2*CopA*(a^2/(a^2+(h-w)^2)).*D3A;
    A  = [-MiA*Ct1A, -MiA*KA; eye(nA), zeros(nA) ] ;
    B  = [-ones(nA,1) ; zeros(nA,1)] ;
    C  = [-MiA*Ct1A, -MiA*KA] ;
    D  = zeros(nA,1) ;
    cos1=a^2/(a^2+(h-w)^2);

    disp('*** Equivalent modal properties of the damped structure:')

    damp(A);
    if i==1
        [Y3A,X3A] = lsim(A,B,C,D,Xg,t);
        V3A  = KA(1)*X3A(:,nA+1)/1e3;
        umA = max( abs(X3A(:,nA+1:2*nA)) );
        VmA = max( abs(V3A) );
    else
        [Y6A,X6A] = lsim(A,B,C,D,Xg,t);
        V6A  = KA(1)*X6A(:,nA+1)/1e3;
        um6A = max( abs(X6A(:,nA+1:2*nA)) );
        Vm6A = max( abs(V6A) );
    end

end

% Structure B

for i=1:2
    D3B=diag(ones(1,nB));
    CopB=Cd1B(i);
    Ct1B=MB* PhiB* diag(2*(0.02+0.03).*wsB)* PhiB'* MB+2*CopB*(a^2/(a^2+(h-w)^2)).*D3B;
    A  = [-MiB*Ct1B, -MiB*KB; eye(nB), zeros(nB) ] ;
    B  = [-ones(nB,1) ; zeros(nB,1)] ;
    C  = [-MiB*Ct1B, -MiB*KB] ;
```

```
    D  = zeros(nB,1) ;
    cos1=a^2/(a^2+(h-w)^2);

    disp('*** Equivalent modal properties of the damped structure:')

    damp(A);
    if i==1
       [Y3B,X3B] = lsim(A,B,C,D,Xg,t);
       V3B  = KB(1)*X3B(:,nB+1)/1e3;
       umB = max( abs(X3B(:,nB+1:2*nB)) );
       VmB = max( abs(V3B) );
    else
       [Y6B,X6B] = lsim(A,B,C,D,Xg,t);
       V6B  = KB(1)*X6B(:,nB+1)/1e3;
       um6B = max( abs(X6B(:,nB+1:2*nB)) );
       Vm6B = max( abs(V6B) );
    end

end

% figure; plot(t,zeros(1,nt),t,X1(:,n) ); grid on; axis tight;
% title(['Relative velocity of top floor Uncontrolled - Earthquake:',nom,' record']);
% xlabel('Time [sec]'); ylabel('Velocity [in/sec]');

% Relative displacement of top floor structure A

figure; plot(t(1:round(np/2)),X1A(1:round(np/2),2*nA),'--b','LineWidth',1),hold on
          plot(t(1:round(np/2)),X6A(1:round(np/2),2*nA),'.-r','LineWidth',0.3),hold on
          plot(t(1:round(np/2)),UdnA(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III','Controlled VDSA',1);
title(['Relative displacement of the top floor structure A: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1A(1:round(np/2),2*nA),'-b','LineWidth',0.3),hold on
         plot(t(1:round(np/2)),X6A(1:round(np/2),2*nA),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III',1); title(['Relative displacement
of the top floor structure A: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1A(1:round(np/2),2*nA),'-b','LineWidth',0.3),hold on
          plot(t(1:round(np/2)),UdnA(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of the
top floor structure A: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X6A(1:round(np/2),2*nA),'-b','LineWidth',0.3),hold on
          plot(t(1:round(np/2)),UdnA(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend(' Fixed Damper Case III','Controlled VDSA',1); title(['Relative
displacement of the top floor structure A: ',nom,' record']); grid on ;axis tight;
```

```
% Relative displacement of top floor structure B

figure; plot(t(1:round(np/2)),X1B(1:round(np/2),2*nB),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),X6B(1:round(np/2),2*nB),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),UdnB(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III','Controlled VDSA',1);
title(['Relative displacement of the top floor structure B: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1B(1:round(np/2),2*nB),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),X6B(1:round(np/2),2*nB),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III',1); title(['Relative displacement
of the top floor structure B: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1B(1:round(np/2),2*nB),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),UdnB(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of the
top floor structure B: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X6B(1:round(np/2),2*nB),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),UdnB(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend(' Fixed Damper Case III','Controlled VDSA',1); title(['Relative
displacement of the top floor structure B: ',nom,' record']); grid on ;axis tight;

% Relative displacement of first floor structure A

figure; plot(t(1:round(np/2)),X1A(1:round(np/2),nA+1),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),X6A(1:round(np/2),nA+1),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Ud1A(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III','Controlled VDSA',1);
title(['Relative displacement of the first floor structure A: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1A(1:round(np/2),nA+1),'-b','LineWidth',0.3),hold on
      plot(t(1:round(np/2)),X6A(1:round(np/2),nA+1),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III',1); title(['Relative displacement
of the first floor structure A: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1A(1:round(np/2),nA+1),'-b','LineWidth',0.3),hold on
      plot(t(1:round(np/2)),Ud1A(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of the
first floor structure A: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X6A(1:round(np/2),nA+1),'-b','LineWidth',0.3),hold on
      plot(t(1:round(np/2)),Ud1A(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend(' Fixed Damper Case III','Controlled VDSA',1); title(['Relative
displacement of the first floor structure A: ',nom,' record']); grid on ;axis tight;
% Relative displacement of first floor structure B
```

```
figure; plot(t(1:round(np/2)),X1B(1:round(np/2),nB+1),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),X6B(1:round(np/2),nB+1),'.-r','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Ud1B(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III','Controlled VDSA',1);
title(['Relative displacement of the first floor structure B: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1B(1:round(np/2),nB+1),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),X6B(1:round(np/2),nB+1),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled',' Fixed Damper Case III',1); title(['Relative displacement
of the first floor structure B: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X1B(1:round(np/2),nB+1),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Ud1B(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of the
first floor structure B: ',nom,' record']); grid on ;axis tight;

figure; plot(t(1:round(np/2)),X6B(1:round(np/2),nB+1),'-b','LineWidth',0.3),hold on
        plot(t(1:round(np/2)),Ud1B(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Displacement [in]'),legend(' Fixed Damper Case III','Controlled VDSA',1); title(['Relative
displacement of the first floor structure B: ',nom,' record']); grid on ;axis tight;

% Relative displacement of all floor structure A

for j=1:nA

   figure; plot(t(1:round(np/2)),X1A(1:round(np/2),j+nA),'-b','LineWidth',0.3),hold on
   plot(t(1:round(np/2)),matrA(1:round(np/2),j),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
   ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of
the  ', int2str(j), '  floor structure A: ',nom,' record']); grid on ;axis tight;

end

% Relative displacement of all floor structure B

for j=1:nB

   figure; plot(t(1:round(np/2)),X1B(1:round(np/2),j+nB),'-b','LineWidth',0.3),hold on
   plot(t(1:round(np/2)),matrB(1:round(np/2),j),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
   ylabel('Displacement [in]'),legend('Uncontrolled','Controlled VDSA',1); title(['Relative displacement of
the  ', int2str(j), '  floor structure B: ',nom,' record']); grid on ;axis tight;

end

% Total base Shear structure A

figure; plot(t(1:round(np/2)),V1A(1:round(np/2)),'--b','LineWidth',1),hold on
        plot(t(1:round(np/2)),V6A(1:round(np/2)),'.-r','LineWidth',0.3),hold on
```

```
    plot(t(1:round(np/2)),V2A(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Fixed Damper Case III','Controlled VDSA',1); title(['Total base
shear structure A: ',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1A(1:round(np/2)),'-b','LineWidth',0.3),hold on
    plot(t(1:round(np/2)),V6A(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Fixed Damper Case III',1); title(['Total base shear structure A:
',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1A(1:round(np/2)),'-b','LineWidth',0.3),hold on
    plot(t(1:round(np/2)),V2A(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Controlled VDSA',1); title(['Total base shear structure A: ',nom,'
record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V6A(1:round(np/2)),'-b','LineWidth',0.3),hold on
    plot(t(1:round(np/2)),V2A(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Fixed Damper Case III','Controlled VDSA',1); title(['Total base shear structure
A: ',nom,' record']); grid on;axis tight;

% Total base Shear structure B

figure; plot(t(1:round(np/2)),V1B(1:round(np/2)),'--b','LineWidth',1),hold on
    plot(t(1:round(np/2)),V6B(1:round(np/2)),'.-r','LineWidth',0.3),hold on
    plot(t(1:round(np/2)),V2B(1:round(np/2)),'-g','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Fixed Damper Case III','Controlled VDSA',1); title(['Total base
shear structure B: ',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1B(1:round(np/2)),'-b','LineWidth',0.3),hold on
    plot(t(1:round(np/2)),V6B(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Fixed Damper Case III',1); title(['Total base shear structure B:
',nom,' record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V1B(1:round(np/2)),'-b','LineWidth',0.3),hold on
    plot(t(1:round(np/2)),V2B(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Uncontrolled','Controlled VDSA',1); title(['Total base shear structure B: ',nom,'
record']); grid on;axis tight;

figure; plot(t(1:round(np/2)),V6B(1:round(np/2)),'-b','LineWidth',0.3),hold on
    plot(t(1:round(np/2)),V2B(1:round(np/2)),'-r','LineWidth',1.6),hold on; xlabel('Time [sec]');
ylabel('Force [kip]'),legend('Fixed Damper Case III','Controlled VDSA',1); title(['Total base shear structure
B: ',nom,' record']); grid on;axis tight;

% Displacement for different case structure A

piso=[0:1:nA];
case1=[0,um4A];
case2=[0,um5A];
```

```
case3=[0,um6A];
contro=[0,umcA];
nocon=[0,um1A];

figure; plot(case1,piso,'--rs','MarkerSize',5,'LineWidth',1),hold on
        plot(case2,piso,'.-b','MarkerSize',15,'LineWidth',1.5),hold on
        plot(case3,piso,'-xb','MarkerSize',10,'LineWidth',1.5),hold on
        plot(contro,piso,'*--r','MarkerSize',5,'LineWidth',1),hold on
        plot(nocon,piso,'o-g','MarkerSize',5,'LineWidth',1.6),hold on; xlabel('Maximum Displacement [in]');
ylabel('Floor'),legend('fixed Damper Case I','fixed Damper Case II','fixed Damper Case III','Controlled
VDSA','Uncontrolled',4); title(['Maximum Displacement structure A: ',nom,' record']); grid on;axis tight;

% Displacement for different case structure B

piso=[0:1:nB];
case1=[0,um4B];
case2=[0,um5B];
case3=[0,um6B];
contro=[0,umcB];
nocon=[0,um1B];

figure; plot(case1,piso,'--rs','MarkerSize',5,'LineWidth',1),hold on
        plot(case2,piso,'.-b','MarkerSize',15,'LineWidth',1.5),hold on
        plot(case3,piso,'-xb','MarkerSize',10,'LineWidth',1.5),hold on
        plot(contro,piso,'*--r','MarkerSize',5,'LineWidth',1),hold on
        plot(nocon,piso,'o-g','MarkerSize',5,'LineWidth',1.6),hold on; xlabel('Maximum Displacement [in]');
ylabel('Floor'),legend('fixed Damper Case I','fixed Damper Case II','fixed Damper Case III','Controlled
VDSA','Uncontrolled',4); title(['Maximum Displacement structure B: ',nom,' record']); grid on;axis tight;

% figure; plot(X1(1:np,2*n),V1(1:np),X4(1:np,2*n),V4(1:np),Udn(1:np),V2(1:np) ); grid on;
xlabel('Displacement [in]');
% ylabel('Force Elastic [kip]'),legend('Uncontrolled',' fixed Damper','Controlled VDSA',1); title('Elastic');

% figure; plot( Udn(1:np),dampi(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Damping [lb]'),legend('Controlled VDSA',2); title('Damping');

% figure; plot( Udn(1:np),V2(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Elastic [lb]'),legend('Controlled VDSA',2); title('Elastic');

% figure; plot( U(1:np),eladam(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Elastic + Damping [lb]'),legend('Controlled VDSA',2); title('Elastic + Damping');

% figure; plot( X1(1:np,2*n),damnc(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Damping [lb]'),legend('Uncontrolled',2); title('Damping');

% figure; plot( X1(1:np,2*n),V1(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Elastic [lb]'),legend('Uncontrolled',2); title('Elastic');
```

```
% figure; plot( X1(1:np,2*n),eladamnc(1:np) ); grid on; xlabel('Displacement [in]');
% ylabel('Elastic + Damping [lb]'),legend('Uncontrolled',2); title('Elastic + Damping');

% figure; plot( t,X1(1:np,n),t,Up ); grid on; xlabel('Time [sec]');
% ylabel('Velocity [in/sec]'),legend('Uncontrolled','Controlled',1); title(['Relative velocity of top floor -
Earthquake:',nom,' record']);

% En1=cumsum(abs(V1(1:np)))*dt;
% En2=cumsum(abs(V2(1:np)))*dt;
% En3=cumsum(abs(dampi(1:np)))*dt;
% En4=cumsum(abs(damnc(1:np)))*dt;
% En6=En1+En4;
% En5=En2+En3;
% En5=cumsum(abs(eladam(1:np)))*dt;
% En6=cumsum(abs(eladamnc(1:np)))*dt;

% figure; plot(t,En1,t,En4,t,zeros(1,np) ); grid on; axis tight; xlabel('Time [sec]');
% ylabel('Energy'),legend('Elastic','Damping','Elastic + Damping',4); title(['Energy Force Uncontrolled -
Earthquake:',nom,' record']);
%
% figure; plot(t,En2,t,En3,t,zeros(1,np) ); grid on; axis tight; xlabel('Time [sec]');
% ylabel('Energy'),legend('Elastic','Damping','Elastic + Damping',4); title(['Energy Force Controlled -
Earthquake:',nom,' record']);
```