# A GRAPHICAL TOOL FOR THE ANALYSIS OF RESPONSE SURFACE EXPERIMENTS: A THREE-DIMENSIONAL APPROACH

By

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A project submitted in partial fulfillment of the requirements for the degree of

MASTER OF ENGINEERING
in
MANAGEMENT SYSTEM ENGINEERING

UNIVERSITY OF PUERTO RICO
MAYAGÜEZ CAMPUS
2007

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**ABSTRACT**

General and statistical software with Design of Experiments (DOE) tools are limited in their capability of allowing the user a full control on independent factors representation. They are trustworthy on statistical analysis, but lack graphical interfaces that support DOE's powerful capabilities. This project presents the development of a graphical user interface application that allows the user to visualize the results of a design of experiments in a 3-D display. The application was developed in Matlab 13® programming software and is capable of analyzing Full Factorial, Box-Behnken and Central Composite Designs. The application was verified against other generally available statistical software and applied to a case study of a local electronics assembly industry.

**RESUMEN**

Actualmente existen aplicaciones de computadora, tanto generales como especializadas en el campo estadístico, que exhiben deficiencias en la representación gráfica de los factores independientes en el diseño de experimentos. Estas herramientas son muy efectivas en análisis estadísticos, pero carecen de las alternativas que permitan aprovechar al máximo las capacidades que ofrece el campo de diseño de experimentos. Este proyecto presenta el desarrollo de una herramienta grafica que permite visualizar los contornos que se obtienen en diseño de experimentos en una perspectiva tridimensional. La herramienta se desarrolló en la aplicación de programación Matlab 13® y es capaz de analizar experimentos Factoriales Completos (Full Factorial), Box-Behnken y Centrales Compuestos (Central Composite Designs – CCD). La aplicación fue validada para cada escenario disponible y los resultados fueron comparados con otra conocida aplicación estadística. Finalmente, la aplicación fue utilizada en un caso de estudio de una compañía local de productos electrónicos.

# DEDICATORY

This work is dedicated in first place to God, for blessing my life the awesome way He has done. To my husband Juan, a special man, for his love, support and patience all these years. To my parents, Julio and Migdalia, thanks for your love, support and overall for teaching that success is at the reach even for those who start from scratch. To my brother Julio and my sisters Frances and Vanessa, I really appreciate your love and faith on me.

# ACKNOWLEGDMENTS

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# 1    INTRODUCTION

## 1.1 Justification

At the first decades of previous century, the methods of Design of Experiments (DOE) lay virtually dormant due to the complex and time consuming hand calculations required. After some technological advances come into view, mainframe computers requiring programming skills beyond most statisticians' scope chugged through complicated DOE equations. It was not until room-sized computers became desktop PCs that affordable DOE software for the no-statistician first appeared [1].

DOE has become a highly versatile technique very useful for a diverse range of industries. DOE methods are useful anywhere people need to learn about a system's inputs and it effects in the expected output. Because of DOE's incredible efficiency and ability to uncover synergistic interactions, its potential is virtually unlimited. DOE tools have impressive capability and flexibility, so much that sometimes the selection of DOE software that properly maximizes this flexibility is not an easy task.

Everyday analytical tasks are commonly built with general-purpose statistical software. Doubtlessly, the power and ease-to-use of a dedicated tool dramatically simplifies the job. Nevertheless, general and statistical software with DOE tools are limited in their capability to allowing the user to have a full control on independent factors representation. They are trustworthy on statistical analysis, but lack graphical interfaces that support DOE's powerful capabilities. For example, many DOE software provide just a basic contour plot over a rectangular grid while others allow color filled or

shaded contours. Generally, software are limited to two factors interactions. Therefore, users working with more than two factors have to perform 2-dimensional or two-factor representations at a time keeping the other factors at some specific levels.

These limitations justify the development of a graphical application to provide the analysts with a versatile, yet reliable statistical tool. This project proposes an alternate analysis tool that combines existing Matlab 13® software programming capabilities with a powerful graphic interface. The suitability of this tool will be shown using an actual case study. The case study scenario summarized in Chapter 7, is based on an actual problem faced by research and development engineers on a local electronics assembly company.

## 1.2 Objectives

The availability of a user-friendly DOE analysis software program with the possibilities of maximizing DOE's capabilities is not a fact. The main objective of this project is to develop a graphical user interface Matlab 13® tool and was conceived to help analyze wave soldering experiments at a local electronics assembly company. In addition, the tool should have the capability for analyzing a wide variety of DOE problems by means of Full Factorial, Box-Behnken, and Central Composite Design approaches. This tool will bring a graphical aid to show and understand multiple factors effect on a specific response through a 3-dimensional visualization of the contours. The application will bring a more practical visual approach on performing contour plots of iteration responses than the well known 2-dimensional approach. The software's add-in will provide a user-friendly alternative for displaying 3-dimensional representations of

actual design of experiments conditions, whether there are three or more independent factors.

Therefore, the objective is to develop a graphical user interface based tool for DOE analysis that will provide the user the alternative of analyzing DOE problems by 3-dimensional visualization of the contours.

## 1.3 Organization

Chapter 2 summarizes the basic concepts and the literature related to the scope of this project. The chapter includes literature related to Design of Experiments (DOE) in subjects such as Full Factorial Experiments, Box Behnken Designs and Central Composite Designs. It also compares current statistical analysis software and their capabilities, such as JMP, Minitab, S-Plus, and DOEpack. Chapter 3 provides a methodology flowchart that describes the path followed for the completion of the project. It also presents the application conceptual that summarizes the tool general concept and application scope. Chapter 4 includes a description of the tool windows and a general tool use algorithm. Chapter 5 summarizes the verification procedure used to demonstrate the validity of the tool. Chapter 6 introduces the applied case study background, the 3-D DOE Contour Plot output, and the verification of the results with other statistical software. Chapter 7 presents the general conclusions and recommendations for future work. References and Appendices containing supplemental information are attached at the end of the document.

# 2 LITERATURE REVIEW

This chapter introduces relevant concepts used in this project with a summarized explanation of them.

## 2.1 Design of Experiments

### 2.1.1 Definition

In an experiment, we deliberately change one or more process variables (or factors) in order to observe the effect that those changes have on one or more response variables. Design of experiments (DOE) is an efficient procedure for planning experiments so that the data obtained can be analyzed to yield valid and objective conclusions. In other words, DOE may be defined as a structured and organized method for determining the relationship between factors (X's) affecting a process and its output (Y) [2]. With this information, the optimum balance between factors can be developed and major improvements in quality, cost, and productivity can be achieved. Some of the common questions uppermost in the minds of the experimenters are:

1. For what values $X_1$, $X_2$ … and $X_i$ will the responses $Y_1$, $Y_2$ …$Y_i$ be optimum?
2. What factors have maximum influence on the response(s)?
3. What factors have no significant contribution on the response(s)?
4. Which are the factors that have no significant contribution on any response, hence can be safely excluded from further studies?

As the cost of conducting experiments increases, the need to answer the above questions with a few trials assumes significance. Answers to all these questions lies at the capabilities of DOE methodology.

By using DOE,

1. Appropriate data can be collected and analyzed using statistical methods so as to obtain objective conclusions from the data.
2. The number of runs for the experiments can be minimized thereby minimizing the cost of experiments.

DOE provides information of the best setting of factors to achieve optimal conditions and predict values of responses for given settings of factors in a model. DOE is extensively used for improving performance of processes and design and/or to develop a new process or a product. In other words, DOE is extensively used in industrial research, development, and production due to its robust capabilities for testing methods, products, screening and identifying vital factors and optimizing processes.

### 2.1.2 Factorial Experiments

The two-level factorial designs are a class of designs that are of great practical importance. To perform the general factorial experiment, an investigator selects a fixed number of levels for each of a number of factors and then runs all the possible combinations and analyzes them simultaneously.

The treatment combinations are randomly assigned to the pool of experimental units with an equal number of units in each treatment. The number of experimental units assigned to each treatment is referred to as the number of replications.

A common experimental design is one with all input factors set at two levels each. These levels are called "high" and "low" or "+1" and "-1", respectively. If there are $k$ factors, each at 2 levels, a full factorial design has $2^k$ runs. Because the number of runs (n) increases exponentially as the number of variables (k) increases, testing all combinations of factor levels becomes too expensive and time-consuming with a large number of factors [2]. In circumstances where there are over five factors it is recommended an alternate experiment design such as fractional factorial or a Plackett-Burman design.

In industry, two-level full and fractional factorial designs are often used to "screen" for the really important factors that influence the process output measures or product quality. Screening (process characterization) is used to reduce the number of input variables by identifying the key input variables or process conditions that affect product quality. This reduction allows focusing the process improvement efforts on the few really important variables. Screening may also suggest the "best" or optimal settings for these factors, and indicates whether or not curvature exists in the responses [1]. Optimization experiments can then be done to determine the best settings and define the nature of the curvature. These designs are useful for fitting first-order models (which detect linear effects), and can provide information on the existence of second-order effects (curvature) when the design includes center points.

### 2.1.2.1   Factor Model Specification

The factors or independent variables considered in any experiment may be quantitative or qualitative factors [1]. In a quantitative factor its levels are associated to a numerical scale. Whereas that in a qualitative factor its levels cannot be arranged in order of magnitude. Some examples of each type of factor are listed on Table 1.

**Table 1: Types of Experimental Factors**

| Quantitative Factors | Qualitative Factors |
|---|---|
| Weight | Equipment (Machine 1 or Machine2) |
| Temperature | Shifts |
| Speed | Batches or Lots |

Commonly, low-order polynomial models are used to establish the functional relationship between the factors and the response. This relationship is explained fitting a linear regression model. Residuals are estimates of experimental error obtained by subtracting the observed responses from the predicted responses. The following equation is a representation of the General Linear Model [4]:

$$\mathbf{Y} = \mathbf{X} * \beta + \varepsilon, \quad \varepsilon \sim N_n(\mathbf{0}, \sigma_\varepsilon^2) \tag{2.1}$$

In expression 2.1, $\mathbf{Y}$ is the vector representation of a set of $n$ numerical observations of a dependable variable. The model parameters, typically represented as $\beta$, are a vector of coefficients for the functions in $\mathbf{X}$, called the design matrix. $\mathbf{X}$ is a matrix with each column representing the numerical value of a function of one or more factors.

In any fitted model there is an uncertainty represented by $\varepsilon$, the random error term. After some matrix operations the following solution derives:

$$\hat{\beta} = \left( \mathbf{X'X} \right)^{-1} \mathbf{X'y} \qquad\qquad \textbf{2.2}$$

The $\hat{\beta}$ coefficients are the least squares estimates for $\boldsymbol{\beta}$, and they minimize the sum of squared differences between components of the predicted value vector $\hat{\mathbf{y}}$ and the actual values observed in the experiment $\mathbf{y}$. Any regression model that is linear in the parameters, $\boldsymbol{\beta}$, is a linear regression model, regardless of the shape of the response surface that it generates [2].This statement is the start point for the main computations to be used on the graphical tool to be developed.

First order general lineal models are just a starting point; there are also quadratic or cubic model fittings when the relationship between factors is not linear. Often the curvature in the true response surface is strong enough that the first-order model (even with the interaction term included) is inadequate. A second-order model will likely be required in these situations.

### 2.1.3 Box-Behnken

If a first-order model demonstrates lack of fit because of the presence of surface curvature a better approximation may be expected from a quadratic model [5]:

$$\mathbf{Y} = \beta_0 + \sum_{i=1}^{k} \beta_i X_i + \sum_{i=1}^{k} \beta_{ii} X_i^2 + \sum \sum_{i>j=1}^{k} \beta_{ij} X_i X_j + \varepsilon \qquad\qquad \textbf{2.3}$$

At this model the first term, $\beta_0$, is a constant term followed by $k$ first-order terms, $k$ quadratic terms, and $k*(k-1)/2$ interaction terms. The total number of terms is given by [2]:

$$p = \frac{(k+1)*(k+2)}{2} \qquad\qquad \textbf{2.4}$$

This model would likely be useful as an approximation to the true response surface in a relatively small region. The second-order model is widely used in response surface methodology for several reasons. One of these reasons is the second-order model is very flexible. It can take on a wide variety of functional forms, so it will often work well as an approximation to the true response surface [6]. A second reason is that it is easy to estimate the parameters (the $\beta$'s) in the second-order model. The method of least squares can be used for this purpose.

Many possible second-order designs may be used to obtain the data for such a model fitting. A very efficient design for fitting the second order model is the Box-Behnken design (BBD). BBD's are a class of three-level designs resulting from the combination of two-level factorial with incomplete block designs [7]. Two main characteristics of BBD's are:

- BBD's rotatability and orthogonal blocking properties are fixed for a specific $k$ and

- BBD's can be expected to have poorer prediction ability on the corners of the cube, because unlike central composite (face centered) designs they do not include points at the corners of the cube.

Box-Behnken designs are usually very efficient in terms of number of runs. An example of a Box-Behnken design is presented below for a $k = 3$ factors.
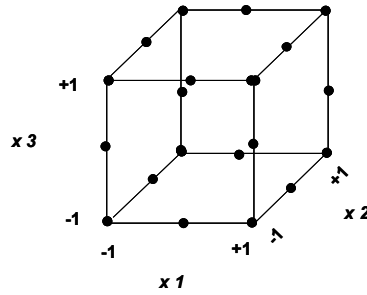


**Figure 1: Box-Behnken Design for k = 3**

Box-Behnken designs have been very popular, especially for a small number of factors. For 3 factors Box-Behnken designs requires only 12 runs, plus a recommended $n_c = 3$ center point runs. Box and Behnken (1960) contains only 10 designs, one each for $t = 3, 4, 5, 6, 7, 9, 10, 11, 12,$ and 16. An earlier technical report (Box and Behnken 1958) contains seven additional designs, including a couple of designs for $t = 8$, although these additional designs have large redundancy factors, defined by BB as the ratio of the number of factorial runs to the number of parameters [8].

### 2.1.4 Central Composite Design (CCD)

Central Composite Designs, introduced by Box and Wilson (1951) and followed up by Box and Hunter (1957), are one of the most popular and commonly used classes of experimental designs for quadratic regression [9].

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The CCD is $2^k$ factorial designs, full or fractional factorial, of resolution V or higher [7]. The central composite design for $k$ factors comprises $n_F$ factorial points taken from a full $2^k$ factorial at levels $\pm$ 1, $2k$ axial points at locations $\pm$ $\alpha$, and $n_C$ center points at the origin [10]. The overall efficiency of the resulting design depends on the quality of the factorial points and number of center points. Next figure illustrates CCD's for two and three factors.
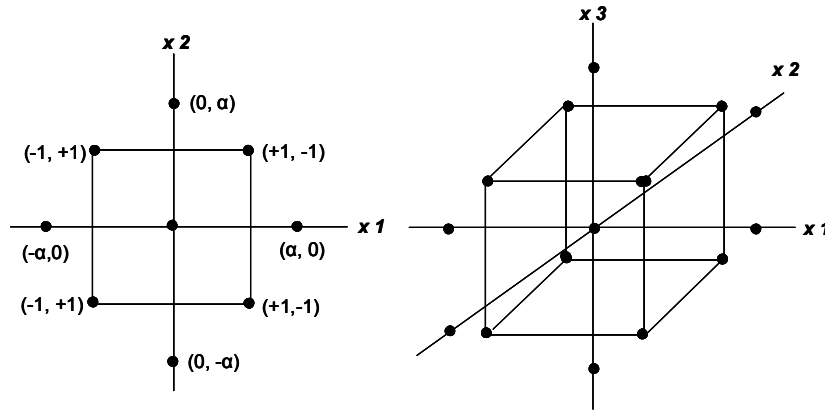


**Figure 2: Central Composite Design for k = 2 & k = 3**

One advantage often cited for the central composite design is the potential to implement it in two or more stages, beginning with an orthogonal two level plan for screening of factors and preliminary estimation of linear effects. Axial runs are executed conditioned to a first-order model lack of fit. The axial level points depend on the number of experimental runs in the factorial portion of the experiment. The appropriate selection of the axial level points yield a rotatable CCD's, which means that the variance of the predicted response at point of interest **x** is reasonably constant and stable. In other words, the variance of the predicted response depends only on the distance from the center of the design [2]. For a CCD to be rotatable [10]:

$$\alpha = \left[ \text{number of factorial runs} \right]^{\frac{1}{4}} \qquad \qquad \textbf{2.5}$$

### 2.1.5 Model Adequacy

The principle of least squares provides a general methodology for fitting straight-line models to regression data. For a multiple linear regression model

$$\mathbf{Y} = \mathbf{X} * \beta + \varepsilon, \quad \varepsilon \sim N_n(\mathbf{0}, \sigma_\varepsilon^2) \qquad \qquad \textbf{2.6}$$

the following four model assumptions are established:

- Independence – The response variables $\mathbf{Y_i}$ are independent. Independence assumption on the response variables is subject to the design of the study and the way the data have been collected.

- Normality – The response variables $\mathbf{Y_i}$ are normally distributed

- Homoscedasticity – The response variables $\mathbf{Y_i}$ all have the same variance $\sigma^2$

- Linearity – The true relationship between the mean of the response variable E(Y) and the explanatory variables $x_1, \ldots, x_k$ is a straight line.

This relationship is explained fitting a linear regression model. Residuals are estimates of experimental error obtained by subtracting the observed responses from the predicted responses. It is recommended that rather than checking these four assumptions on the response variables directly, it is convenient to re-express the assumptions in terms of the random error, and check the assumptions on the random errors instead [2].

$$\varepsilon = \mathbf{Y} - \mathbf{X} * \beta, \ \varepsilon \sim N_n(\mathbf{0}, \sigma_\varepsilon^2), \qquad \qquad \textbf{2.7}$$

The four assumptions on the random errors are equivalent to the assumptions on the response variables. If all the four assumptions are satisfied, the random errors $\varepsilon_i$ are independent, identically distributed random variables with distributions:

$$\varepsilon \sim N_n(\mathbf{0}, \sigma_\varepsilon^2), \quad i = 1, 2, ..., n \qquad\qquad \mathbf{2.8}$$

The need for a model-independent estimate of the random variation means that replicate measurements made under identical experimental conditions are necessary to carry out a lack-of-fit test [20]. When no replicate measurements are available a baseline estimate of the random process variation to compare with the results from the model is not possible.

Assumptions on the random errors can be checked (and thereby the assumptions on the response variables) by analyzing an observed sample of the random errors. Various model inadequacies and violations of the assumptions can be revealed through the study of the residuals. Although statistical test of the hypothesis that the model is sufficient may be helpful, no single test can provide the rich feedback to the user that a graphical analysis of the residuals can provide.

### 2.1.5.1 Normality Assumption

A histogram plot of the residuals helps when checking normality. The histogram plot should look as a sample from a normal distribution centered at zero for the normality assumption to be true. It is possible to observe significant fluctuations when sample sizes are small. In such a case, a moderate departure from normality does not necessarily imply normality assumption is violated.

Histograms intention is to graphically summarize the distribution of a univariate data set. This plot illustrates: location (centering) of the data, spread, skewness, presence of outliers, and multiple modes in the data. All these characteristics help distinguish the adequate data distributional model.

The normal probability plot is a graphical technique for assessing whether or not a data set is approximately normally distributed. The data are plotted against a theoretical normal distribution in such a way that the points should form an approximate straight line. Departures from straight line indicate departures from normality.

Normal probability plots of the residuals are very functional. Small to moderate departures from normality are of little concern for fixed effects analysis of variances [2]. It is of more concern a residuals distribution with thicker or thinner tails than a skewed residuals distribution. An analysis of variance is considered as robust because the F test is the only slightly affected. Normal probability plots allow visualizing outliers. Outliers are residuals that are very much larger than any of the others. Just one outlier can affect analysis of variance results.

## 2.1.5.2 Residuals vs. Time Sequence

A good tool to determine residuals correlation or dependency is a plot of residuals in time sequence. A positive correlation is represented by runs of positive and negative residuals which translate as a violation to the independence assumption on the errors. Therefore, when the error terms are independent, it is expected the residuals to fluctuate in a more or less random pattern around the base line. The residuals versus time sequence plot is strongly recommended whenever data is collected in time order. Proper

randomization minimizes probability of collecting dependant data. When the error terms are independent, we expect the residuals to fluctuate in a more or less random pattern around the base line 0.

### 2.1.5.3 Residuals vs. Fitted Values

A residuals plot versus the fitted values should exhibit no particular structure. A plot with no obvious pattern indicates residuals are unrelated to any other variable. A residuals plot versus the fitted values layout is directly related to the constant variance assumption. It is possible to detect a non-constant variance when the residual versus the fitted values plot displays observations that increase as the magnitude of the observations increases in a plot that resembles an outward-opening funnel shape. This pattern generally occurs when the error (residuals) are a constant percentage of the size of the observations. Non-normal, skewed data tends to be a function of the mean. Hence, it is common to have non-constant variance plots for this type of data.

### 2.2 GRAPHICAL TOOLS

The graphical representation of the DOE analysis and outcomes has practical importance for analysts, specifically when analyzing a problem with factorial designs. Using graphical tools allow understanding many of the main concepts in the design and analysis of experiments. It is virtually easy to plot the settings for the design factors in each run when there are only two factors associated to the design, but for three or more factors it may be complex. If there are more than three factors it is not possible to graphically represent the design factors for a particular run corresponding to the coordinates of a point in the space [4]. Therefore, it will need a more practical method.

Since factorial design points lay on a regular grid, whenever there is a third dimension it can only be represented on a two-dimensional surface. This two-dimensional representation is possible using a perspective drawing or an orthogonal projection. When a fourth and up dimensions appear, the design plot is possible with a hierarchical representation of the coordinates [7].

Figure 3 displays the factorial design plots for: a) two, b) three, c) four and d) five factors. The factors levels are labeled as "-1" for the low level and "+1" for the high level. Also, all the designs plotted are orthogonal arrays (OA's). The OA's are widely used in industrial experiments to screen important factors from a large number of potential factors and for detecting interactions among a subset of active factors. Orthogonality is interpreted in the combinatory sense that is, for any pair of columns, all combinations of factor levels occur and they occur an equal number of times [8].
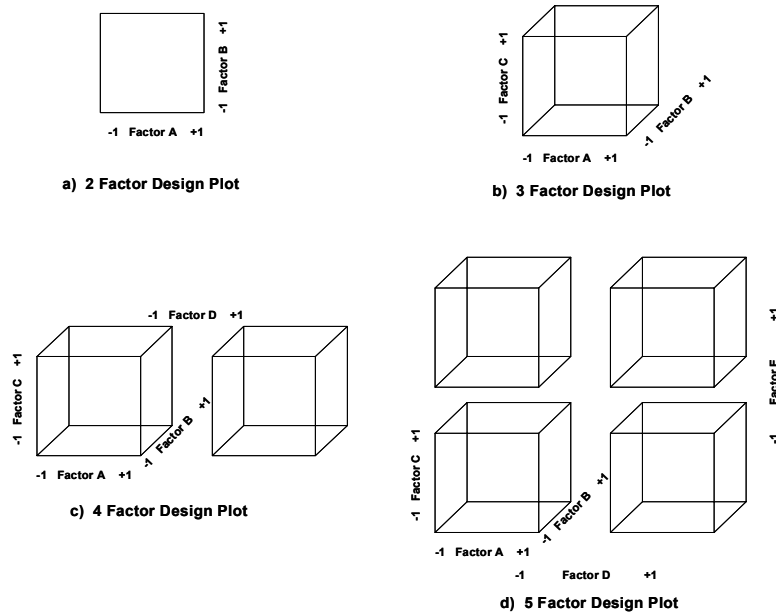


**Figure 3: a) Two-Factor Design Plot; b) Three-Factor Design Plot; c) Four-Factor Design Plot; d) Five Factor Design Plot**

The geometric properties of 3-D plots make easier the communication of design properties to non-statisticians, especially once the experiment run is completed. When data from designed experiments is analyzed, the specialized graphical displays most widely used and discussed are those constructed after an ANOVA model has been fitted. In particular, those graphical displays focused on standard plots of residuals and normal plots of fitted effects. In general, design of experiments interpretation tools such as interaction plots, surface plots, boxplots, and contour plots are available among a vast selection of statistical softwares. There are a few more graphical alternatives for the user to visualize numerical results in an easier perspective.

Minitab 14®, JMP V5.1®, S-Plus 7® and DOEpack 2000® are some of the commercially available software with DOE and statistical capabilities. Some of those softwares offer a wide selection of tools other than the ones mentioned above. The basic ones may just provide with a selection of type of experiment and simple planar scatter plots. Others bring the choice of a more complex representation such as surface plots in a three-dimensional field that may let customize colors, textures, titles, etc. Next section will briefly introduce the capabilities of the DOE softwares mentioned above with the intention of clearly identify the expected advantages of the tool to be designed with this project over the current available alternatives.

## 2.2.1   Minitab 14®

Minitab 14® software is one of the most complete analysis packages available for statistical purposes. Among its multiples functions, it generates two-level full and fractional factorial designs, and Plackett-Burman designs. Minitab 14® calculates

statistics in a single response column from experimental repeats or replicates. Among the vast selection of features Minitab 14 has to offer the user it:

1. Displays main interaction plots;

2. Displays main effects, interactions, and cube plots for two-level full and fractional factorial designs, and Plackett-Burman designs;

3. Calculates a numerical optimal solution and draws an interactive plot to help identify the combination of factor settings that jointly optimize a set of responses;

4. Searches for a combination of input variables that jointly optimize a set of responses by satisfying the requirements for each response in the set with the Response Optimizer desirability function.

5. And draws of a contour plot and a three dimensional response surface plot among many others.

On Figure 4, Figure 5, and Figure 6 there are some examples of Minitab's graphical outputs for these and other application representations. Cube plots, are similar to design plots, but they differ on cube plots by illustrating the responses at each of the treatment combinations considered. Surface plots show a three-dimensional response behavior considering two independent variables over a fixed range and one dependent variable. The main drawback is that any additional independent factor has to be set on hold in order for the surface plot to be generated. A similar situation arises when displaying contour plots in Minitab 14®, because responses traces are only represented

on a two-dimensional plane no matter the number of independent factors associated in the experiment.
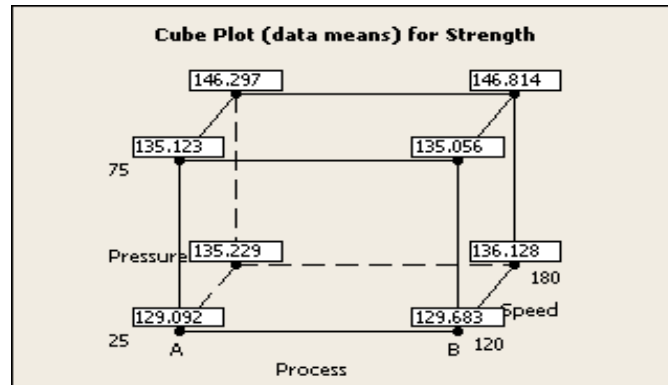


**Figure 4: Minitab 14® - Main Effects Plot**
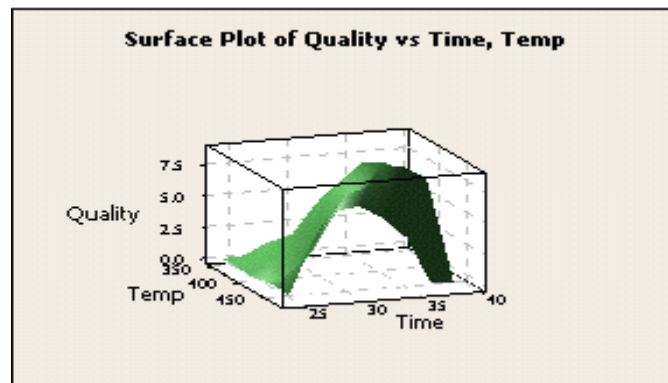


**Figure 5: Minitab 14® - Surface Plot**
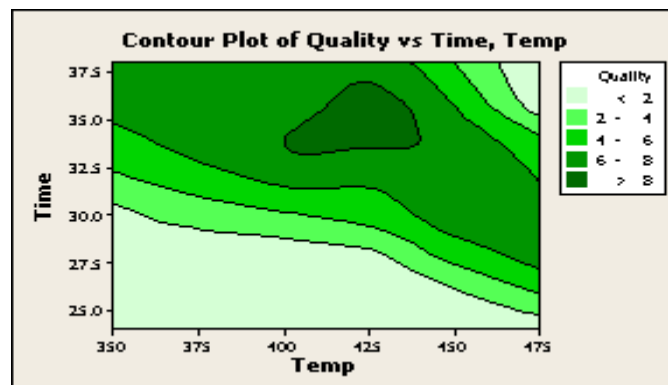


**Figure 6: Minitab 14® - Contour Plot**

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### 2.2.2 JMP V5.1®

JMP V5.1® software is a tool that offers a design choice for almost every situation. The selection ranges from custom designs that bring the most flexibility, because let selecting roles as continuous, categorical, blocking, mixture, or covariate for factors, to screening, response surface and full factorial designs. A graphical comparison of the designs is possible with the Prediction Variance Profiler, shown below in Figure 7. With the Profiler, is possible to determine how much variance in the expected response with the different factor settings. As the number of runs increases, the prediction curve (prediction variance) decreases. The dotted red lines can be moved around to figure out how different levels of the factors affect the variance.
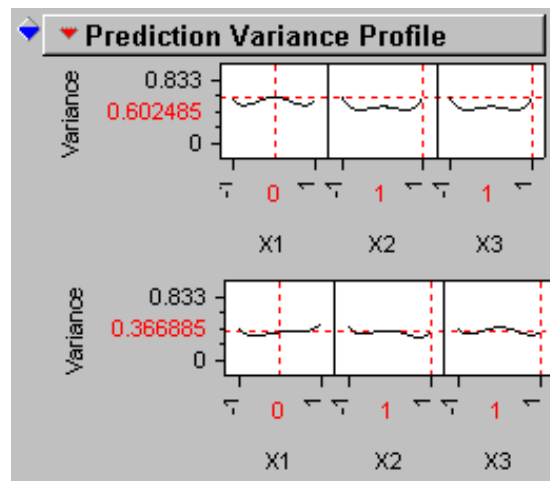


**Figure 7: JMP V5.1® - Prediction Variance Profile**

Desirability functions that specify whether you are trying to maximize, minimize, aim for target values, or satisfy constraints can also be added as in Minitab 14®. The Contour Profiler is useful for optimizing response surfaces graphically especially when there are multiple responses. Options on the title bar include setting grid density,

requesting a surface plot, and adding contours at specified intervals. There are also sliders that allow to set factors at certain levels. Figure 8 show that even though JMP V5.1® software offers diverse options to the user it also has limitations at the time of manipulating a non two-dimensional field.  Those limitations can compare to the limitations found in softwares such as Minitab 14®. When displaying contour plots, user has to fix the levels for any other factors over any two selected.
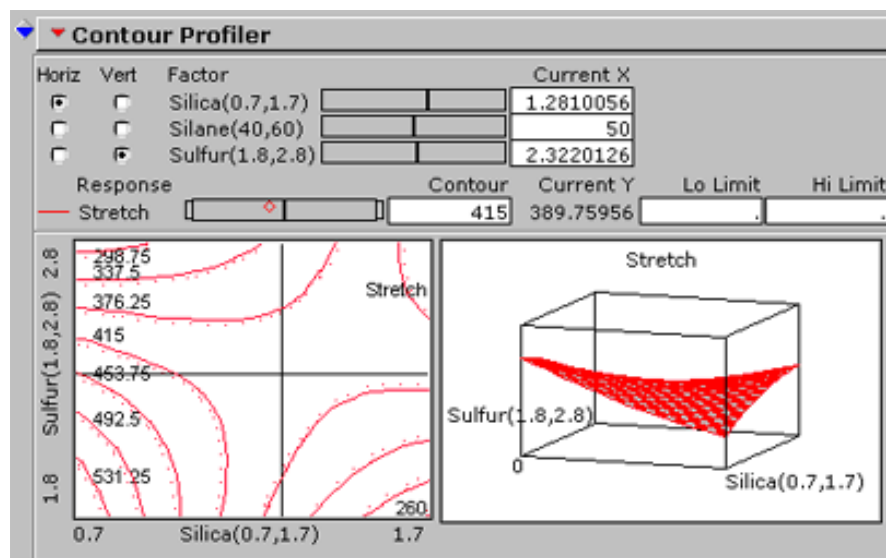


**Figure 8: JMP V5.1® - Contour Profiler**

### 2.2.3   S-Plus®

Insightful Inc. offers an S-Plus® software family which is an enhanced, extended, and supported version of AT&T Bell Laboratories' New S Language and System. S is a language and an iterative programming environment for data analysis and graphics. The S language is a very high-level language for specifying computations. They claim the S-

Plus® data analysis tool is focused on integrating statistics where and how they are needed.

The S-Plus® system allows the user the entry of data as vectors, matrices, and time-series. It will do various types of data analysis, including statistical and numerical along with various data manipulations. The graphics capabilities include scatter plots, bar graphs, histograms, pie charts, and box plots. There is a huge selection of three-dimensional perspective graphs among standard features such as ANOVA analysis. Figures 8 and 9 illustrate some of these graphics.
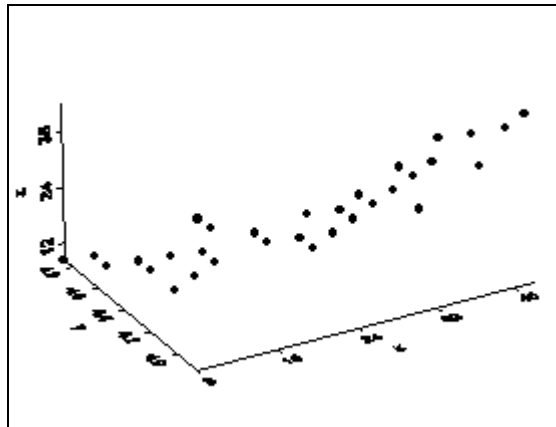


**Figure 9: S-Plus® - Histogram and Boxplot**
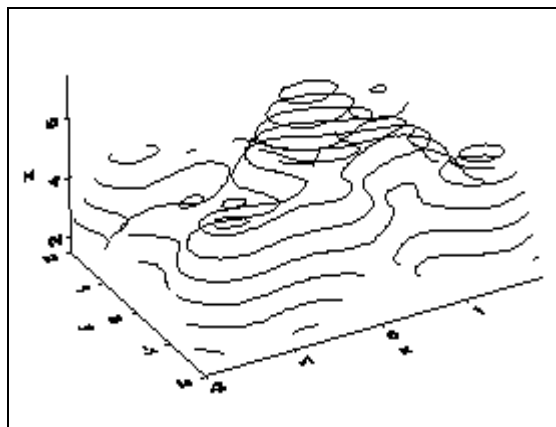


**Figure 10: S-Plus® 3-D Contour Plot**

It is fair to mention that S-Plus® offers the analyst a feature not available on most popular statistical softwares. It is possible to create any kind of graph with a simple "drag and place" action. User may have a data spreadsheet and S-Plus® is capable to understand that a drag of data to a specific graph selection means chart creation or even adding a panel variable to any existing graph.

### 2.2.4 DOE-Pack 2000®

DOEpack 2000®, as previous softwares, offers the availability of an easy-to-use program that guides trough a logical process for planning, designing, implementing, and interpreting effective experimental designs. Among the software capabilities, it provides help to optimize responses and identify winning combinations on process improvement initiatives. The tools for data analysis are comparable to those already discussed; similar capabilities and disadvantages. Specifically, it presents the limitations that justify the development of the proposed graphical tool: poor manipulation of data analysis for multiple factors in plots such as contour plots. Figure 11 shows an example of a DOEpack 2000® response surface plot.
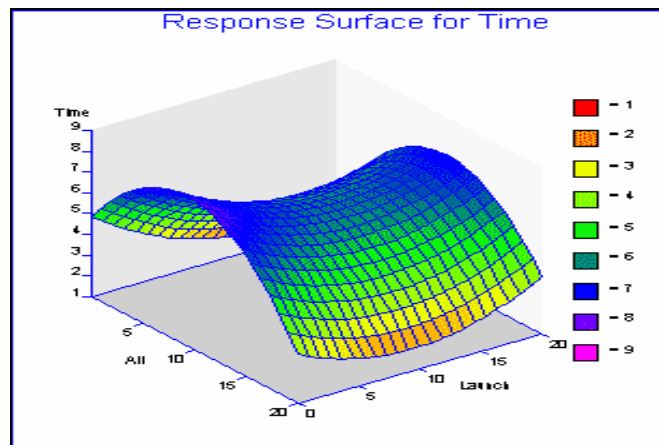


**Figure 11: DOEpack 2000® - Response Surface Plot**

Minitab14®, JMP V5.1®, S-Plus® and DOEpack 2000® are just few examples from a vast diversity of statistical programs currently available with toolboxes for experiment designing. Others that may be mentioned are Statistica®, SAS®, Stat-Graphics®, etc. This project scope focuses on the pros and cons of software capabilities when performing three-dimensional data analysis and the opportunity of bringing an easy-to-use and versatile application.

## 2.3 GRAPHICAL USER INTERFACE (GUI's)

In the above sections it was clearly stated that most popular statistical analysis software lack of 3-dimensional display of contours for DOE applications. Many others software commercially available share the same opportunities of improvement. In order to provide an alternative to experiment analysts, this project presents a graphical user interface (GUI) program developed within the Matlab 13® graphics system. Matlab 13® is a robust and versatile programming tool with an extended capability limited by the imagination and creativity of the programmer.

A graphical user interface is a pictorial interface to a program. GUI's allow to graphically generating customized functional applications without needing complex coding. The environment of GUI's contains pushbuttons, toggle buttons, lists, menus, text boxes, and so forth. Any program developed with GUI's needs inputs, mouse clicks or keyboard inputs, and responds to events so it is called an event driven program [5].

To create a Matlab 13® GUI three principal elements are required:

1. Components – Types of components include graphical controls (pushbuttons, lists, edit boxes, etc), static elements (frames and text strings), menus, and axes.

2. Figures – Figures is a window in a computer screen. Empty figures can be created to hold any combination of components.

3. Callbacks – The callback is the code executed in response of an event.

A good GUI makes an application easy, practical, and efficient to use. Unfortunately, it is not an easy task to define easy, practical, and efficient when developing a GUI application. Marketplace has attempted to address these characteristics, but even after over 10 years of GUI development a good GUI still not definitely defined. There are several screen design guidelines available, though a lack of empirical studies confirming them. In a narrower sense, available empirical studies draw attention to three of many human factor concepts basic on good GUI design: visual acuity, limits to absolute memory, and the principle of grouping (Gestalt Principle) [16]. These three factors have a direct effect on three GUI design areas: the amount of presented information, the grouping of information, and the placement of this information on the screen.

Visual acuity is defined as the ability of the human eye to resolve detail. It has been demonstrated that the retina of eye can only focus on a very small section of computers' display at one time [17]. Visual acuity decreases by half at a distance greater than 2.5 degrees from the point of fixation. As a result, a circle of radius of 2.5 degrees around the point of fixation is what the user can see clearly. This relationship is called the Rule of 1.7 [16]. Not appropriate GUI designs force users to constantly move their eyes across the screen to focus causing unnecessary and tiring eye movement.

There is a limit to the amount of information a user can process at a time once a fixation point is selected. It has been established the range of options or alternatives not to exceed five or six [18]. Absolute identification using one-dimensional criteria is about seven items, plus or minus two; limitation that held for memory span as well [16]. By

expanding identification criteria from one to more dimensions the user is not only able to handle more choices but will remember more. This concept lead to consider that users group information, a statement that directly relates to the number of menu items and icons.

A principle that influences how graphical information is organized on the screen is the Gestalt Principle. This principle states that people use a top-down approach to organize data [16]. Adequate information grouping results in a necessary redundancy of information selection that helps the user. If users know where one item in a group is on a screen, it will be expected similar or related items to be there too.

### 2.3.1 GUI Design Considerations

In view of the above psychological factors, it could be wrongly concluded that it is easy to extrapolate these factors to GUI design. The Rule of 1.7 could lead to think that a good GUI design should hold a lot of small icons. However, too many randomly placed icons are against the limits of absolute memory factor. Furthermore, because of the Gestalt Principle, like items could be grouped together using color without considering that too many colors destroy the global visual grouping of items. Designs with those deficiencies lead the users to concentrate on the GUI, and any primary cognitive task attention committed to the GUI interface possibly will interfere with the tool primary task [19]. Some basis GUI standards from basic human factors have been derived related to the presentation of information, the grouping of information and the information sequencing.

The most basic consideration in GUI design is the amount of information that is presented in a screen. Screen clarity and readability are improved with less crowded designs. The designed screen display should be as simple as possible showing only the amount of information that is needed to perform the current operation. Studies show that limiting the amount of information resulted in fewer errors and less execution times. It will always be necessary to analyze the expected tool functionality to match it to the appropriate information to be displayed. General approaches to maximize screen space usage are:

- an appropriate use of abbreviations - It is recommended using complete words whenever possible, but this is not always doable because of screen sizing constraints. Abbreviations should not only be contextual and consistent but also keep to a minimum.

- use of familiar formats – When users are facing familiar format screens will noticeably need less time to perform a tasks than if layout is completely unexpected. A simple example of this technique is the standard USA address format: street, city, state, and zip code.

- use of tabular formats and headings – Related data could be efficiently labeled and arranged with tabular formats. A simple design decision of splitting items stacked on a single long column into shorter multiple columns could impact productivity by around 20% [18].

Information could be displayed in numerous ways, but only adequate information grouped displays improve information readability. It is appropriate to design displays

considering the limits of visual acuity. This meaning that user needs to be capable to perceive the different portions of information from the screen at a glimpse. Two best predictors related to the ease of use of a GUI screen are density and item alignment [16]. It has been demonstrated that search time significantly increases as size of displayed information groups exceed 5 degrees of arc and the number of groupings surpass five [17].

Practical techniques to effectively grouping screen information include:

- Color – GUI designs that incorporate color positively impact screen usage productivity. Utilizing color variations enhance information display by allocating some degree of grouping among the elements of the same color. Attention has to be paid to the fact that overuse of color coding null the effectiveness of the technique.

- Graphical boundaries – A general GUI design practice for grouping information is using graphical boundaries around elements. It is a very explicit information grouping signal for the user to understand.

Screen display should be displayed considering how easy will be for the user to search and find any information on it. Modern GUIs adhered to one of three de facto GUI screen standards: Apple Macintosh, the IBM SAA, and the MIT X-Windowing System [16]. The reason for this is users commonly expect certain modes of operation in all GUIs. One general expectation users have could be the top of screen to contain the headings for the pull-down menus. Also, a window title is usually displayed at the top of

the screen. One more common expectation users typically have is an icon for closing the current window located at the top left.

There are studies that demonstrate most users firstly examine the screen starting at the upper-left corner of the window. This indirectly means this corner should be the obvious starting point for the applications to be executed within the window. Doing so will allow a smooth reading path from left-to-right and top-to-bottom, just as in standard Western cultures. Effective screen presentations result from multiple considerations, among those:

- Sequence of use – Information should be provided in the order users will probably utilize it.

- Conventional usage – GUI designs should continue using conventional designs in general usage. Just to mention one, the file option icon located at the top, far left at a menu bar.

- Alphabetical or Chronological – If there is no required rule for ordering information elements a technique like alphabetical or chronological listing should be used. Selection time decreases for GUI designs with alphabetical listings compared to any other information ordering.

When developing graphical user interfaces for interactive statistical analysis an iterative optimization process with the potential user or people involved in the process is indispensable in order to optimize the interface. Examples of existing interactive statistical graphics software show that the complex tasks of manipulation interfaces can not be performed with trivial controls [16]. It can not be expected that an optimal

interface will cope with all tasks. Most interfaces are optimized for a specific task, and that is the case with the tool presented in this project. The main focus is to develop a response surface analysis tool in a 3-dimensional view that facilitates visualization of experiment treatments behavior over a specific region.

This chapter presented a detailed look to what DOE designs unlimited capabilities are (Section 2.1). It also highlights current commercially available statistical software offerings, specifically for DOE contours analysis. Actual statistical software lack of 3-dimensional plotting capabilities for analyzing  designs that contain qualitative and quantitative factors in a single experiment is available. What has been identified as a drawback on commercially available statistical software has indeed been foreseen as an area of opportunity for the development of a graphical user interface application. The purpose of this application is to provide analysts with a tool for exploring experiment contour areas in a 3-dimensional view. In addition, it is this project objective for this application to be able to analyze designs consisting of quantitative and qualitative levels. For such accomplishment, DOE theoretical knowledge will be combined with appropriate application design techniques discussed in Section 2.3

# 3 METHODOLOGY AND APPLICATION CONCEPTUAL

## 3.1 Methodology

As mentioned in the previous chapter DOE theoretical knowledge was combined with GUI's design theory in order to develop a Matlab 13® application that will ease experiments contour plots analyses. The methodology followed for this initiative is presented in Figure 12.

The first step toward the project completion once the necessity for a DOE contour plot analysis tool was identified, was to define project objectives and scope. Full Factorial's, Box-Behnken's and Central Composite's experiments contour plots analyses were considered within project scope. The limitations and default settings assumed for tool development purposes will be discussed later throughout the project content. Literature review was performed on key DOE theoretical concepts including, but not limited to, Full Factorial Experiments, Box-Behnken Experiments, Central Composite Designs, Residuals Analysis, and Model Fitting. An evaluation was conducted to explore current applications capabilities and areas of improvement of commercially available software. Literature Review was completed on efficient graphical user interface applications development. Most relevant findings for each one of these topics were outlined in Chapter 2.
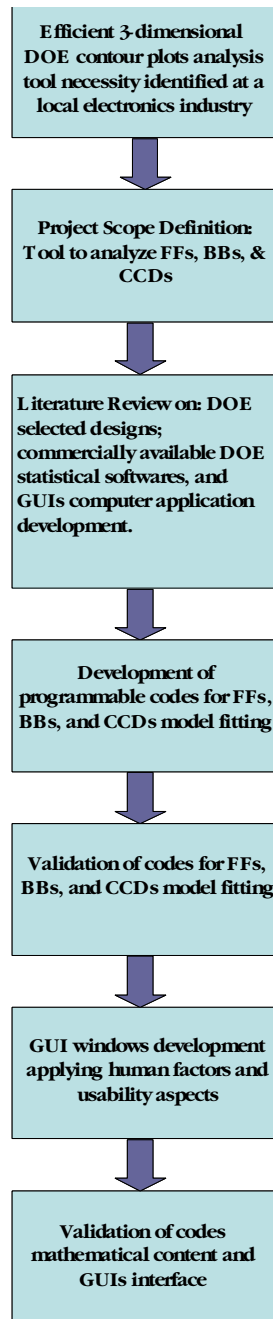
**Figure 12: 3-D DOE Contour Plot Design Project Methodology**

Model fitting and 3-dimensional contour plots codes were developed for each one of the selected experimental designs. Codes were validated against Minitab 14® statistical software and textbook examples. A validation run consisted of comparing the

model coefficients, residuals or contour plots resulting from the 3-D application against those from Minitab14®. A graphical user interface was developed that contains buttons, editable boxes, axes and lists. These GUI windows provide the user with an easy to use DOE tool. GUI windows will be discussed in detail later in the project.

Last step consisted in validating the DOE program codes developed and GUIs as a whole. This validation ensured correspondence between experiment description provided by the user at the GUIs windows and programs executable paths and results.

## 3.2    Application Concept

The methodology described above facilitated the project completion. Once literature reviews and evaluations were performed, next phase was creating a code program for a graphical user interface application for analyzing DOE contour plots in a 3-dimensional perspective. The resulting tool has the flexibility to handle large amounts of data from the most simple to a significantly complex combination of factors, as specified on Table A1-1 in Appendix 1, and to project data in an easy and useful 3-dimensional way. Other functionalities of the graphical interface are basic statistical computations such as residual plots. There are four possible residual plots including histogram, normal probability plots, residuals versus fitted values and residuals versus time sequence. These capabilities are complementary to the main purpose of this tool which is the capability of full rotation of design blocks created by the contour levels of factor combinations. It is possible to visualize multiple factors at multiple levels no matter the nature of factor type, quantitative or qualitative levels.

An example of regression analysis of a $2^3$ factorial design [2] was executed with the simplest code to illustrate the basic concept of the 3-D graphical tool functionality. All the three factors are quantitative factors. Figure 13, shows the six 2-dimensional planes plotted in Matlab 13®. This figure is just a reproduction of market available statistical analysis software capabilities.
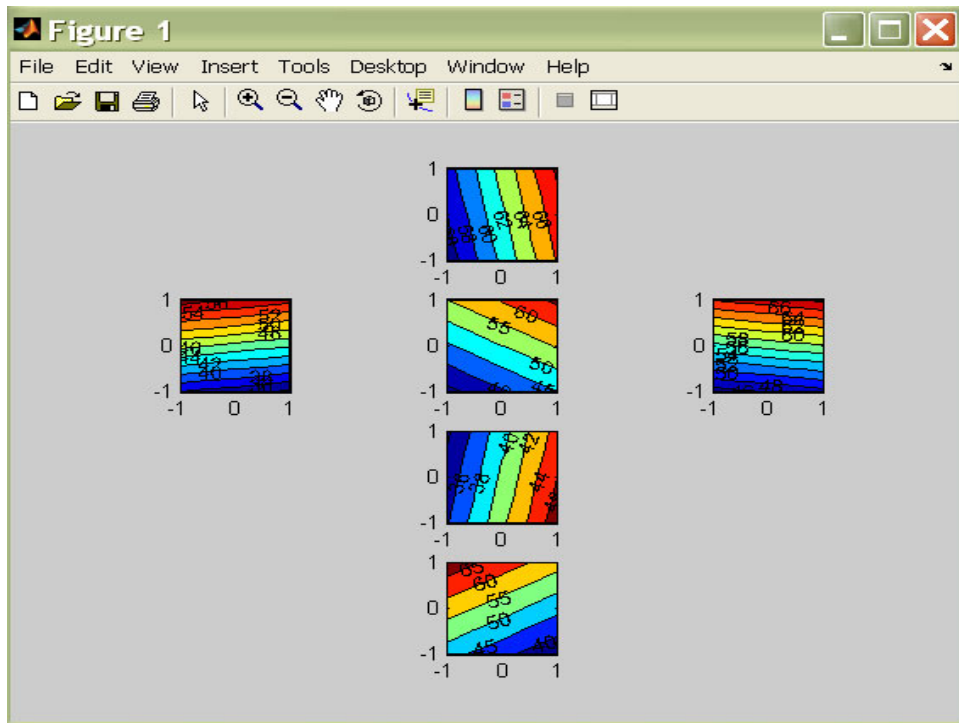


**Figure 13: Six 2-dimensional contour plots**

This project's real contribution consists of contour surfaces for factor levels that instead of being plotted on six 2-dimensional planes are plotted on six faces of a 3-dimensional contour cube as shown in Figure 14. The way this tool is developed allows the user to fully rotate the contour plot cube in all directions for a better visualization of response behavior within the experimental region. Specifically, Matlab 13®'s capability will be exploited in goodwill of design of experiment needs.
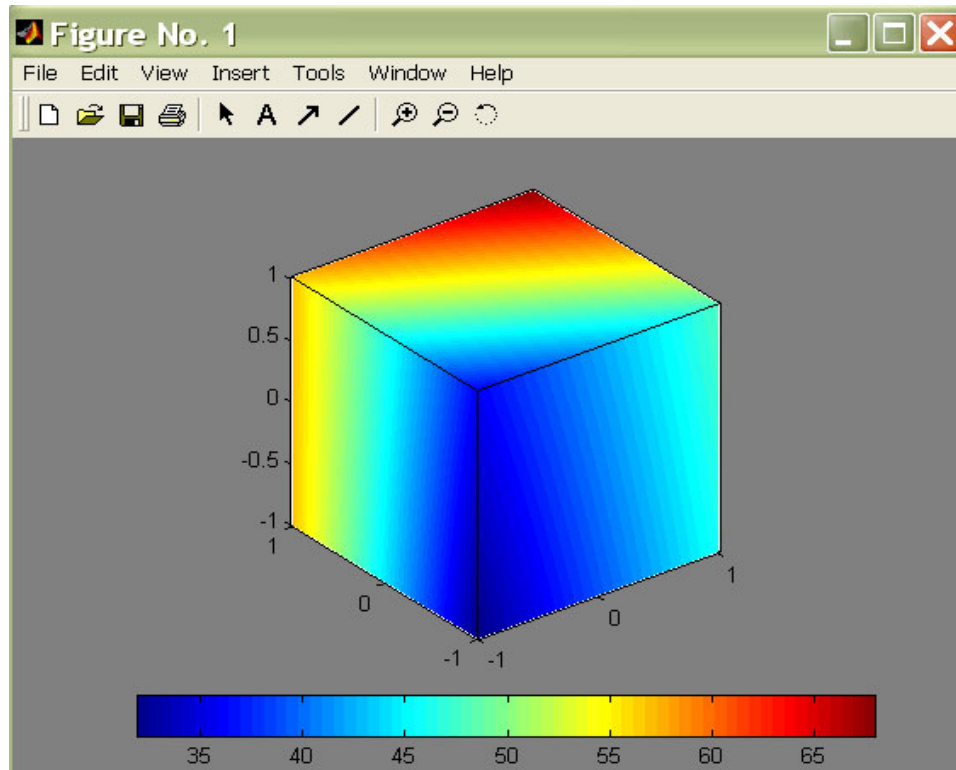
**Figure 14: 3-D Contour Plot example**

## 3.3 Experiment Description Window

For any of the three available experiment selections, Full Factorial, Box-Behnken, or CCD the 3-D DOE Contour Plot application inquires the user to enter experiment parameters. These parameters will permit the program to develop the contour plot cubes. The 3-D DOE Contour Plot experiment description windows for each of these designs will be presented in Chapter 5: Computer Application (Refer to Figure 21, 22, and 23, respectively). Depending of the experiment parameters the user has the opportunity to select the display residuals plots and residual values in addition to the display of betas or model coefficients.

Input parameters may include: number of quantitative factors, number of qualitative levels, number of replicates, number of central points, and response. The maximum number of factors the tool is able to analyze is directly dependant of the combination of quantitative factors and qualitative levels being considered in the experiment. Editable input fields are discussed in detail in Chapter 5.

This section will address a brief description of the analysis algorithm the application follows to provide the 3-dimensional displays and a detail description of contour plot cubes figures output. Depending on the number of quantitative factors entered is the cube layout to be displayed. The possible quantitative factors/qualitative level combinations the tool is capable of analyzing are listed in Table A1- 1 in Appendix 1.

## 3.4 Algorithm Description

Independently of the type of experiment selected a general algorithm that describes application analysis could be described as follows:

1. Application is initialized from the Matlab 13® command window typing "mainfigure".

2. An Experiment Selection window is prompted. Selection of type of experiment consists of mutually exclusive options: Full Factorial, Box-Behnken or CCD.

3. An Experiment Description window is opened corresponding to the option selected on the previous figure.

a.  User provides information in editable fields located at the left hand section of the window. Input parameters are described in detail in Chapter 5.

b.  After populating input parameters the "Analyze Design" button executes the analysis.

4.  The "Analyze Design" initiates certain evaluations:

a.  Depending on inputs the application calls the corresponding design matrix.

b.  Evaluates responses and calculates estimated coefficients for the fitted model) through linear regression. Estimated coefficients could be displayed selecting "Coefficients" button.

c.  Fitted responses are compared to observed responses and residuals are calculated. Calculated residuals can be displayed selecting "Residuals" button.

5.  Graphical outputs are generated:

a.  3-dimensional contour cube plots are displayed automatically after pressing the "Analyze Design" button in a new figure.

b.  There are four possible residual plots activated with the Residual Plots buttons located in the middle section of the experiment description windows.

## 3.5    3-D DOE Contour Plot Cubes Display

This section addressed a detail description of contour plot cubes figures output. Depending on the number of quantitative factors entered is the cube layout to be displayed. As stated earlier possible quantitative factors/qualitative level combinations the tool is capable to analyze are listed in Table A1- 1 in Appendix 1.

### 3.5.1    Experiments with No Qualitative Levels

If a combination of zero ("0") qualitative levels and three ("3") quantitative factors are selected, then a Full Factorial experiment, a Box-Behnken experiment or a CCD experiment with three independent quantitative factors could be being described. As a result contour plot graph will show a single cube for factors response changing from low level to high level for each of the three quantitative factors specified (labeled at the display as A, B, and C).

The value that will be displayed at the extremes of the cube and that represent the low level and high level of each factor will depend on the type of experiment selected. For Full Factorial it ranges from -1 to +1, for Box-Behnken it ranges from -1 to +1 with points plotted at the middle of the range (0), and for CCD it ranges from $-\alpha$ to $+ \alpha$. Figure 15 shows an example of this type of display.
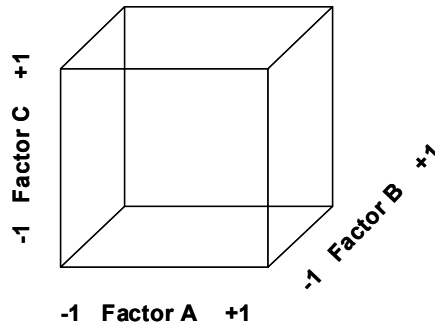
**Figure 15: 3-D Contour Plot for Three Quantitative Factors**

On the other hand, if input parameters are as follows: zero ("0") qualitative levels and four ("4") quantitative factors, the resulting new figure layout will show two contour plot cubes. The cube more to the left will represent contours for three of the factors labeled A, B, and C varying along the range of low level and high level with a fourth factor, labeled as D, hold at its low level as represented in Figure 16. The cube at the right will represent contours for three of the factors labeled A, B, and C varying along the range of low (-1) and high (+1) level with the fourth factor, D, hold at its high level (+1).
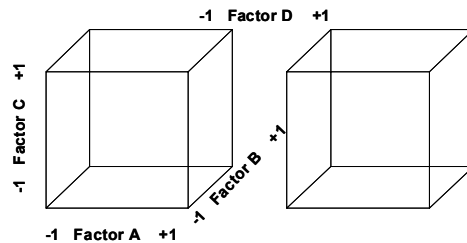


**Figure 16: 3-D Contour Plot for Four Quantitative Factors**

For a combination representing a $2^5$ full factorial, zero ("0") qualitative levels and five ("5") quantitative factors the graphical output results in four contour plot cubes. Similarly, contour plots display for a full factorial experiment with no qualitative levels and six ("6") quantitative factors will consist of two figures with a layout of four cubes per figure. In other words, two figure windows will open and each one will look similar

to Figure 17. Incorporating a sixth factor, F, implies having the factor hold at it low level on the first window and hold at it high level on the second window to open.
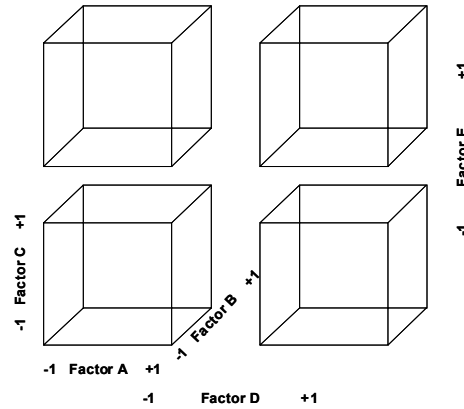


**Figure 17: 3-D Contour Plot for Five Quantitative Factors**

Analyzing more than three quantitative factors brings a particular implication. The 3-dimensional capability provided with any of the three experiment selections presented in this tool (Full Factorial, Box-Behnken, and CCD) is that when more than three quantitative factors are analyzed remaining factors above three will be displayed fixed at their low and high levels individually. It is not possible to vary four factors at the same time within the low to high level that will imply a fourth dimension (4–D). The user have the alternative to rearrange data so any factor of interest that initially was displayed fixed on one of the extremes become a factor in one of the three axes that changes through the complete experiment range. These varying axes are the ones labeled at the cubes as factors A, B, and C. A detailed example of how to handle this snag is discussed in subsequent chapters.

### 3.5.2 Experiments with Quantitative factors and Qualitative Levels

The 3-D DOE Contour Plots tool is designed to handle design of experiments that contains limited amounts of both, quantitative factors and qualitative levels. Plot displays are very similar to contour plots with no qualitative levels displays described above. When an experiment with qualitative levels is specified, analysis will consider an individual Full Factorial, Box-Behnken, or CCD at each of these qualitative levels, depending which experiment is being analyzed.

For example, for an experiment described by: quantitative factors = 3 and qualitative levels = 3, as shown in Figure 18, the tool will consider it as three different experiments, one individual experiment for each qualitative level. For this specific experiment description the 3-D Contour Plot will display a figure with three contour cubes layout at the graphical output.
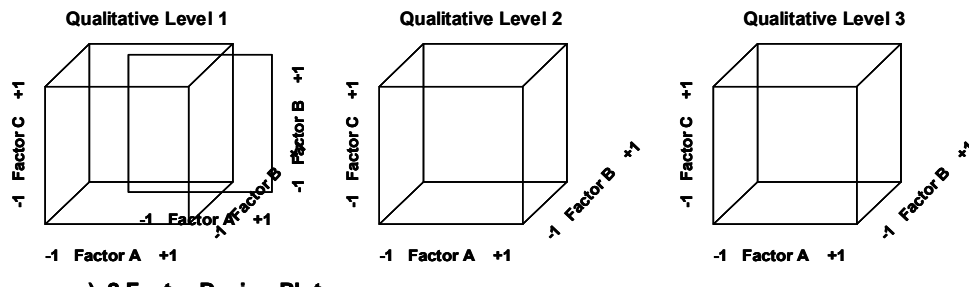


**Figure 18: 3-D Contour Plot for Three Quantitative Factors at Three Qualitative Levels**

Another example is an experiment with quantitative factors = 5 and qualitative levels = 2. The graphical output will consist of two figures. The first figure that opens will display the five quantitative factors for the qualitative level 1. The second figure will display the five quantitative factors for the qualitative level 2.

# 4  COMPUTER APPLICATION

The main objective of this project is the development of a visual user interface tool with the capability of displaying 3-dimensional contour plot cubes for different methods of design of experiments. This chapter describes a general user guide for the tool developed for Full Factorials, Box Behnken, and CCD designs.

Figure 19 shows user process flow for the 3-D DOE Contour Plot. Selection window and Description Windows mentioned in the process flow will be detailed in the following sections.
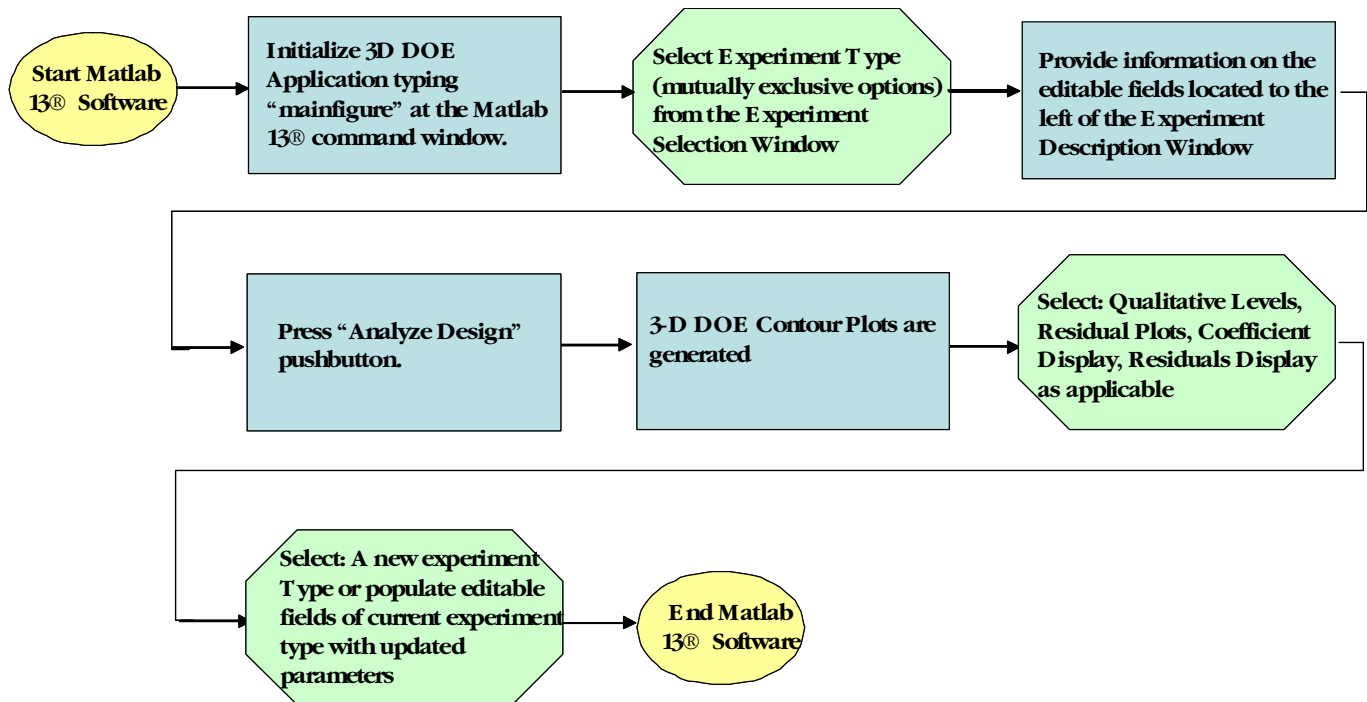
**Figure 19: 3-D DOE Contour Plot Application Use Process Flow**

These three experiment methods are available from main figure screen shown in Figure 20.
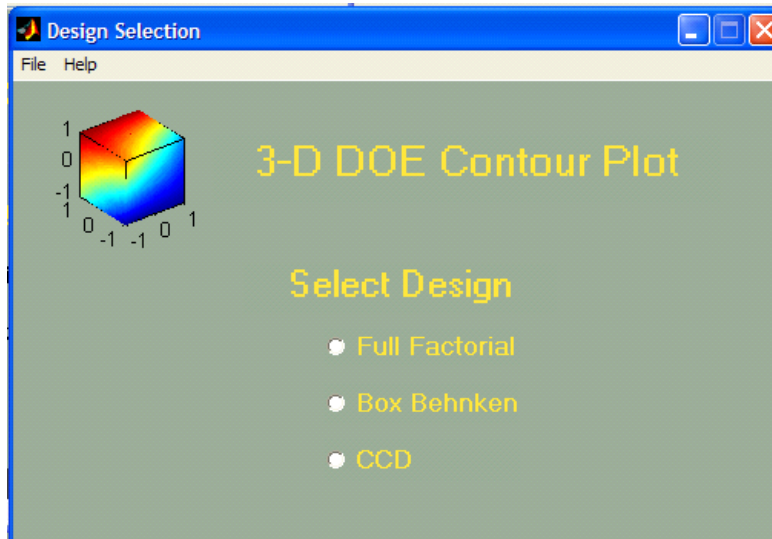


**Figure 20: 3-D DOE Contour Plot Application Main Window**

Chapter 2 presented GUI design elements to take into consideration when developing interactive applications. Some of these elements were incorporated to the 3-D DOE Contour Plot interface design. All the application windows presented in the next pages make use of familiar formats: left-to-right and top-to-bottom screen reading layout. Data is arranged in tabular format and labeled with the intention of maximizing efficiency when using the application. Color is used to enhance information without overusing color coding: labels and stationary text is highlighted in yellow, editable text boxes are grey, ad pushbutton (action buttons) are blue. Graphical boundaries group information at the description windows presented in Figures 21, 22, and 23. The boxes at the descriptive windows indicate the user the boundaries between the parameters input, output selection, and graphical field.

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## 4.1    Full Factorial Experiments: Application User Guide

This help guide section provides the user with detailed correct input data format for the application to execute appropriately. Figure 21 shows a screenshot of the Full Factorial Experiment Description Window.



**Figure 21: Full Factorial Experiment Description Window**

The left hand section of the window holds the editable textboxes where the user defines the Full Factorial experiment to be analyzed.

- ***Number of Quantitative Factors*** – Number of continuous independent factors to be evaluated. . For an experiment design that does not contain any qualitative level, a single $2^k$ full factorial design, and the minimum number of quantitative

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factors is 3 and the maximum number of quantitative factors is 6. The minimum number of factors is limited by the nature of a 3-dimensional approach; at least 3 factors are needed to form a cube. The maximum number of factors was established having in consideration:

- o the necessity for an easy to handle tool – it gets complicated handling more than two or three windows simultaneously to compare graphs

- o the graphical capabilities of system where the tool is to be operated – as the number of factors increase better graphical system are required

- o the fact that commonly full factorial experiments do not contain more than five or six factors – as the number of factors increase the number of runs increase, too. Associated experiment costs, project length, and resources needed are impacted as well.

- *Number of Qualitative Levels* – Number of qualitative levels to be evaluated. As for the number of quantitative factors, the qualitative levels could vary from 3 up to 6 depending on how many quantitative factors are considered.

Table A1- 1 attached to Appendix 1 presents experimental number of quantitative factors and qualitative levels the 3-D DOE Contour Plot application is capable to analyze.

- *Number of Replicates* – The user may specify any integer positive number at this editable field. For a single replicate design the user should enter 1 at the editable text box.

- *Number of Center Points* – For a design with no center points the user should enter 0 at the editable text box.

- *Response (Y)* – This editable text box allows the user to specify a .txt file were responses have been saved. User will have to save response values beforehand in a separate notepad file (.txt extension) contained in the same directory the application executable files are. It is very important to discuss data order at this .txt file. Saving a response data file in a format different from the one discussed here will not allow the application execute the analysis.

Data response values should be entered at the notepad file as follows:

- Data values corresponding to a particular response should be stored in a row (no "Enter" spaces). Refer to Figure A2- 1 in Appendix 2 for an example of a $2^3$ Full Factorial.

- Data values corresponding to multiple responses (i.e. multiple qualitative levels) should be stored as one row per response. Two "Enter" spaces should be allowed after the last value of each row. See Figure A2- 2 in Appendix 2 for an example of a $2^3$ Full Factorial with three (3) qualitative levels.

- Data values corresponding to multiple replicates should be stored in a single row containing values for all replicates. See Figure A2- 3 in Appendix 2 for an example of a $2^3$ Full Factorial with 2 replicates.

How to store response vector file has been described above. Moreover, data should comply with a specific order for the application to analyze the design of experiment. Data order is determined by the design matrix (X). Appendix 3 summarizes design matrices that show the user correct response vector sequence.

For a $2^3$ example the X matrix (for A, B, and C) and the response vector (Y) looks as follows:

**Table 2: $2^3$ Full Factorial Experiment Matrix**

| A | B | C | Y |
|---|---|---|---|
| - 1 | - 1 | - 1 | 51 |
| + 1 | - 1 | - 1 | 63 |
| - 1 | + 1 | - 1 | 89 |
| + 1 | + 1 | - 1 | 76 |
| - 1 | - 1 | + 1 | 81 |
| + 1 | - 1 | + 1 | 61 |
| - 1 | + 1 | + 1 | 71 |
| + 1 | + 1 | + 1 | 103 |

- *"Analyze Design" pushbutton* – Once the user entered all required fields as described above, the Analyze Design button activation calculates the appropriate contour plot cubes. The 3-D DOE Contour Plots appear in a new figure. The user will always have the opportunity to close/minimize the new figure(s) to return to the experiment description window.

The middle section of the Full Factorial Experiment Description window provides the user with GUIs that execute four different plots as long as the design contains replicates or center points. If any of these two assumptions is not met, a note will prompt below plotting axes. Also, there are two more buttons at the middle bottom section of the window that allows the user to request the estimated model coefficients

and the calculated residuals display. By pressing "Coefficients" and "Residuals" buttons automatically create excel files that contain the values displayed in the list boxes. A file named "betas.xls" is created to store the coefficients. When more than one coefficient column is generated by the experiment analysis user needs to select "Data > Text Column > Delimited > Space" at the excel menu bar. This will create as many coefficients columns as displayed on the GUI window. A file named "residuals.xls" is created to store experiment residuals. For an experiment with multiple qualitative levels the file stores residuals values for the qualitative level selected and its content refreshes each time "Residuals" button is pressed. The purpose of these files is to made the experiment results available for further analysis or display.

## 4.2 Box-Behnken Experiments – Application User Guide

Box-Behnken Experiment Description Window behaves very similar to Full Factorial Experiment Description Window. This help guide section provides the user with the information for appropriately perform Box-Behnken designs analysis using the 3-D DOE Contour Plot application. Figure 22 shows a screenshot of the Box-Behnken Experiment Description Window.
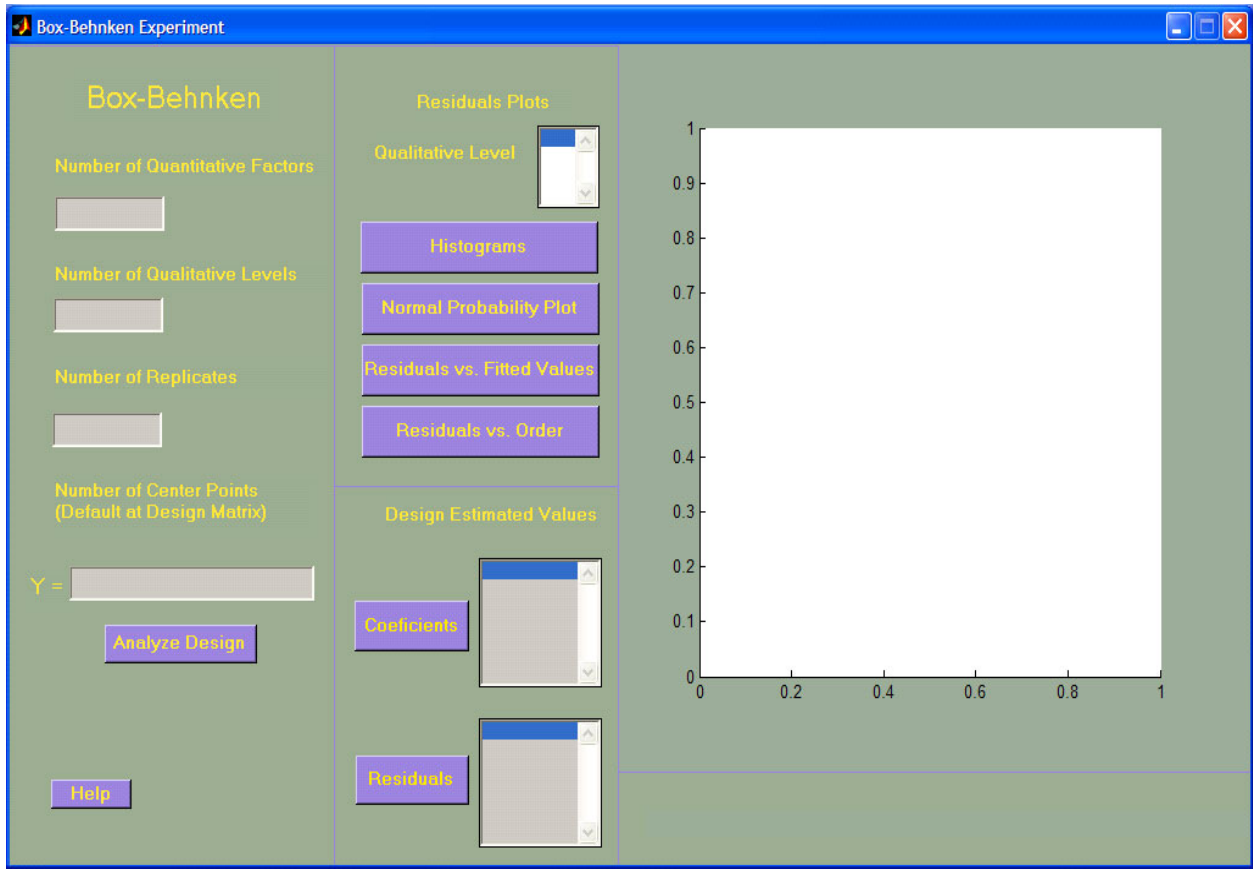
**Figure 22: Box-Behnken Experiment Description Window**

The left hand section of the window practically remains unchanged from what was previously presented for Full Factorial Experiments. It does hold the editable textboxes where the user defines the experiment to be analyzed. Although, there is a difference for the Box-Behnken Center Points input parameter. There is no input field for center points at the Box-Behnken Experiment Description window. Default number of center points are defined at the design matrices, X. These matrices are presented in Appendix 4 and establish the order data should be stored at the .txt file the user should enter at "Y", the response field.

## 4.3 CCD Experiments – Application User Guide

The CCD Experiment Description window (Figure 23) has all the main input parameters presented at the Box-Behnken Help above:
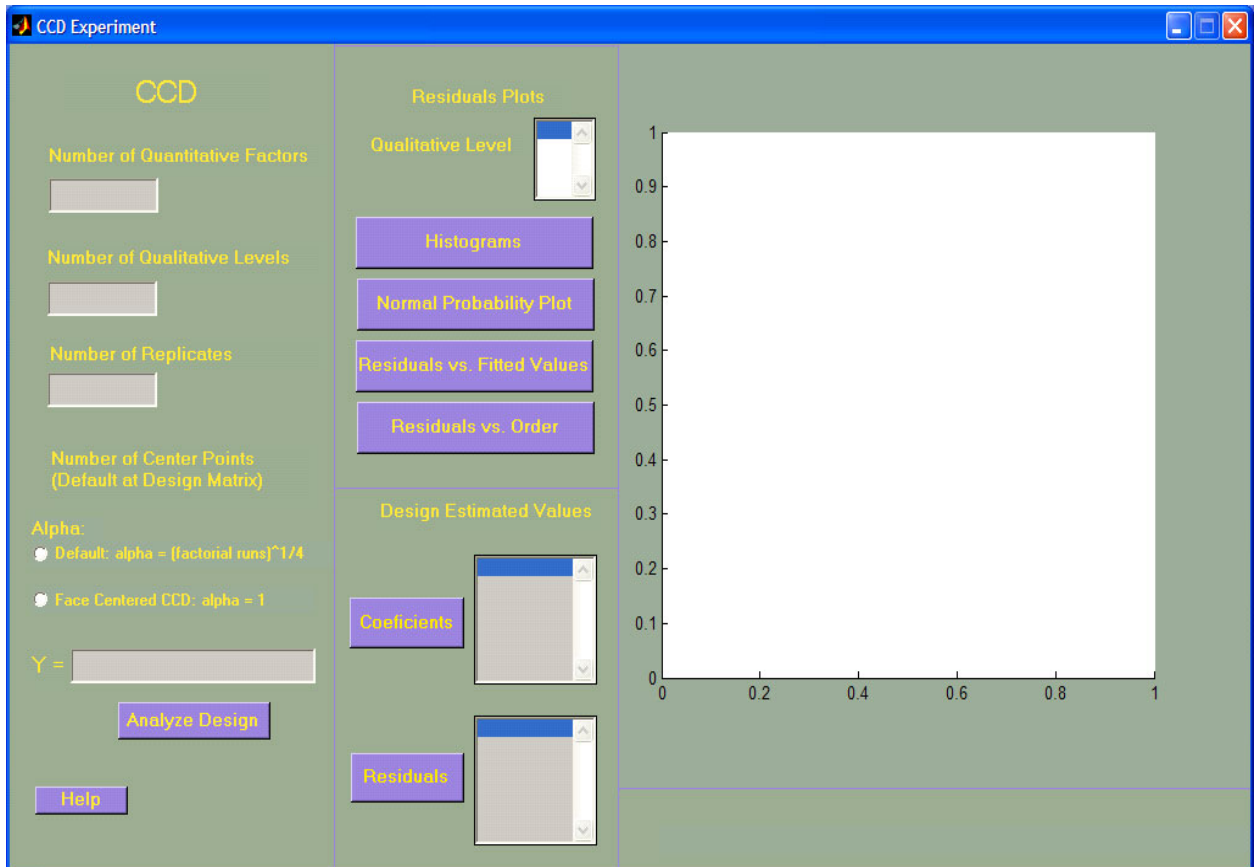


**Figure 23: CCD Experiment Description Window**

In addition, this window contains two additional options. These options are associated to the alpha or location of axial points in the experiment. The user may choose between: a default alpha value or a face centered design.

- Default: *alpha = (number of factorial runs)* $^{1/4}$

- Face centered CCD: *alpha = 1*

Design matrices, Xs, presented in Appendix 5 are shown with default alpha values. If the Face Centered option is selected the application will automatically place an alpha value of +1 or -1, respectively. The user only needs to select desired CCD axial points at the input window.

CCD window also provides the user icons for selecting desired residual plot, displaying estimated model coefficients and displaying a list of the experiment residuals. These last three pending the experiment contains replicates or center points.

**4.4    Contour Plot Cubes Figure – Application User Guide**

Independently of the experiment design selection, Full Factorial, Box-Behnken, or CCD, the graphical output window where contour cubes are display has the same functionalities. Previous sections provided a detailed explanation of the window layout. Furthermore, there are two additional functionalities available at the contour plot windows. These are the cube capability to rotate $360^0$ in any direction and the optional display of a color bar besides each contour cube. These two functionalities are standard at the main toolbar of the figure.

# 5 VALIDATION TESTING AND RESULTS

The purpose of this chapter is to highlight validation procedure prior case study application. All the possible scenarios were properly challenged and validated prior to project completion. Nevertheless, due to wide amount of possible scenarios some suitable examples were properly documented as part of this chapter. The following sections summarize validation testing and results for selected experiment descriptions. Different scenarios are presented at each type of experiment (Full Factorial, Box-Behnken, and CCD) and compared against results obtained using a commonly available statistical software (Minitab 14®).

## 5.1 Full Factorial Experiments

### 5.1.1 $2^3$ Full Factorial

This first exercise tests and validates a Full Factorial $2^3$ experiment with no center points and no replicates. Therefore, there are 8 experimental runs. Experiment objective was to maximize observed response within the low and high levels three continuous independent factors. To demonstrate tool adequacy the experiment was run in Minitab 14® and in the 3-D DOE Contour Plots tool developed for this project.

Contour plots in Minitab 14® for a $2^3$ Full Factorial result in six 2-dimensional plots. Each of these six plots varies two of the factors and the third one is hold at either one of the levels, low (-1) or high (+1). This section demonstrates 3-D DOE Contour Plot application capability to reproduce experiment analysis Minitab 14® performs. But, results instead of being plotted in a 2-dimensional plane are plotted in a 3-dimesional

cube. Each one of the six faces of the cube will be equivalent to one of the six 2-dimensional plots obtained with Minitab14®. This equivalency is demonstrated in detail in the next pages. Results are as follows:

- ***Minitab 14®***

**Table 3: Experimental Treatments and Response for the 50% board thickness measure**

| Factor A | Factor B | Factor C | Response (y) |
|---|---|---|---|
| -1 | -1 | -1 | 51 |
| 1 | -1 | -1 | 63 |
| -1 | 1 | -1 | 89 |
| 1 | 1 | -1 | 76 |
| -1 | -1 | 1 | 81 |
| 1 | -1 | 1 | 61 |
| -1 | 1 | 1 | 71 |
| 1 | 1 | 1 | 103 |

```
Estimated Effects and Coefficients for y (coded units)
Term      Effect    Coef
Constant            74.375
A          2.750    1.375
B         20.750   10.375
C          9.250    4.625
A*B        6.750    3.375
A*C        3.250    1.625
B*C       -4.750   -2.375
A*B*C     19.250    9.625
```
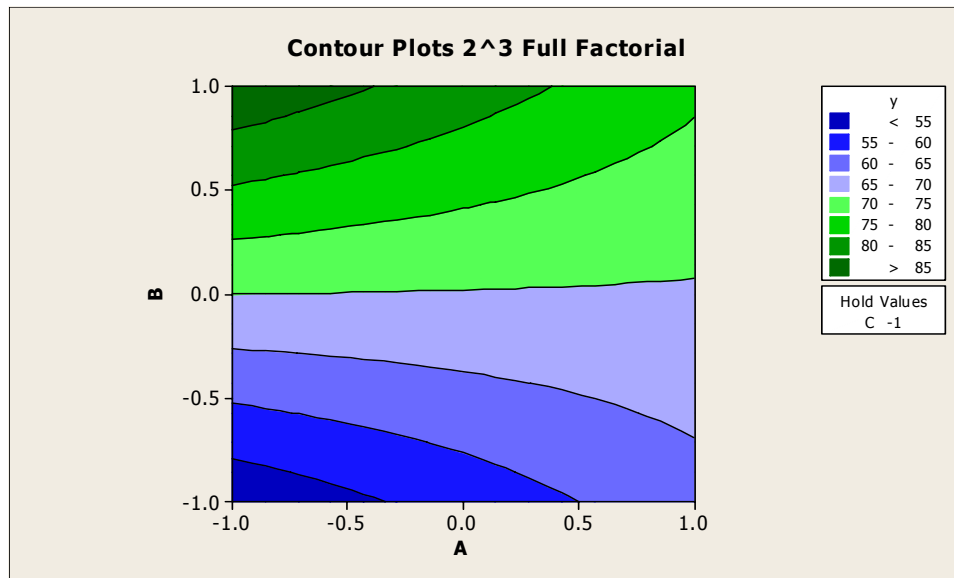
**Figure 24: $2^3$ Full Factorial (A & B varying, C = -1)**
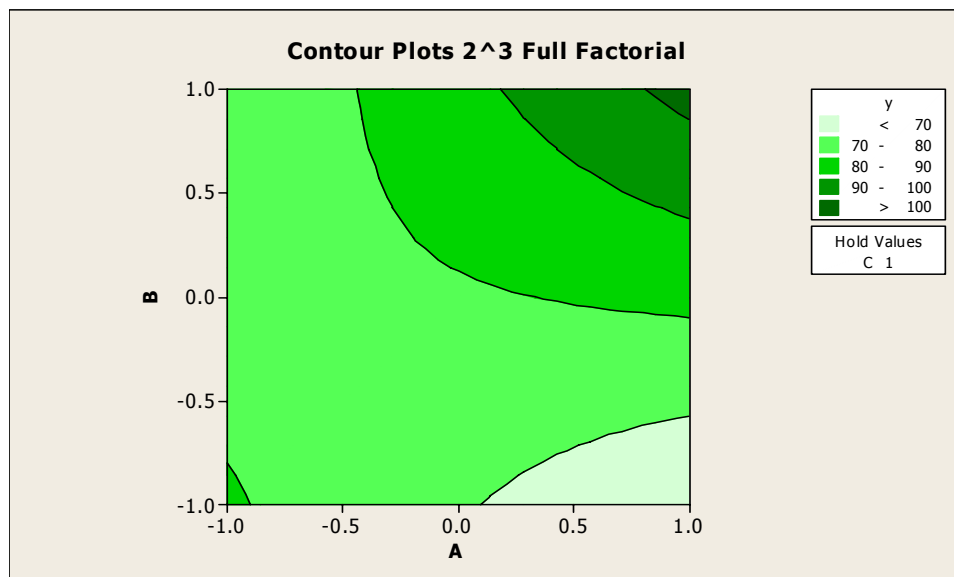


**Figure 25: $2^3$ Full Factorial (A & B varying, C = 1)**

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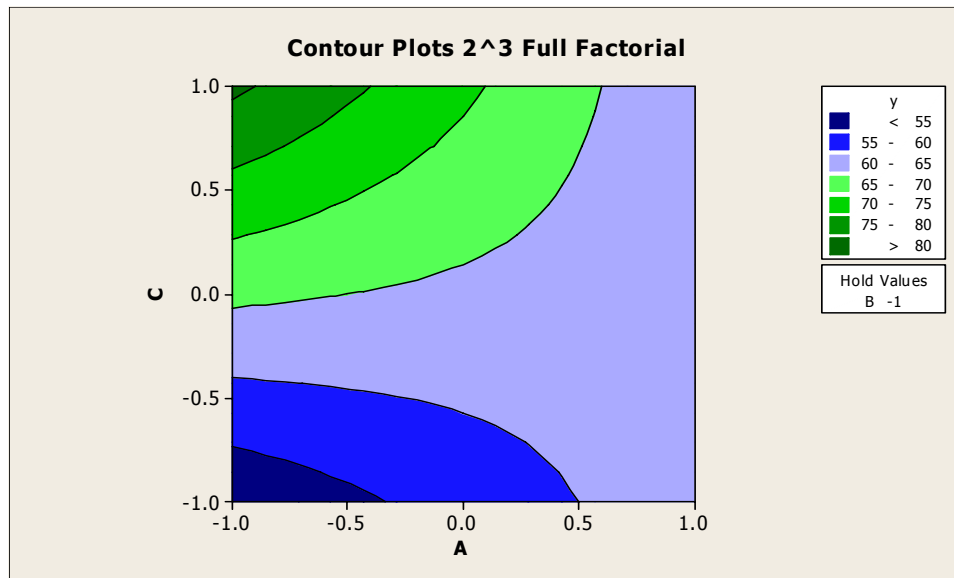
**Figure 26: $2^3$ Full Factorial (A & C varying, B = -1)**
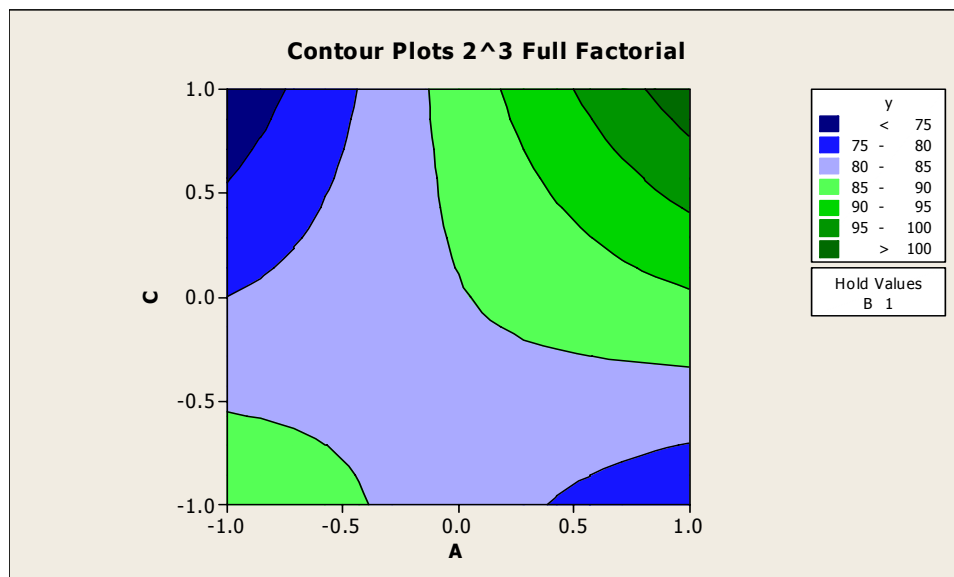


**Figure 27: $2^3$ Full Factorial (A & C varying, B = 1)**
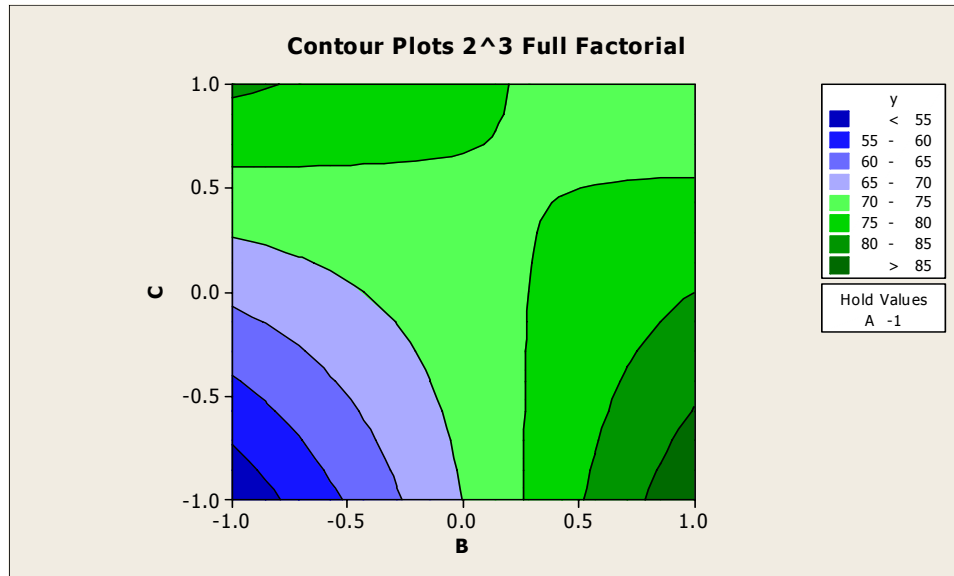
**Figure 28: $2^3$ Full Factorial (B & C varying, A = -1)**
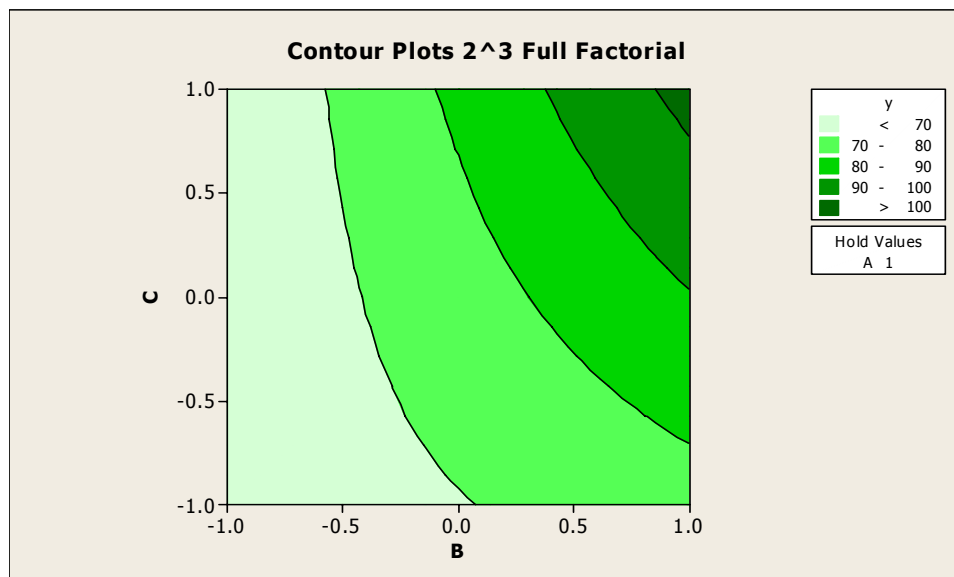


**Figure 29: $2^3$ Full Factorial (B & C varying, A = 1)**

- ***3-D Contour Plots***

Results for this experiment using the 3-D DOE Contour Plot are as follows. Figure 30 shows snapshots of beta coefficients calculated by the 3-D DOE Contour Plot

using linear regression as the Coefficients list box is scrolled down. Recall that betas coefficients are displayed as per order stated by design matrix presented in Appendix A.
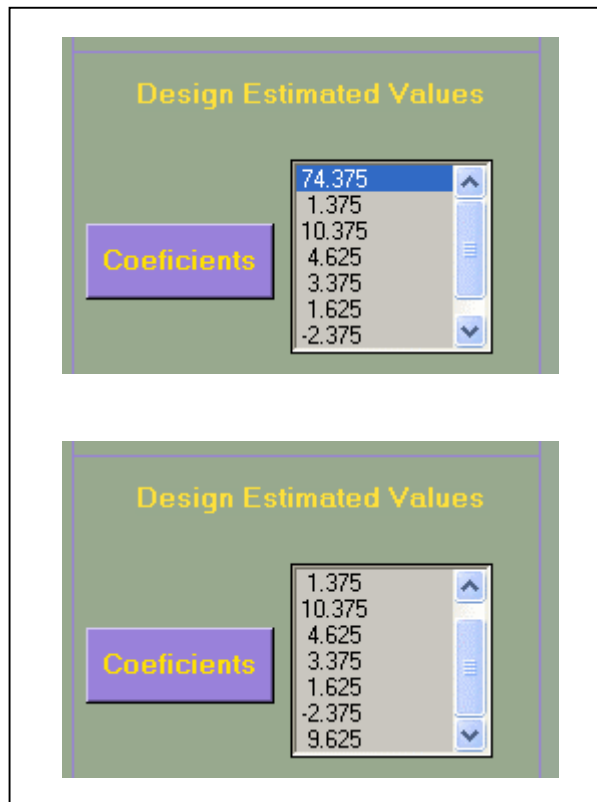


**Figure 30: Estimated Coefficients (with 3-D DOE Contour Plot)**


The 3-dimensional contour plot cube for this experiment is presented in the figure below.
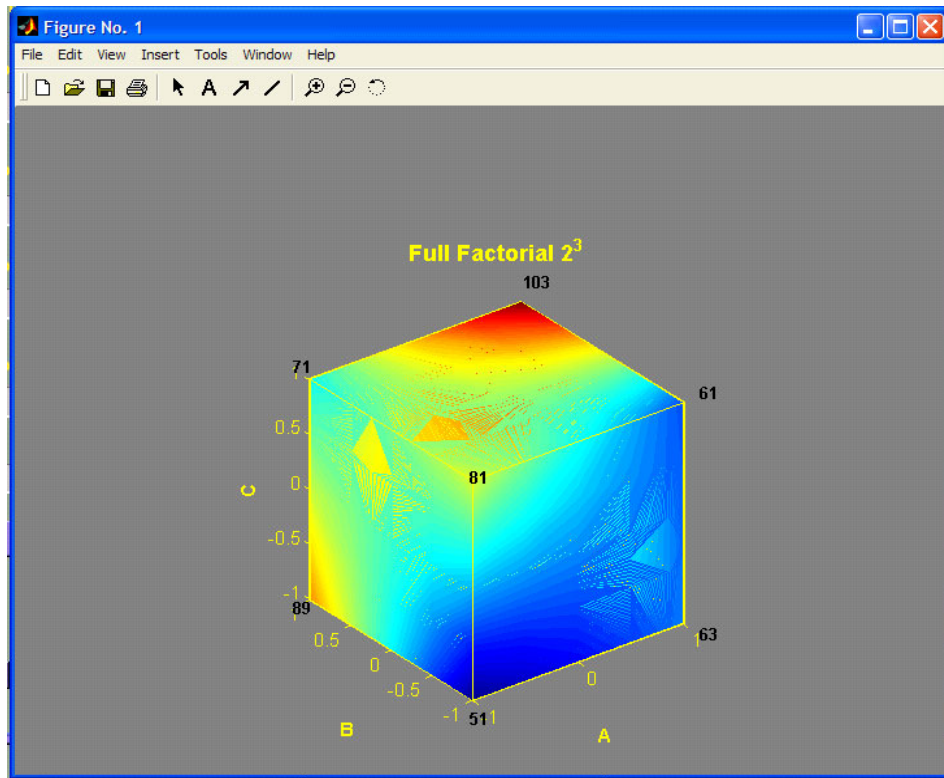
**Figure 31: $2^3$ Full Factorial Contour Plots (with 3-D Contour Plot application)**

Both applications returned the same betas or estimated coefficient values. As mentioned earlier, Minitab 14®'s 2-dimensional contour plots are equivalent to the six faces of the 3-dimensional faces produced by the 3-D DOE Contour Plot. Figure 31 demonstrate 3-D Contour Plot application capability to reproduce experiment analysis Minitab 14® performs. For example, Minitab 14® 2-dimensional contour plot for factors A and C varying within -1 and +1 and B = -1 matches with its correspondent face at the 3-D Contour Plot application. Considering each one of the corners identified on the equivalent face of the 3-dimensional cube, as illustrated in Figure 32, it is clear the corners were equally identified over the Minitab contour plot. If one goes over the color key presented at the Minitab plot, the green color shaded at Corner #1 indicates a value >80. This same corner at the 3-dimensional cube indicates a value of 81. Consistency is

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present for all the three corners, as well. Furthermore, color patterns from both images look very similar. They both show a change of color palette (resembling a funnel opening to the right) that clearly highlights change in response as factors A and C approximate the left upper corner.
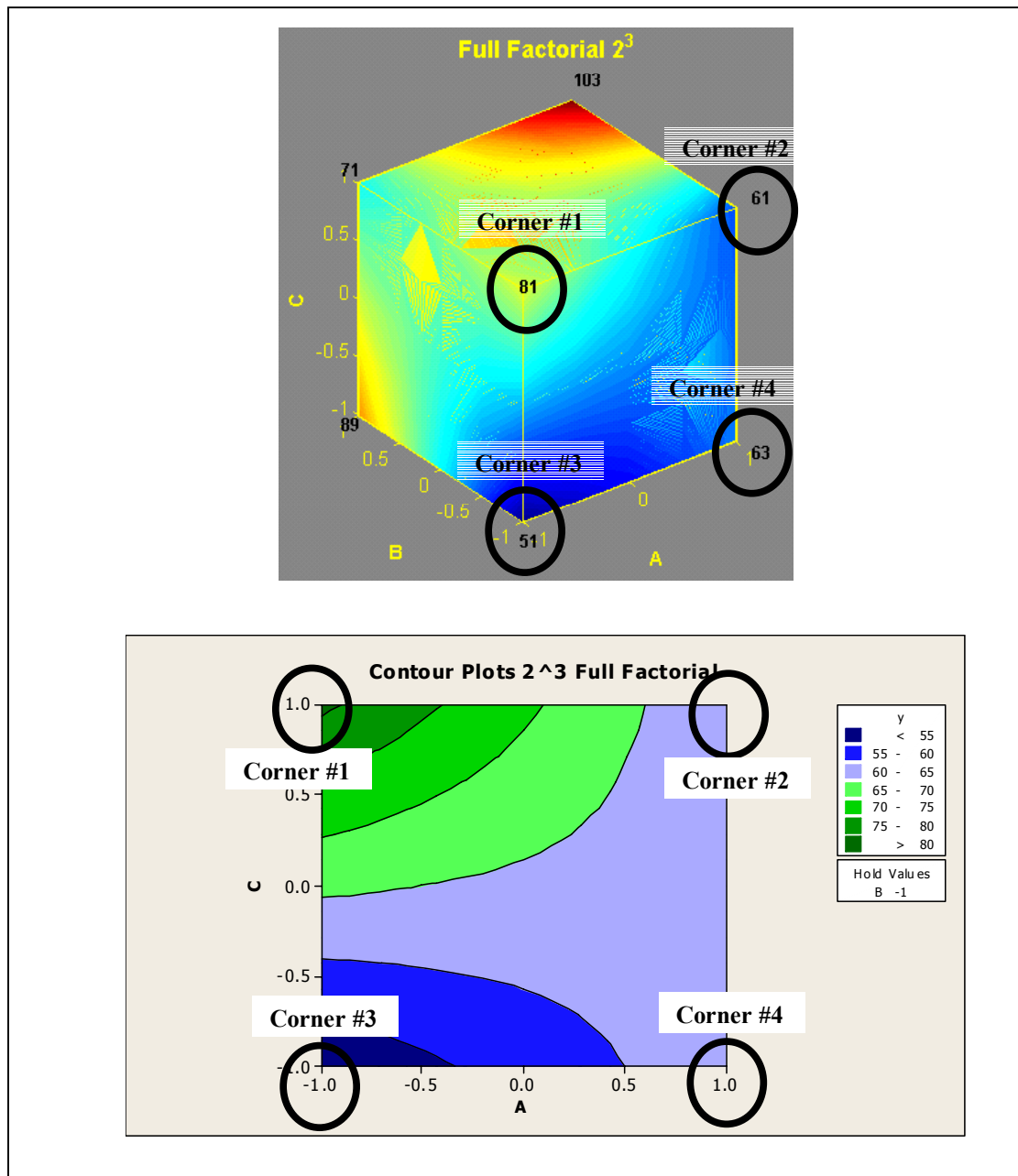


**Figure 32: $2^3$ Contour Plots comparison (Minitab 14® vs. 3-D Contour Plot Application)**

The same approach was performed for each of the six 2-dimensional plots resulting from Minitab 14® against each one of the faces of the 3-dimensional cube. One important fact to discuss is that three of the faces of the 3-dimensional cube are in fact a mirror image of the plots obtained with Minitab 14®. These three faces are: A & B (C = -1), A & C (B = +1), and B & C (A = +1). The reason for this plotting difference is because of Minitab's 14® contour plots being 2-dimensional all the six faces are presented from a front view perspective. When the faces are put together to form a cube the user is no longer capable to see through the faces that are plotted as in Minitab 14®. For example, front cube face (A & B with C =+1 verified above) does not allows seeing back cube face (A & B with C =+1) through it. The user will have to rotate the cube to see the back face, and that will yield a mirror image of what Minitab 14® presents. Next figure shows what is explained on previous lines.
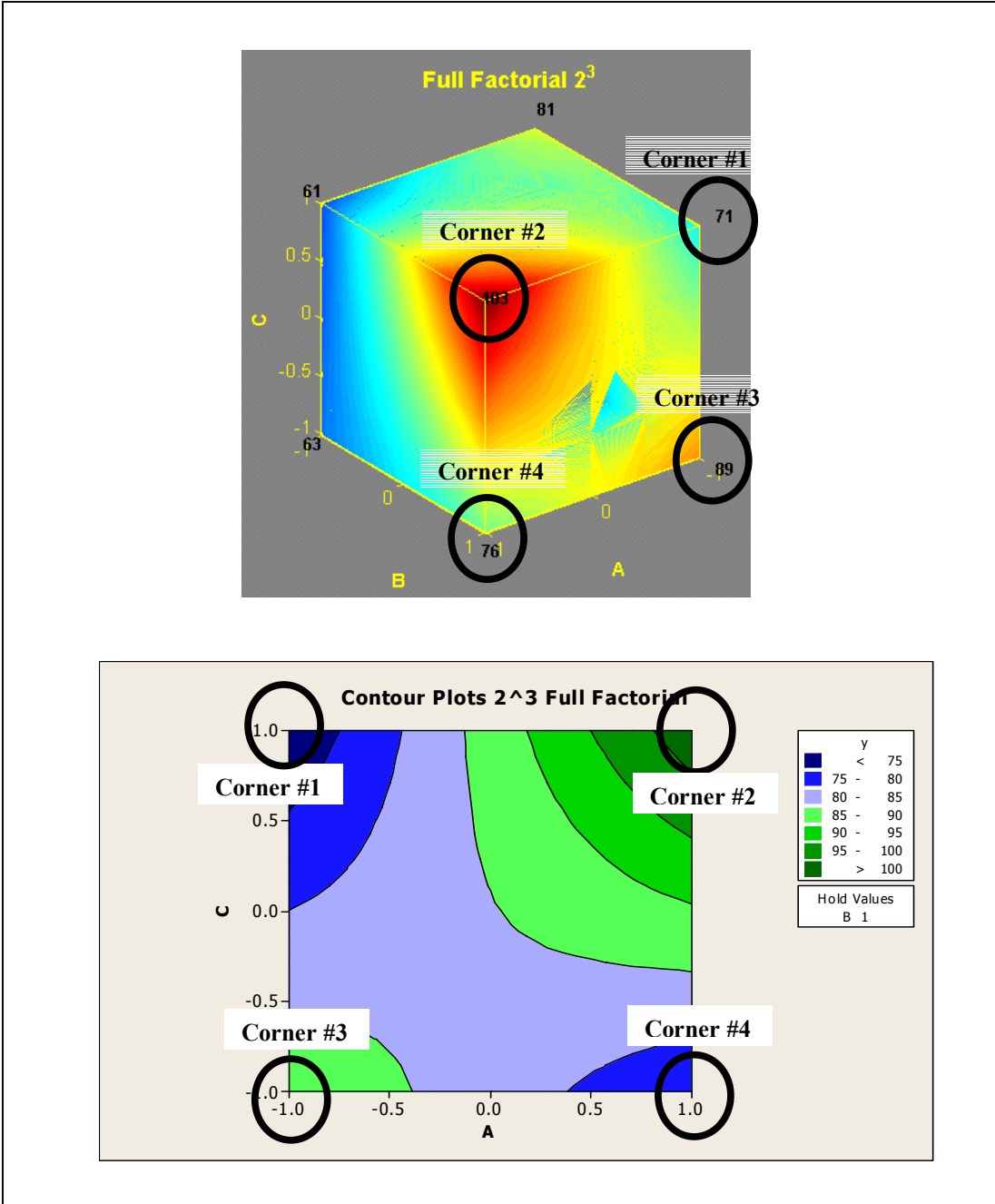
**Figure 33: $2^3$ Contour Plots – Mirror Effect (Minitab 14® vs. 3-D Contour Plot Application)**

Doubtlessly, this simple example graphically displays what the purpose of this tool is. If a decision were to be made it would be easier to visualize the corner of the cube that yields optimal responses than look at six different contours individually.

It can be concluded that the 3-D Contour Plot application is validated and yield acceptable results when compared to one of the most common available statistical analysis software (Minitab 14®).

## 5.1.2  $2^4$ Full Factorial with 2 replicates

Similarly a $2^4$ Full Factorial experiment with 2 replicates was validated comparing Minitab 14® and 3-D Contour Plot graphs, model estimated coefficients and calculated residuals. It is important to discuss this exercise because it introduces how to handle data and plots when there are more than three quantitative factors in an experiment. It is known that at a 3-dimensional perspective only three variables could be displayed varying within their minimum and maximum levels. For this experiment results are as follows:

- ***Minitab 14®***

The estimated coefficients calculated in Minitab 14® are:

```
Estimated Effects and Coefficients for y (coded units)

Term        Effect     Coef  SE Coef      T      P
Constant             67.406    1.483  45.44  0.000
A           18.688    9.344    1.483   6.30  0.000
B            2.187    1.094    1.483   0.74  0.472
C            4.438    2.219    1.483   1.50  0.154
D           15.937    7.969    1.483   5.37  0.000
A*B         -3.687   -1.844    1.483  -1.24  0.232
A*C        -11.688   -5.844    1.483  -3.94  0.001
A*D         16.313    8.156    1.483   5.50  0.000
B*C          2.063    1.031    1.483   0.70  0.497
B*D         -0.688   -0.344    1.483  -0.23  0.820
C*D          1.813    0.906    1.483   0.61  0.550
A*B*C        1.187    0.594    1.483   0.40  0.694
A*B*D        2.437    1.219    1.483   0.82  0.423
A*C*D       -1.313   -0.656    1.483  -0.44  0.664
B*C*D       -3.062   -1.531    1.483  -1.03  0.317
A*B*C*D     -0.437   -0.219    1.483  -0.15  0.885
```

This experiment objective was to minimize observed response. Contour plots are given by Figures34 and 33. By just looking at the 12 2-dimensional contour plots is not obvious what specific factor combinations accomplish the desired output. What is more, Minitab 14® contour plots lack an important feature. Minitab 14® plots change their color scheme within plots of the same experiment. Color legend for Figure 34 is different from Figure 35, so visual appreciation becomes more difficult especially if response data have a big range within the different treatments. If user decides to plot each of the combinations individually instead of six plots within the same panel there is a possibility to end with up to twelve different color legends for a single experiment.
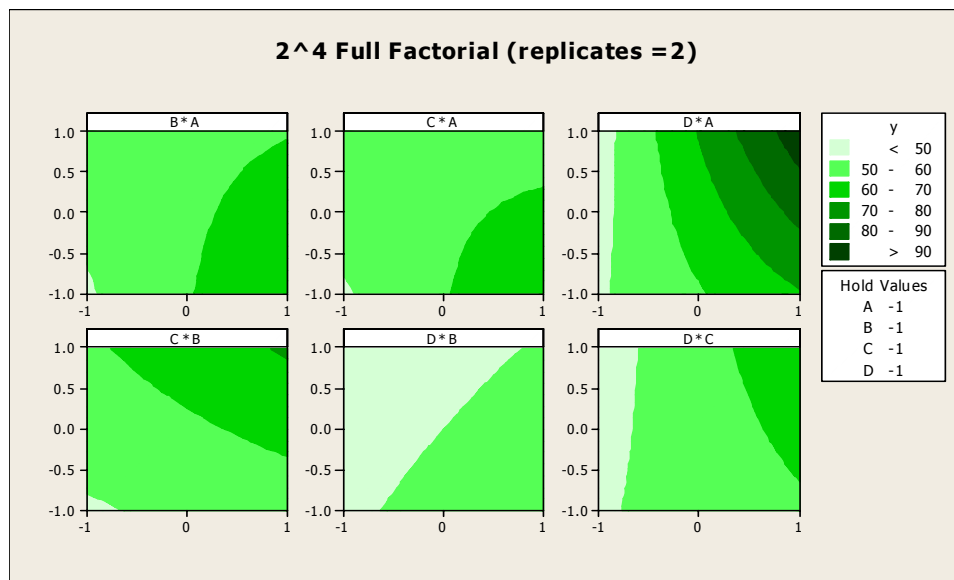


**Figure 34: 2⁴ Contour Plots – Low Settings (with Minitab 14®)**
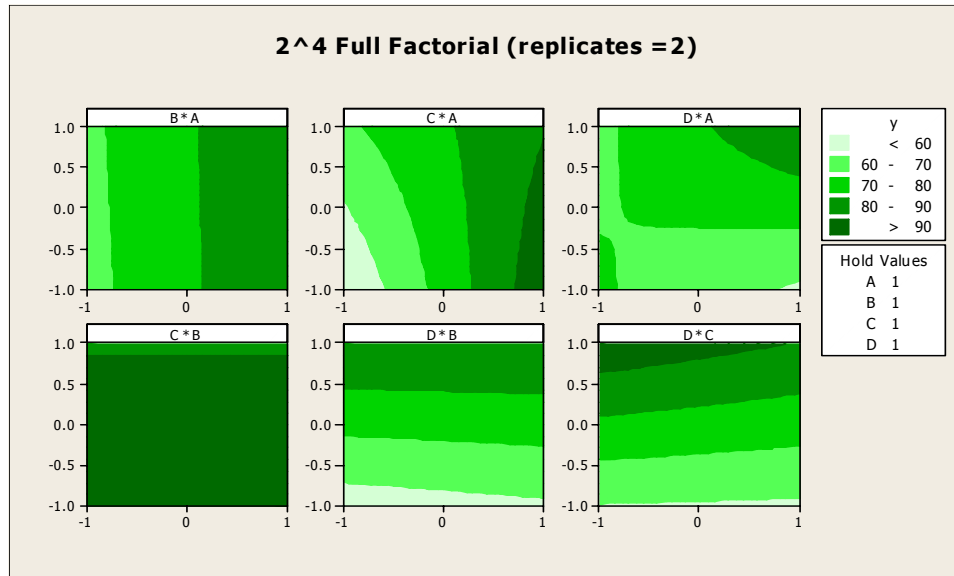
**Figure 35: $2^4$ Contour Plots – High Settings (with Minitab 14®)**

- ***3-D Contour Plot***

Results for the same experiment using the 3-dimensional application developed for this project are as follows:
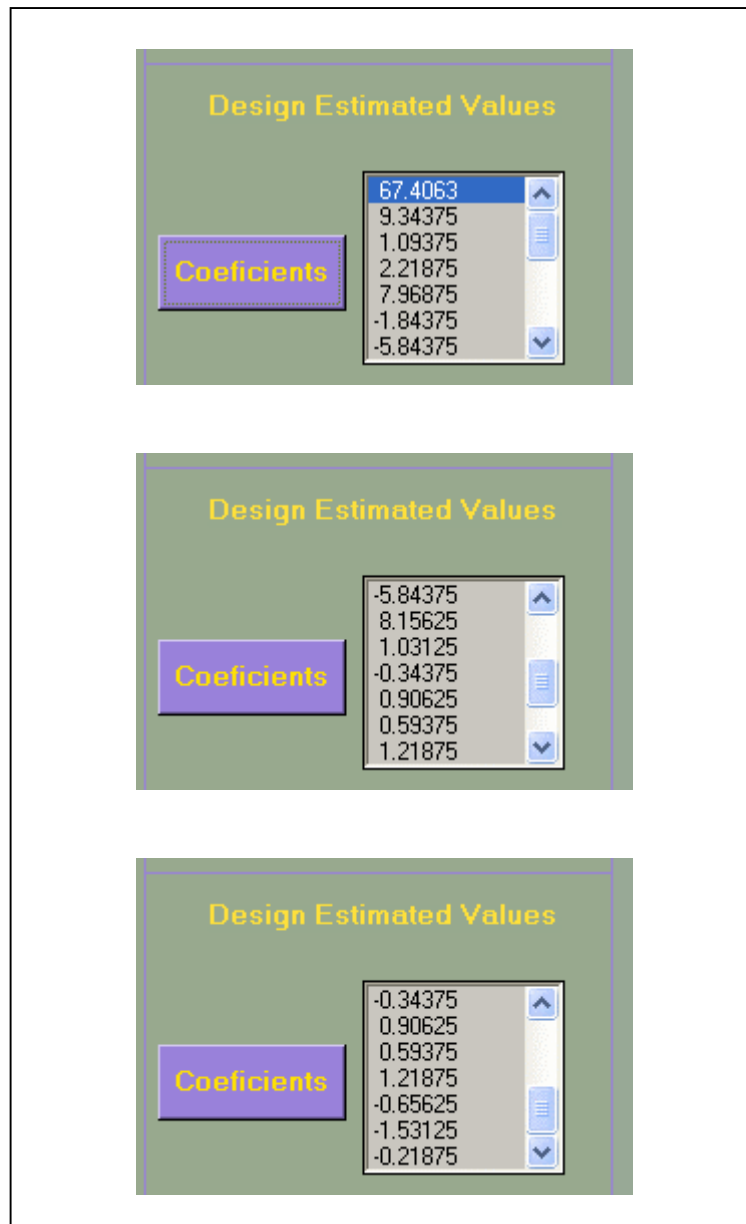
**Figure 36: $2^4$ Estimated Betas (with 3-D Contour Plot application)**

Expected coefficients showed in Figure 36 matched perfectly with expected coefficients calculated by Minitab 14®.

**Table 4: $2^4$ Residuals Comparison (Minitab 14® vs. 3-D Contour Plot application)**

| Residuals | |
|---|---|
| **Minitab** | **Matlab 13®** |
| -4 | -4 |
| 1.5 | 1.5 |
| -6.5 | -6.5 |
| 5.5 | 5.5 |
| 9.5 | 9.5 |
| 5 | 5 |
| 9 | 9 |
| 6.5 | 6.5 |
| -3 | -3 |
| 4 | 4 |
| -5.5 | -5.5 |
| 7.5 | 7.5 |
| 8 | 8 |
| -3.5 | -3.5 |
| 2 | 2 |
| 6.5 | 6.5 |
| 4 | 4 |
| -1.5 | -1.5 |
| 6.5 | 6.5 |
| -5.5 | -5.5 |
| -9.5 | -9.5 |
| -5 | -5 |
| -9 | -9 |
| -6.5 | -6.5 |
| 3 | 3 |
| -4 | -4 |
| 5.5 | 5.5 |
| -7.5 | -7.5 |
| -8 | -8 |
| 3.5 | 3.5 |
| -2 | -2 |
| -6.5 | -6.5 |

As for expected coefficients, residuals resulted identical for both applications. In addition, contour plots properly matched when studied one plot at a time.
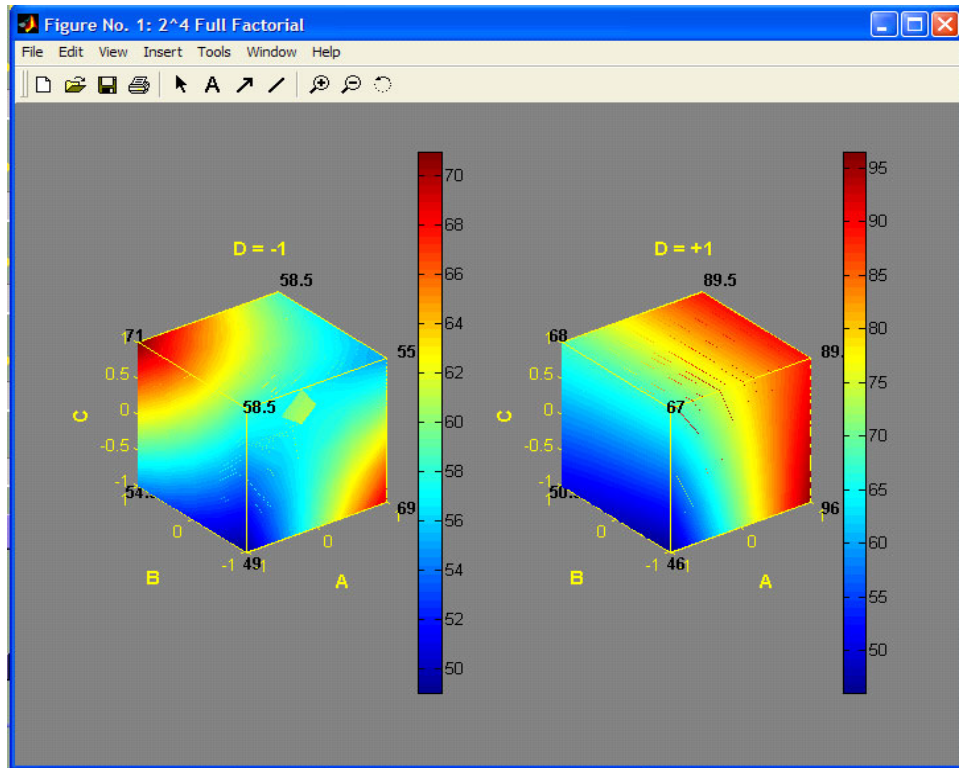
**Figure 37: 2⁴ Full Factorial Contour Plots (different color schemes)**

The 3-dimensional contour plots for this experiment are displayed in two cubes. At first sight, it is not so easy for the user to distinguish factor combinations that lead to desired minimum responses when compared to the twelve 2-dimensional Minitab's 14® contours. Color bars were added to Figure 37 contour cubes. These color bars are optional and available at the Insert option in the figure Menu bar. These color bars show an important fact, contour cubes could not be visually compared because color scales used vary from one cube to the other as was discussed earlier for Minitab 14® contours. Drawing conclusions based on visual perception may result in wrong process conditions.

The 3-D Contour Plot application goes a step further in this direction and provides the user with graphical displays with matching color schemes within a single experiment

analysis. Figure 38 presents how the 3-D Contour Plot cubes behave after upgrading this capability. It is evident that to meet the experimental objective, minimization of response, experimental conditions with all factors at their high level should be avoided, specifically most of the treatments containing factor D at its high level. Having a 3-dimensional contour plot application that in addition considers value variations on its graphical display definitely maximizes DOE capabilities.
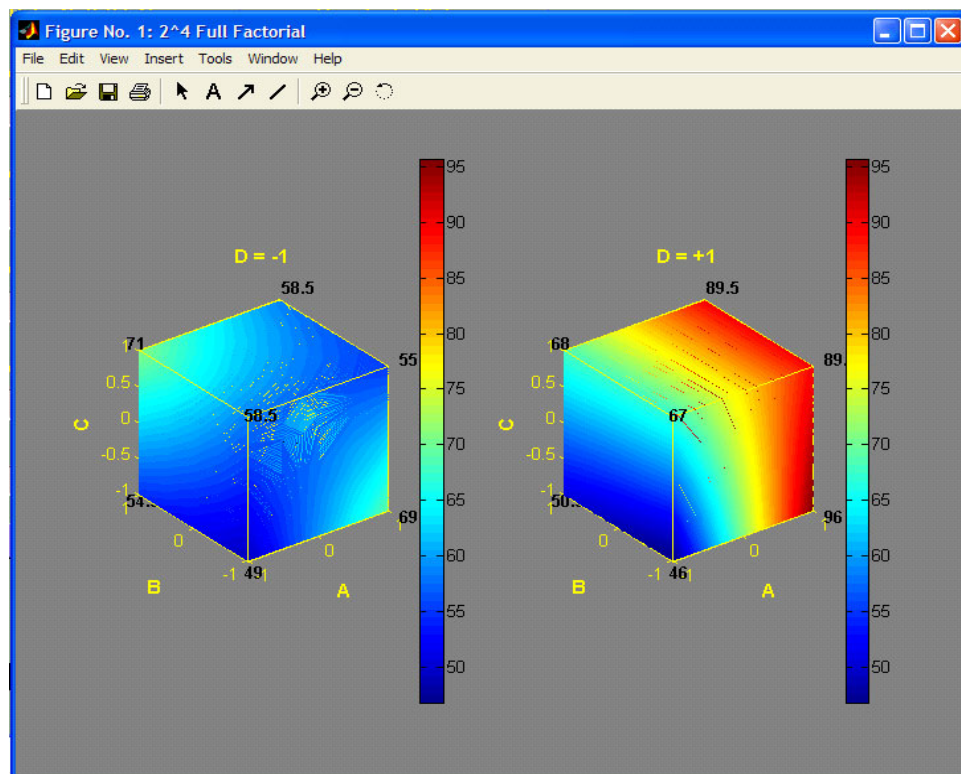


**Figure 38: $2^4$ Full Factorial Contour Plots**

As for the $2^3$ Full Factorial, the $2^4$ Full Factorial with replicates = 2 contour plots obtained with the 3-D DOE Contour Plot match those obtained using Minitab 14®. Each of the cubes represents three factors (labeled as A, B, and C) varying within the specified limits, whereas the remaining factor (labeled as D) remains fixed on its extreme levels (-1 and +1). If the user wants current factor D varying within the high and low levels data

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will need to be stored in a different file and application run again. Factor D that was fixed could be switch over to appear on the axis currently being labeled as A, B, or C. Data will need to be rearranged to match test experiment matrix X.

Any of the factors initially presented at one of either fixed limits will become a factor varying from high to low. It will appear labeled as A, B, or C, depending on which order data was entered, but technically cube plot will be representing behavior for the selected factor.

## 5.2    Box-Behnken Experiments

All possible Box-Behnken combinations were validated and this section contains one example described in detail. The example is a Box-Behnken experiment with 3 quantitative factors. The exercise was resolved using Minitab 14® and the 3-D DOE Contour Plot application.

### 5.2.1    Box-Behnken: Quantitative Factors = 3

The Box-Behnken experiment was analyzed using Minitab 14® and the 3-D DOE Contour Plot tool. The design challenged on this section has 3 factors at two levels (high = +1, low = -1).

To demonstrate tool adequacy the experiment was run in Minitab 14® and in the 3-D DOE Contour Plots tool developed for this project. Results are as follows:

- *Minitab 14®*

## Response Surface Regression: y versus A, B, C

```
Term        Coef
Constant  12.5667
A          0.0125
B         -0.0875
C          0.0750
A*A       -0.3083
B*B       -0.3583
C*C       -0.4333
A*B        0.2500
A*C       -0.3250
B*C       -0.1250
```
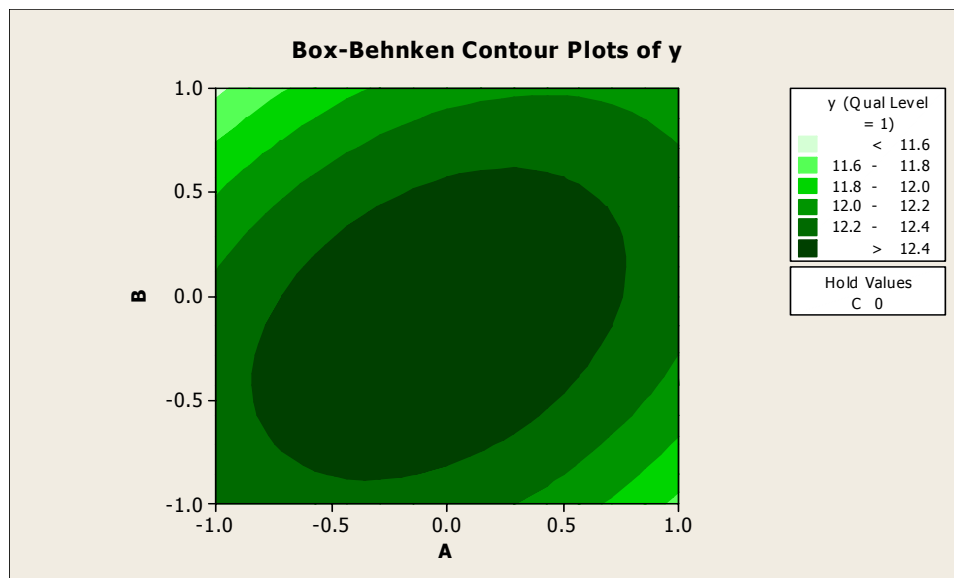


**Figure 39: Box-Behnken Contour Plots, Quantitative Factors = 3 (C = 0)**
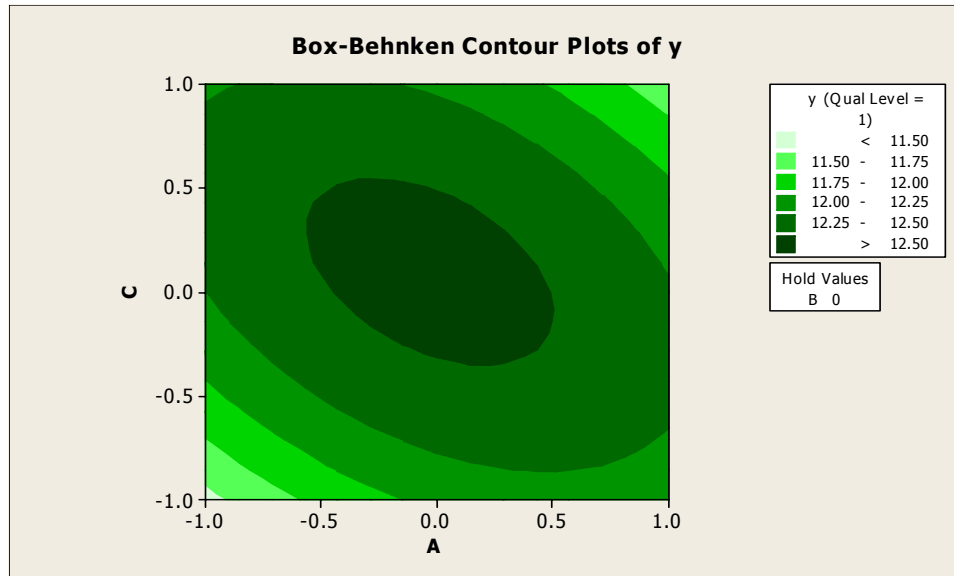
**Figure 40: Box-Behnken Contour Plots, Quantitative Factors = 3 (B = 0)**
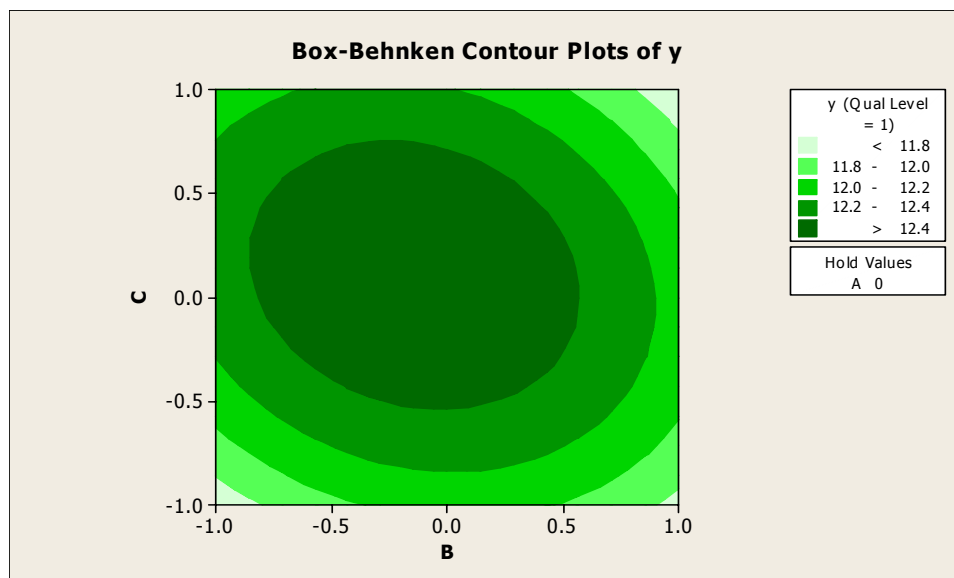


**Figure 41: Box-Behnken Contour Plots, Quantitative Factors = 3 (A = 0)**

Experiment residual plots are grouped in the following figure.
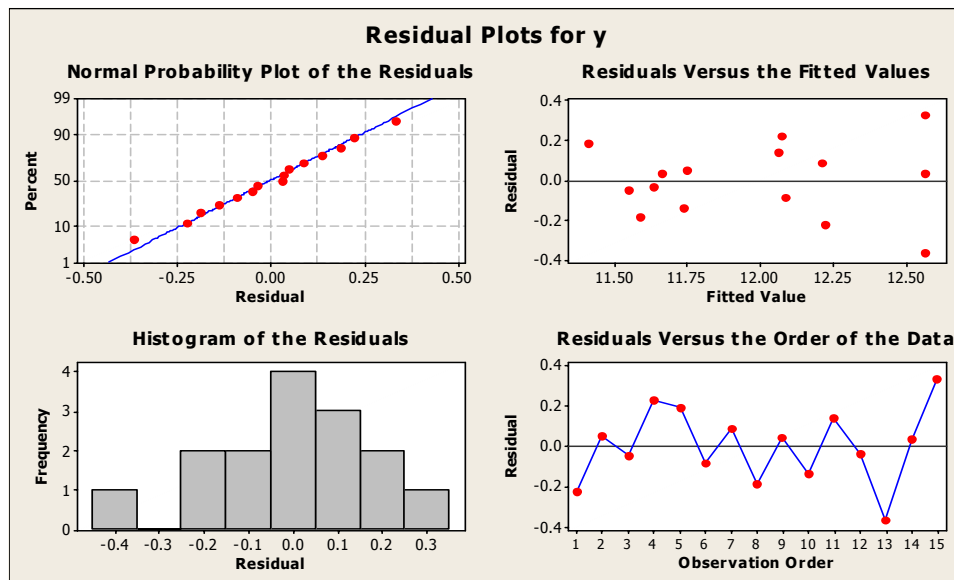


**Figure 42: Three variable Box-Behnken Residual Plots (with Minitab 14®)**

- *3-D DOE Contour Plot*
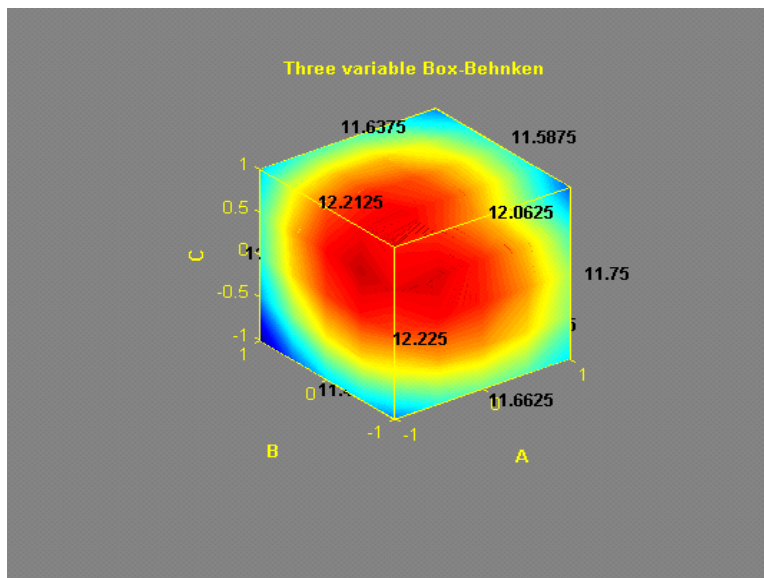


**Figure 43: Three variable Box-Behnken Contour Plots**

Taking advantage of the application main functionality the cube was rotated and middle edges, highlighted in black numbers in Figure 43, were compared to corners of Minitab 14® contour plots. As for previous examples, both statistical applications output match as expected.

If cube displayed in Figure 41 is rotated -90° it is easier to confirm (refer to Figure 44) that treatment combination A=-1, B=0, C=-1 yields lower responses. Whereas treatment combination A=-1, B=-1, C=0 yields higher responses.
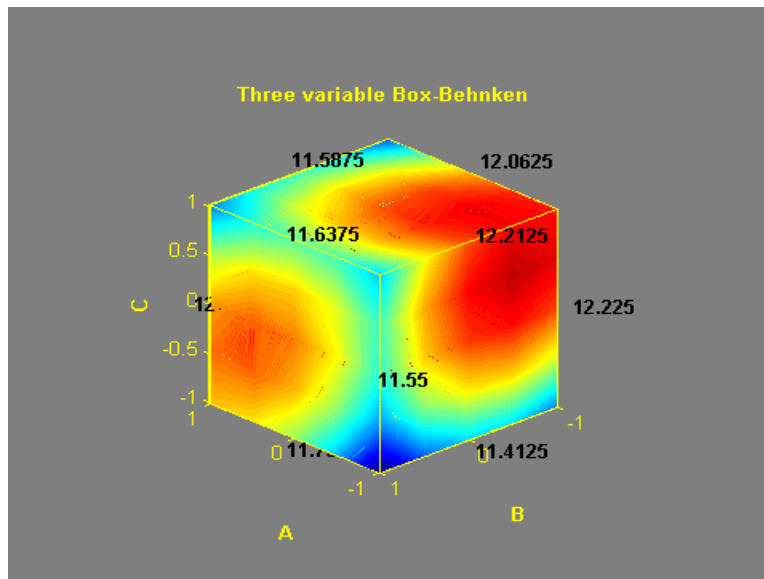


**Figure 44: Three variable Box-Behnken Contour Plots (rotated -90°)**

Coefficients obtained with 3-D DOE Contour Plot are identical to coefficients obtained with Minitab 14® for this same experiment. The following figure shows coefficients calculated by 3-D DOE Contour Plot.
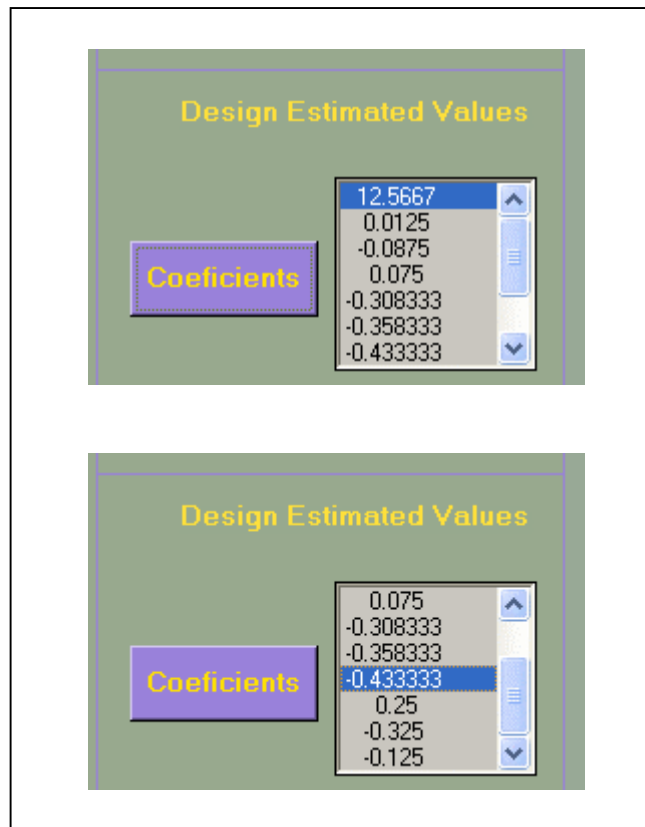
**Figure 45: Three variable Box-Behnken Coefficients (with 3-D Contour Plot application)**

The 3-D DOE Contour Plot provides the user for plotting residual plots when applicable. Moreover, residual plots from both statistical tools result the same. Figures 46 to 49 illustrate those plots and confirm the validity of the developed tool after comparing results with Minitab 14® plots presented earlier.
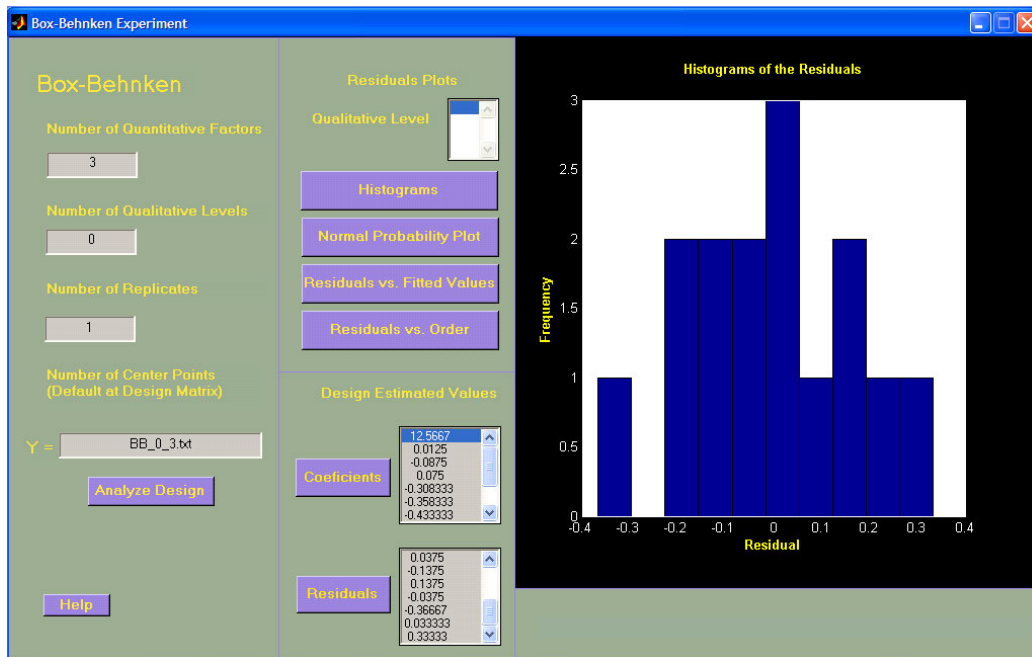
**Figure 46: BoxBehnken Histogram of Residuals (Quantitative Factors = 3)**
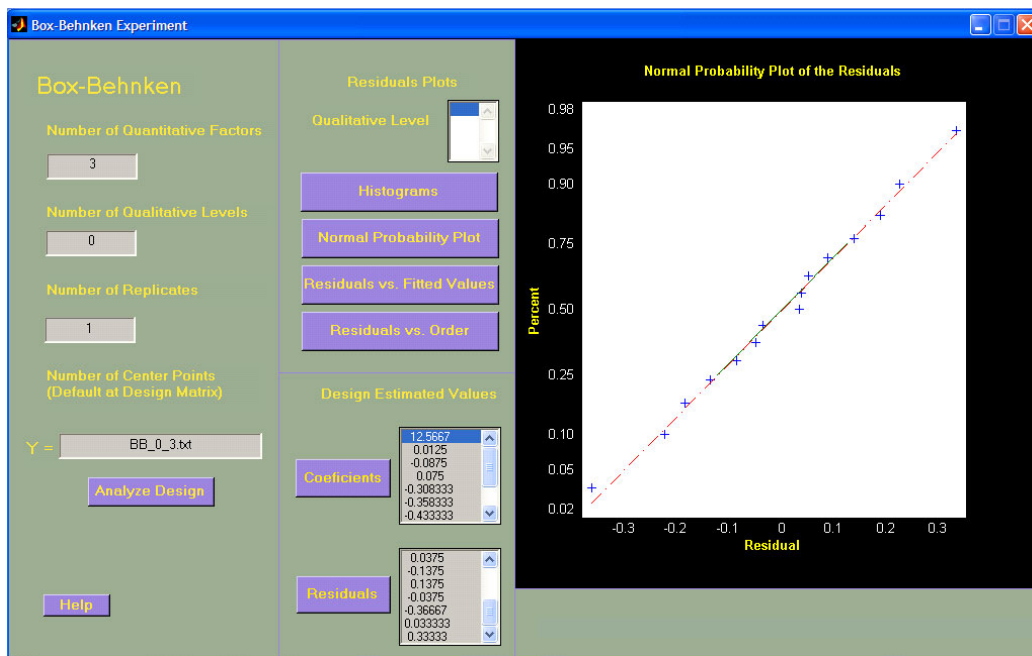


**Figure 47: Box-Behnken Normal Probability Plot (Quantitative Factors = 3)**
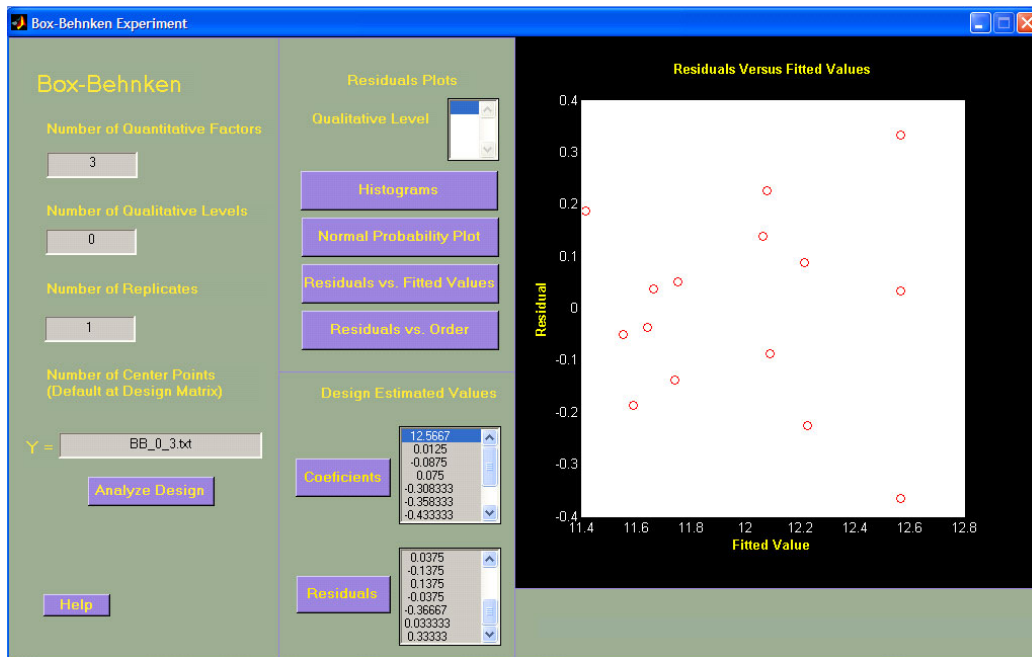
**Figure 48: Box-Behnken Residuals vs Fitted Values (Quantitative Factors = 3)**
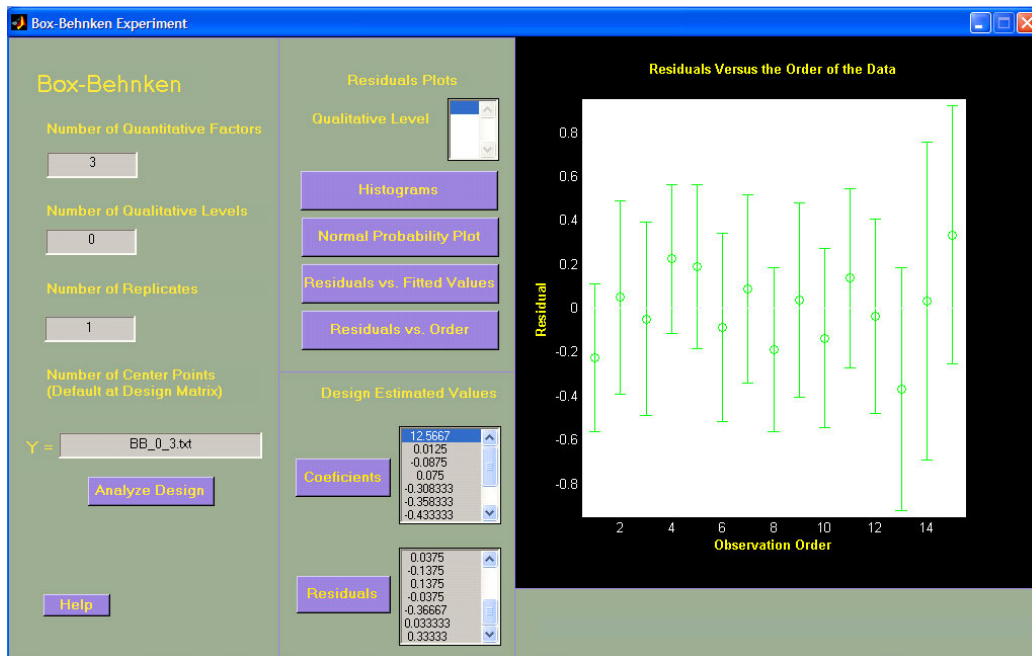


**Figure 49: Box-Behnken Residual vs Order (Quantitative Factors = 3)**

## 5.3 CCD Experiments

As for Full Factorial and Box-Behnken, all CCD experiment scenarios were validated. All possible combinations were properly tested and this section illustrates one of those CCD experiments, a CCD with four quantitative factors two replicates.

- *__Minitab 14®__*

**Response Surface Regression: y versus A, B, C, D**

```
Estimated Regression Coefficients for y using data in uncoded units

Term        Coef
Constant  9.2665
A         0.5763
B         0.1743
C         0.0655
D        -0.2774
A*A      -0.2242
B*B      -0.2062
C*C       0.0445
D*D      -0.1808
A*B      -0.4300
A*C       0.3904
A*D      -0.2131
B*C      -0.2040
B*D      -0.0145
C*D      -0.6553
```
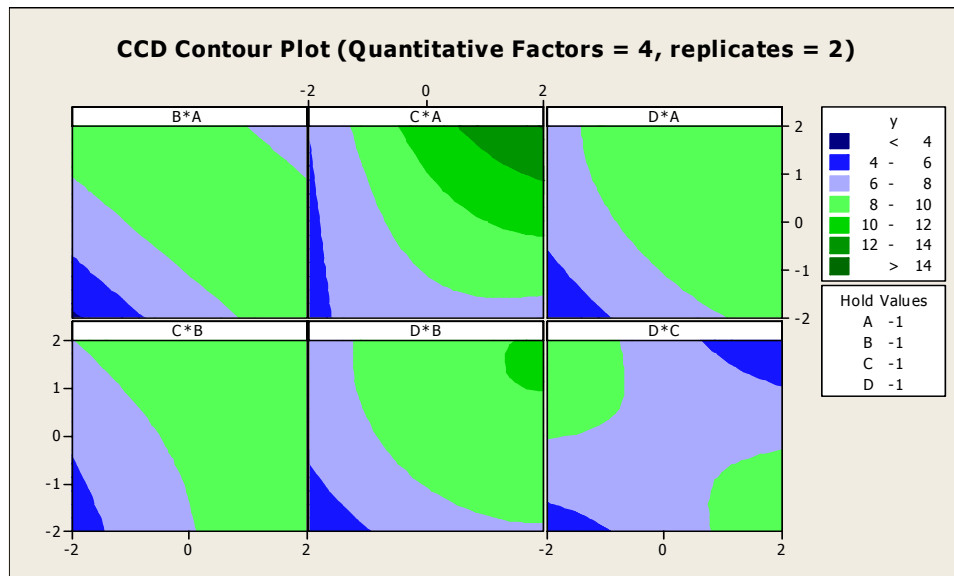


**Figure 50: CCD Contour Plots Quantitative Factors = 4 (Low Settings)**
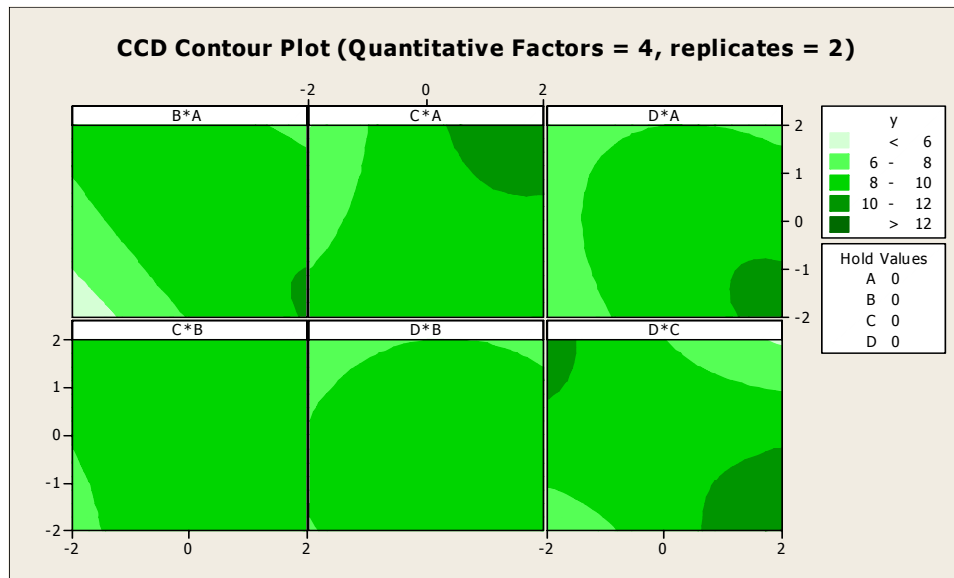
**Figure 51: CCD Contour Plots Quantitative Factors = 4 (Middle Settings)**
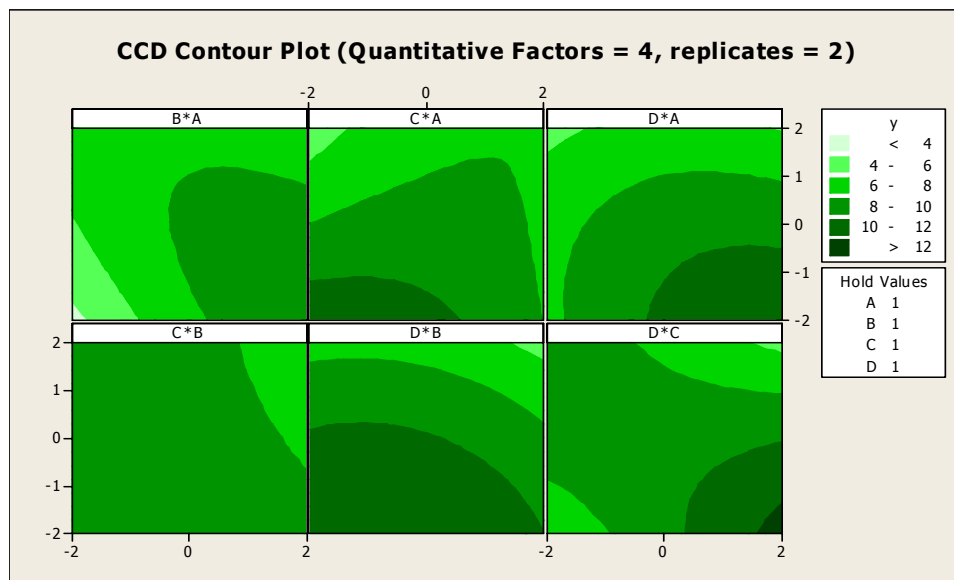


**Figure 52: CCD Contour Plots Quantitative Factors = 4 (High Settings)**

A CCD experiment that studies response within four different factors could yield up to 18 different 2-dimensional contour plots as shown in Figures 48 – 50. The amount of plots could turn decision taking in a complicated task. Even more, when response

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surfaces changes little within factor levels it could not be so obvious what the best option is as happens for many of the faces of this example. Next pages will illustrate 3-D DOE Contour Plot application advantage of having all faces linked as a cube and with a common color scheme along all the faces and able to rotate to see all possible factors combinations.

- ### *3-D DOE Contour Plot*

Once again, the experiment was evaluated by the developed 3-D DOE Contour Plot tool. Results did not vary from those provided by the Minitab 14® statistical analysis software. Estimated coefficients were the same as those from the Minitab 14® evaluation.
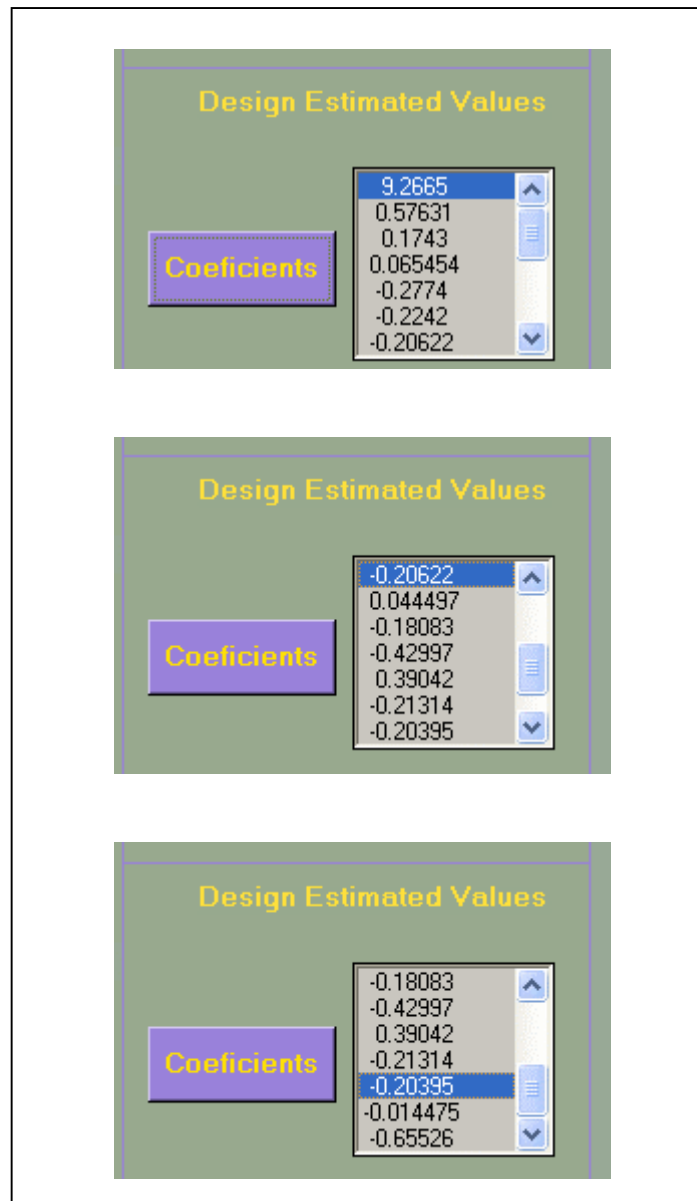
**Figure 53: CCD Expected Coefficients for Quantitative Factors = 4**
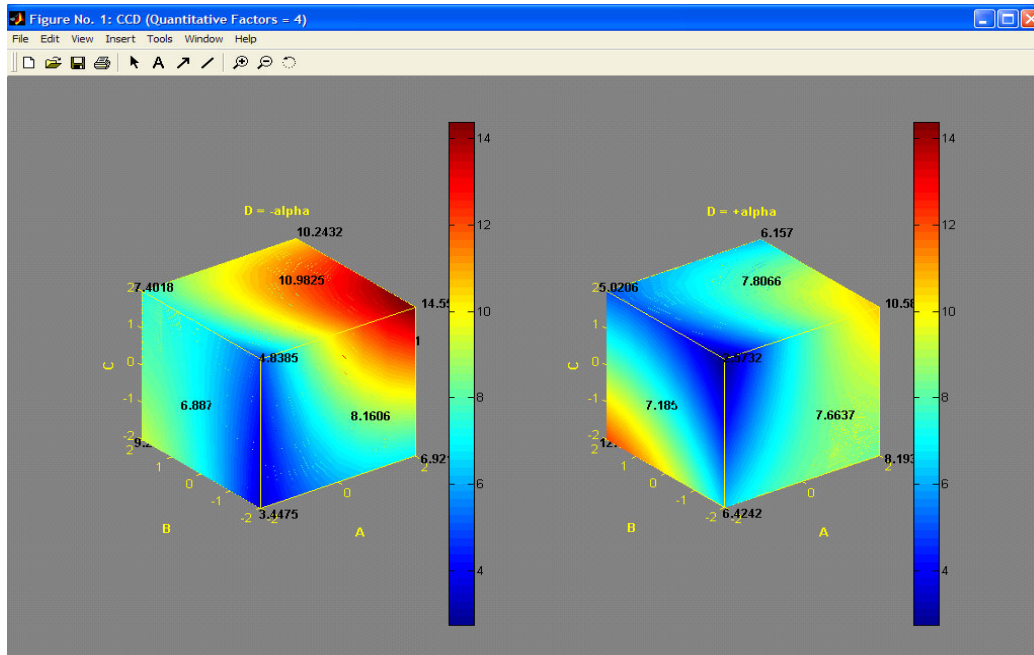
**Figure 54: CCD Contour Plots Quantitative Factors = 4 (with 3-D DOE Contour Plot)**

Default alpha = 2 for CCD experiments with four quantitative factors. As a result, cubes axes are extended to the low and high level of that value as demonstrated on Figure 54. Visualizing slices on the faces address the factorial portion of the experiments, whereas visualizing slices at the middle address the axial portion of the experiment. For all the required points the developed tool matched the Minitab 14® outputs. Doubtlessly, minimum and maximum responses are easier to spot with the 3-dimensional cube rather than with the eighteen 2-dimensional plots provided by Minitab 14®.

# 6 APPLIED CASE STUDY

## 6.1 Wave Soldering at a local Electronics Industry

Electronics assembly industry has been forced to explore new wave soldering processes because of a couple of reasons. Even though wave soldering is one of preferred assembly operations it also has disadvantages associated with it, especially when considering complex assemblies [11]. There are two main reasons that raised limitations in wave soldering process. One is the introduction of Organic Solder Preservative boards (OSP). OSP board is designed to produce a thin, uniform, protective layer on the copper surface of the PCB's. This coating protects the circuitry from oxidation during storage and assembly operations. OSP has been around for some time but is only recently gaining popularity with the search of lead free techniques and fine pitch solutions. It is precisely the search of lead-free techniques what is considered the second drawback with wave soldering processes. Interest in lead-free environmentally friendly assembly processes responds to regulatory and consumer pressure from Europe and Japan for a coming implementation of lead-free alloys in electronic goods. As a result, the electronics assembly industry continuously seeks for areas of opportunity to improve their processes, which translates in manufacturing high quality products.

A local electronics assembly industry conducted a series of experiments with the purpose of getting aligned with new market tendencies. The purpose of the experiments was to select the best no clean flux for their assembly process. Along with flux selection another critical factor that control wave soldering is flux application. There are various types of soldering fluxes and technologies. The case study scenario considered

comprehends no-clean and water soluble fluxes with high activity/solid content flux technologies.

## 6.2 Case Study Experiment Description

A first experiment was executed for testing five no-clean, low solid wave fluxes. Results showed poor performance with current no-clean flux technologies and forced the engineers to find out the best flux type to manage this process change [12]. A second experiment purpose was to select best two fluxes from a total of seven new pre-selected fluxes (Fluxes: A, B, C, D, E, F, G). After a preliminary screening three out of seven of the pre-selected fluxes were eliminated from the study. Therefore, there were still four pre-selected fluxes from where to choose the best no-clean flux (Fluxes: B, C, E, F). The idea of developing the 3-dimensional DOE contour plots tool in Matlab 13® came up because of the opportunities foreseen by the team during the execution of the second experiment.

The expected results of the experiments were to optimize weld thickness for the solder paste barrel fill. Barrel fill weld thickness is a variable response dependant on several factors. Among these factors are:

- Flux type – consists of a selection of no-clean/ water soluble and alcohol/ water based fluxes.

- PCB metallization – could either be Organic Coated Copper (OCC) or Electrolytic Nickel/ Electrolytic Gold (Type 1).

- PCB thickness – Even though there are two different test board thicknesses for test purposes only one was considered and will be identified as Thickness 1.

- Flux application pressure consists of two different levels Low and High.

- Process Condition – The two remaining factors mentioned above: preheat temperature (Low, High) and conveyor speed (Low) were combined into a factor considered Process Condition. Considered combinations resulted in two process conditions: Profile 1 (Low Temperature & Low conveyor speed) and Profile 2 (High temperature & Low conveyor speed).

Test method for the first screening round on the second experiment summarizes as follows: a general full factorial with a total of 112 runs for each of two alloys, SAC and SNPB. The 112 runs are the result of 7 fluxes x 2 process conditions x 2 PCB metallization x 2 flux pressures. An automated inspection x-ray machine measures the barrel fill thickness through a laminography process that focuses the measures at different z-axis positions [11]. That thickness parameter is a number which relates to the amount of solder present at the x-ray focus level. Therefore, smaller thicknesses are represented by smaller readings. The automated inspection machine took a measurement for every joint in each board for a total of over one thousand data points for each of the 112 experimental runs.

A second screening round was executed based on results of the first round described above. Test method for this second screening round on the second experiment summarizes as follows: a general full factorial with a total 32 runs. This time only SAC

alloy was tested. The 32 runs are the result of 4 fluxes (B, C, E, F) x 2 process conditions x 2 PCB metallization x 2 flux pressures. Even experimental conditions were reduced, automated inspection methodology and measured response remains the same. Tables 5, 6, and 7 summarize experiment conditions and observed responses.

Engineers defined an index that represents the flux behavior depending on the treatment combination being tested. For each of the fluxes being observed the higher the index response the better the flux is for the specified process characterization.

**Table 5: Experimental Treatments and Response for the 50% board thickness measure**

| | 50% | | | |
|---|---|---|---|---|
| | Factor A | Factor B | Factor C | Response |
| Flux | Metallization | Flux Pressure | Process Condition | Index |
| Flux B | -1 | -1 | -1 | 0.944 |
| Flux B | 1 | -1 | -1 | 1.245 |
| Flux B | -1 | 1 | -1 | 0.600 |
| Flux B | 1 | 1 | -1 | 1.279 |
| Flux B | -1 | -1 | 1 | 1.082 |
| Flux B | 1 | -1 | 1 | 1.418 |
| Flux B | -1 | 1 | 1 | 0.855 |
| Flux B | 1 | 1 | 1 | 1.453 |
| Flux C | -1 | -1 | -1 | 0.861 |
| Flux C | 1 | -1 | -1 | 1.386 |
| Flux C | -1 | 1 | -1 | 0.846 |
| Flux C | 1 | 1 | -1 | 1.407 |
| Flux C | -1 | -1 | 1 | 0.763 |
| Flux C | 1 | -1 | 1 | 1.476 |
| Flux C | -1 | 1 | 1 | 0.462 |
| Flux C | 1 | 1 | 1 | 1.518 |
| Flux E | -1 | -1 | -1 | 1.226 |
| Flux E | 1 | -1 | -1 | 1.234 |
| Flux E | -1 | 1 | -1 | 1.260 |
| Flux E | 1 | 1 | -1 | 1.186 |
| Flux E | -1 | -1 | 1 | 1.466 |
| Flux E | 1 | -1 | 1 | 1.459 |
| Flux E | -1 | 1 | 1 | 1.486 |
| Flux E | 1 | 1 | 1 | 0.961 |
| Flux F | -1 | -1 | -1 | 1.247 |
| Flux F | 1 | -1 | -1 | 1.187 |
| Flux F | -1 | 1 | -1 | 1.299 |
| Flux F | 1 | 1 | -1 | 1.253 |
| Flux F | -1 | -1 | 1 | 0.783 |
| Flux F | 1 | -1 | 1 | 1.327 |
| Flux F | -1 | 1 | 1 | 0.808 |
| Flux F | 1 | 1 | 1 | 1.371 |

**Table 6: Experimental Treatments and Response for the 75% board thickness measure**

| | 75% | | | |
| --- | --- | --- | --- | --- |
| | Factor A | Factor B | Factor C | Response |
| Flux | Metallization | Flux Pressure | Process Condition | Index |
| Flux B | -1 | -1 | -1 | 0.425 |
| Flux B | 1 | -1 | -1 | 0.626 |
| Flux B | -1 | 1 | -1 | 0.354 |
| Flux B | 1 | 1 | -1 | 0.663 |
| Flux B | -1 | -1 | 1 | 0.801 |
| Flux B | 1 | -1 | 1 | 1.240 |
| Flux B | -1 | 1 | 1 | 0.498 |
| Flux B | 1 | 1 | 1 | 1.309 |
| Flux C | -1 | -1 | -1 | 0.383 |
| Flux C | 1 | -1 | -1 | 0.759 |
| Flux C | -1 | 1 | -1 | 0.433 |
| Flux C | 1 | 1 | -1 | 0.749 |
| Flux C | -1 | -1 | 1 | 0.589 |
| Flux C | 1 | -1 | 1 | 1.290 |
| Flux C | -1 | 1 | 1 | 0.372 |
| Flux C | 1 | 1 | 1 | 1.306 |
| Flux E | -1 | -1 | -1 | 0.470 |
| Flux E | 1 | -1 | -1 | 0.549 |
| Flux E | -1 | 1 | -1 | 0.510 |
| Flux E | 1 | 1 | -1 | 0.630 |
| Flux E | -1 | -1 | 1 | 0.970 |
| Flux E | 1 | -1 | 1 | 1.181 |
| Flux E | -1 | 1 | 1 | 0.999 |
| Flux E | 1 | 1 | 1 | 0.871 |
| Flux F | -1 | -1 | -1 | 0.503 |
| Flux F | 1 | -1 | -1 | 0.689 |
| Flux F | -1 | 1 | -1 | 0.541 |
| Flux F | 1 | 1 | -1 | 0.632 |
| Flux F | -1 | -1 | 1 | 0.622 |
| Flux F | 1 | -1 | 1 | 1.160 |
| Flux F | -1 | 1 | 1 | 0.672 |
| Flux F | 1 | 1 | 1 | 1.242 |

**Table 7: Experimental Treatments and Response for the 90% board thickness measure**

| | 90% | | | |
| --- | --- | --- | --- | --- |
| | **Factor A** | **Factor B** | **Factor C** | **Response** |
| **Flux** | **Metallization** | **Flux Pressure** | **Process Condition** | **Index** |
| Flux B | -1 | -1 | -1 | 0.288 |
| Flux B | 1 | -1 | -1 | 0.428 |
| Flux B | -1 | 1 | -1 | 0.274 |
| Flux B | 1 | 1 | -1 | 0.428 |
| Flux B | -1 | -1 | 1 | 0.562 |
| Flux B | 1 | -1 | 1 | 0.899 |
| Flux B | -1 | 1 | 1 | 0.395 |
| Flux B | 1 | 1 | 1 | 0.961 |
| Flux C | -1 | -1 | -1 | 0.254 |
| Flux C | 1 | -1 | -1 | 0.501 |
| Flux C | -1 | 1 | -1 | 0.296 |
| Flux C | 1 | 1 | -1 | 0.495 |
| Flux C | -1 | -1 | 1 | 0.424 |
| Flux C | 1 | -1 | 1 | 0.951 |
| Flux C | -1 | 1 | 1 | 0.309 |
| Flux C | 1 | 1 | 1 | 0.916 |
| Flux E | -1 | -1 | -1 | 0.307 |
| Flux E | 1 | -1 | -1 | 0.358 |
| Flux E | -1 | 1 | -1 | 0.353 |
| Flux E | 1 | 1 | -1 | 0.432 |
| Flux E | -1 | -1 | 1 | 0.567 |
| Flux E | 1 | -1 | 1 | 0.790 |
| Flux E | -1 | 1 | 1 | 0.609 |
| Flux E | 1 | 1 | 1 | 0.660 |
| Flux F | -1 | -1 | -1 | 0.337 |
| Flux F | 1 | -1 | -1 | 0.454 |
| Flux F | -1 | 1 | -1 | 0.360 |
| Flux F | 1 | 1 | -1 | 0.413 |
| Flux F | -1 | -1 | 1 | 0.449 |
| Flux F | 1 | -1 | 1 | 0.856 |
| Flux F | -1 | 1 | 1 | 0.518 |
| Flux F | 1 | 1 | 1 | 0.944 |

In experiment analysis there are visual graphical aids for quantitative (continuous contours) and qualitative (discrete contours). The 3-D DOE Contour Plot tool intends to comply with both categories characteristics in order to analyze the applied case study necessity that originates this project.

## 6.3    Case Study – Minitab 14® Results

Full factorial experiment data contained on Tables 5, 6, and 7 was analyzed on Minitab 14® statistical software. Response for each of the three different board thicknesses was individually analyzed. That is, a full factorial $2^3$ experiment was run for each of the four fluxes (B, C, E, and F) at 50%, 75%, and 90%. Minitab 14® result for the 50% response is discussed in the following pages. Results for 75% and 90% are attached to Appendix 6.

### 6.3.1    Contour Plots at Board Thickness = 50%

#### 6.3.1.1    Minitab 14® Results

##### *6.3.1.1.1    Flux B*

The $2^3$ Full Factorial experiment data for Flux B resulted in the following coefficients. The six 2-dimensional contour plots for the 50% Response Index for Flux B are shown in figures 55 and 56.

**Factorial Fit: Flux B 50% Response Index versus A, B, C**

```
Estimated Effects and Coefficients for Flux B 50% Response Index
Term        Effect      Coef
Constant               1.10950
A           0.47850    0.23925
B          -0.12550   -0.06275
C           0.18500    0.09250
A*B         0.16000    0.08000
A*C        -0.01150   -0.00575
B*C         0.02950    0.01475
A*B*C      -0.02900   -0.01450
```
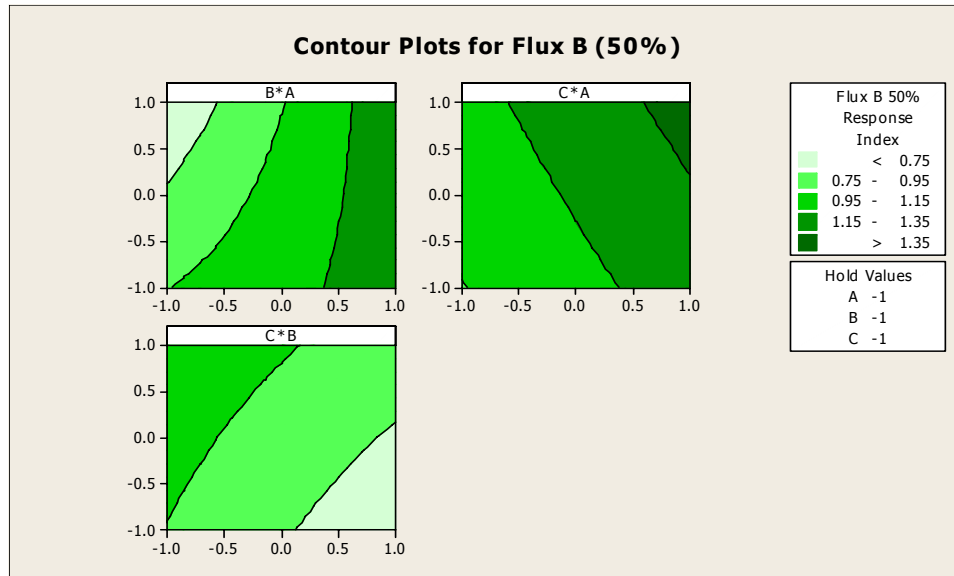
**Figure 55: Flux B 50% (Low Settings)**
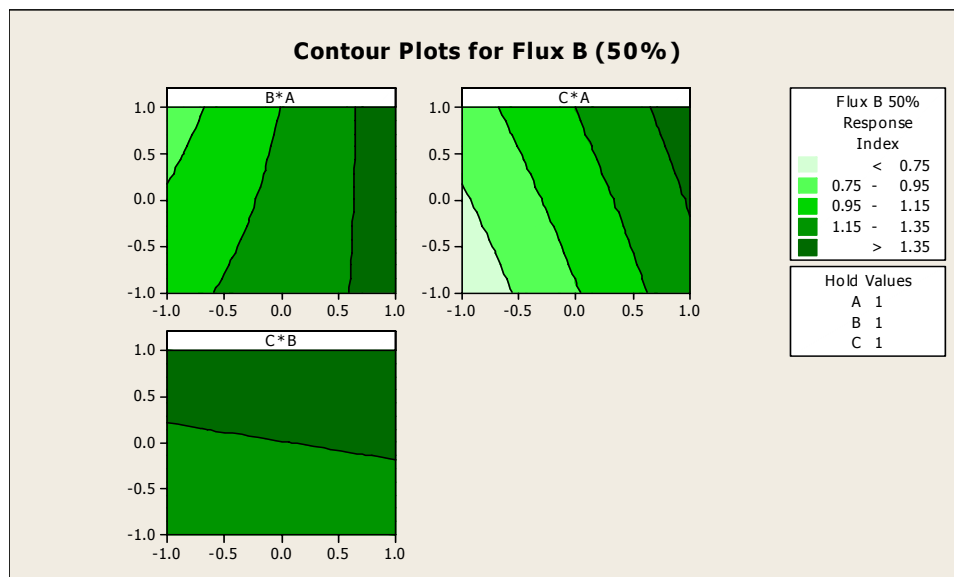


**Figure 56: Flux B 50% (High Settings)**

### 6.3.1.1.2    Flux C

The $2^3$ Full Factorial experiment data for Flux C resulted in the following coefficients. The six 2-dimensional contour plots for the 50% Response Index for Flux C are shown in Figures 57 and 58.

**Factorial Fit: Flux C 50% Response Index versus A, B, C**

```
Estimated Effects and Coefficients for Flux C 50% Response Index

Term        Effect       Coef
Constant               1.08988
A          0.71375    0.35687
B         -0.06325   -0.03162
C         -0.07025   -0.03512
A*B        0.09475    0.04738
A*C        0.17075    0.08538
B*C       -0.06625   -0.03313
A*B*C      0.07675    0.03838
```
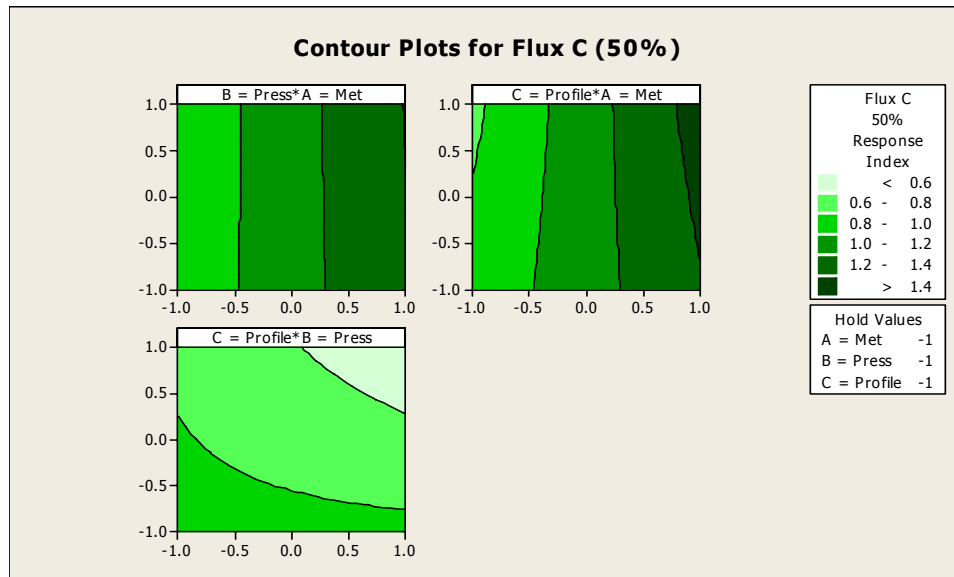


**Figure 57: Flux C 50% (Low Settings)**

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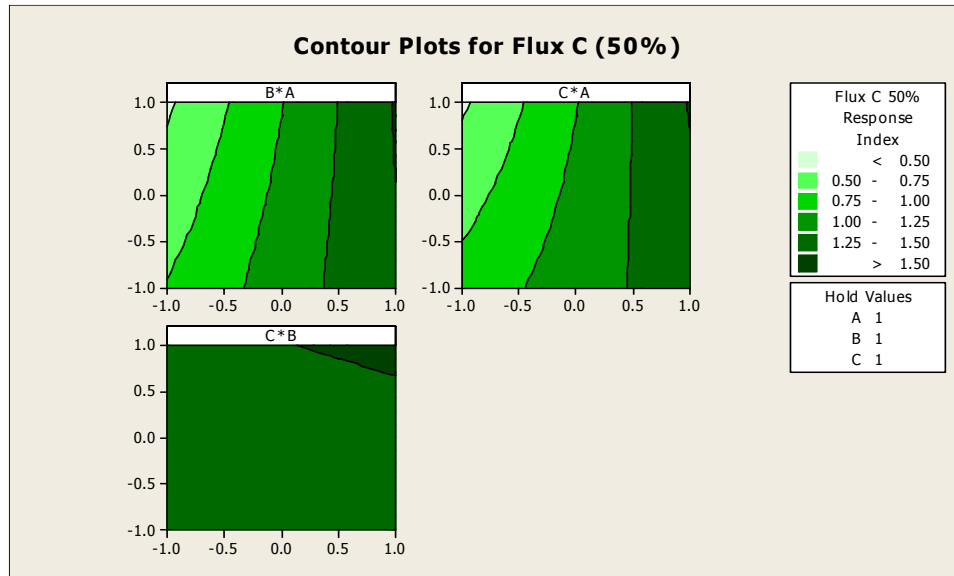
**Figure 58: Flux C 50% (High Settings)**

### 6.3.1.1.3　　Flux E

The $2^3$ Full Factorial experiment data for Flux E resulted in the following coefficients. The six 2-dimensional contour plots for the 50% Response Index for Flux E are shown in figures 59 and 60.

**Factorial Fit: Flux E 50% Response Index versus A, B, C**

```
Estimated Effects and Coefficients for Flux E 50% Response Index

Term          Effect        Coef
Constant                  1.28475
A           -0.14950    -0.07475
B           -0.12300    -0.06150
C            0.11650     0.05825
A*B         -0.15000    -0.07500
A*C         -0.11650    -0.05825
B*C         -0.11600    -0.05800
A*B*C       -0.10900    -0.05450
```
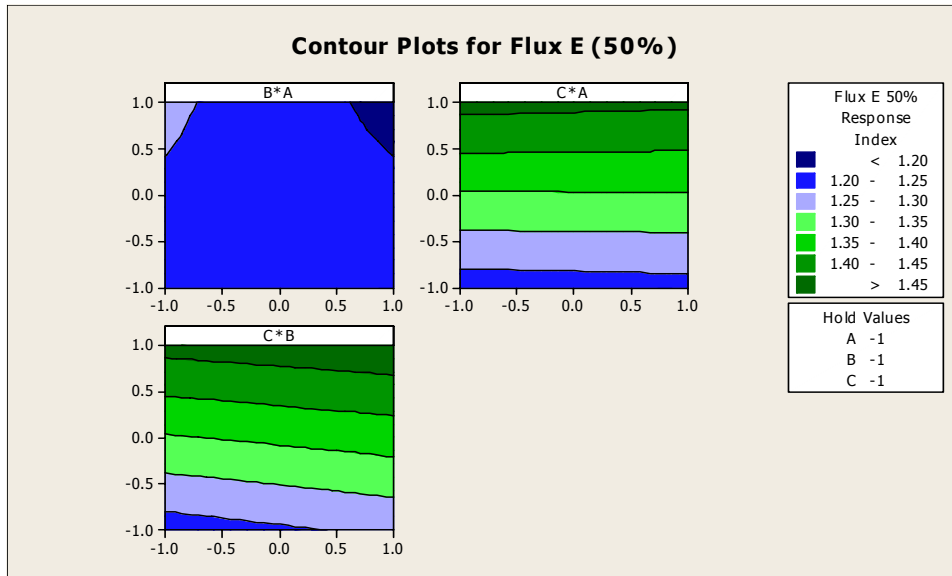
**Figure 59: Flux E 50% (Low Settings)**
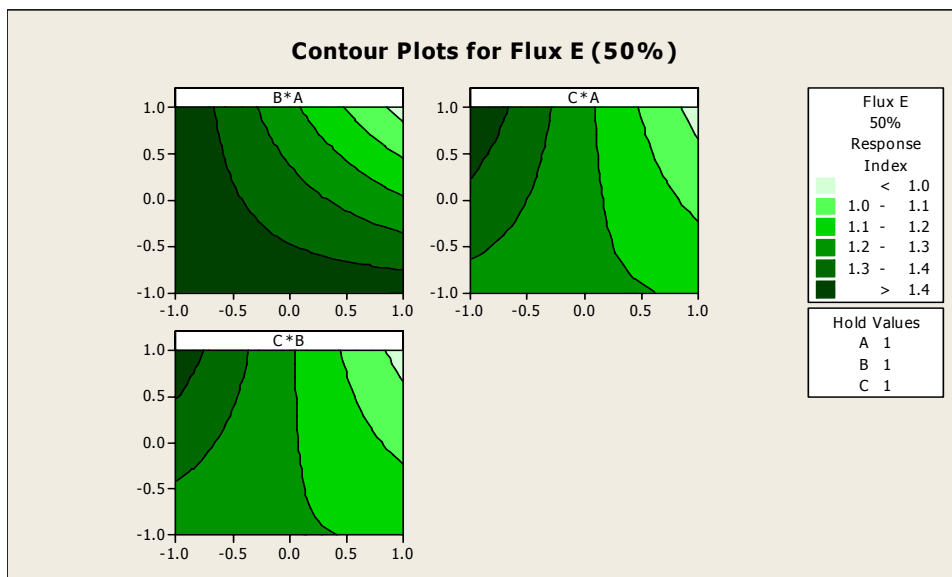


**Figure 60: Flux E 50% (High Settings)**

### 6.3.1.1.4 *Flux F*

The $2^3$ Full Factorial experiment data for Flux F resulted in the following coefficients. The six 2-dimensional contour plots for the 50% Response Index for Flux F are shown in figures 61 and 62.

**Factorial Fit: Flux F 50% Response Index versus A, B, C**

```
Estimated Effects and Coefficients for Flux F 50% Response Index

Term        Effect       Coef
Constant              1.15938
A          0.25025    0.12512
B          0.04675    0.02337
C         -0.17425   -0.08713
A*B        0.00825    0.00413
A*C        0.30325    0.15163
B*C       -0.01225   -0.00612
A*B*C      0.00125    0.00062
```
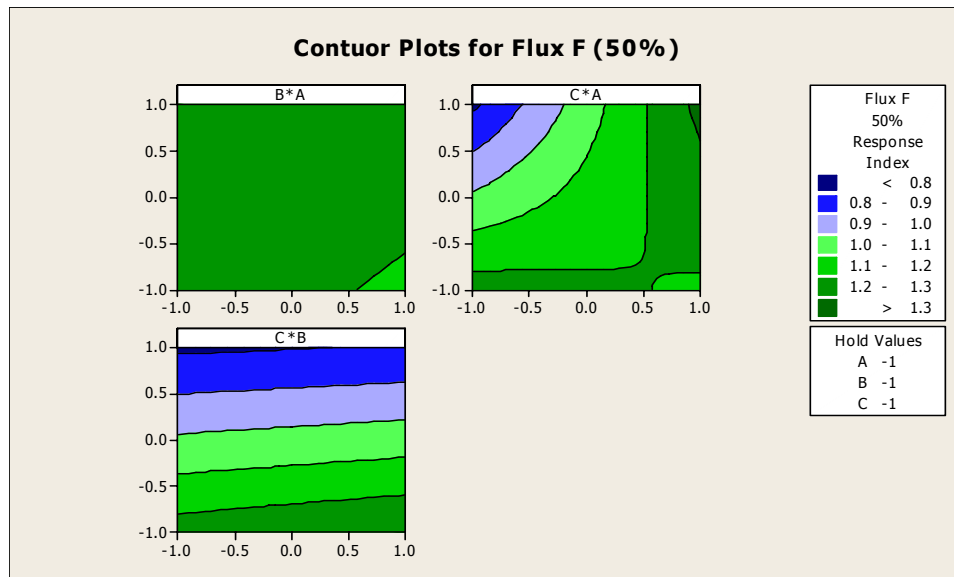


**Figure 61: Flux F 50% (Low Settings)**
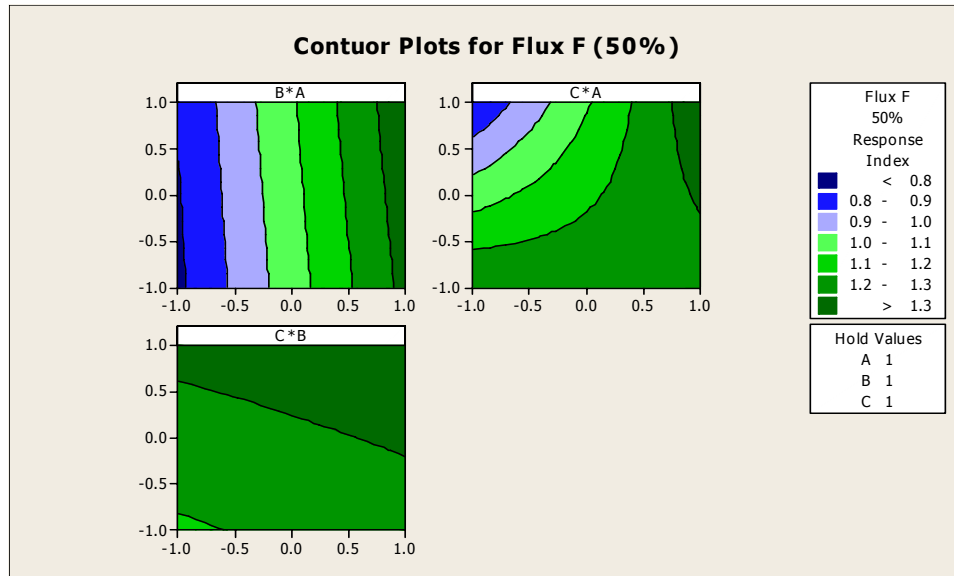
**Figure 62: Flux F 50% (High Settings)**

Analyzing Response Index at 50% board thickness for the four fluxes resulted in a total of 24 2-dimensional contour plots. The huge amount of 2-dimensional plots resulting from the four fluxes makes difficult establishing a conclusion. As mentioned in a previous chapter, Minitab 14® color scheme is not consistent between the different plots. It implies the user needs to identify values at the desired experimental conditions to evaluate. Minitab 14® plots can not be used as a visual tool alone. This same experiment was evaluated with the 3-dimensional graphing tool to challenge the application capabilities and to compare differences between both visualization methods.

### 6.3.1.2    3-D DOE Contour Plot (50%)

The experiment description window for a Full Factorial Experiment with: Quantitative Factors = 3 and Qualitative Levels = 4 looks as in Figure 63.
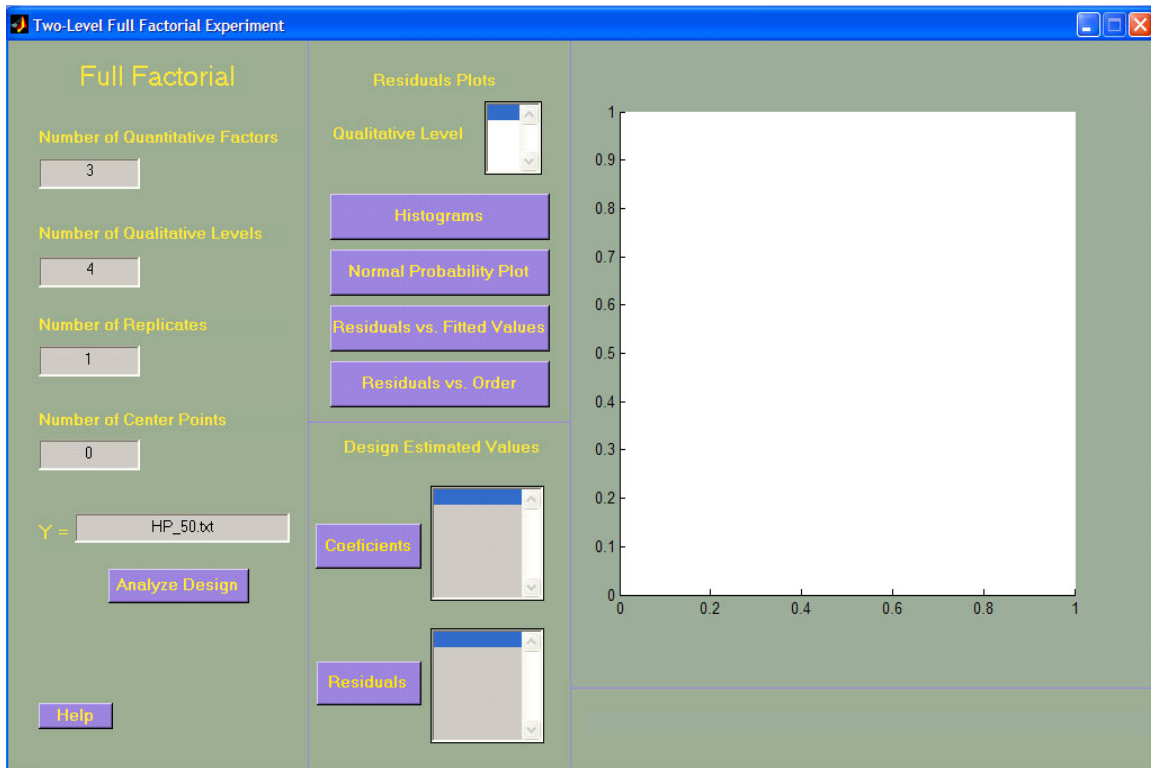
**Figure 63: Applied Case Study Experiment Description – Response Index at 50%**

Selecting the Analyze Design button generates the 3-dimensional contour plots for the four fluxes in a single figure. Each cube is identified as Qualitative Level 1 = Flux B, Qualitative Level 2 = Flux C, Qualitative Level 3 = Flux E, and Qualitative Level 4 = Flux F, respectively.

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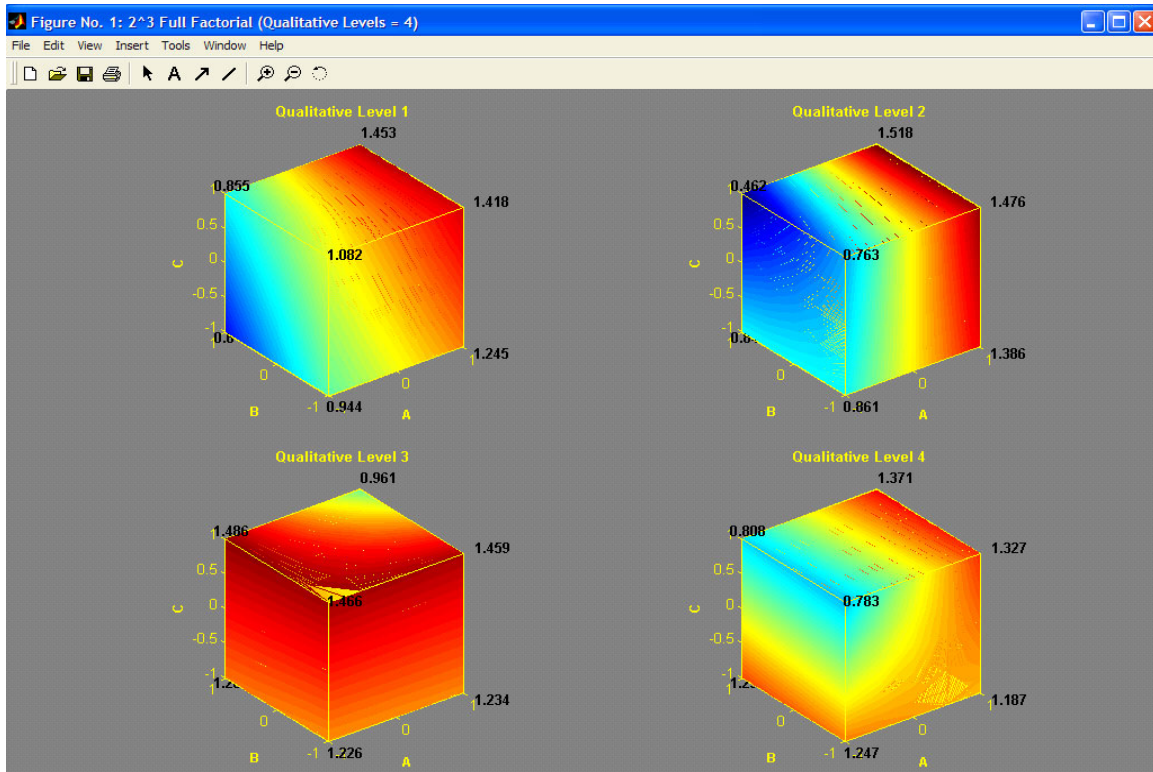
**Figure 64: 3-dimensional contour plots (Qualitative Levels = 4, Response Index at 50%)**

After verifying all cube faces it is confirmed the 3-dimensional contour plots cubes values are consistent with the 24 2-dimensional contour values generated by Minitab 14®. Moreover, it is evident which cube (qualitative level) exhibits better performance at the 50% level. Qualitative Level = 3, representing Flux E, shows in general higher response indexes compared to the other three fluxes. It is also easy to visualize that Qualitative Level = 2 (Flux C) has the worst response index with the A=-1, B=+1, C=+1 treatment. Another conclusion that derives from Figure 64 is the ability of all the four fluxes to achieve significant good performance for treatment conditions where factors A and C are at their high level.

In addition to compare the contour plots patterns it is possible to verify estimated model coefficients, for each of the four qualitative levels. Coefficients obtained with Minitab 14® and displayed earlier matches beta values obtained with the 3-D DOE tool developed for this project.

Table 8: 3-D DOE Contour Plot Coefficients for Qualitative Levels (Response Index at 50%)

| 50% | | | |
|---|---|---|---|
| Flux B | Flux C | Flux E | Flux F |
| 1.1095 | 1.0899 | 1.2847 | 1.1594 |
| 0.2393 | 0.3569 | -0.0747 | 0.1251 |
| -0.0628 | -0.0316 | -0.0615 | 0.0234 |
| 0.0925 | -0.0351 | 0.0583 | -0.0871 |
| 0.0800 | 0.0474 | -0.0750 | 0.0041 |
| -0.0058 | 0.0854 | -0.0582 | 0.1516 |
| 0.0148 | -0.0331 | -0.0580 | -0.0061 |
| -0.0145 | 0.0384 | -0.0545 | 0.0006 |

The same verification was performed with Response Index at 75% and 90%. Contour plots for both levels are attached to Appendix 6. The 3-D DOE Contour Plot application makes it easier to identify treatments that promote better responses based on selected factors levels. It is not so straightforward to draw conclusions from those plots, especially because analyst does not have a common scale to compare with between the different plots. In the other hand, for Response Index at 75% contour cubes demonstrate that Qualitative Level = 3 that was favorite on Response Index 50% is no longer that attractive (refer Appendix 6, Figure 74). Although, all the four factors once again resulted in higher responses at factors A and C at their high level.

The same approach for Response Index at 90% demonstrates that definitely Qualitative Level = 3 is not the best flux to choose if the engineers performing the

experiment want to maximize Response Index. Higher results are obtained with fluxes B, C, and F and those results are very close from each other (refer to Appendix 6 Figure A6-18). Even that general conclusion could not be effortlessly derived from Minitab 14® 2-dimesional contours presented in Appendix 6. Minitab 14® plots not only make it difficult to imagine treatment levels from 2-dimesional to a 3-dimensional perspective, but also add the barrier of color non-standardization between plots even from a single $2^3$ Full Factorial experiment.

# 7 GENERAL DISCUSSION, CONCLUSIONS & FUTURE WORK

## 7.1 General Discussion

The purpose of this work was to develop a graphical user interface Matlab 13® based tool that help analyze wave soldering experiments at a local electronics assembly company in an easy and useful 3-dimensional way. In addition, the tool has the capability of analyze a wide variety of DOE problems by means of Full Factorial, Box-Behnken and Central Composite Design approaches. After validating the 3-D DOE Contour Plots tool with test cases, the wave soldering experiment of a local electronics assembly company was analyzed. The results from 3-D DOE Contour Plots tool were compared with the results of the well-know statistical tool Minitab 14®. Overall results have been found equal to the result of Minitab 14®. Nevertheless, more useful visualization was obtained by the 3-D DOE Contour Plots tool in the majority of the cases. The 3-dimensional contour plots approach allow the user to fully rotate the cubes in order to visualize and relate the factors and responses behavior within the experimental region.

Doubtlessly, the 3-D DOE Contour Plot application is an innovative tool in an area of design of experiments that have not been explored so deeply yet. However, a good design of experiment is crucial in order for this or any other tool to be useful. Proper ranges should be set for screening factors and appropriate number of runs/replicates should be considered. Even when all previous conditions have been considered, it is the responsibility of the user to complement enhanced capabilities of multiple analysis methods in order to establish appropriate conclusions.

## 7.2 Conclusion

This work presented the development of a user-friendly 3-D DOE Contour Plots application. The problem addressed was a wave soldering design of experiment at the electronics industry. The tool was programmed using Matlab 13® and the results were compared with the well-know statistical software Minitab 14®. Functionalities of basic statistical computations such as residual plots such as histogram, normal probability plots, residuals versus fitted values and residuals versus time sequence were implemented in the tool. These capabilities were complementary to the main purpose of this tool which was the capability of full rotation of design blocks containing the contour levels of factor combinations. The rotation capability of the cubes is cleverly complemented with the application potential to standardize contours color schemes within a single experiment analysis. This so far has not been provided by other commercially available statistical software such as Minitab 14®. Those two features doubtlessly take advantage of what the DOE versatility is about.

In addition, the visualization of multiple factors at multiple levels no matter the nature of factor type, quantitative or qualitative levels, was possible. In general, the results obtained show the effectiveness, flexibility, and applicability of the 3-D DOE Contour Plots tool for the specific wave soldering problem at a local electronics industry. This project definitely sets milestones in terms of possibilities in the DOE world and opens doors for applications improvement in goodwill of analysis of experiments.

## 7.3    Future Work

The 3-D DOE Contour Plot graphical user interface developed in this project was designed to deal with a specific necessity in a local electronics assembly industry. The application offers an innovative, user-friendly approach of analyzing experiments contours under a varied range of experiments types (Full Factorial, Box-Behnken, and CCD) and a diverse combination of factors (quantitative factors and qualitative levels). However, because being a new concept on DOE analysis it also does have room for improvement. Among possible future work it could be considered including additional types of experiments to the ones already validated. Experiments such as fractional factorials are very common in screening experiments. Improvements in axes and title handling can also be listed as open actions for future work. It could be practical to add the capability for the user to label cube titles and axes as desired. Complementary general analysis such as ANOVAs, boxplots, and main effects can be added to improve application offerings.

A capability that would certainly add value to the tool will be to provide the user the option to display slices from a particular experiment cube. Currently the 3-D DOE Contour Plot Tool cubes plot the contours at the high and low level settings of the factors levels. Because of the nature of the code programmed for this tool, vectors of estimated values are calculated over the full range of each factor level. That means that the cubes display for each analysis are indeed solid cubes. Providing users with editable texts boxes where they can specify at which factor levels they want a contour slice could definitely

be a feature that would capture analysts' interest. It will allow a better understanding of responses for the full range as estimated by the fitted model.

In addition, a new perspective could be given to the experiment incorporating human factors aspects to the application's development. Executing surveys or conducting observed studies related to the application capabilities and appearance could help define new standards or make significant contributions to 3-dimensional approach in DOE.

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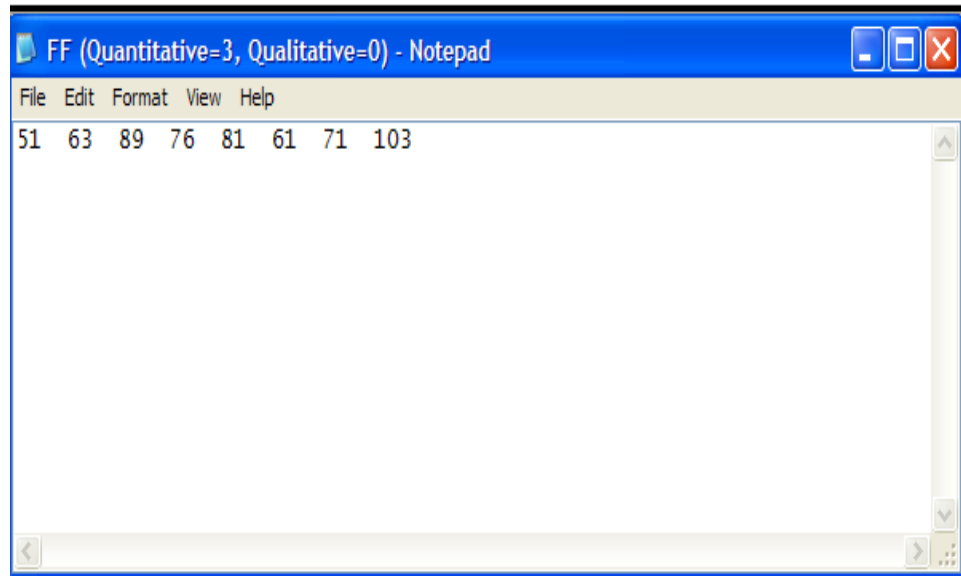
# Appendix 1: 3-D DOE Contour Plot Application Analysis Capabilities

**Table A1- 1: 3-D DOE Contour Plot Application Analysis Capabilities**

| Scenario | Number of Qualitative (Discrete) Levels | Number of Quantitaive (Continous) Factors |
|---|---|---|
| **1** | 0 | 3 |
| | | 4 |
| | | 5 |
| | | 6 |
| **2** | 1 | 3 |
| | | 4 |
| | | 5 |
| | | 6 |
| **3** | 2 | 3 |
| | | 4 |
| | | 5 |
| **4** | 3 | 3 |
| | | 4 |
| **5** | 4 | 3 |
| | | 4 |
| **6** | 5 | 3 |
| **7** | 6 | 3 |

\
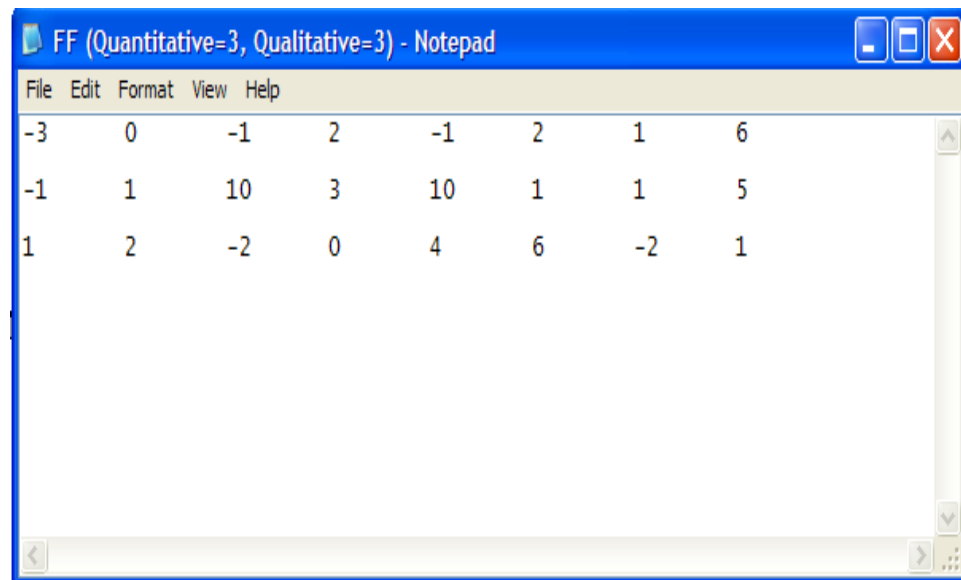
# Appendix 2: Data Files Format



**Figure A2- 1: Response Data Format - Quantitative Factors**



**Figure A2- 2: Response Data Format - Quantitative Factors and Qualitative Levels**
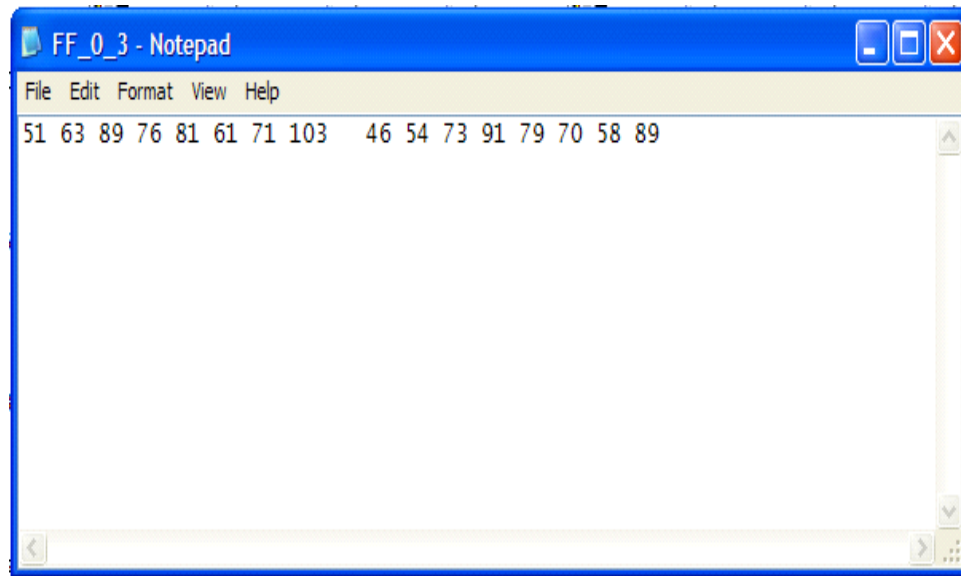
**Figure A2- 3: Response Data Format – Experiment with Replicates**

# Appendix 3: Full Factorial Design Matrices (Xs)

**Table A3- 1: Full Factorial Design Matrix (X) for Quantitative Factors = 3**

| A | B | C | AB | AC | BC | ABC |
|---|---|---|---|---|---|---|
| -1 | -1 | -1 | 1 | 1 | 1 | -1 |
| 1 | -1 | -1 | -1 | -1 | 1 | 1 |
| -1 | 1 | -1 | -1 | 1 | -1 | 1 |
| 1 | 1 | -1 | 1 | -1 | -1 | -1 |
| -1 | -1 | 1 | 1 | -1 | -1 | 1 |
| 1 | -1 | 1 | -1 | 1 | -1 | -1 |
| -1 | 1 | 1 | -1 | -1 | 1 | -1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

**Table A3- 2: Full Factorial Design Matrix (X) for Quantitative Factors = 4**

| A | B | C | D | AB | AC | AD | BC | BD | CD | ABC | ABD | ACD | BCD | ABCD |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 |
| 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 |
| -1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | -1 |
| 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 |
| -1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 |
| 1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | 1 |
| -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 |
| 1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 |
| -1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 |
| 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 |
| -1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 |
| 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 |
| -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | 1 |
| 1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 |
| -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

**Table A3- 3: Full Factorial Design Matrix (X) for Quantitative Factors = 5**

| A | B | C | D | E |
|---|---|---|---|---|
| -1 | -1 | -1 | -1 | -1 |
| 1 | -1 | -1 | -1 | -1 |
| -1 | 1 | -1 | -1 | -1 |
| 1 | 1 | -1 | -1 | -1 |
| -1 | -1 | 1 | -1 | -1 |
| 1 | -1 | 1 | -1 | -1 |
| -1 | 1 | 1 | -1 | -1 |
| 1 | 1 | 1 | -1 | -1 |
| -1 | -1 | -1 | 1 | -1 |
| 1 | -1 | -1 | 1 | -1 |
| -1 | 1 | -1 | 1 | -1 |
| 1 | 1 | -1 | 1 | -1 |
| -1 | -1 | 1 | 1 | -1 |
| 1 | -1 | 1 | 1 | -1 |
| -1 | 1 | 1 | 1 | -1 |
| 1 | 1 | 1 | 1 | -1 |
| -1 | -1 | -1 | -1 | 1 |
| 1 | -1 | -1 | -1 | 1 |
| -1 | 1 | -1 | -1 | 1 |
| 1 | 1 | -1 | -1 | 1 |
| -1 | -1 | 1 | -1 | 1 |
| 1 | -1 | 1 | -1 | 1 |
| -1 | 1 | 1 | -1 | 1 |
| 1 | 1 | 1 | -1 | 1 |
| -1 | -1 | -1 | 1 | 1 |
| 1 | -1 | -1 | 1 | 1 |
| -1 | 1 | -1 | 1 | 1 |
| 1 | 1 | -1 | 1 | 1 |
| -1 | -1 | 1 | 1 | 1 |
| 1 | -1 | 1 | 1 | 1 |
| -1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

**Table A3- 4: Full Factorial Design Matrix (X) for Quantitative Factors = 6**

| A | B | C | D | E | F |
|---|---|---|---|---|---|
| -1 | -1 | -1 | -1 | -1 | -1 |
| 1 | -1 | -1 | -1 | -1 | -1 |
| -1 | 1 | -1 | -1 | -1 | -1 |
| 1 | 1 | -1 | -1 | -1 | -1 |
| -1 | -1 | 1 | -1 | -1 | -1 |
| 1 | -1 | 1 | -1 | -1 | -1 |
| -1 | 1 | 1 | -1 | -1 | -1 |
| 1 | 1 | 1 | -1 | -1 | -1 |
| -1 | -1 | -1 | 1 | -1 | -1 |
| 1 | -1 | -1 | 1 | -1 | -1 |
| -1 | 1 | -1 | 1 | -1 | -1 |
| 1 | 1 | -1 | 1 | -1 | -1 |
| -1 | -1 | 1 | 1 | -1 | -1 |
| 1 | -1 | 1 | 1 | -1 | -1 |
| -1 | 1 | 1 | 1 | -1 | -1 |
| 1 | 1 | 1 | 1 | -1 | -1 |
| -1 | -1 | -1 | -1 | 1 | -1 |
| 1 | -1 | -1 | -1 | 1 | -1 |
| -1 | 1 | -1 | -1 | 1 | -1 |
| 1 | 1 | -1 | -1 | 1 | -1 |
| -1 | -1 | 1 | -1 | 1 | -1 |
| 1 | -1 | 1 | -1 | 1 | -1 |
| -1 | 1 | 1 | -1 | 1 | -1 |
| 1 | 1 | 1 | -1 | 1 | -1 |
| -1 | -1 | -1 | 1 | 1 | -1 |
| 1 | -1 | -1 | 1 | 1 | -1 |
| -1 | 1 | -1 | 1 | 1 | -1 |
| 1 | 1 | -1 | 1 | 1 | -1 |
| -1 | -1 | 1 | 1 | 1 | -1 |
| 1 | -1 | 1 | 1 | 1 | -1 |
| -1 | 1 | 1 | 1 | 1 | -1 |
| 1 | 1 | 1 | 1 | 1 | -1 |
| -1 | -1 | -1 | -1 | -1 | 1 |
| 1 | -1 | -1 | -1 | -1 | 1 |
| -1 | 1 | -1 | -1 | -1 | 1 |
| 1 | 1 | -1 | -1 | -1 | 1 |
| -1 | -1 | 1 | -1 | -1 | 1 |
| 1 | -1 | 1 | -1 | -1 | 1 |
| -1 | 1 | 1 | -1 | -1 | 1 |
| 1 | 1 | 1 | -1 | -1 | 1 |
| -1 | -1 | -1 | 1 | -1 | 1 |
| 1 | -1 | -1 | 1 | -1 | 1 |
| -1 | 1 | -1 | 1 | -1 | 1 |
| 1 | 1 | -1 | 1 | -1 | 1 |
| -1 | -1 | 1 | 1 | -1 | 1 |
| 1 | -1 | 1 | 1 | -1 | 1 |
| -1 | 1 | 1 | 1 | -1 | 1 |
| 1 | 1 | 1 | 1 | -1 | 1 |
| -1 | -1 | -1 | -1 | 1 | 1 |
| 1 | -1 | -1 | -1 | 1 | 1 |
| -1 | 1 | -1 | -1 | 1 | 1 |
| 1 | 1 | -1 | -1 | 1 | 1 |
| -1 | -1 | 1 | -1 | 1 | 1 |
| 1 | -1 | 1 | -1 | 1 | 1 |
| -1 | 1 | 1 | -1 | 1 | 1 |
| 1 | 1 | 1 | -1 | 1 | 1 |
| -1 | -1 | -1 | 1 | 1 | 1 |
| 1 | -1 | -1 | 1 | 1 | 1 |
| -1 | 1 | -1 | 1 | 1 | 1 |
| 1 | 1 | -1 | 1 | 1 | 1 |
| -1 | -1 | 1 | 1 | 1 | 1 |
| 1 | -1 | 1 | 1 | 1 | 1 |
| -1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

# Appendix 4: Box-Behnken Design Matrices (X's)

**Table A4- 1: Box- Behnken Design Matrix (X) for Quantitative Factors = 3**

| A | B | C | AA | BB | CC | AB | AC | BC |
|---|---|---|----|----|----|----|----|----|
| -1 | -1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | -1 | 0 | 1 | 1 | 0 | -1 | 0 | 0 |
| -1 | 1 | 0 | 1 | 1 | 0 | 0 | -1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| -1 | 0 | -1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 1 | 0 | -1 | 1 | 0 | 1 | 0 | -1 | 0 |
| -1 | 0 | 1 | 1 | 0 | 1 | 0 | -1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | -1 | -1 | 0 | 1 | 1 | 0 | 0 | 1 |
| 0 | 1 | -1 | 0 | 1 | 1 | 0 | 0 | -1 |
| 0 | -1 | 1 | 0 | 1 | 1 | 0 | 0 | -1 |
| 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

**Table A4- 2: Box- Behnken Design Matrix (X) for Quantitative Factors = 4**

| A | B | C | D | AA | BB | CC | DD | AB | AC | AD | BC | BD | CD |
|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
| -1 | -1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | -1 | 0 | 0 | 1 | 1 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 |
| -1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | -1 | -1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | -1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | -1 |
| 0 | 0 | -1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | -1 |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| -1 | 0 | -1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | -1 | 0 | 1 | 0 | 1 | 0 | 0 | -1 | 0 | 0 | 0 | 0 |
| -1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | -1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | -1 | 0 | -1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | -1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | -1 | 0 |
| 0 | -1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | -1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| -1 | 0 | 0 | -1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | -1 | 1 | 0 | 0 | 1 | 0 | 0 | -1 | 0 | 0 | 0 |
| -1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | -1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | -1 | -1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | -1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | -1 | 0 | 0 |
| 0 | -1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | -1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

**Table A4- 3: Box- Behnken Design Matrix (X) for Quantitative Factors = 5**

| A | B | C | D | E | AA | BB | CC | DD | EE | AB | AC | AD | AE | BC | BD | BE | CD | CE | DE |
|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| -1 | -1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | -1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | -1 | -1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | -1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 |
| 0 | 0 | -1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | -1 | 0 | 0 | -1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | -1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 | 0 |
| 0 | -1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| -1 | 0 | -1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | -1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | -1 | -1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | -1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| 0 | 0 | 0 | -1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | -1 | -1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | -1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 |
| 0 | -1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| -1 | 0 | 0 | -1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | -1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | -1 | 0 | -1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | -1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 |
| 0 | 0 | -1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| -1 | 0 | 0 | 0 | -1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | -1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 |
| -1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | -1 | 0 | -1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | -1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 |
| 0 | -1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

**Table A4- 4: Box- Behnken Design Matrix (X) for Quantitative Factors = 6**

| A | B | C | D | E | F | AA | BB | CC | DD | EE | FF | AB | AC | AD | AE | AF | BC | BD | BE | BF | CD | CE | CF | DE | DF | EF |
|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| -1 | -1 | 0 | -1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | -1 | 0 | -1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | -1 | 0 | -1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -1 | 1 | 0 | -1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | -1 | 0 | 1 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | -1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | -1 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -1 | -1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | -1 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | -1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | -1 | 0 | 1 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | -1 | 0 | -1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | -1 | -1 | 0 | -1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | -1 | 0 | -1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | -1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | -1 | 1 | 0 | -1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 1 | 0 | 0 | -1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | -1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | -1 | 0 | 0 | -1 | 0 | 0 | 0 | 0 |
| 0 | -1 | -1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | -1 | 0 | 0 | -1 | 0 | 0 | 0 | 0 |
| 0 | 1 | -1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 1 | 0 | 0 | -1 | 0 | 0 | 0 | 0 |
| 0 | -1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | -1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | -1 | -1 | 0 | -1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 0 | 1 | -1 | 0 | -1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | -1 | 0 | 1 | 0 |
| 0 | 0 | -1 | 1 | 0 | -1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 1 | 0 | -1 | 0 |
| 0 | 0 | 1 | 1 | 0 | -1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | -1 | 0 | -1 | 0 |
| 0 | 0 | -1 | -1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | -1 | 0 | -1 | 0 |
| 0 | 0 | 1 | -1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 1 | 0 | -1 | 0 |
| 0 | 0 | -1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | -1 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| -1 | 0 | 0 | -1 | -1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | -1 | -1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | -1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| -1 | 0 | 0 | 1 | -1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | -1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 |
| 1 | 0 | 0 | 1 | -1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 |
| -1 | 0 | 0 | -1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 |
| 1 | 0 | 0 | -1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | -1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 |
| -1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | -1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | -1 | 0 | 0 | -1 | -1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | -1 | -1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | -1 | 0 | 0 | 1 | -1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | -1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | -1 | 0 | 0 | -1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | -1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | -1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| -1 | 0 | -1 | 0 | 0 | -1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | -1 | 0 | 0 | -1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | -1 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| -1 | 0 | 1 | 0 | 0 | -1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | -1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | -1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 | 0 |
| -1 | 0 | -1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 | 0 |
| 1 | 0 | -1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | -1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 | 0 |
| -1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | -1 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

# Appendix 5: CCD Design Matrices (alpha = default)

**Table A5- 1: CCD Design Matrix (X) for Quantitative Factors = 3**

| A | B | C |
|---|---|---|
| **-1** | -1 | -1 |
| 1 | -1 | -1 |
| -1 | 1 | -1 |
| 1 | 1 | -1 |
| -1 | -1 | 1 |
| 1 | -1 | 1 |
| -1 | 1 | 1 |
| 1 | 1 | 1 |
| -1.68179 | 0 | 0 |
| 1.68179 | 0 | 0 |
| 0 | -1.68179 | 0 |
| 0 | 1.68179 | 0 |
| 0 | 0 | -1.68179 |
| 0 | 0 | 1.68179 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

**Table A5- 2: CCD Design Matrix (X) for Quantitative Factors = 4**

| A | B | C | D |
|---|---|---|---|
| -1 | -1 | -1 | -1 |
| 1 | -1 | -1 | -1 |
| -1 | 1 | -1 | -1 |
| 1 | 1 | -1 | -1 |
| -1 | -1 | 1 | -1 |
| 1 | -1 | 1 | -1 |
| -1 | 1 | 1 | -1 |
| 1 | 1 | 1 | -1 |
| -1 | -1 | -1 | 1 |
| 1 | -1 | -1 | 1 |
| -1 | 1 | -1 | 1 |
| 1 | 1 | -1 | 1 |
| -1 | -1 | 1 | 1 |
| 1 | -1 | 1 | 1 |
| -1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| -2 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 0 | -2 | 0 | 0 |
| 0 | 2 | 0 | 0 |
| 0 | 0 | -2 | 0 |
| 0 | 0 | 2 | 0 |
| 0 | 0 | 0 | -2 |
| 0 | 0 | 0 | 2 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |

**Table A5- 3: CCD Design Matrix (X) for Quantitative Factors = 5**

| A | B | C | D | E |
|---|---|---|---|---|
| -1 | -1 | -1 | -1 | -1 |
| 1 | -1 | -1 | -1 | -1 |
| -1 | 1 | -1 | -1 | -1 |
| 1 | 1 | -1 | -1 | -1 |
| -1 | -1 | 1 | -1 | -1 |
| 1 | -1 | 1 | -1 | -1 |
| -1 | 1 | 1 | -1 | -1 |
| 1 | 1 | 1 | -1 | -1 |
| -1 | -1 | -1 | 1 | -1 |
| 1 | -1 | -1 | 1 | -1 |
| -1 | 1 | -1 | 1 | -1 |
| 1 | 1 | -1 | 1 | -1 |
| -1 | -1 | 1 | 1 | -1 |
| 1 | -1 | 1 | 1 | -1 |
| -1 | 1 | 1 | 1 | -1 |
| 1 | 1 | 1 | 1 | -1 |
| -1 | -1 | -1 | -1 | 1 |
| 1 | -1 | -1 | -1 | 1 |
| -1 | 1 | -1 | -1 | 1 |
| 1 | 1 | -1 | -1 | 1 |
| -1 | -1 | 1 | -1 | 1 |
| 1 | -1 | 1 | -1 | 1 |
| -1 | 1 | 1 | -1 | 1 |
| 1 | 1 | 1 | -1 | 1 |
| -1 | -1 | -1 | 1 | 1 |
| 1 | -1 | -1 | 1 | 1 |
| -1 | 1 | -1 | 1 | 1 |
| 1 | 1 | -1 | 1 | 1 |
| -1 | -1 | 1 | 1 | 1 |
| 1 | -1 | 1 | 1 | 1 |
| -1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| -2.3784 | 0 | 0 | 0 | 0 |
| 2.37841 | 0 | 0 | 0 | 0 |
| 0 | -2.3784 | 0 | 0 | 0 |
| 0 | 2.37841 | 0 | 0 | 0 |
| 0 | 0 | -2.3784 | 0 | 0 |
| 0 | 0 | 2.37841 | 0 | 0 |
| 0 | 0 | 0 | -2.3784 | 0 |
| 0 | 0 | 0 | 2.37841 | 0 |
| 0 | 0 | 0 | 0 | -2.3784 |
| 0 | 0 | 0 | 0 | 2.37841 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

**Table A5- 4: CCD Design Matrix (X) for Quantitative Factors = 6**

| A | B | C | D | E | F |
|---|---|---|---|---|---|
| -1 | -1 | -1 | -1 | -1 | -1 |
| 1 | -1 | -1 | -1 | -1 | -1 |
| -1 | 1 | -1 | -1 | -1 | -1 |
| 1 | 1 | -1 | -1 | -1 | -1 |
| -1 | -1 | 1 | -1 | -1 | -1 |
| 1 | -1 | 1 | -1 | -1 | -1 |
| -1 | 1 | 1 | -1 | -1 | -1 |
| 1 | 1 | 1 | -1 | -1 | -1 |
| -1 | -1 | -1 | 1 | -1 | -1 |
| 1 | -1 | -1 | 1 | -1 | -1 |
| -1 | 1 | -1 | 1 | -1 | -1 |
| 1 | 1 | -1 | 1 | -1 | -1 |
| -1 | -1 | 1 | 1 | -1 | -1 |
| 1 | -1 | 1 | 1 | -1 | -1 |
| -1 | 1 | 1 | 1 | -1 | -1 |
| 1 | 1 | 1 | 1 | -1 | -1 |
| -1 | -1 | -1 | -1 | 1 | -1 |
| 1 | -1 | -1 | -1 | 1 | -1 |
| -1 | 1 | -1 | -1 | 1 | -1 |
| 1 | 1 | -1 | -1 | 1 | -1 |
| -1 | -1 | 1 | -1 | 1 | -1 |
| 1 | -1 | 1 | -1 | 1 | -1 |
| -1 | 1 | 1 | -1 | 1 | -1 |
| 1 | 1 | 1 | -1 | 1 | -1 |
| -1 | -1 | -1 | 1 | 1 | -1 |
| 1 | -1 | -1 | 1 | 1 | -1 |
| -1 | 1 | -1 | 1 | 1 | -1 |
| 1 | 1 | -1 | 1 | 1 | -1 |
| -1 | -1 | 1 | 1 | 1 | -1 |
| 1 | -1 | 1 | 1 | 1 | -1 |
| -1 | 1 | 1 | 1 | 1 | -1 |
| 1 | 1 | 1 | 1 | 1 | -1 |
| -1 | -1 | -1 | -1 | -1 | 1 |
| 1 | -1 | -1 | -1 | -1 | 1 |
| -1 | 1 | -1 | -1 | -1 | 1 |
| 1 | 1 | -1 | -1 | -1 | 1 |
| -1 | -1 | 1 | -1 | -1 | 1 |
| 1 | -1 | 1 | -1 | -1 | 1 |
| -1 | 1 | 1 | -1 | -1 | 1 |
| 1 | 1 | 1 | -1 | -1 | 1 |
| -1 | -1 | -1 | 1 | -1 | 1 |
| 1 | -1 | -1 | 1 | -1 | 1 |
| -1 | 1 | -1 | 1 | -1 | 1 |
| 1 | 1 | -1 | 1 | -1 | 1 |
| -1 | -1 | 1 | 1 | -1 | 1 |
| 1 | -1 | 1 | 1 | -1 | 1 |
| -1 | 1 | 1 | 1 | -1 | 1 |
| 1 | 1 | 1 | 1 | -1 | 1 |
| -1 | -1 | -1 | -1 | 1 | 1 |
| 1 | -1 | -1 | -1 | 1 | 1 |
| -1 | 1 | -1 | -1 | 1 | 1 |
| 1 | 1 | -1 | -1 | 1 | 1 |
| -1 | -1 | 1 | -1 | 1 | 1 |
| 1 | -1 | 1 | -1 | 1 | 1 |
| -1 | 1 | 1 | -1 | 1 | 1 |
| 1 | 1 | 1 | -1 | 1 | 1 |
| -1 | -1 | -1 | 1 | 1 | 1 |
| 1 | -1 | -1 | 1 | 1 | 1 |
| -1 | 1 | -1 | 1 | 1 | 1 |
| 1 | 1 | -1 | 1 | 1 | 1 |
| -1 | -1 | 1 | 1 | 1 | 1 |
| 1 | -1 | 1 | 1 | 1 | 1 |
| -1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| -2.8284 | 0 | 0 | 0 | 0 | 0 |
| 2.82843 | 0 | 0 | 0 | 0 | 0 |
| 0 | -2.8284 | 0 | 0 | 0 | 0 |
| 0 | 2.82843 | 0 | 0 | 0 | 0 |
| 0 | 0 | -2.8284 | 0 | 0 | 0 |
| 0 | 0 | 2.82843 | 0 | 0 | 0 |
| 0 | 0 | 0 | -2.8284 | 0 | 0 |
| 0 | 0 | 0 | 2.82843 | 0 | 0 |
| 0 | 0 | 0 | 0 | -2.8284 | 0 |
| 0 | 0 | 0 | 0 | 2.82843 | 0 |
| 0 | 0 | 0 | 0 | 0 | -2.8284 |
| 0 | 0 | 0 | 0 | 0 | 2.82843 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |

# Appendix 6: Case Study results for 75% and 90% Response Indexes

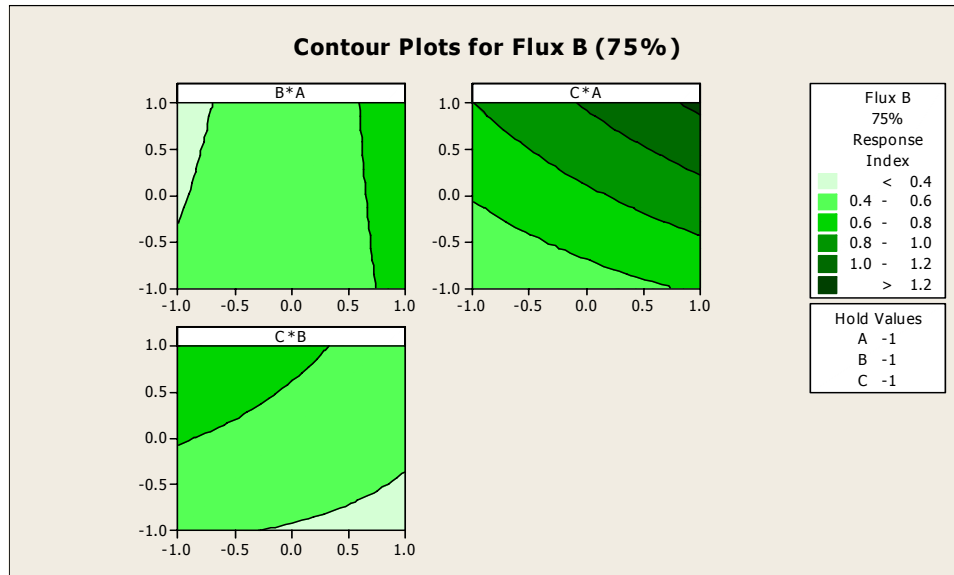## Minitab 14® Contour Plots at Board Thickness = 75%

- *(Flux B)*
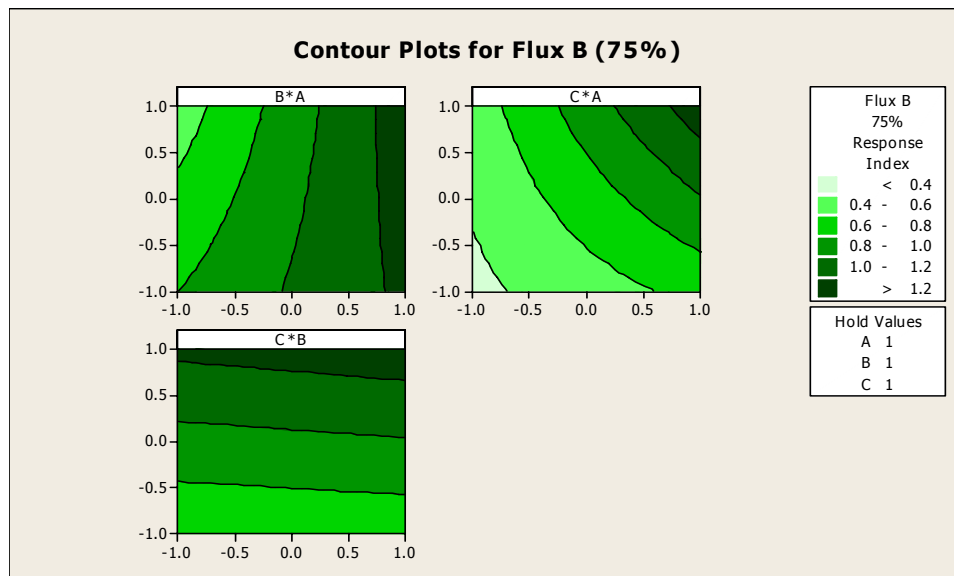


**Figure A6- 1: Flux B 75% (Low Settings)**
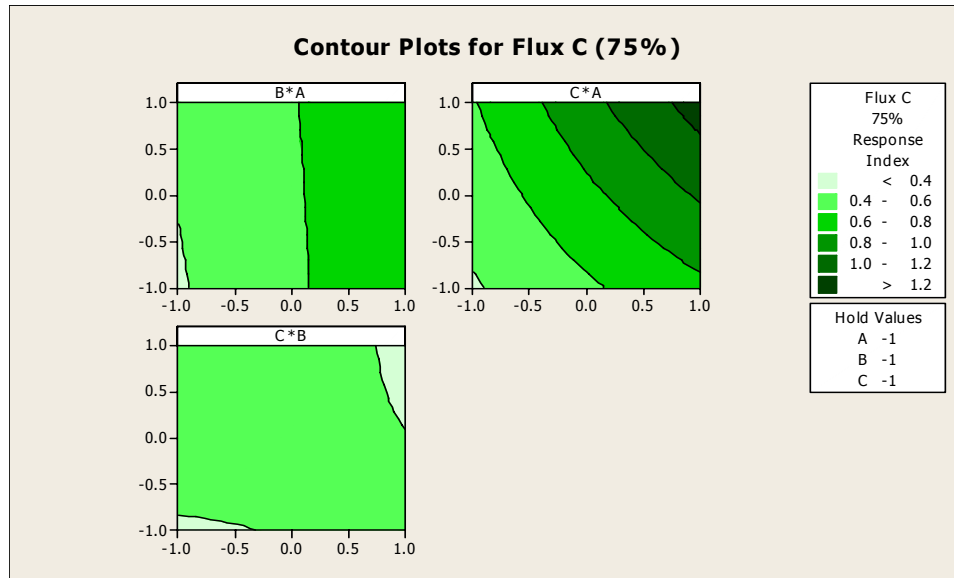


**Figure A6- 2: Flux B 75% (High Settings)**

- *Flux C*
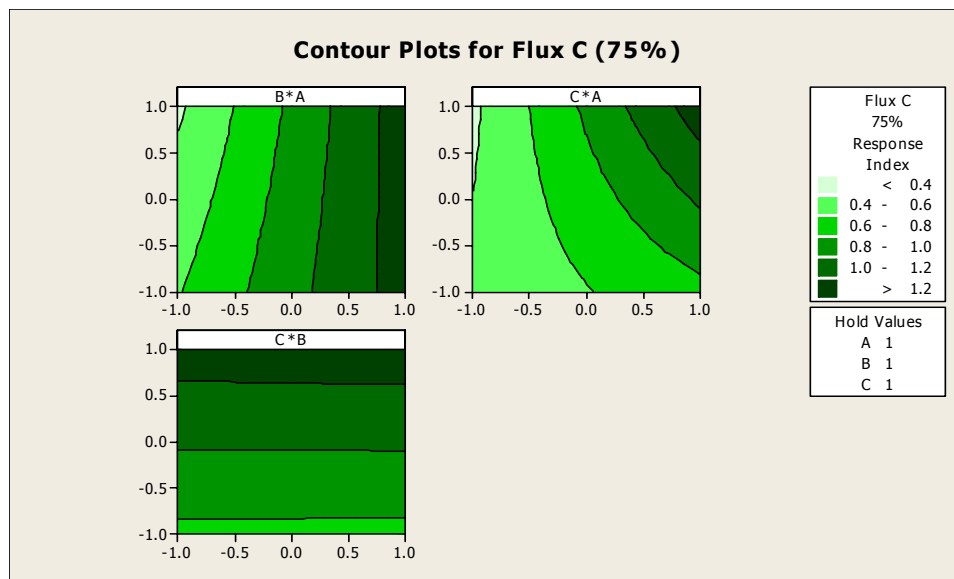


**Figure A6- 3: Flux C 75% (Low Settings)**



**Figure A6- 4: Flux C 75% (High Settings)**
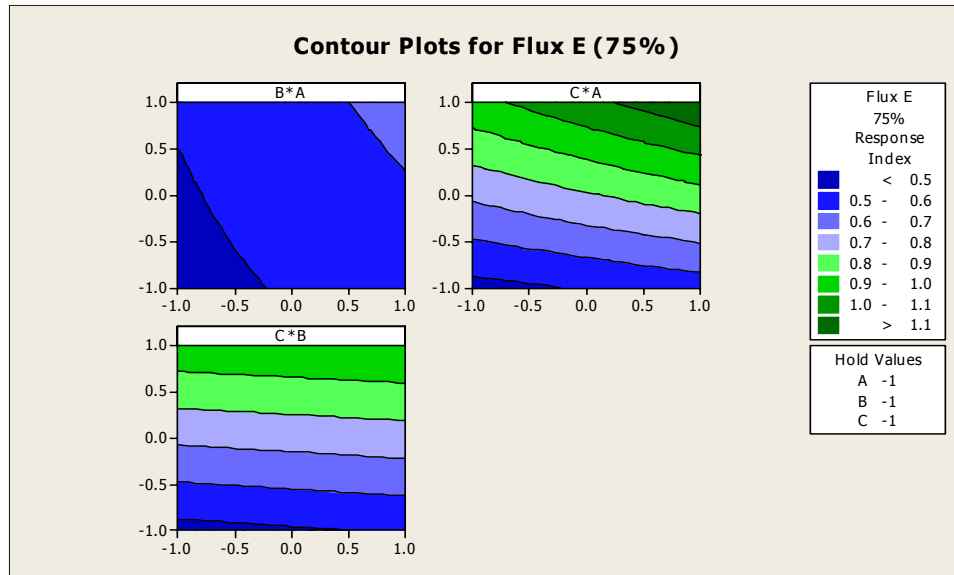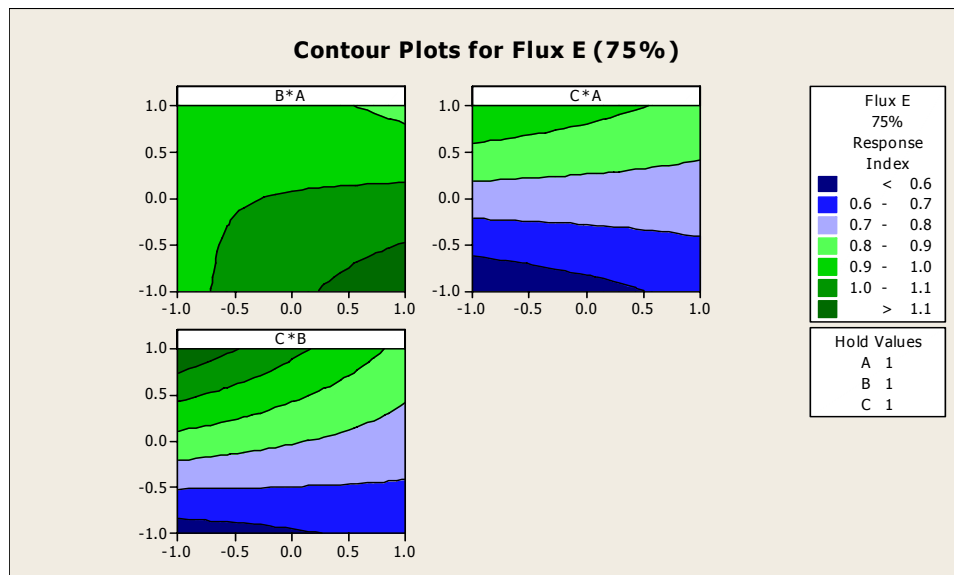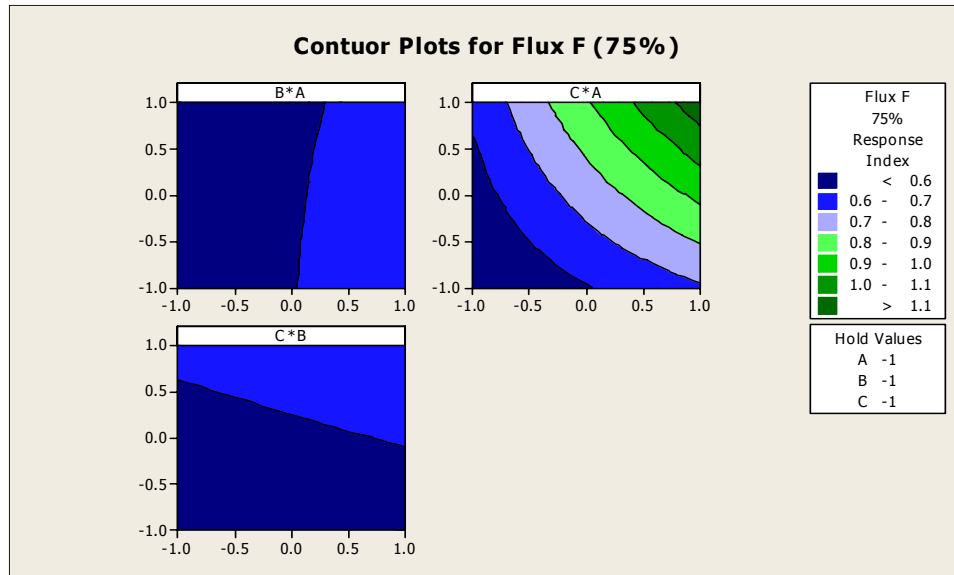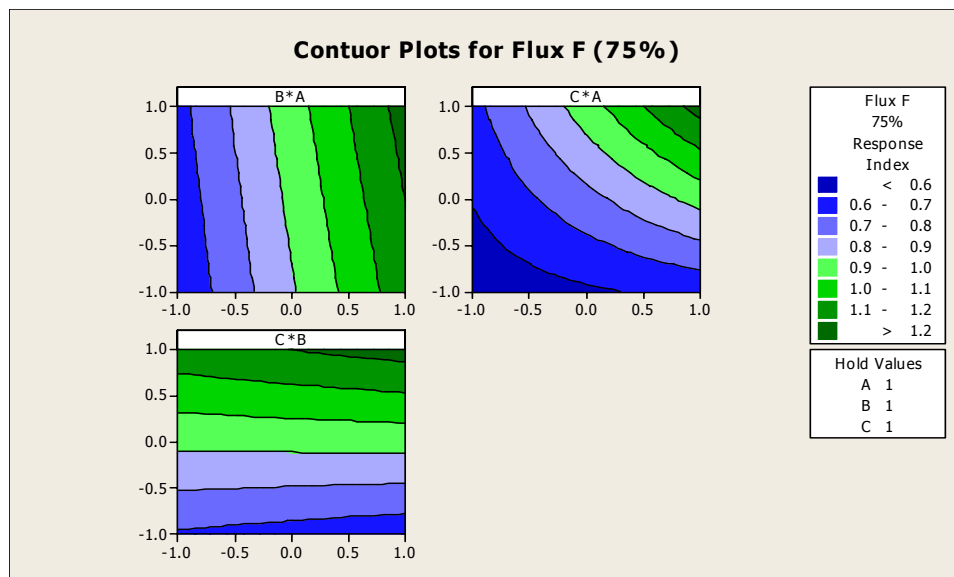
- *Flux E*



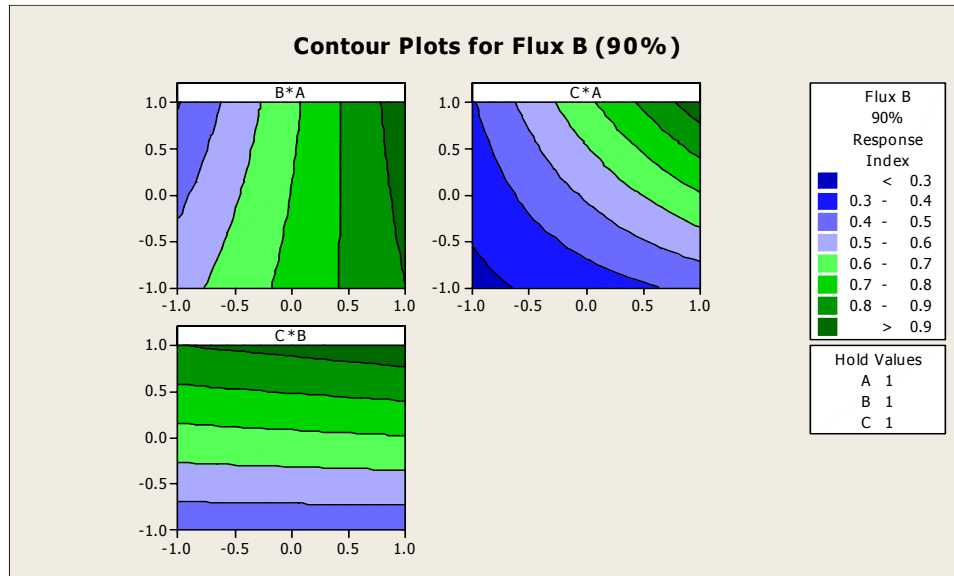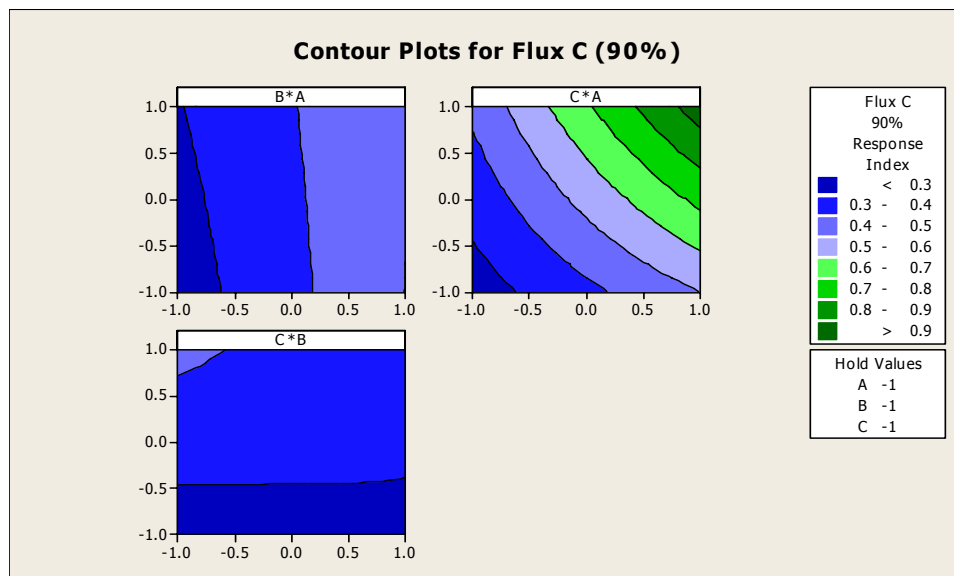**Figure A6- 5: Flux E 75% (Low Settings)**



**Figure A6- 6: Flux E 75% (High Settings)**

- *Flux F*



**Figure A6- 7: Flux F 75% (Low Settings)**



**Figure A6- 8: Flux F 75% (High Settings)**

**3-D DOE Contour Plot at Board Thickness = 75%**
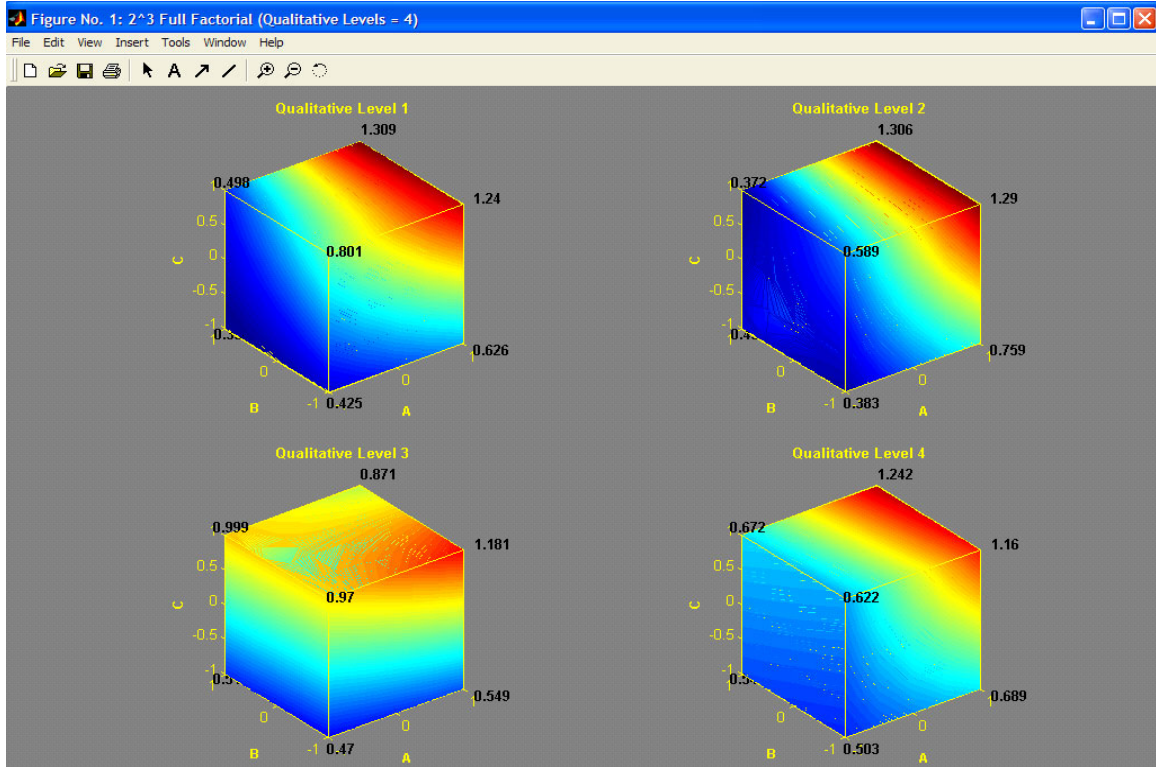


**Figure A6- 9: 3-dimensional contour plots (Qualitative Levels = 4, Response Index at 75%)**

**Table A6- 1: Case Study Minitab 14® Coefficients for Response Index at 75%**

**Coefficients**

| Term | Flux B | Flux C | Flux E | Flux F |
|---|---|---|---|---|
| Constant | 0.7395 | 0.73513 | 0.7725 | 0.757625 |
| A | 0.22 | 0.29087 | 0.03525 | 0.173125 |
| B | −0.0335 | −0.02012 | −0.02 | 0.014125 |
| C | 0.2225 | 0.15413 | 0.23275 | 0.166375 |
| A*B | 0.06 | 0.02163 | −0.03725 | −0.007875 |
| A*C | 0.0925 | 0.11788 | −0.0145 | 0.103875 |
| B*C | −0.025 | −0.03013 | −0.05025 | 0.018875 |
| A*B*C | 0.033 | 0.03663 | −0.0475 | 0.015875 |

**Table A6- 2: Case Study 3-DOE Contour Plot Coefficients for Response Index at 75%**

| 75% | | | |
|---|---|---|---|
| **Flux B** | **Flux C** | **Flux E** | **Flux F** |
| 0.7395 | 0.7351 | 0.7725 | 0.7576 |
| 0.2200 | 0.2909 | 0.0353 | 0.1731 |
| -0.0335 | -0.0201 | -0.0200 | 0.0141 |
| 0.2225 | 0.1541 | 0.2327 | 0.1664 |
| 0.0600 | 0.0216 | -0.0373 | -0.0079 |
| 0.0925 | 0.1179 | -0.0145 | 0.1039 |
| -0.0250 | -0.0301 | -0.0503 | 0.0189 |
| 0.0330 | 0.0366 | -0.0475 | 0.0159 |

## Minitab 14® Contour Plots at Board Thickness = 90%

- *Flux B*
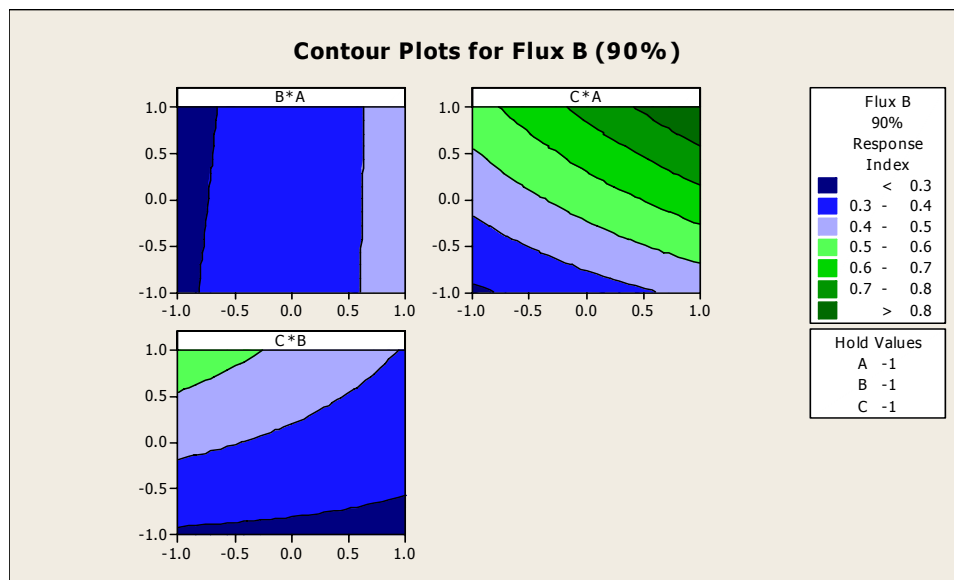


**Figure A6- 10: Flux B 90% (Low Settings)**

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**Figure A6- 11: Flux B 90% (High Settings)**

- *Flux C*



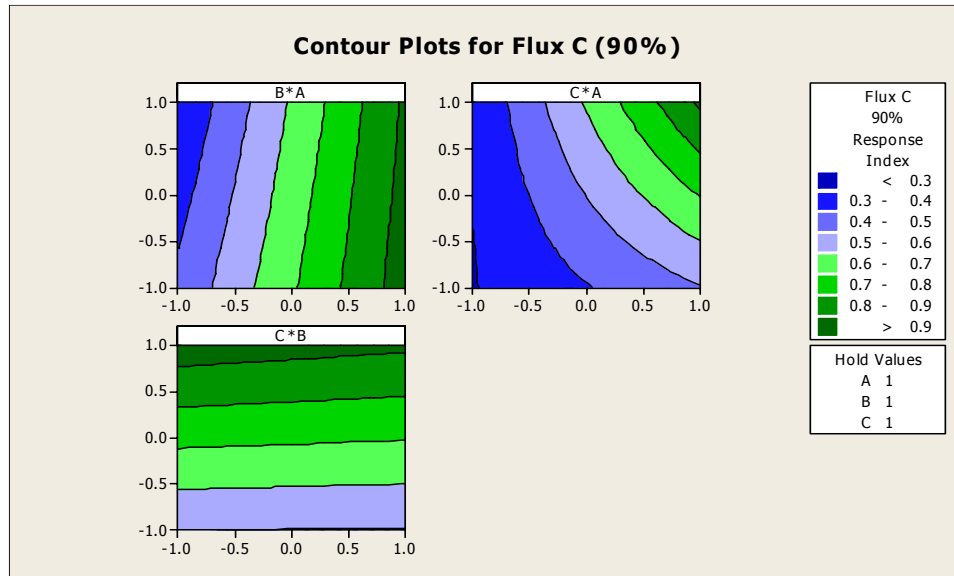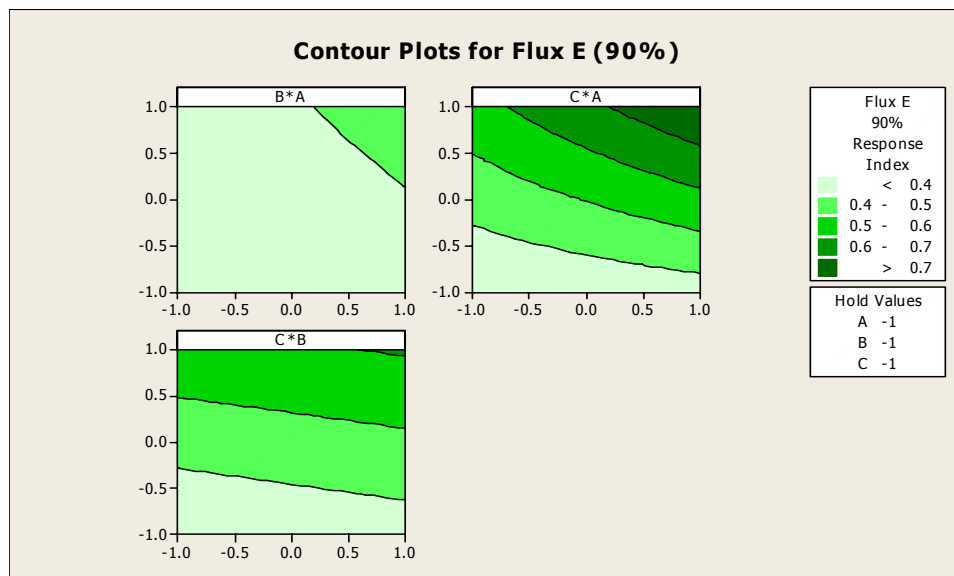**Figure A6- 12: Flux C 90% (Low Settings)**

**Figure A6- 13: Flux C 90% (High Settings)**

- *Flux E*
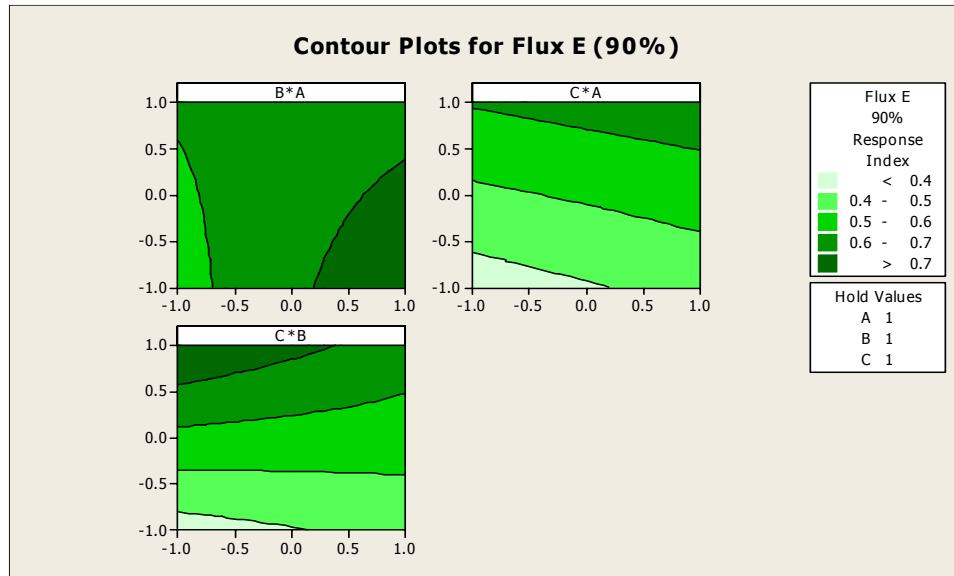


**Figure**

**Figure A6- 14: Flux E 90% (Low Settings)**

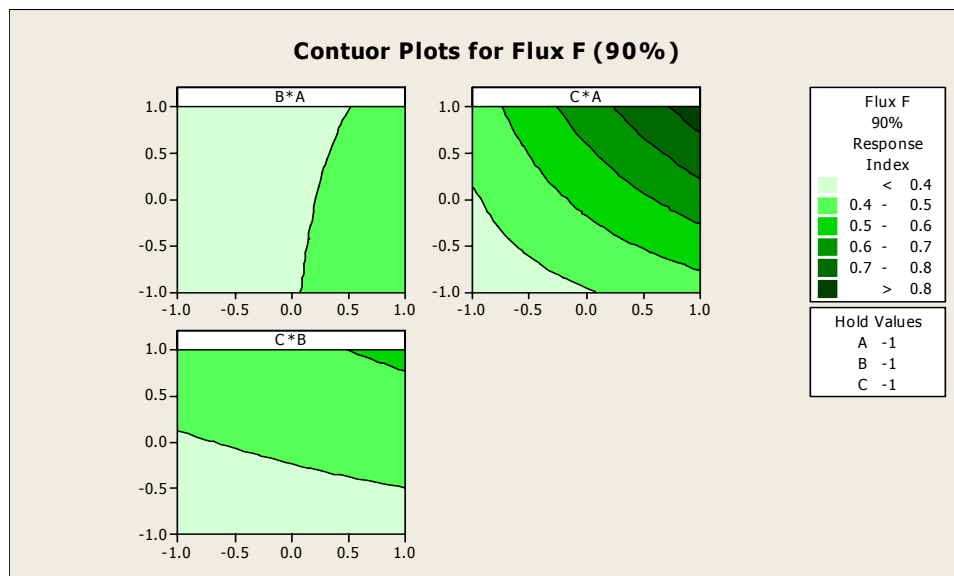**Figure A6- 15: Flux E 90% (High Settings)**

- *Flux F*



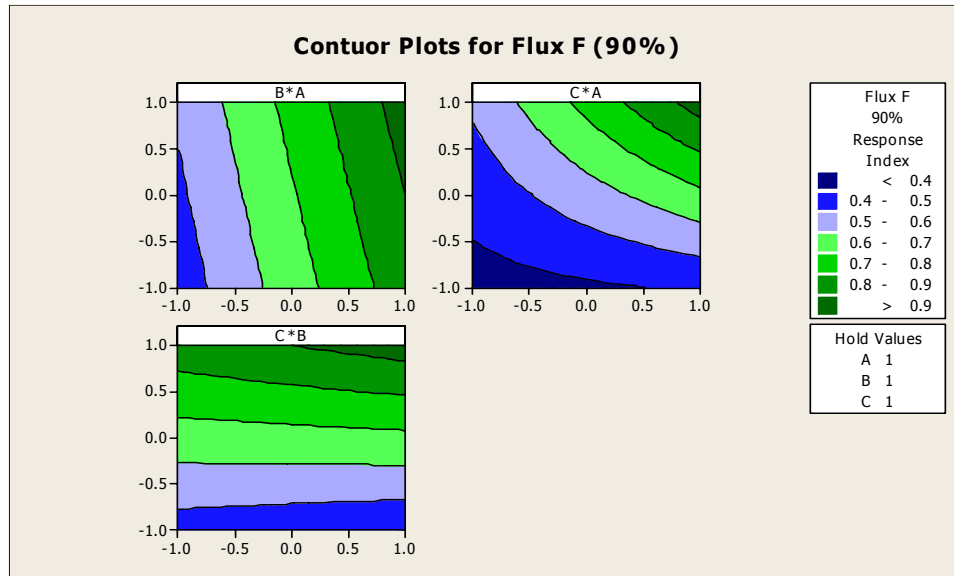**Figure A6- 16: Flux F 90% (Low Settings)**

**Figure A6- 17: Flux F 90% (High Settings)**

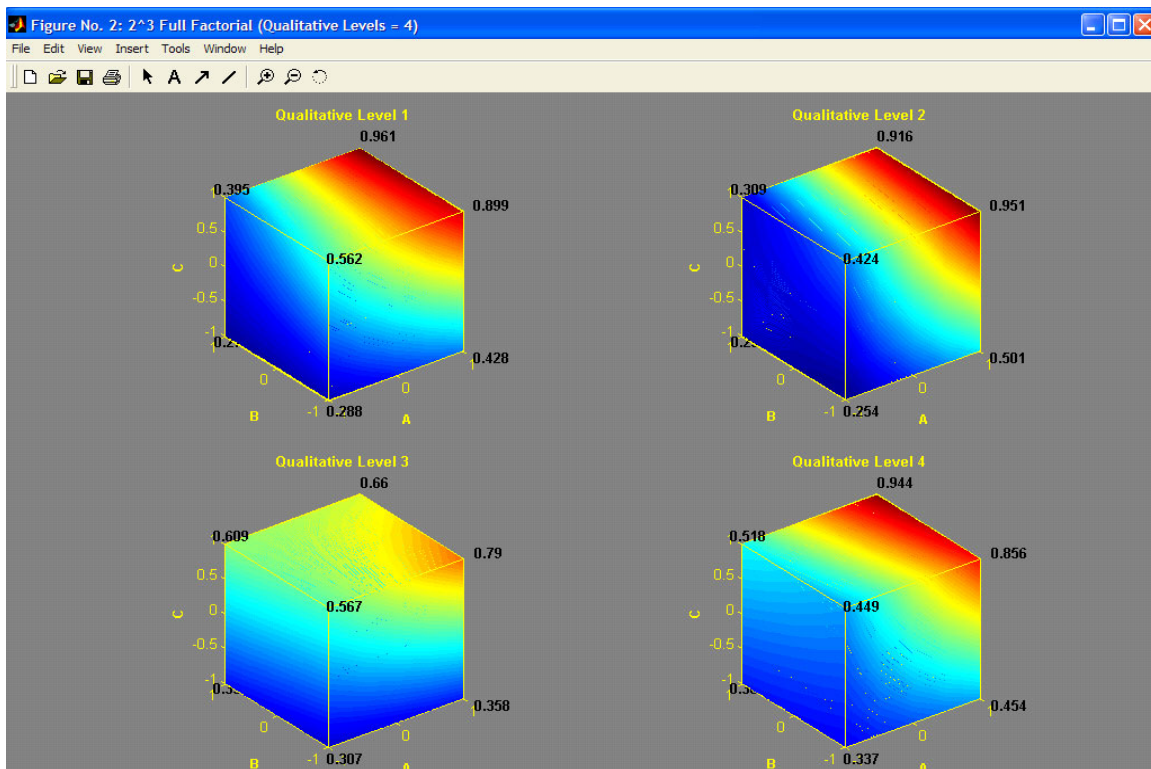## 3-D DOE Contour Plot at Board Thickness = 90%



**Figure A6- 18: 3-dimensional contour plots (Qualitative Levels = 4, Response Index at 90%)**

**Table A6- 3: Case Study Minitab 14® Coefficients for Response Index at 90%**

```
                        Coefficients
  Term      Flux B      Flux C      Flux E      Flux F
Constant    0.52938     0.51825     0.5095     0.541375
   A        0.14962     0.1975      0.0505     0.125375
   B       -0.01487    -0.01425     0.004      0.017375
   C        0.17488     0.13175     0.147      0.150375
  A*B       0.03037     0.004      -0.018     -0.005625
  A*C       0.07613     0.086       0.018      0.082875
  B*C      -0.01138    -0.02325    -0.026      0.021875
 A*B*C      0.02687     0.016      -0.025      0.010375
```

**Table A6- 4: Case Study 3-DOE Contour Plot Coefficients for Response Index at 90%**

| 90% | | | |
|---|---|---|---|
| **Flux B** | **Flux C** | **Flux E** | **Flux F** |
| 0.5294 | 0.5182 | 0.5095 | 0.5414 |
| 0.1496 | 0.1975 | 0.0505 | 0.1254 |
| -0.0149 | -0.0142 | 0.0040 | 0.0174 |
| 0.1749 | 0.1317 | 0.1470 | 0.1504 |
| 0.0304 | 0.0040 | -0.0180 | -0.0056 |
| 0.0761 | 0.0860 | 0.0180 | 0.0829 |
| -0.0114 | -0.0232 | -0.0260 | 0.0219 |
| 0.0269 | 0.0160 | -0.0250 | 0.0104 |