ITERATIVE MULTIPLE CRITERIA SIMULATION OPTIMIZATION IN MANUFACTURING

by

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A thesis submitted in partial fulfillment of the requirements for the degree of

MASTERS OF SCIENCE
in
INDUSTRIAL ENGINEERING

UNIVERSITY OF PUERTO RICO
MAYAGÜEZ CAMPUS
2016

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ABSTRACT

This work approaches the multiple criteria simulation optimization problem in manufacturing in an iterative fashion. Such problem entails using an optimization strategy to manipulate the parameters of a simulation model to arrive to the best possible configurations in the presence of several performance measures in conflict. The system can be a real manufacturing setting or a computer simulation of it, albeit the latter is the emphasis of this work. Pareto Efficiency conditions are used in an iterative framework based on experimental design and pairwise comparison. In particular, this work improves upon the use of Data Envelopment Analysis to determine the efficient frontier as well as the use of a single-pass algorithm previously proposed by our research group. The results show a rapid convergence to a more precise characterization of the Pareto-efficient solutions. The algorithm is illustrated by a series of cases involving simulation of manufacturing systems as well as prototyping by 3D printing.

RESUMEN

Este trabajo se enfoca en la optimización de problemas múltiple criterio a través de simulación en manufactura. Este tipo de problemas implica la utilización de una estrategia de optimización para manipular los parámetros de un problema de simulación para llegar a las mejores configuraciones posibles en la presencia de varias medidas de desempeño en conflicto. El sistema puede ser los parámetros de una manufactura real o una simulación por computadora de esta, aunque la ultima es el énfasis de este trabajo. Las condiciones de Pareto son usadas en un marco iterativo basado en diseño experimental y comparación por pares. En particular este trabajo es una mejora sobre el uso del algoritmo basado en Análisis Envolvente de Datos para determinar la frontera eficiente previamente propuesto por nuestro grupo de investigación, así como del uso de un algoritmo de una sola iteración también propuesto anteriormente en nuestro grupo. Los resultados muestran una rápida convergencia a una caracterización más precisa de las soluciones Pareto eficientes. El algoritmo es ilustrado por una serie de casos que involucran la simulación de sistemas de manufactura, así como también la impresión de prototipos 3D.

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1 Introduction

The use of optimization is critical in manufacturing to approach three decision-making problems: design, control and improvement of processes and systems. The underlying optimization objective in all three can be casted as finding values for decision variables that most competitively meet several performance measures (PMs) or criteria simultaneously. Although the use of a single PM has been a popular approach to all three, decision models that involve multiple conflicting PMs simultaneously and explicitly more closely reflect manufacturing reality. These latter models fall in the realm of Multiple Criteria Optimization **[1-4]**.

Nowadays, it is a prevailing practice to rely on computer simulation to estimate the performance of manufacturing processes and systems. Computer simulation is, obviously, a lot more convenient than carrying out experiments with actual systems. With ever-increasing computing power, this practice will only become stronger in the future. The concatenation of computer simulation and the optimization objective described previously has resulted in the field known as Simulation-Optimization **[5]**. In a manufacturing context, Simulation Optimization is commonly applied to approach the decision-making problems of design, control or improvement of processes and systems. It follows then that –in this context- considering the simultaneous optimization of multiple criteria can contribute to make simulation-optimization closer to manufacturing reality. Therefore, it is of interest to study multiple criteria simulation optimization (MCSO) problems **[6]**.

Incorporating the ability to deal with multiple criteria in conflict greatly enhances simulation-optimization. In order to fully exploit the power of a computer simulation model, however, it

becomes paramount that simulation optimization methods be capable to help decide upon the values of tens of variables at a time in a convenient manner. The contribution of these variables should be assessed not only in their linear contribution to the PMs of interest, but also at least in their quadratic contribution and their second order interactions to be more useful for decision making. Indeed, nonlinearity and interaction are more rules than exceptions in manufacturing **[7-8]**.

This work presents a MCSO strategy that is capable to incorporate tens of variables at a time and uses their linear, quadratic and second-order interactions to approach design, control and improvement of manufacturing processes and systems **[9]**. The use of the strategy is demonstrated through the fine tuning of a theoretical simulated manufacturing line with 50 decision variables and 2 PMs in conflict.

On the other hand MCO has been used widely in designing mechanical components and other engineering systems. Recent advances in 3D printing technologies bring a wide range of applications from fast prototyping to product manufacturing. New advances in additive manufacturing, such as breakthroughs in 3D printing, present an opportunity to contribute to recycling **[10, 11]**. Additive manufacturing could possibly be utilized to recycle waste material, such as PET water bottles, and deliver a new product or even better construct an innovative design that can be used to construct several products with different useful material. The Proposed strategy is illustrated for the development of a 3D printer prototype; specifically an interlocking device designed with two performance measures, max-load and minimize mass, with different conflicting optimization goals **[12-14]**.

1.1 Motivation

The use of Simulation Optimization is critical in manufacturing problems; every day it is necessary to make decisions, hopefully in an optimal manner, to satisfy different design specifications or process performance. One of the primary advantages is that they are able to provide users with practical feedback when designing real world systems. This allows the user to determine the correctness and efficiency of a system. The user may explore different alternatives without actually physically building the systems. By this reason the use of simulation optimization methods to solve manufacturing problems has been increasing in recent years. However optimization techniques are normally developed to optimize a single criterion. On the other hand, in most applications two or more criteria are relevant. In the presence of conflict of multiple performance measures, making decisions falls in the realm of multiple criteria optimization. Efficient methods to deal with this decision making problem are highly desirable. In this thesis an iterative method is proposed to find the efficient frontier in Multiple Criteria problems.

1.2   Objective

The objective of this thesis is to arrive to a framework of simulation optimization capable to handle realistic engineering problems. To this end, it is proposed that such framework be capable to handle multiple optimization criteria simultaneously as well as a tens of decision variables at a time. Three very important aspects of this framework is that (i) it does not require a large number of runs, (ii) it is transparent to the user, and (iii) it does not require any preference structure a priori.

In addition, it is sought for this framework to be capable to characterize the entire Efficient Frontier of a discretized problem, as opposed to just the convex part of it afforded by previously used Banker-Charnes-Cooper Model Data Envelopment Analysis (DEA) model.

1.3   Thesis Organization

This thesis is structured as follows: in Chapter 2 is a review of the most relevant methods developed to approach optimize multiple criteria optimization problems, Chapter 3 shows the path to the MCSO strategy that can be traced back to a series of manufacturing papers of our research group, Chapter 4 shows the proposed method; and an illustrative example is presented to illustrate our approach in Chapter 5. Chapter 6 illustrates how a multiple criteria optimization problem of 50 variables and 2PMs is approached with the proposed method, Chapter 7 a second case study was developed. This case study focuses on the design of a 3D printed prototype. As an important note, three refereed conference proceeding related to this thesis -in which this research is a coauthor- are included in the Appendix section. The first paper shows the best compromises between two criteria using Pareto Efficient conditions applied to injection molding. The second paper uses the proposed strategy to deal with tens of decisions variables in simulation, and forms

the basis of Chapter 6. The last paper is about the interlocking device proposed by our group but alternatively using desirabilty functions, which can be seen as a reference strategy with preferences defined a priori for this work.

2   Literature Review

The advances in computing power over the last decade have opened up the possibility of optimizing complex simulation models. This development offers a big opportunity in simulation, this is an area of interest in the academic world as in practical settings, by this reason several strategies can be found in the literature as reviewed by Carson and Maria in 1997 and Fu in 2015 [1, 2] . It is evident, however, that the vast majority has focused on the use of a single criterion optimization. Regarding multiple criteria simulation optimization, a recent review by Andradóttir in 2015 [3] evidences how Genetic Algorithms (GA) have become a popular to approach problems of this nature, as exemplified by Al-Aomar; Ding, Benyoucef and Xie. GAs are heuristic in nature, thus optimality cannot be guaranteed as a result. It is then understandable, then, that optimality certainty be a worthy objective [4-6].

The work of Mollaghasemi and Evans, falls into the category of iterative multiple criteria optimization. The proposed method is illustrated with an example involving the optimization of a manufacturing system. The simulation optimization algorithm developed in the research involves a progressive articulation of preferences through the use of an iterative method, although their approach favors the definition of a preference structure among PMs a priori. The decision maker provides some local information regarding his preferences over the multidimensional outcome space, which departs from the non-parametric point of view advocated in this work [7]. The works of Zakerifar, Biles, and Evans in 2011; Couckuyt, Deschrijver, and Dhaene in 2012; Dellino, Kleijnen, and Meloni in 2012, approach multiple criteria simulation optimization models using Kriging models with various degrees of success, adding evidence to the soundness of using

metamodeling strategies to support the determination of competitive solutions in the presence of conflicting PMs **[8, 15, 16]**.

Indeed, there seems to be a significant interest in the assessment of multiple criteria using simulation in different production applications such as planning and scheduling. In the paper by Duvivier, the authors developed an optimization model based on the hybridization of a classical hill-climber meta-heuristic with Promethee II multicriteria method. More specifically, a generic optimization and simulation framework, to quickly generate good compromises between conflicting objectives on scheduling nonpreemptable jobs, this case study describes the practical possibilities of three hybrid models within this framework. However, hybrid methods might provide some useful tools for addressing real-life problems in a small amount of time, but do not guarantee an optimal solution **[17]**. According to Mortazavi and Arshadi khamseh, simulation modeling is considered as efficient tool for modeling of retailer multiproduct inventory system. They proposed a surrogate model for a robust and multiobjective inventory system based on discrete simulation, design of experiments (DOE), and multiple attribute decision making (MADM). Due to the stochastic nature of the objective function, they employed principal component analysis (PCA) as statistical method to improve the MADM'S performance. PCA is applied for more realistic weighting of objective function values on their statistical influence on improvement the other objectives. The correct determination of the feasible solution set is a major challenge in engineering optimization problems. In order to construct the feasible solution set **[18]**. In the paper by Statnikov in 2009, the authors developed a method called PSI (Parameter Space investigation), this work made a contribution on the approximation of the feasible solution set. Other metaheuristic method is Particle Swam Optimization (PSO) an evolutionary algorithm to determine a Pareto set **[19]**. In the paper by Stöbener, the authors used this method to determine

parameters of intermolecular interaction potentials [20]. Derringer and Suich, developed a procedure that uses desirability functions. This approach assigns a value from 0 to 1, according to how the response for a combination of design variables lies between the desirable limits. One and two-sided desirability functions are used depending on whether the response is to be maximized, minimized or set to a specific target. As the response reaches the upper limit for the case of maximization, the desirability of the response function increases. The same happens when the response reaches the lower limit for the case of minimization or reaches the objective for the target goal case. The method depends on the researchers own priorities and criteria for the response values, indicating the subjectivity associated with the procedure. The solution strategy consists on expressing each objective (response variable) as individual desirability functions and the expected solution consists on maximizing the geometric mean of the desirability functions [21]. Martínez developed a simplex search method for experimental optimization with multiple objectives. He presents a novel concept of maximizing the concordance of desirability functions of all responses. A multi-directional simplex method is proposed by the author, using Kendall's coefficient of concordance W to provide statistical characterization of optimality in multi-objective setting. The goal of the multi-directional statistical simplex algorithm is to construct a sequence of best vertices that converges to a maximizer of the concordance index W. To achieve this, the algorithm requires that the values of W for the best vertex be monotonically increasing [22]. Mart and Coello proposed an algorithm for dealing with nonlinear and unconstrained multi-objective optimization problems (MOPs). The proposed algorithm adopts a nonlinear simplex search scheme in order to obtain multiple approximations of the Pareto optimal set. The search is directed by a well distributed set of weighted vectors [23]. Each weighted vector defines a scalarization problem, which is solved by deforming a simplex according to the movements described by Nelder and

Mead's in 1965 method. The simplex is constructed with a set of solutions, which minimize different scalarization problems defined by a set of neighbor-weighted vectors. The solutions found in the search are used to update a set of solutions considered to be the minima for each separate problem. In this way, the proposed algorithm collectively obtains multiple trade-offs among the different conflicting objectives, while maintaining a well distributed set of solutions along the Pareto front **[24].**

The evolutionary multi-objective optimization algorithms have demonstrated their ability in solving complicated multiple objective problems. Toscano and Coello presented a popular evolutionary algorithm is the genetic algorithm (GA) that uses the principle of natural selection to evolve a set of solutions towards an optimal solution. Genetic algorithms (GA) are population-based algorithms and they can efficiently handle non-linear problems with discontinuities and many local minima **[25]**. In the paper by Wright and Farmani in 2001, the authors used GA for simultaneous optimization of building construction, HVAC system size and the control strategy.

The most significant area of opportunity is the capability in iteratively dealing with tens of variables at a time, aided by saturated second-order regression models, using a Pareto-efficiency scheme of exact nature to approach multicriteria simulation optimization problems **[26]**. In many cases in the literature, the number of variables being investigated is less than a dozen, in fact, in most cases it is only three or four factors **[27-31]**.

In addition, Chapter 7 of this thesis presents the use of multiple criteria optimization for the design of a 3D printing based prototype, which, up to the time of writing this thesis was a problem not approached in the literature to the best extent of our acknowledge.

3   Background

The path to the MCSO strategy shown in this manuscript can be traced back to a series of manufacturing papers of our research group that have built upon each other as detailed next. The first idea related to manufacturing simulation-optimization in our group is presented by Cabrera-Ríos, where the design of a manufacturing cell was approached through discrete-event stochastic simulation and the maximization of profit as the sole PM. The optimization task was not iterative, thus the strategy in this work can be classified as a single-pass, single criterion, simulation optimization one **[32]**.

The second relevant work by Cabrera-Ríos, where design and process variables were included to meet multiple criteria modeled as a single composite objective function in the context of reactive polymer processing. The simulation type in this case relied on finite-element techniques, so it was continuous and deterministic in nature. Again the strategy was single-pass, single-criterion, and simulation optimization **[33]**.

The next step was to include multiple PMs in parallel. Using continuous physics-based simulation. The cases under study were in the context of polymer processing ranging from in-mold coating, to compression molding and injection molding. In these cases, the strategy was single-pass, multiple criteria, simulation optimization. An additional characteristic in these works was the use of Data Envelopment Analysis (DEA) to solve the associated multiple criteria optimization problem. The DEA model adopted for such means was based on linear optimization and could detect all solutions that were in the convex part of the Pareto-efficient frontier of the problem; however, solutions that were in the non-convex part escaped it. It was, thus, deemed necessary to

find an effective way to detect all solutions, those in the convex and in the nonconvex parts of the efficient frontier **[34-37]**.

The possibility to detect the entire set of solutions belonging to the efficient frontier came along in the shape of a full pairwise comparison scheme reported by Rodríguez-Yañez, where the improvement of an injection molding process was approached with a single-pass, multiple criteria, simulation optimization strategy **[38]**.

The first iterative simulation-optimization schemes in our group were reported for polymer injection molding (continuous simulation) and control/improvement of a painting line for automotive parts (discrete event simulation), respectively **[39-40]**. These were iterative, single-criterion, simulation optimization schemes. An iterative algorithm capable to deal with multiple criteria using DEA was subsequently developed. This is, then, an iterative, multiple criteria, and simulation optimization scheme **[39]**.

The present work introduces an iterative MCSO strategy capable to detect both the convex and nonconvex parts of the efficient frontier through the adoption of the scheme reported by Rodríguez-Yañez, so it improves and replaces the use of DEA **[38]**. It also incorporates the possibility of analyzing tens of variables through an economic experimental design proposed by Méndez-Vázquez in 2014 which ensures the possibility of estimating full quadratic regression models, that is, regression models that include linear, quadratic and second-order interaction terms **[9]**. A collection of experimental designs with these capabilities was described by Méndez-Vázquez **[40]** and can be found online in:

http://yaileenmendez.wix.com/experimentaldesignlv

3.1   Design Of Experiments (DOE)

Design of experiments, referred to as DOE, is a systematic approach to understanding how process and product parameters affect response variables such as process ability, physical properties, or product performance **[41]**. In engineering, experimentation has an important role in simulation models. We can define an experiment as a test or series of tests in which purposeful changes are made to the controllable variables of a process or systems **[41].** It allows us to observe and identify the reasons for change that may be observed in the performance measures. **Figure 1** is a general diagram of a process or system.

An example is the 3D printer process. The input is the interlocking device design (CAD), the three controllable factors are Cylinder Diameter, Cylinder depth and Thickness. The uncontrollable factors are ambient temperature and the force applied to assembly the interlocking device. The output is the 3D part and the response measures are max-load and minimize the mass.
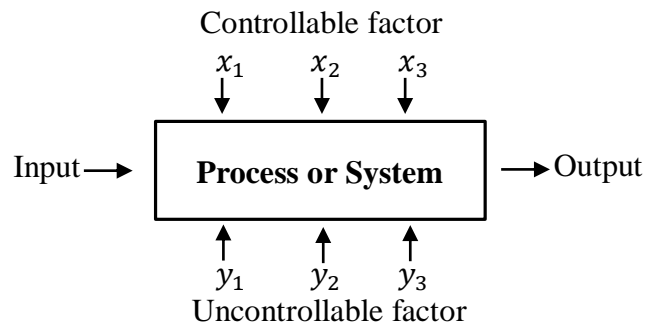
Controllable factor

x_1 x_2 x_3

Input → **Process or System** → Output

y_1 y_2 y_3
Uncontrollable factor

Figure 1. General Model of a Process or System

3.1.1 Response Surface Methodology

Response surface methodology can be defined as a collection of statistical and mathematical techniques used for developing, improving, and optimizing processes; processes in which a response of interest is influenced by several variables. RSM has a significant application in the design, development and formulation of new products, as well as in the improvement of existing product design. It defines the effect of the independent variables, alone or in combination, **[42]**. The objective of the methodology is to model a response variable as a function of design variables, performing an analysis to determine areas on the surface that provides optimal results according to the goal for the response: maximize, minimize or target. When multiple design variables and multiple response variables are being considered, the optimal function includes more than one response; which means that the optimal combination values of design variables that satisfies one response do not necessarily provides optimality conditions for the other responses.

3.1.2 Multiple Criteria Optimization

The use of optimization is critical in manufacturing problems; every day it is necessary to make decisions, hopefully in an optimal manner, to satisfy different design specifications or process performance. Almost all optimization problems in real life involve more than one performance measures to be optimized and are usually in conflict with each other. When the optimization procedure involves more than one objective function it is not recommendable to optimize each one in a separate way, because each objective function may arrive to a different solution, which could be in conflict.

Multiple Criteria Optimization (MCO) is a useful tool to identify the best compromising solutions in the presence of multiple performance measures **[43]**. These best compromising solutions form a so-called Pareto-efficient frontier or non-dominated solutions. In this thesis, the solutions that form the Pareto-Efficient Frontier will be found through the direct application of the dominance cone formed by the linear convex combinations of the desired directions establishes a region of dominance, the solution is found when it originates an empty cone. This method was explored by Rodríguez-Yañez in 2012 and Camacho-Caceres in 2015, in manufacturing process and Biological applications **[44, 45]**. In Rodríguez-Yañez in 2014, the said full pairwise comparison scheme was developed to improve upon DEA's constraint of finding only the convex portion of the efficient frontier. This full pairwise comparison finds the entire efficient frontier, both the convex and non-convex parts **[37]**.

In Camacho- Caceres in 2015, an example of multiple criteria optimization with two conflicting performance measures (PMs) is shown in **Figure 2**. In this figure, a set of eight candidate points, characterized by their values on both performance measures are shown. The aim is to minimize F_1 and F_2 simultaneously. The problem is to find those candidate points that dominate all of the other points in both performance measures. Applying the dominance cone, this will result in a group of candidates in the **Figure 2**, the triangle points are the Pareto-Efficient solutions. Note that the general problem involves at least two performance measures to be optimized, where only the case with two performance measures has a convenient graphical representation. An MCO problem, however, can include as many dimensions (or performance measures) as necessary **[45]**. A general optimization problem with one performance measure is as follows:

Find x to

$$\text{Minimize } f_j(x) \quad j = 1, 2, \dots, J \tag{1}$$

Here we want to find the values of the controllable variables $x^k, (k = 1, 2, \dots, |K|)$ between all the possible solutions, which can be evaluated in the J performance measures to result in values $f_j(x^k)$. That is the kth combination of values for the decision variables evaluated in the jth objective function. The example in **Figure 2** follows the discretization with $J=2$ performance measures and $|K|=7$ solutions.

The MCO formulation under such discretization is, then as follows:

$$\text{Find } x^k (k \in K) \text{ to}$$

$$\text{Minimize } f_j(x^k) \quad j = 1, 2, \dots, J \tag{2}$$

The solutions to (2) are, then, the Pareto-efficient solutions of the discretized MCO problem. Considering formulation (2), a particular combination x^0 with evaluations $f_j(x^0)$ will yield a Pareto-Efficient solution to (2) if and only if no other solution x^ψ exists that meets two conditions, from this point on called Pareto-optimality conditions:

$$f_j(x^\psi) \leq f_j(x^0) \,\forall j \qquad \text{(Condition 1)}$$

$$f_j(x^\psi) < f_j(x^0) \text{ in at least one } j \qquad \text{(Condition 2)}$$

Conditions (1) and (2) imply that no other solution x^ψ dominates the solution under evaluation, x^0, in all performance measures simultaneously.
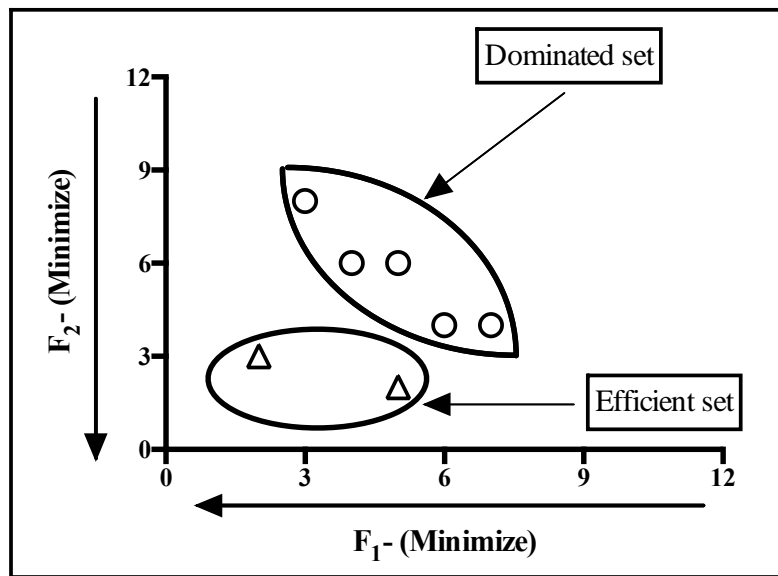
Figure 2. Representation of a Multiple Criteria Optimization Problem with two Performance Measures

4   Proposed Method

The proposed strategy integrates the use of experimental design, simulation and metamodelling techniques to solve multiple criteria simulation optimization problems. **Figure 3** schematically shows the proposed method which is also described below.

The method begins with an experimental design (DOE) where a simulation is performed at each design point (1-2) and an initial incumbent solution is obtained (3). The incumbent solution here corresponds to the Pareto-efficient frontier that represents the set of best compromises between all performance measures in the experiment. With the simulated experiment, one metamodel per performance measure is obtained (4) and used to make predictions in the discretized experimental region (5). Using the Pareto analysis, potentially Pareto efficient points are detected (6-8). These predicted points are then simulated and joined with the incumbent solution to determine if a new Pareto efficient frontier has been found (9). If the Pareto-efficient frontier does not change, then the incumbent solution is reported and no more iterations are performed. Otherwise the incumbent solution is updated and the newly-simulated points are added to the set available points and a new iteration begins with the constructions of new metamodels (4). The method can be understood following the ensuing notation:

Constants

n Number of simulation runs in the initial experimental design (DOE)

L Number of decision variables

J Number of criteria or Performance Measures (PMs)

Indices

j	Criterion or PM (*i=1,2,3,...,J)*
k	Iteration counter
l	Decision variable *(l=1,2,3,...,L)*

Metamodels

$y_j[k]$	Metamodel for the jth criterion at the kth iteration

Variables

G_l	Number of discrete points for the lth decision variable for prediction purposes
n_k	Total number of sampling points for metamodel prediction at the kth iteration

Matrices

D_0	Initial DOE with *n* simulation runs
D_k	Set of additional simulation runs generated at the kth iteration
$D_k^{available}$	Set of total simulation runs available at the end of the kth iteration
I_0	Set of initial incumbent Pareto-efficient solutions (simulation runs)
I_k	Set of incumbent Pareto-efficient solutions (simulation runs) at the kth iteration
C_k^I	Set of candidate solutions (simulation runs) to be evaluated for Pareto-optimality at the kth iteration
P_k	Set of solutions predicted by the metamodels at the kth iteration
P_k^E	Set of Pareto-efficient solutions predicted by the metamodels at the kth iteration

The method, then, is as follows:

Initialization

- Set counter $k = 0$

- Initial DOE: Perform a first design of experiments D_0 with n simulation runs considering all decision variables (*L* variables) and all criteria (*J* Criteria). Each controllable variable

and performance measure must be scaled to fall between -1 and 1 to avoid dimensionality problems.

- Select incumbent: Analyze D_0 to determine which of its points are Pareto Efficient. I_0 now contains the Pareto Efficient points of D_0.

  A Pareto efficient solution will be found when, in the full pairwise comparison with the rest of the solutions, there is no other solution that dominates it in all PMs simultaneously.

- Set $D_0^{available} = D_0$

Main Iteration

- Update counter: $k = k+1$
- Use $D_{k-1}^{available}$ to fit all J metamodels, $y_j[k]$. (For a brief description of metamodeling, see Appendix B)
- Use metamodels to predict the values of all J objective functions using a grid of $n_k = [G_1 \times G_2 \times ... \times G_l \times ...G_L]$ points, where G_l is the number of equidistant discrete points for the lth variable. This implies choosing a discretization size Δ to form such grid. If a complete enumeration on the n_k points is impossible, then a statistical sample on this grid must be used, with the consequence of loosing certainty on the optimality of the final efficient frontier, as discussed in the next section of this manuscript. Store these points in a matrix P_k with dimensions $[n_k \times (L+J)]$.
- Analyze P_k to determine which of its points are Pareto Efficient. Store the efficient points in P_k^E.
- Simulate all points in P_k^E. Store the simulated results in a matrix D_k.
- Set $C_k^I = I_{K-1} \cup D_k$

Analyze C_k^I to determine which of the points are Pareto Efficient. Store the efficient points in I_k.

Termination

Evaluate the stopping Criteria. If $I_k = I_{k-1}$, then terminate the algorithm and present the incumbent solution I_k . Otherwise, update $D_k^{available} = D_{k-1} \cup D_k$ and reiterate.
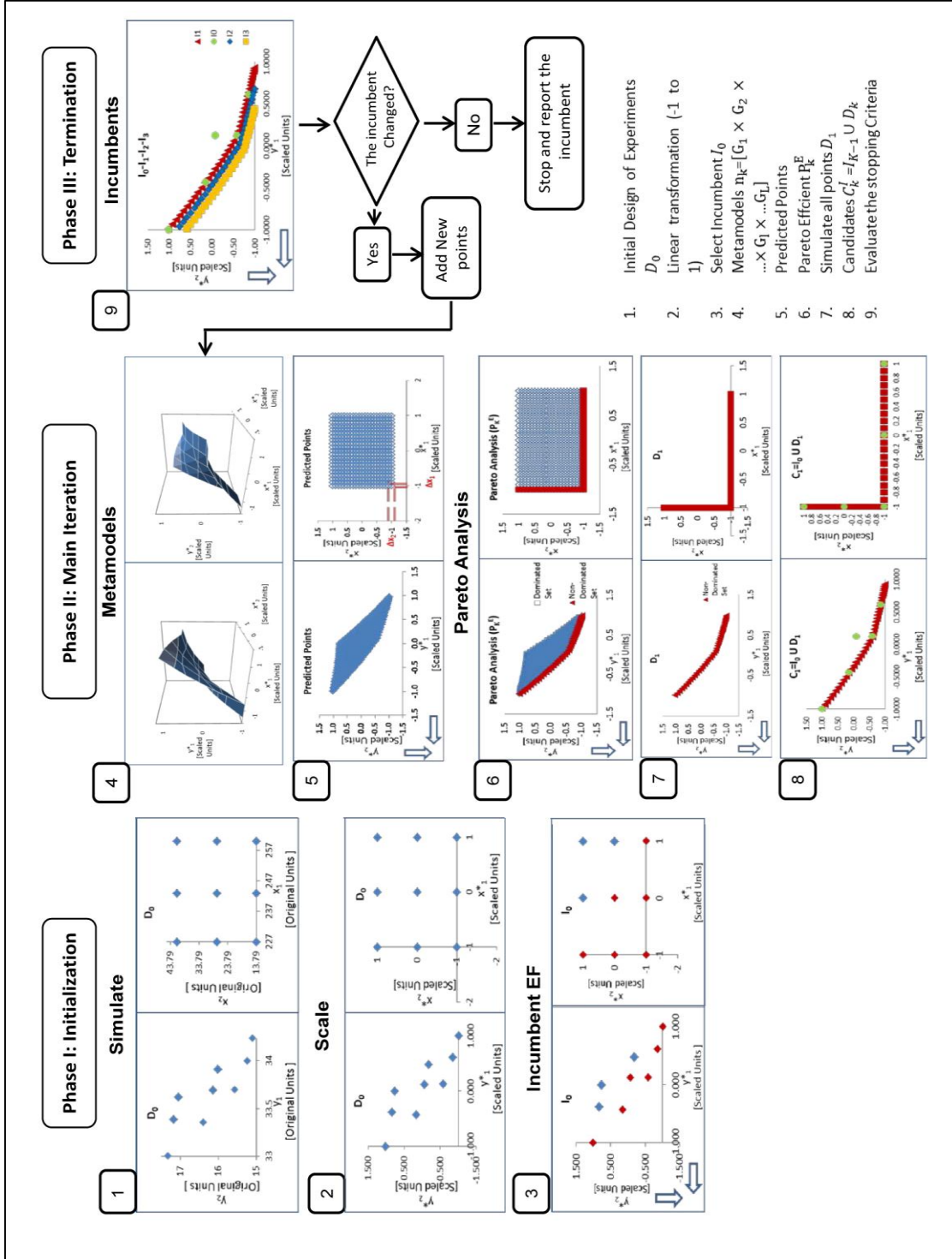
Figure 3. Proposed Multicriteria Simulation Optimization Method

5   Illustrative Example

In order to illustrate our approach, let us consider the case study where $f_1(x)$ and $f_2(x)$ are our performance measures and x is our controllable variable. An example of performance measures in conflict in a minimization problem with two objectives in conflict is shown in **Figure 4**.

$$\text{Minimize } f_1(x) = \sqrt{x} + 1 \text{ and Minimize } f_2(x) = x^2 - 4x + 5$$

Statement of the problem
Find x to

To: min $f_1(x) = \sqrt{x} + 1$

min $f_2(x) = x^2 - 4x + 5$ St. $0 \leqslant x \leqslant 4$
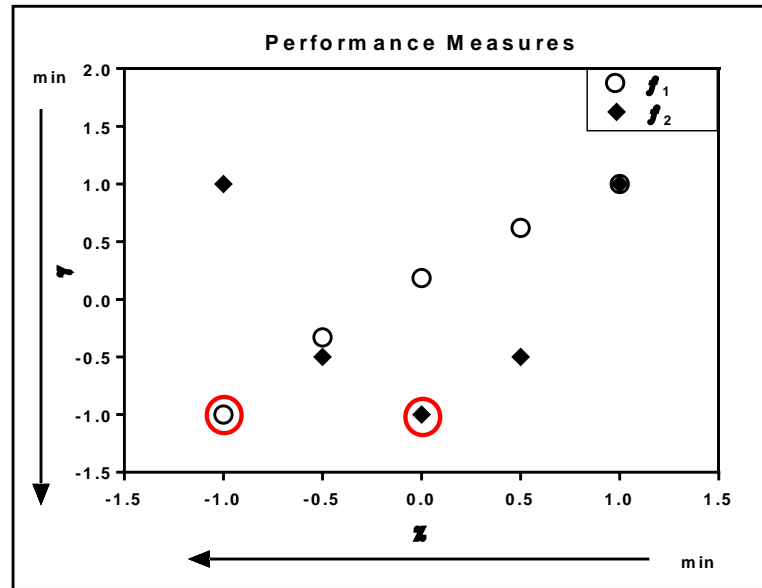
Figure 4. Performance Measures in Conflict

Step 1: Initial DOE: Run (n=5) a first design of experiments D_0 with n simulation runs considering all variables (I =1) and all objectives (J=2), where D stands for Design. The initial experimental design is shown in **Figure 5**. Each controllable variable and performance measure must be scaled to fall between -1 and 1 to avoid dimensionality problems. The linear transformation is shown in **Figure 6**
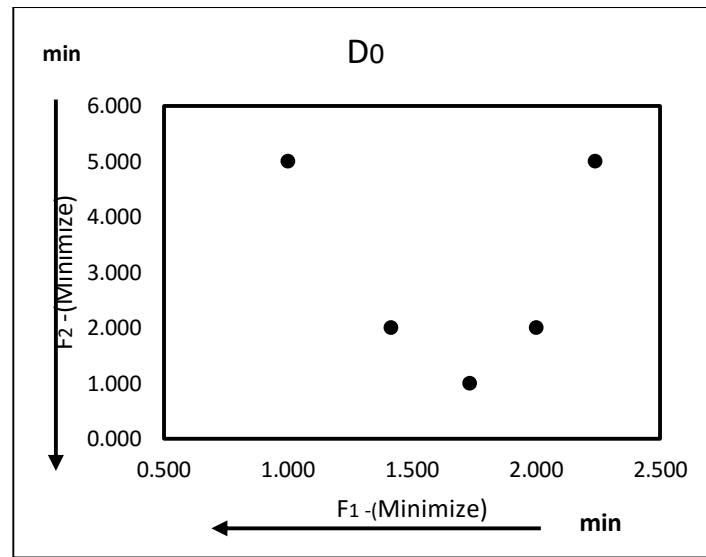
Figure 5. Initial Experimental Design D_0
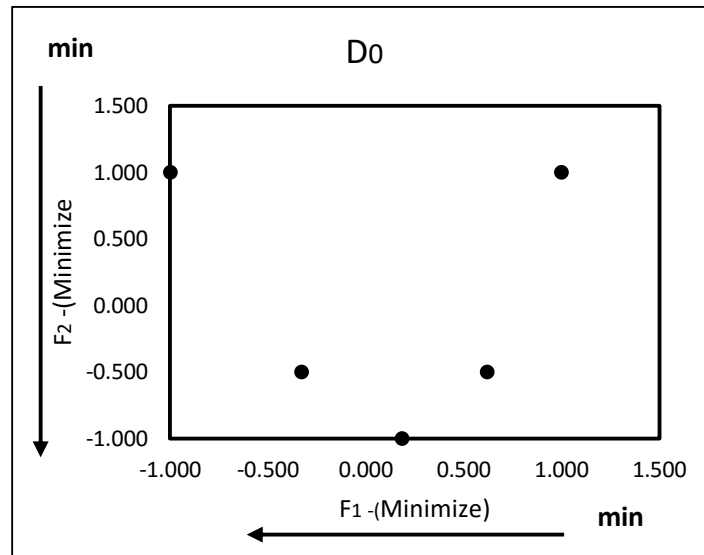
Figure 6. Linear Transformation $y = mx + b$

Step 2: Select incumbent: Considering the optimization criterion in the formulation *min* $f_1(x) = \sqrt{x} + 1$ and *min* $f_2(x) = x^2 - 4x + 5$. Analyze D_0 to determine which of its points are Pareto Efficient. I_0 now contains the Pareto Efficient points of D_0. I stands for incumbent. Pareto Efficient frontier is shown in **Figure 7.**
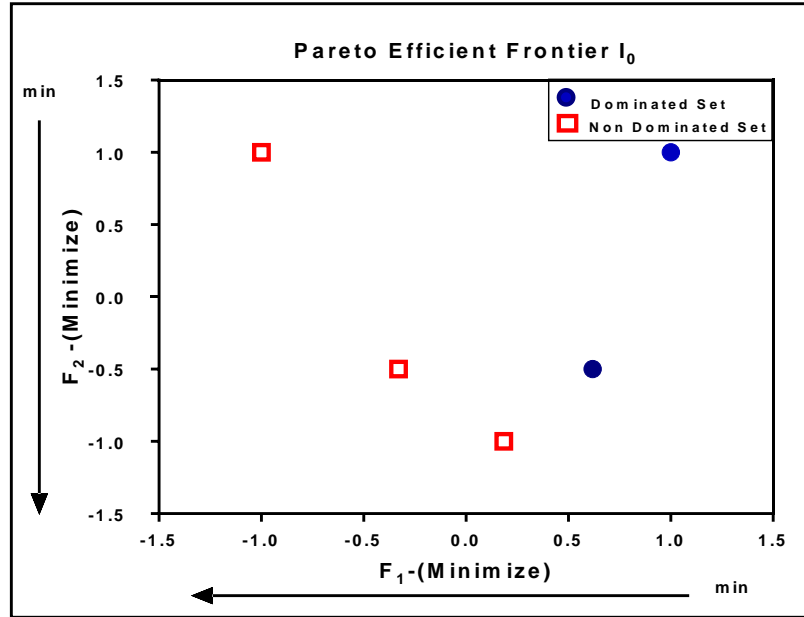
Figure 7. Pareto Efficient Solution I_0

Step 3: Set $D_0^{available} = D_0$ Set of all simulated points that are available at the beginning of

the iteration. The set of all simulated points is shown in **Figure 8**.
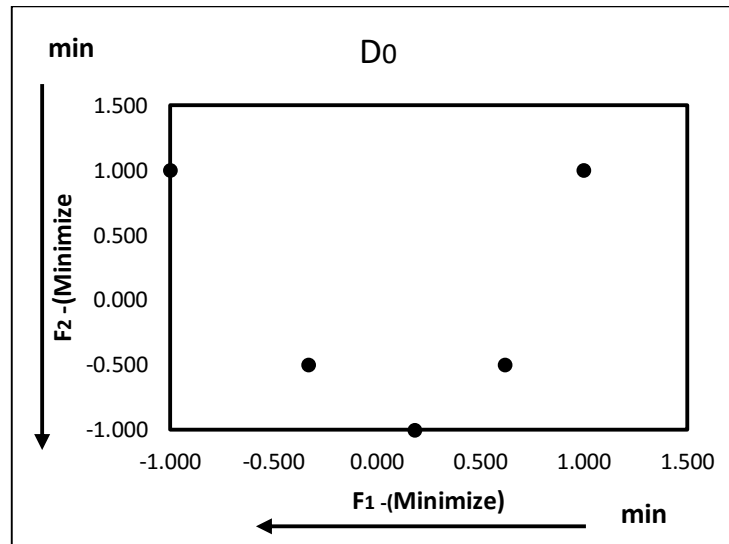
Figure 8. Set of all Simulated Points $D_0^{available} = D_0$

Step 4: Update counter k= k+1

Step 5: Obtain metamodels using the available points, use $D_0^{available}$ to fit all J metamodels, y_j [1].

Iteration	Quadratic Regression (Metamodels)

$$f_1 (x)^* = -0.1878x^2 + 0.9896x + 0.1884 \tag{1}$$

k=1

$$f_2 (x)^* = 2x^2 - 1 \tag{2}$$

Step 6: Predicted points using the metamodel generated in the previous step. Predicted points in iteration k=1 is shown in **Figure 9**.
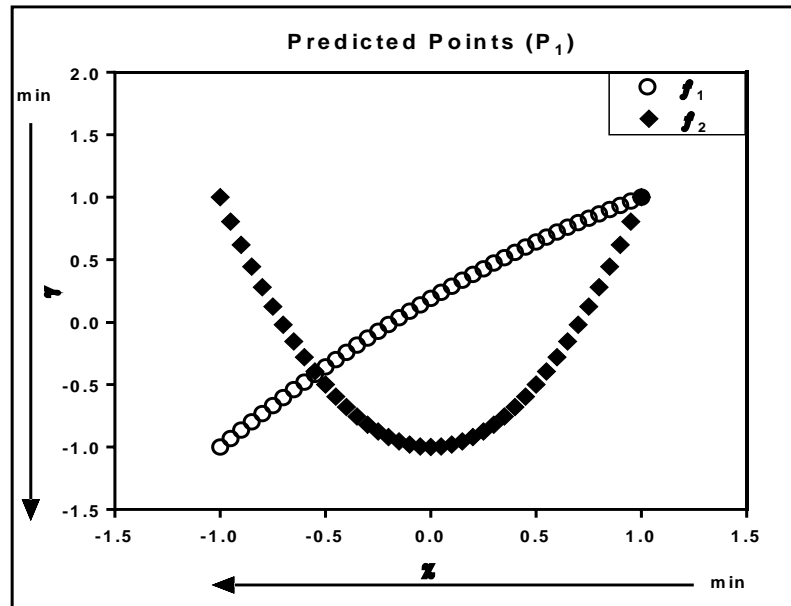
Figure 9. Predicted Points in the k=1

Step 7: Analyze P_1 to determine which of its points are Pareto Efficient. Store the Efficient points in P_1^E (where P stands for predicted and E stands for Efficient) is shown in **Figure 10**.
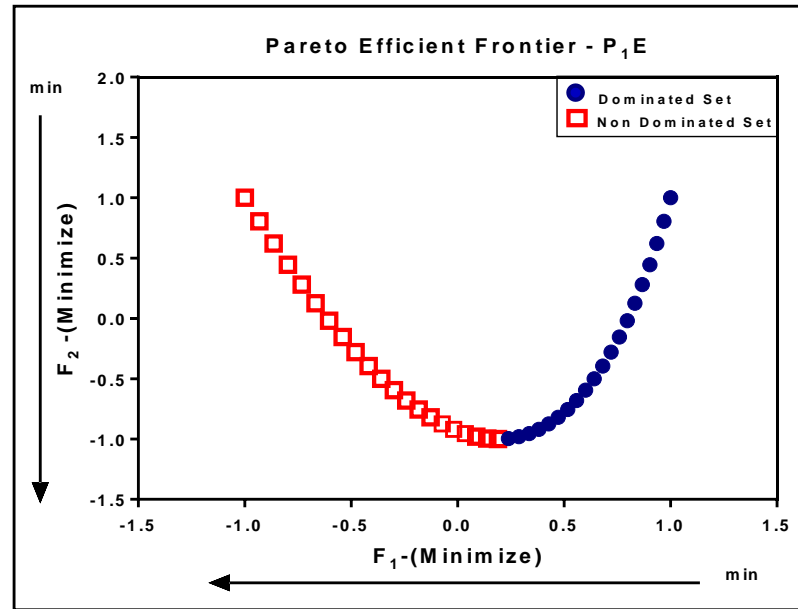
Figure 10. Set of Predicted Efficient Points at the k=1 Iteration

Step 8: Simulate all points in P_1^E. Store the simulated results in a matrix D_1. Predicted points in k=1 is shown in **Figure 11**.
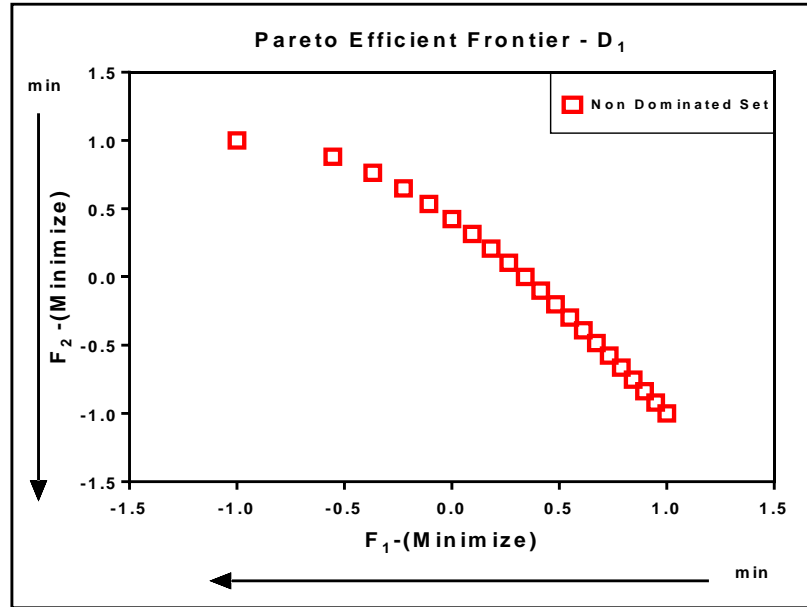
Figure 11. Simulated Points that are Potentially Efficient (D_1)

Step 9: Set $C_1^I = I_0 \cup D_1$, where stands for candidates and I for incumbent. The set of candidate points are shown in **Figure 12**.
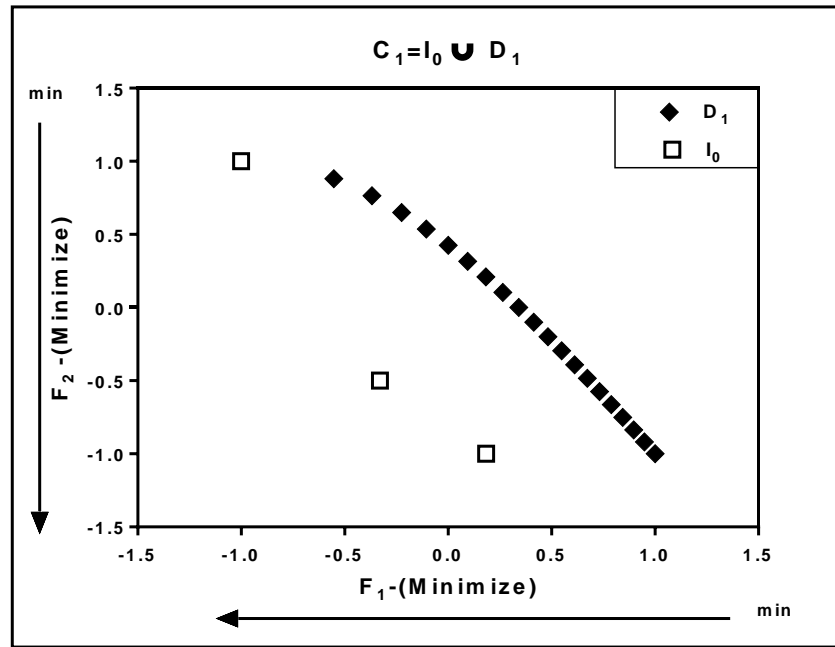
Figure 12. Set of Candidate Points C_1^I to the Incumbent Solution in the k=1

Step 10: Analyze C_1^I to determine which of the points are Pareto Efficient. Store the efficient points in I_k. Incumbent Solutions is shown in **Figure 13**.
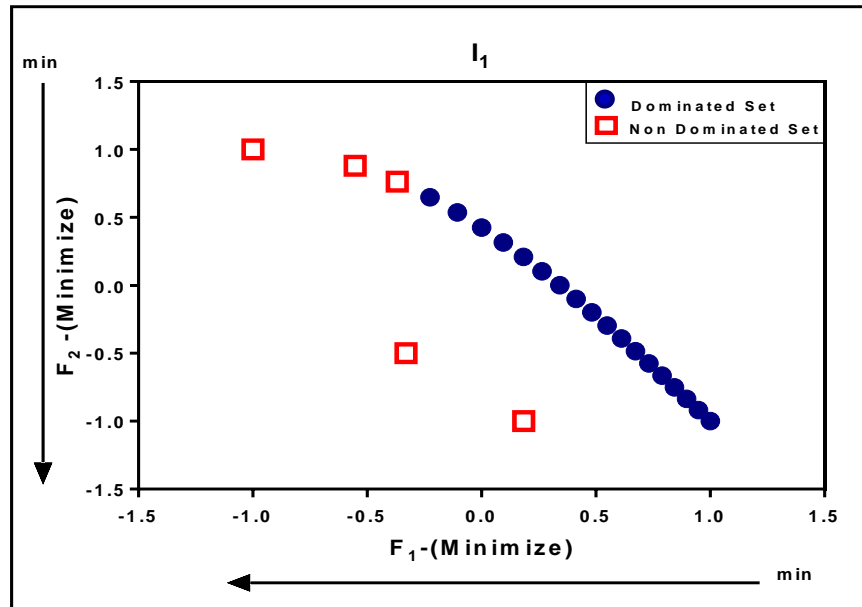
Figure 13. Incumbent Solution (I_1) in the k=1 Iteration

Step 11: Evaluating the stopping Criteria if $I_1 = I_0$, if True, then terminate the algorithm, and report the best solution. Incumbent solutions are shown in **Figure 14.**
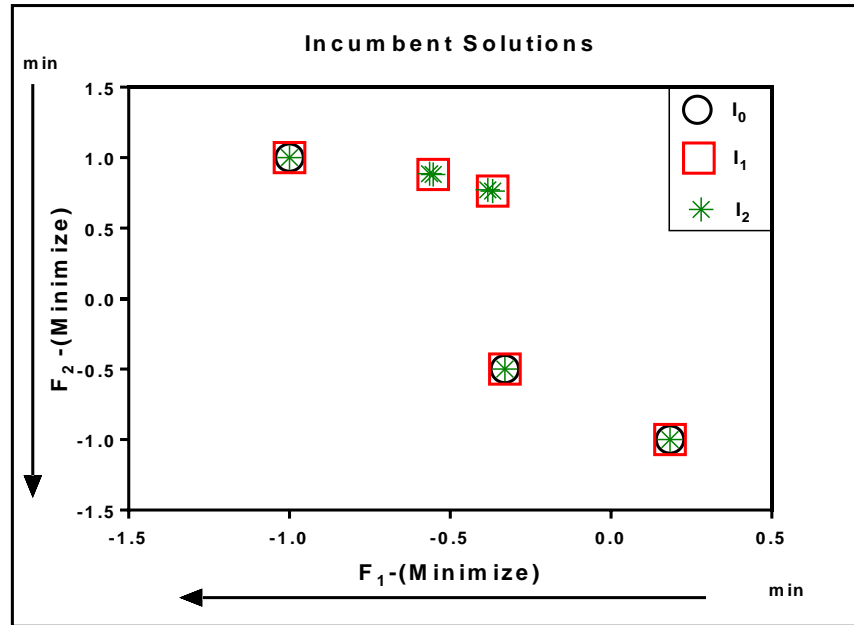
Figure 14. Incumbent Solution (I_2) in the k=2 Iteration

Summary

In the illustrative example presented in this work, one controllable variable and two performance measures were analyzed. With the proposed method, the efficient frontier was found. This work presents a simple method that quickly converges to the Pareto Set a solution with a small number of iteration of the algorithm.

6   Case Study I: Production Line with 50 Workstations

This section illustrates how a multicriteria optimization problem of 50 variables and 2 PMs is approached with the proposed method. Consider a fictitious production line with 50 workstations in series simulated with the software package Simio **[46, 47]**. The simulation is run for 8 hours per day with 10 replicates. The simulation model have an interarrival entity time that follows an exponential distribution with a mean equal to 3 minutes. The simulation parameters of interest were the mean process time on each of the workstations (WS_i). The process time of each workstation was assumed to follow a normal distribution with a mean that varied in three levels and a constant standard deviation of 0.25 minutes. **Figure 15** shows the ranges of values to be explored for the process time of each workstation. It is further assumed that the nominal process time can be chosen by a particular user, so the problem at hand involves deciding upon the nominal process time for each of the fifty workstations. This theoretical problem was presented by Méndez-Vázquez, where it was treated with the iterative single criterion simulation optimization described by Villarreal-Marroquín **[39, 40]**.
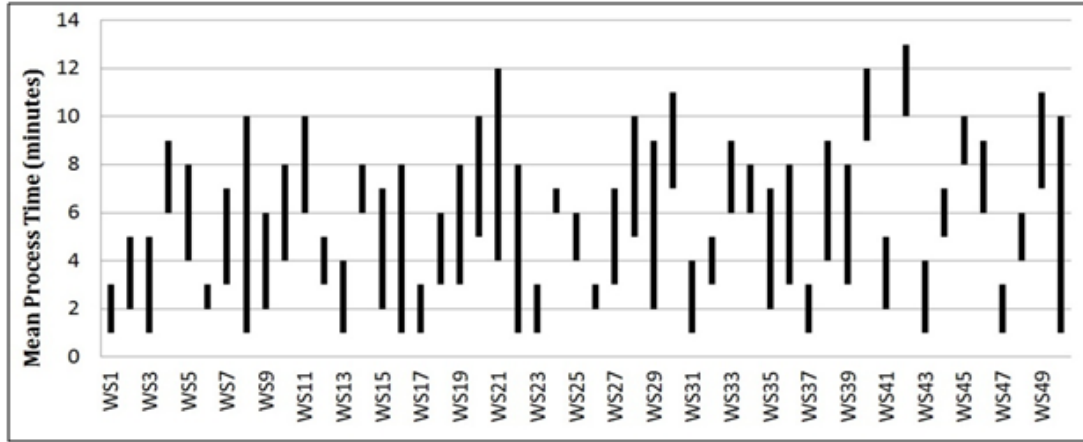
Figure 15. Range of Values for the Workstations' Mean Process Time in Simulation Model

The PMs of interest were the system time (F_1) defined as the period of time elapsed since a raw part to be processed enters the system until it exits as a finished product, and the average utilization of all workstations (F_2). The first one is to be minimized while the second one is to be maximized.

The proposed method begins with the initialization phase where an initial experimental design (D_0) is simulated using the simulation model described previously. The experimental design used for this case is a D-optimal design generated using the statistical software JMP. The experimental design in this case is for 50 variables at three levels each, and has 1327 runs. The number of runs corresponds to the minimum number of necessary runs to estimate a second order model **[9]**.

The natural variables and the simulated values of the PMs are coded using a linear transformation to make them fall in the range of [-1,1] to avoid dimensionality problems. With these coded values the efficient frontier was found, using a MATLAB code available in our group to carry out the full pairwise comparison **[45]**. The found efficient frontier represents the initial

incumbent solution (I_0) as shown in **Figure 16**. Do notice that, for representation purposes, both PMs are shown as minimization cases. Indeed, any maximization case can be turned into an equivalent minimization cases through a suitable linear transformation.

With I_0 at hand, the iterative phase of the algorithm begins. To generate predictions within the experimental region one second order regression metamodel per PM was constructed using D_0. Each metamodel consisted of 1326 terms. Firstly a discretization of the experimental region was performed with an increment (Δ_1) of 0.25 units in the natural values and sampled using 30,000 uniformly distributed data points.

Predictions were then obtained, using the metamodels, in these 30,000 points for both PMs. In turn, these predicted solutions were evaluated to determine the Pareto-efficient ones. Do notices that the number of Pareto Efficient Solutions is expected to be considerably less than the original number of solutions under analysis. The Pareto Efficient Solutions at this point are predictions so an actual simulation must be carried out at these attractive points. With these new points, the incumbent solution I_0 must be revised for Pareto-efficiency. When the comparison was carried out, 3 new points added to the efficient frontier and 12 points of the incumbent solution I_0 were now dominated points in **Table 1**. Consequently, these dominated solutions were deleted from the new incumbent solution I_1 as shown in **Figure 16.** The points of the candidate set D_1 are added to the available points D_0, and the second iteration of the algorithm ensued.

Table 1. Incumbent Solutions of the Pairwise Comparison for the Initial Stage of the Algorithm

Incumbent (I_0)		
Point	System Time (minutes)	Average Utilization (%)
66	329.944	46.466
72	335.054	47.602
344	322.083	42.055
436	329.739	46.257
445	323.619	43.759
525	334.712	46.719
557	326.754	44.973
590	329.829	46.401
871	336.216	48.137
965	313.848	38.032
1022	316.439	40.973
1034	341.130	49.730
1059	338.449	48.448

Incumbent (I_1) - Δ_1		
Point	System Time (minutes)	Average Utilization (%)
965	313.848	38.032
5825	325.217	48.561
23109	316.355	48.261
28268	336.119	51.604

Incumbent (I_2) - Δ_1		
Point	System Time (minutes)	Average Utilization (%)
965	313.848	38.032
5825	325.217	48.561
23109	316.355	48.261
28268	336.119	51.604
11236	329.648	48.755
24070	337.002	52.649
29313	330.588	49.543
29786	328.028	48.700

Incumbent (I_3) - Δ_1		
Point	System Time (minutes)	Average Utilization (%)
965	313.848	38.032
5825	325.217	48.561
23109	316.355	48.261
24070	337.002	52.649
9046	333.239	51.593
18682	326.349	49.596
27619	332.566	50.531
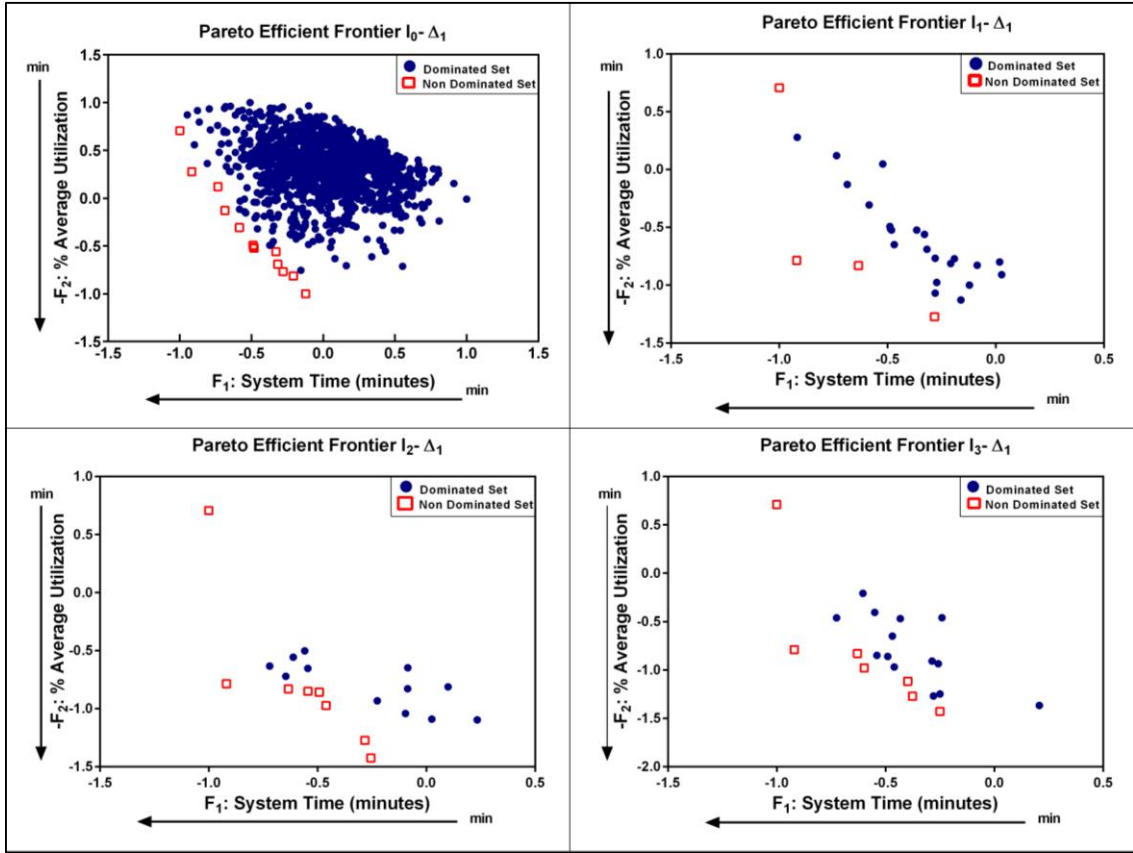
Figure 16. Pairwise Comparison for the Incumbent Solutions (I_1-I_2-I_3) with a Discretization of Δ_1

The second iteration follows the same structure as before, with the creation of a new set of potential solutions D_2. These combinations were then simulated and compared with the incumbent solution I_1 using Pareto conditions as shown in **Figure 16**. Four new point are added to the efficient frontier to obtain a new efficient frontier (I_2) as shown in **Table 1**. The candidate set of solutions D_2 were then added to the available points to calculate the new metamodels in a new iteration. In the third iteration, the simulated values of the new set of potential solution D_3 were obtained and compared with the incumbent solution I_2. As a result 3 new points are added to the efficient frontier and 3 points see **Table 1**. The new incumbent solution I_3 as shown in **Figure 16**. The points of the candidate set of solutions D_3 were added to the available points.

One more iteration was necessary to bring the method to a stop. It must be recalled at this point that the method stops only when no modifications are introduced in the current efficient frontier. The solutions for the initialization and each of the iterations are shown in **Figure 17**. The seven solutions identified by the method represent the best possible tradeoffs between cycle time and average machine utilization. Each of these solutions contain the prescriptive values at which each of the 50 workstations must be set.

When looking into the progression of the method in its two PMs as shown in **Figure 17,** it can be appreciated how the method effectively explored beyond the initial experimental (simulation) samples. It is also clear that the efficient frontier –achieved with a discretization increment $\Delta_1 = 0.250$ -would benefit from a finer exploration in that zone. A refinement stage was then performed using progressively decreasing discretization increments: first by choosing $\Delta_2 = 0.125$ and running the algorithm to an automatic stop, then by choosing $\Delta_3 = 0.0625$ and running the algorithm to an automatic stop.
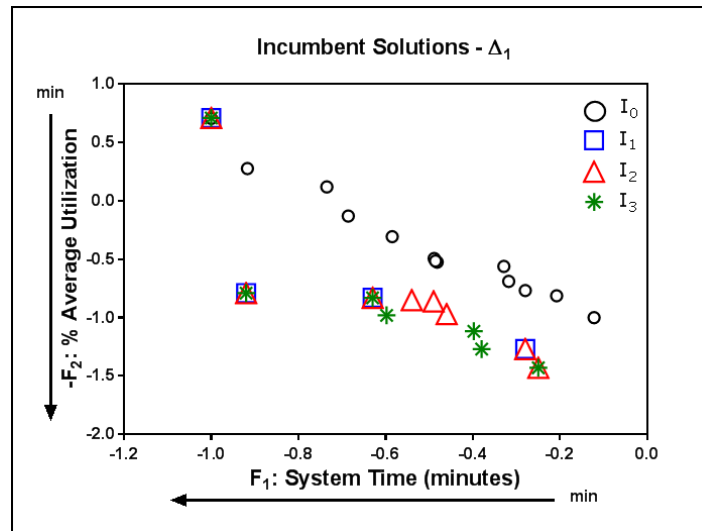

Figure 17. Each of the Incumbent Frontiers (I_0-I_3) of the Problem under Study

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When an increment of $\Delta_2 = 0.125$ was used for the discretization of the experimental region, 30,000 data points were chosen for sampling using a uniform distribution. Following the same structure of the iteration phase previously described, predictions at these points were obtained for both PMs with metamodels constructed using the available points when the first stage of the method ends. These predictions were then evaluated using the Pareto conditions and simulations of these potential candidate solutions D_4 were obtained. When the comparison of D_4 with the incumbent solution I_3 was performed as shown in **Figure 17**, 1 new point was added to the efficient frontier and 1 point was left out as part of the dominated solutions see **Table 2**. The candidate set of solutions D_4 were then added to the available points for the construction of metamodels of the next iteration.

Figure 18. Pairwise Comparison for the Incumbent Solutions (I_4-I_5) with a Discretization of Δ_2

In the second iteration using the increment of Δ_2 for the discretization, predictions with these new metamodels were performed and evaluated using the Pareto conditions to obtain the potential candidate set of solution D_5. Then the simulated values of D_5 were obtained and compared with the incumbent solution I_4 as shown in **Figure 18.** As the result of this comparison 2 new

points were added to the efficient frontier and 1 point was found dominated and consequently deleted of the new incumbent solution I_5 see **Table 2**. The candidate set of solutions D_5 were added to the available points and metamodels for the next iteration were constructed.

In the second iteration using the increment of Δ_2 for the discretization, predictions with these new metamodels were performed and evaluated using the Pareto conditions to obtain the potential candidate set of solution D_5. Then the simulated values of D_5 were obtained and compared with the incumbent solution I_4 as shown in **Figure 18**. As the result of this comparison 2 new points were added to the efficient frontier and 1 pot was found dominated and consequently deleted of the new incumbent solution I_5 see **Table 2**. The candidate set of solutions D_5 were added to the available points and metamodels for the next iteration were constructed.
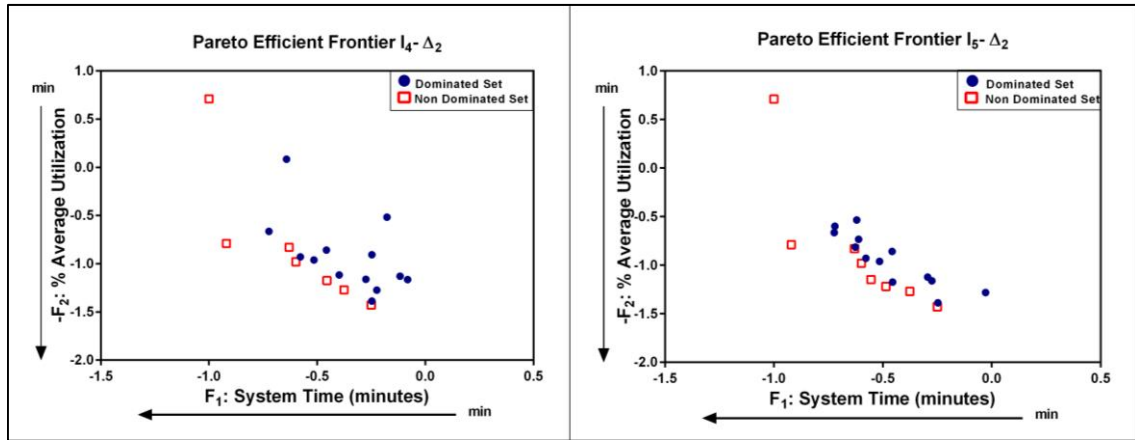
Table 2. Incumbent Solutions of the Pairwise Comparison for the Refinement Stage of the Algorithm using an Increment of $\Delta_2 = 0.125$ Minutes

Incumbent (I_4) - Δ_2			Incumbent (I_5) - Δ_2		
Point	System Time (minutes)	Average Utilization (%)	Point	System Time (minutes)	Average Utilization (%)
965	313.848	38.032	965	313.848	38.032
5825	325.217	48.561	5825	325.217	48.561
23109	316.355	48.261	23109	316.355	48.261
24070	337.002	52.649	24070	337.002	52.649
9046	333.239	51.593	9046	333.239	51.593
18682	326.349	49.596	18682	326.349	49.596
23553	330.786	50.928	21	327.693	50.752
			10959	329.814	51.237

The method iterates until no changes occurs in the efficient frontier. When the simulated values of the potential candidate solutions were compared with the incumbent I_5, no new points were added to the efficient frontier and consequently the algorithm stopped.

In the second step of the refinement stage, a reduction in the increment size for the discretization of the experimental region was applied. Using the available data at this point, a metamodel was constructed for each PM and the algorithm was run with an increment of $\Delta_3 = 0.0625$ units of the natural values for the discretization. The discretized region was sampled with 30,000 data points using a uniform distribution. The second step in the refinement stage followed the same structure previously described.

With the metamodels constructed with the available data, predictions were made in the 30,000 sampled data points, and a comparison using the Pareto conditions was made. Simulations in the resultant potential candidate set of solutions D_6 were obtained and compared with the incumbent solution I_5 as shown in **Figure 18**. As a result of this comparison 2 new points were added to the efficient frontier and 1 of the existing points were now part of the dominated solutions see **Table 3.** The incumbent solution was updated (I_6) and the candidate set of solutions D_6 were added to the available points. Metamodels for the next iteration were constructed.
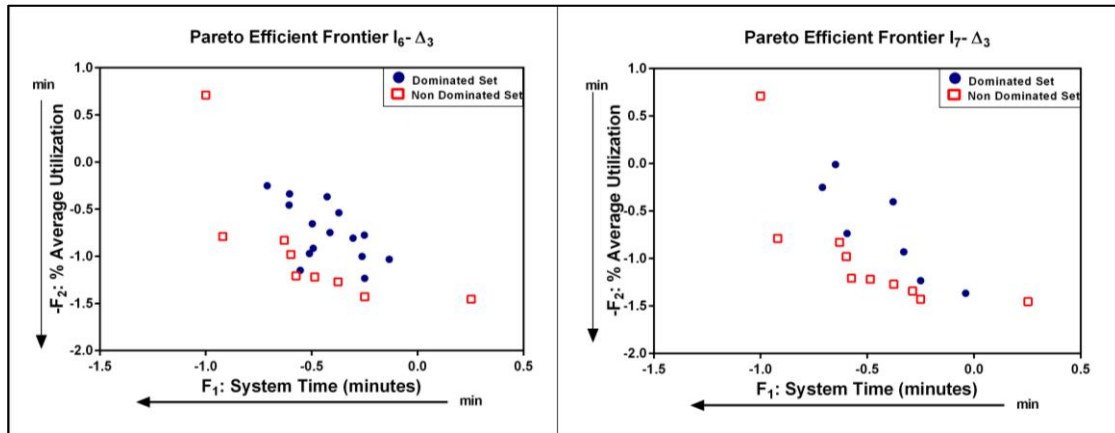

Figure 19. Pairwise Comparison for the Incumbent Solutions (I_6-I_7) with a Discretization of Δ_3

For the second iteration using Δ_3, prediction in the sampled points were made and the potential candidate solutions D_7 were identified. Simulations of the potential set of solutions were obtained and compared with the incumbent solution I_6 as shown in **Figure 19**. In this iteration, 1 new point was added to the existing efficient frontier and 2 were deleted, resulting in a new incumbent solution I_7 see **Table 3**. The potential candidate solutions D_7 were added to the available points and metamodels for each PM were constructed.

In the third iteration, the potential candidate solutions D_8 were obtained from the Pareto analysis of the predictions in the discretized sampled points. When a comparison of the simulated values of D_8 with the incumbent solution I_7 was performed no new points were added to the efficient frontier resulting in an automatic stop.

Table 3. Incumbent Solutions of the Pairwise Comparison for the Refinement Stage of the Algorithm using an Increment of $\Delta_3 = 0.0625$ Minutes

Incumbent (I_6) - Δ_3			Incumbent (I_7) - Δ_3		
Point	System Time (minutes)	Average Utilization (%)	Point	System Time (minutes)	Average Utilization (%)
965	313.848	38.032	965	313.848	38.032
5825	325.217	48.561	5825	325.217	48.561
23109	316.355	48.261	23109	316.355	48.261
24070	337.002	52.649	24070	337.002	52.649
9046	333.239	51.593	9046	333.239	51.593
18682	326.349	49.596	18682	326.349	49.596
10959	329.814	51.237	10959	329.814	51.237
10003	327.078	51.152	16825	335.984	52.080
17652	352.776	52.851			

The refinement stage resulted in a more precise efficient frontier, as it can be appreciated in **Figure 20**. It is, indeed, recommended that a similar strategy be followed for the potential practitioners:

(1) run the algorithm to find a coarse efficient frontier with an aggressively sized discretization increment, to then (2) run the algorithm with finely-sized increments to refine the results.

This work presents a simple method that quickly converges to a solution with a small number of iteration of the algorithm. The method stops automatically when no more changes are found in the efficient frontier. The magnitude and number of the increments for the discretization of the experimental region are defined by the user. With the proposed method the efficient frontier was found for a case of 50 controllable variables considering two performance measures.
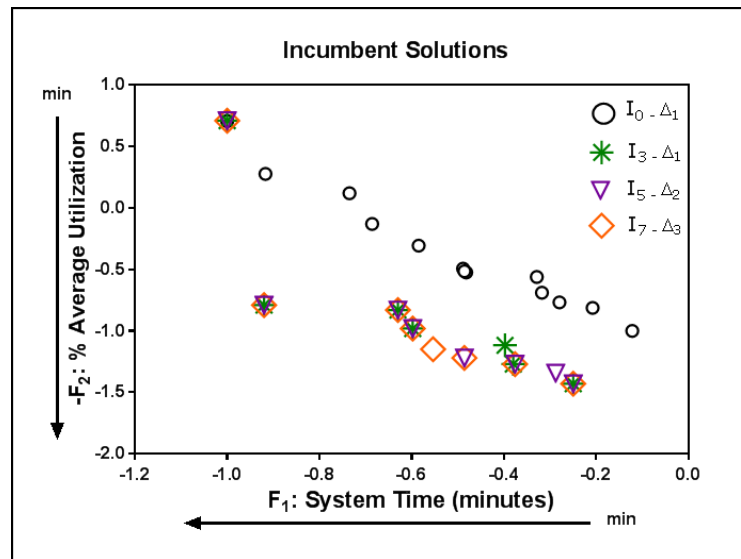
Figure 20. Each of the Incumbent Frontiers for each Discretization step (Δ_1-Δ_2-Δ_3) of the Problem under Study

Summary

In the case study presented in this work, initially the potential sampling for the intended experimental region was very large: 3^{50}, that is, 50 independent variables at three levels. For the initialization of the algorithm an optimal experimental design was used to deal with this situation.

However for the predictions, the use of a sample of 30,000 points was necessary to alleviate the computational effort, rendering the method heuristic.

Results

This thesis presents an iterative multiple criteria simulation optimization strategy capable to handle tens of variables at a time. In its current state, the method is an improvement over single-pass methods as well as the use of Data Envelopment Analysis models as previously proposed by our group. Manufacturing decisions regarding design, control and improvement of processes and systems can greatly benefit from using the proposed optimization strategy from the point of view of its capabilities -multicriteria, multifactorial- as well as its frugality in terms of the number of simulation runs. Future work includes assessing the method's runtime as relevant information for those cases where decision times are short.

7   Case Study II: 3D Printing Prototype

This case study focuses on the design and optimization of an interlocking device to meet multiple needs, in a household item starting from water bottles. This design must incorporate interlocking devices because it is desired that the final product can be assembled without the use of an external fastener. We want to be able to reassemble an existing 3D object and reconfigure it for alternative designs, we emphasize not only the complicated 3D interlocking device requirements, but also the geometrical and dimensional requirements on the single parts. When optimizing designs with polymers, many criteria could be of interest. However, many of them can be in conflict, therefore multiple criteria optimization is explored, and employed by necessity. In the case of the design of the interlocking prototype, which will be evaluated in this work, it is desired to maximize its load strength and minimize the required manufacturing material.

7.1   Experimental Design and Response Surface Methodology

Experimental design and response surface methodologies were used to solve the underlying problem. The most widely used designs for optimization problems are central composite, Box–Behnken, D-optimal and full factorial. These optimization designs allow obtaining experimental data, which are then fitted in a polynomial model of multiple linear regressions with the purpose of characterizing a response surface **[48].** Important restrictions are considered: running cost and time to selected the design of experiments. Box Behnken (BBD) design technique was selected to execute the experimental design. In comparison with central composite design, Box Behnken uses

less runs than the central composite design with three factors Cylinder Diameter (x_1), Cylinder

depth (x_2) and Thickness (x_3) see **Table 4**. It also avoids design corners **[41- 48]**.

Table 4. Response-Surface Designs Most Commonly used for Optimization

Design	Type of factors	Factors	Number of runs
Central composite design (CCD)	Numerical Categorical	5	2k+2k+Cp
Box Behnken (BBD)	Numerical Categorical	3	2k (k-1)+Cp
Full Factorial design	Numerical Categorical	3	3k
D-Optimal	Numerical Categorical	Different for each model. Irregular experimental domains	Select subset of all possible combinations

Box Behnken design is an independent design in that it does not contain an embedded

factorial or fractional factorial design. In this design the treatment combinations are at the

midpoints of the edges of the process space and at the center as shown in **Figure 21**.
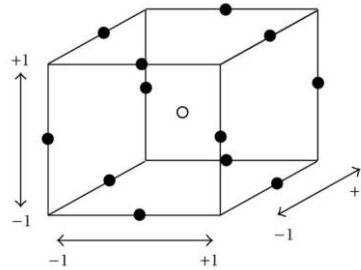

Figure 21. A Box-Behnken Design for Three Factors

7.2   3D Printers

3D printers employ an additive manufacturing process whereby products are built on a layer-by-layer basis, through a series of across- sectional **[49]**. All 3D printers also use 3D CAD software that measures thousands of cross-sections of each product to determine exactly how each layer is to be constructed **[10,11]**. The 3D machine dispenses a thin layer of liquid resin and uses a computer-controlled ultraviolet laser to harden each layer in the specified cross-section pattern. At the end of the process, excess soft resin is cleaned away through use of a chemical bath. 3D printers can produce simple small objects, such as a gear, in less than one hour. They can also develop products with free-moving parts that do not have to be assembled **[12]**. However, 3D printing has an intrinsic limitation: a 3D printer cannot directly print an object whose size is greater than the printer's working volume. This practical limitation has been pointed out recently by Luo L. et al **[13]**, who proposed a solution to partition a given 3D object into parts for 3D printing and then assemble the printed parts together to reconstruct the object. The initial interlocking prototypes described in this case study were done on a 3D printer Replicator 2 (Makerbot) that uses Polylactide (PLA) material.

7.3   Interlocking Device

Interlocking devices can be designed to be held by internal or external constraints. Such devices held by external constraints are called Topological Interlocking materials which are designed in a peculiar shape in order to create coupling while an external force is applied. On the other hand, interlocking devices are those that require internal structural constraints to couple the unit elements **[14]**. These internal constraints can produce stress concentrations on the material which will

diminish structural integrity **[50]**. Although this is what is known about interlocking structures, this project aims to elucidate the structural performance by optimizing the geometry. Another disadvantage of such structure is the manufacturability given that interlocking elements requires high precision machining tools and labor consumption to create them **[50]**. However, using 3D printing labor can be eliminated while achieving manufacturing precision.

The interlocking matrix provides several structural advantages over other types of structure such as resistance to fracture propagation, elimination of binding agent and creation of structure with different materials **[14, 50]**. The design was motivated by the puzzle coupling, which provide internal constrains on the planar axis, designed by César A. Rivera Collazo. Furthermore, it was modified to provide orthogonal constraint too. The planar movement is limited by 6 cylindrical shapes while the orthogonal constrain is provided by domes on each of the device faces. The hexagonal shape of the device will balance the planar stresses applied to the structure. This interlocking structure, if optimized, will not necessarily be affected by the material strength but by the interlock friction. Furthermore, tribology analysis will be needed to understand the structure mechanics. This work will study empirically the performance of the structure, but will require further validation of the physical models. The variation on the dimensional parameters to study structural performance makes this topological material a perfect fit for the optimization study using Box Behnken design (BBD) and Pareto Efficient Frontier. The parameters to optimize were maximum load and structural weight given that it is often needed to have a low material consumption and structural integrity.

7.4   Case Study: 3D Printer Prototype

In order to obtain a design that can satisfy both the criteria of minimum mass and max-load, as a first step a design of experiments was carried out to evaluate three separate factors that were initially considered to have a possible effect on the results of the criteria of interest. **Figure 22** is a representation of the interlocking prototype with its inputs representing the three controllable variables that are initially considered. To carry out the experimental design, a Box-Behnken design was chosen to analyze the three controllable factors. Once the experimental design was executed and the respective data was collected, the next step was to identify a model that fit adequately to each response based on the information gathered during experimentation. Finally we applied the proposed method to determine suitable parameters for each factor that would maximize the load and minimize the mass response. **Figure 23** presents a general representation of the procedure followed to accomplish the analysis and optimization presented in this work. Each phase is described in further detail in the following sub-sections of this case study.
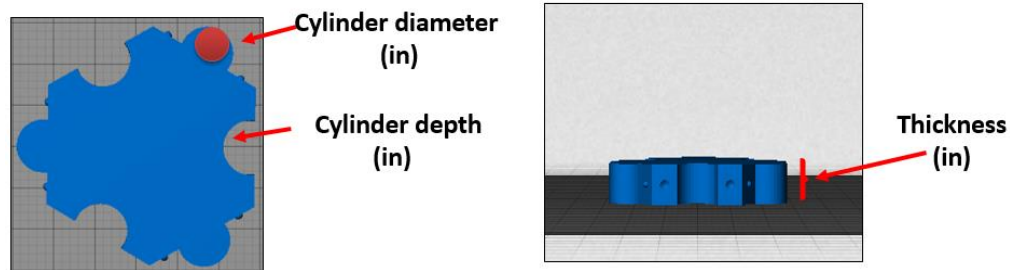

Figure 22. Interlocking Device Controllable Variables

INPUT   PROCESS   OUTPUT

Thickness
Cylinder depth
Cylinder Radius
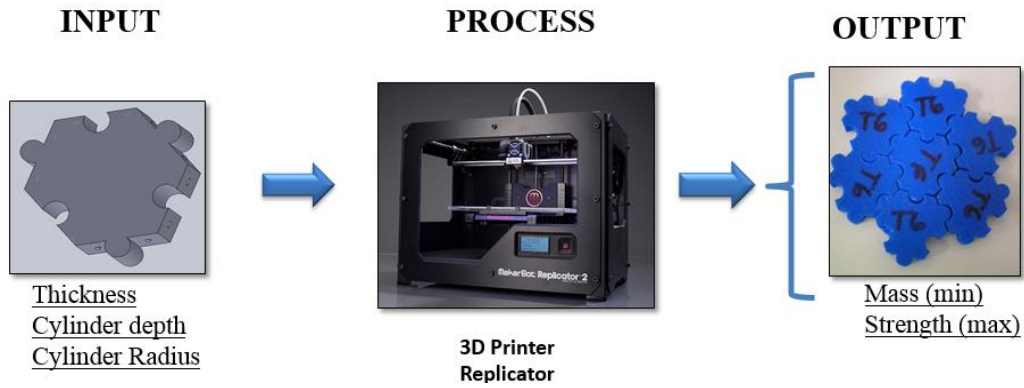
3D Printer
Replicator

Mass (min)
Strength (max)

Figure 23. Interlocking Device Design System Representation

Flexural tests were carried out to determine the max load in (lbf) using the Universal Testing Instruments Machine / 5940 Series Single Column Table Top Systems for Low-Force Mechanical Testing / INSTRON. To evaluate the max load, a distributed load was applied as shown in **Figure 24** and **Figure 25**.
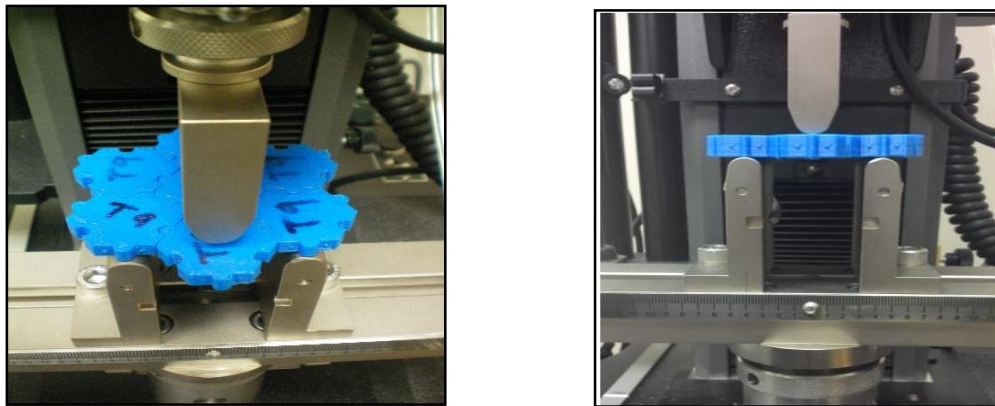

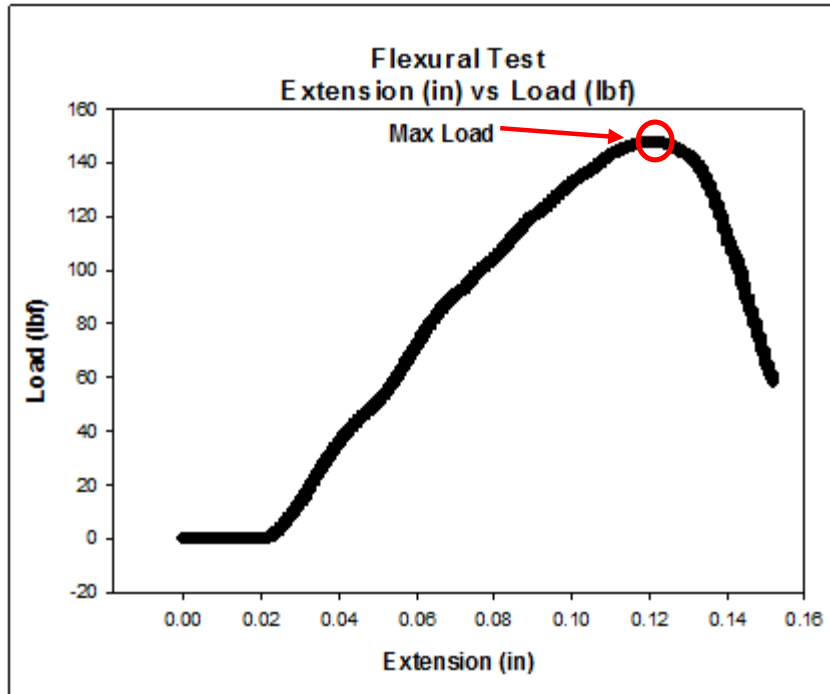**Figure 24. Interlocking Device Flexural Test**

Figure 25. Interlocking Device Max Load

After the max load was measured, the interlocking device was weighted to know the mass in (g) of this, as shown in **Figure 26**.
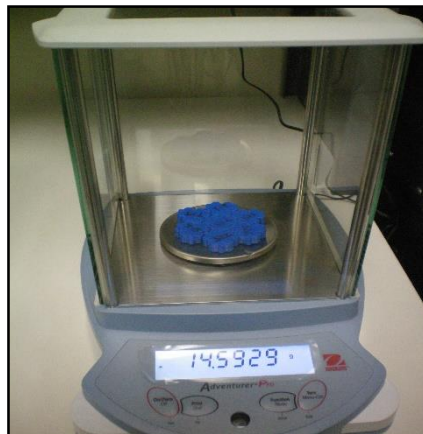

Figure 26. Iterlocking Device Mass

The proposed method begins with the initialization phase where an initial experimental design (D_0) as shown in **Table 5.** The experimental design used for this case is a Box Behnken design generated using the statistical software Minitab as shown in **Figure 27**and **Figure 28**. The experimental design in this case is for 3 variables at three levels each, and has 16 runs.

The natural variables and the experimental values of the PMs are coded using a linear transformation to make them fall in the range of [-1, 1] to avoid dimensionality problems as shown in **Figure 29.** With these coded values the efficient frontier was found, using a MATLAB code available in our group to carry out the full pairwise comparison **[45]**. The found efficient frontier represents the initial incumbent solution (I_0) as shown in **Figure 30**. Do notice that, for representation purposes, both PMs are shown as minimization cases. Indeed, any maximization case can be turned into an equivalent minimization cases through a suitable linear transformation.

Table 5. Initial Design Box-Behnken (Original Units)

Original Units					
Controllable Variables (CV)			Performance Measure (PM)		
Test	Thickness(in)	Cylinder diameter (in)	Cylinder depth (in)	mass (g)	Max Load (lbf)
T1	0.092	0.018	0.345	14.449	1.155
T2	0.23	0.018	0.345	14.545	12.558
T3	0.092	0.041	0.345	14.488	84.699
T4	0.23	0.041	0.345	14.978	66.053
T5	0.092	0.03	0.23	11.22	33.560
T6	0.23	0.03	0.23	11.542	57.421
T7	0.092	0.03	0.46	11.449	49.308
T8	0.23	0.03	0.46	17.919	83.722
T9	0.161	0.018	0.23	11.374	6.480
T10	0.161	0.041	0.23	11.435	28.560
T11	0.161	0.018	0.46	17.648	33.508
T12	0.161	0.041	0.46	17.856	147.733
T13	0.161	0.03	0.345	14.679	45.427
T14	0.161	0.03	0.345	14.225	10.461
T15	0.161	0.03	0.345	14.592	13.110
T16	0.161	0.041	0.46	17.569	104.412

53

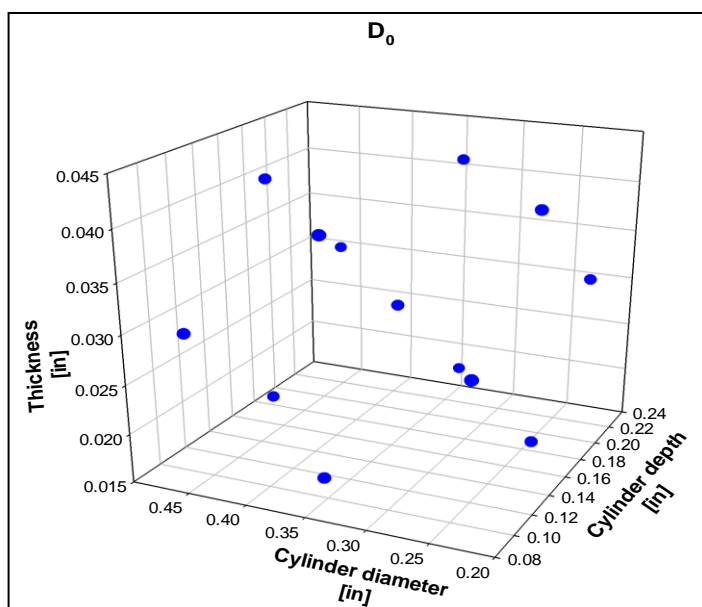**Figure 27. Initial DOE-Controllable Variables (CV)**

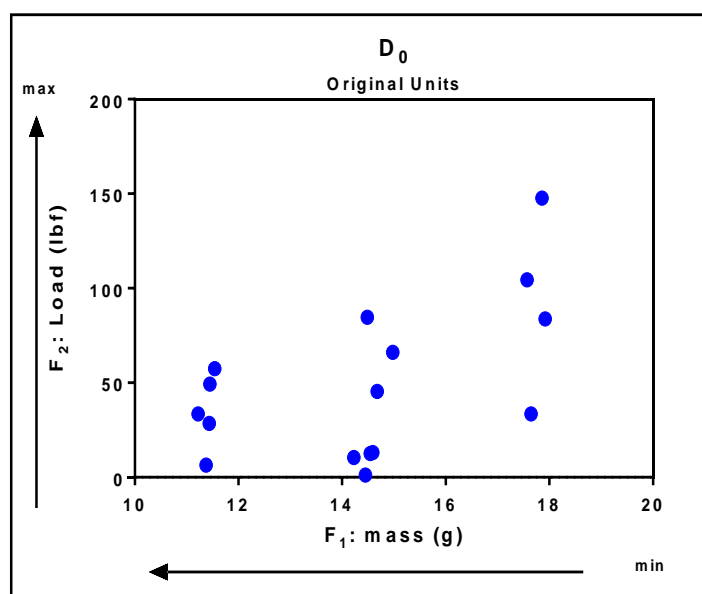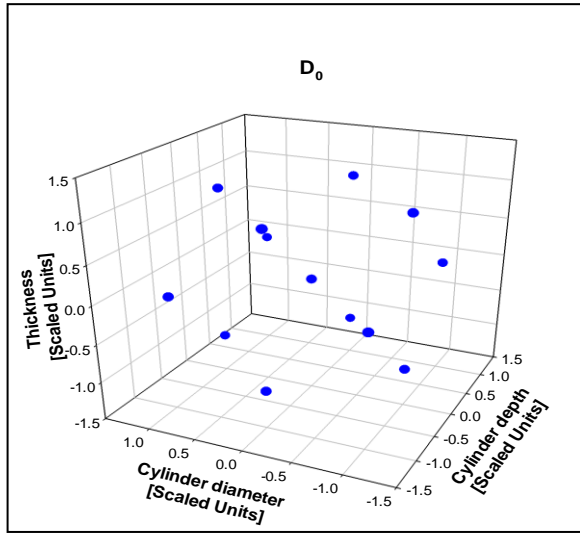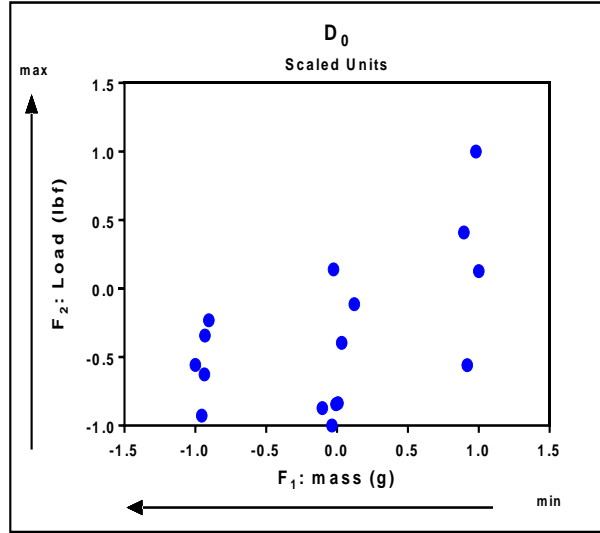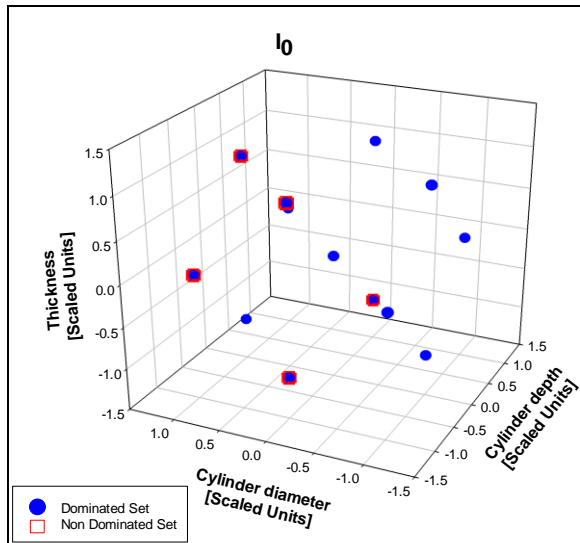**Figure 28. Performance Measures (PM)**
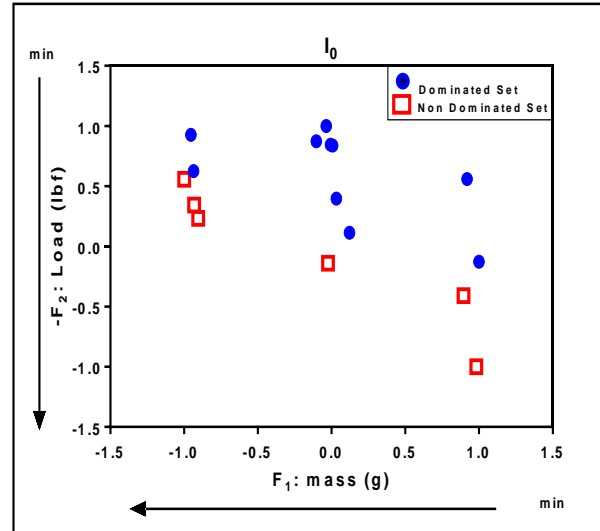
a)   b)

Figure 29. Linear Transformation in the Range [-1, 1], a) Controllable Variables (CV) and b) Performance Measures (PM)

a)   b)

Figure 30. Initial Pairwise Comparison for the Incumbent Solution (I₀), a) Controllable Variables (CV) and b) Performance Measures (PM)

With I_0 at hand, the iterative phase of the algorithm begins. To generate predictions within the experimental region one second order regression metamodel per PM was constructed using D_0. Predictions were then obtained in these 125 points for both PMs as shown in **Figure 31**. In turn, these predicted solutions were evaluated to determine the Pareto-efficient ones as shown in **Figure 32**. The solutions found represent potentially efficient at this point, so an experimentation is carried out at these attractive points, as shown in **Figure 33.** With these new points, the incumbent solution I_0 must be revised for Pareto-efficiency. When the comparison was carried out, 3 new points added to the efficient frontier and 4 points of the incumbent solution I_0 were now dominated points as shown in **Figure 34** Consequently, these dominated solutions were deleted from the new incumbent solution I_1 as shown in **Figure 35**.The points of the candidate set D_1 are added to the available points D_0, and the second iteration of the algorithm ensued.

a)   b)
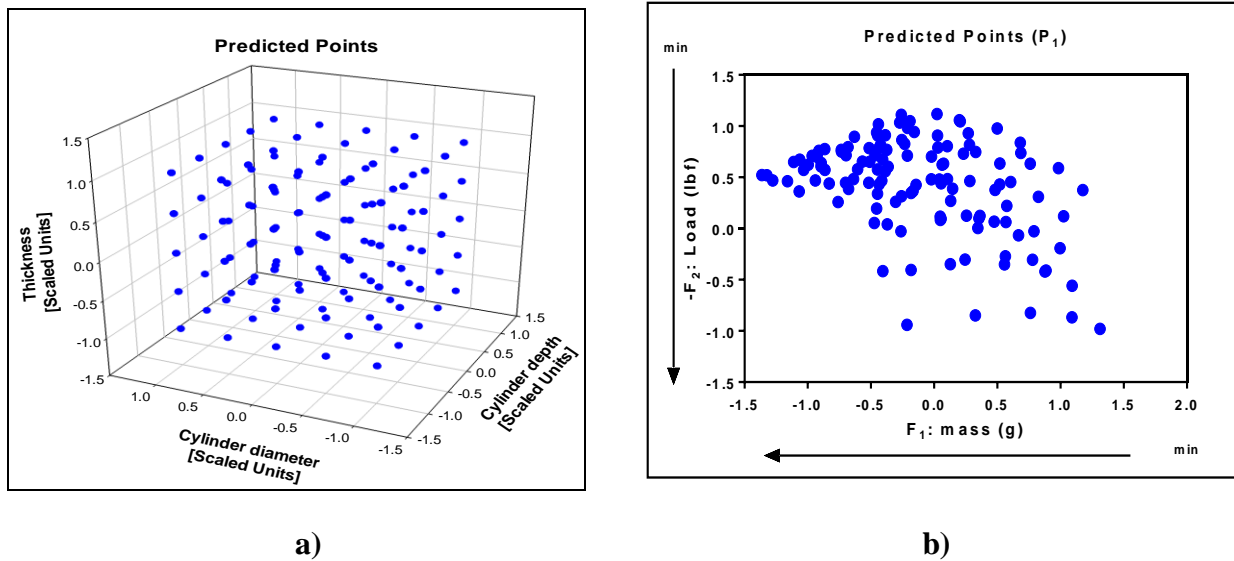
Figure 31. Initial Predicted Points (P_1) a) Controllable Variables (CV) and
b) Performance Measures (PM)
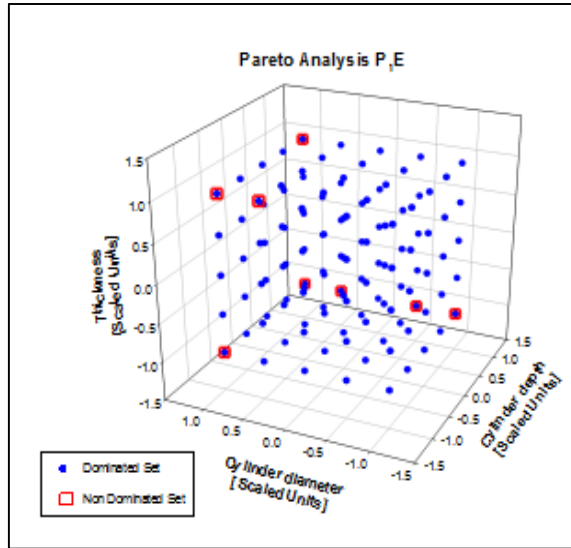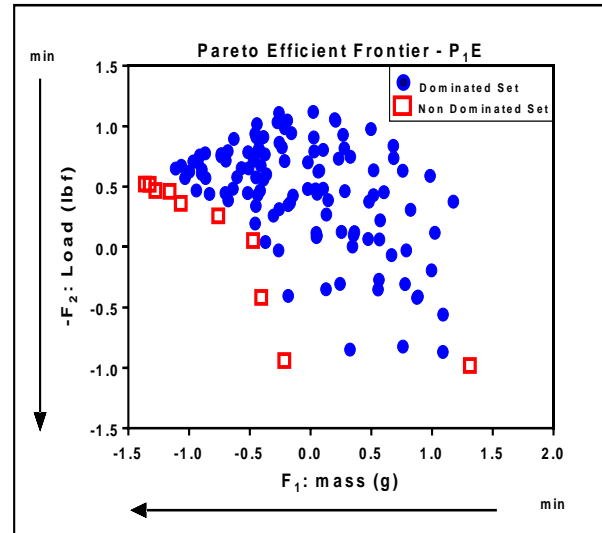
a)   b)

Figure 32. Predicted Points that are Pareto Efficient (P₁E) in the k=1 a) Controllable Variables (CV) and b) Performance Measures (PM)
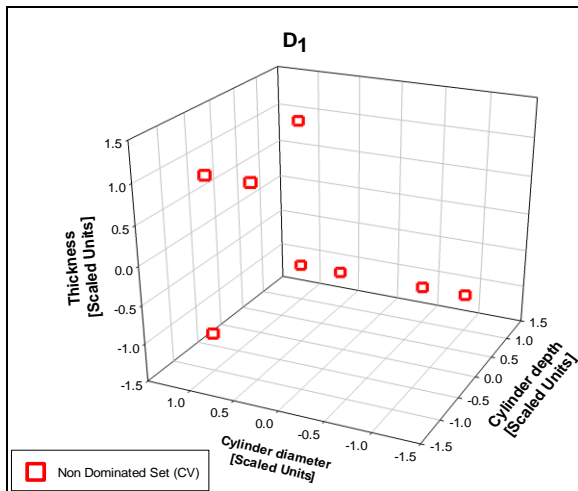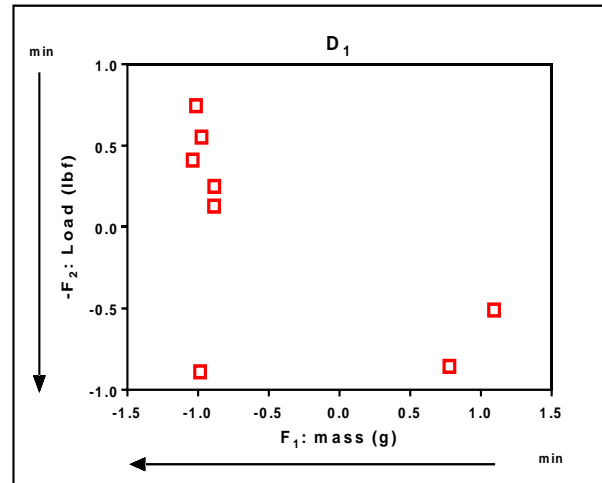
a)   b)

Figure 33. Set of Simulated points that are Potentially Efficient (D₁) a) Controllable Variables (CV) and b) Performance Measures (PM)
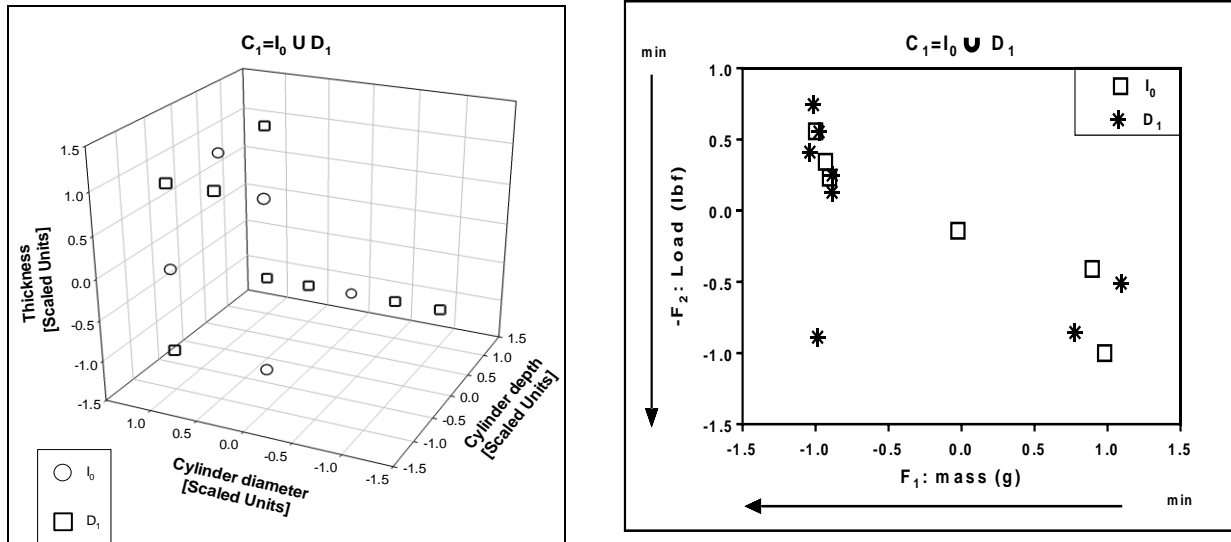
a)

b)

Figure 34. Initial Pairwise Comparison $I_0 \cup D_1$ a) Controllable Variables (CV) and b) Performance Measures (PM)
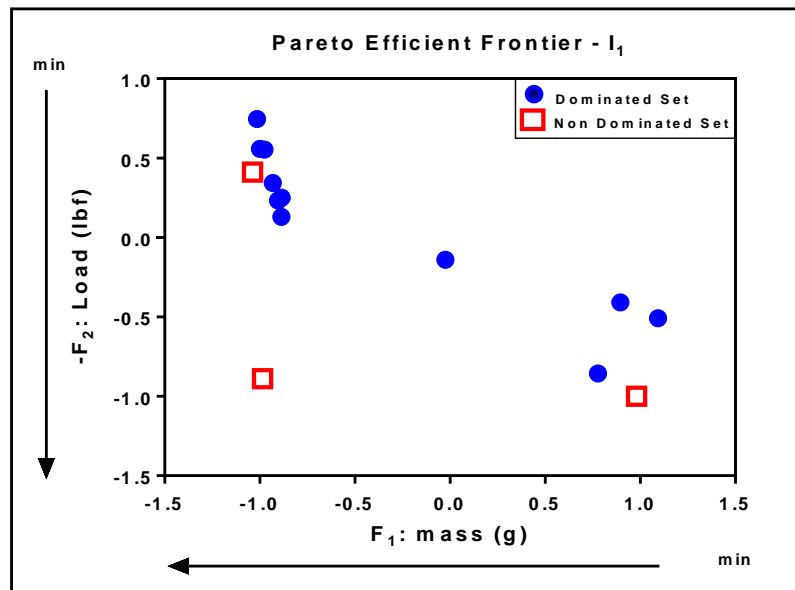
Figure 35. Pairwise Comparison for the Incumbent Solution (I_1)

The second iteration follows the same structure as before, with the creation of a new set of potential solutions D_2. These combinations were then simulated and compared with the incumbent solution I_1 using the Pareto conditions. No more points were added to the Efficient Frontier.

The three solutions identified by the method represent the best possible tradeoffs between mass and Load as shown in **Figure 36**.

a)   b)
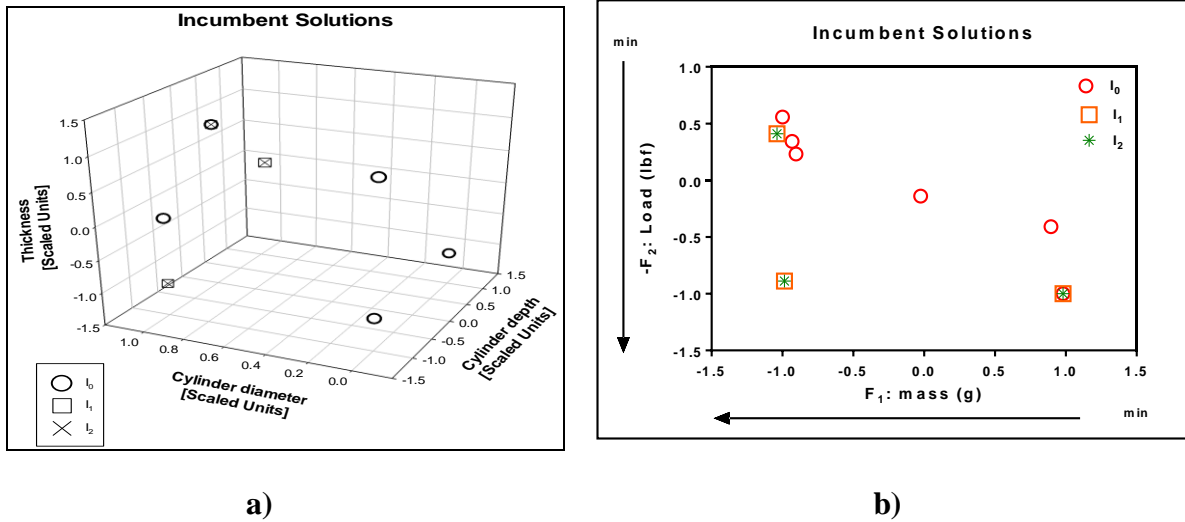
Figure 36. Each of the Incumbent Frontiers (I_0-I_1-I_2) of the Problem under Study. a) Controllable Variables (CV) and b) Performance Measures (PM)

Table 6. Pareto Efficient Frontier

Pareto Efficient Frontier					
Position	Thickness(in)	Cylinder diameter (in)	Cylinder depth (in)	mass (g)	Max Load (lbf)
T12	0.0920	0.041	0.2300	11.2716	139.597
T2-8	0.2300	0.041	0.2300	11.0938	44.333
T2-9	0.161	0.041	0.460	17.856	147.733

Summary

With the proposed method the efficient frontier was found for a case of 3 controllable variables considering two performance measure. We reiterate that this work present a simple method that quickly converges to the Pareto frontier with a small number of iterations. The method stops automatically when no more changes are found in the efficient frontier.

Results

This work proposes a formal optimization strategy for several measures performances in conflict. Preliminary results supporting the use of our optimization strategy is feasible effective and efficient to arrive the best possible solution in few iterations. We can generate 3D parts that are structurally strongly connected by 3D interlocking through the coordinated used of a 3D printer, an initial interlocking device design and the proposed optimization algorithm. A total of 24 assemblies (168 interlocking devices) were tested using our proposed strategy. If this project had been approached with a full factorial design, it would have required a total of 27 assemblies. This shows how efficient the strategy is in terms of number of runs, especially considering that it is solving a multiple criteria optimization problem experimentally.
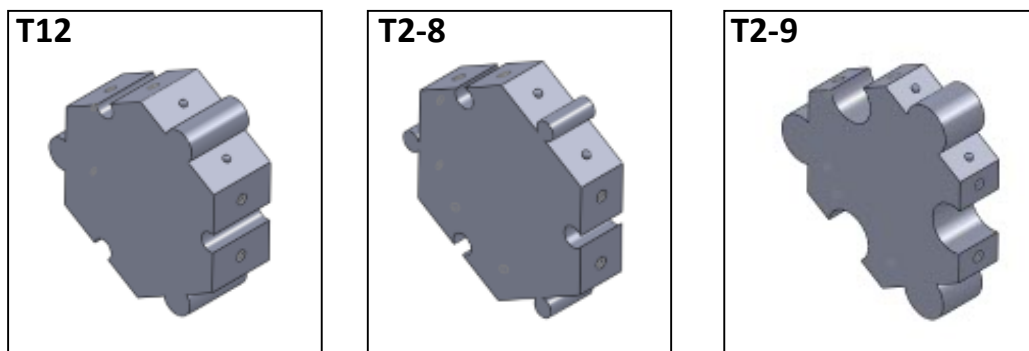
Figure 37. Best Possible Tradeoffs between Mass and Load Interlocking Devices Design, T12, T2-8, T2-9

8   Conclusion

This work proposes a formal simulation optimization strategy for several measures performances in conflict. The results support the use of our strategy as feasible, effective and efficient to arrive the best possible solution in relatively few iterations. The strategy shows a rapid convergence to attractive solutions with a modest number of runs and it is capable to handle tens of variables at a time. This accounts to its ability to approach realistic engineering problems, where the simultaneous consideration of multiple conflicting objectives and tens of decision variables is more the rule than the exception.

In addition, being that the proposed strategy is focused on optimization results with frugality, the case study presented here related to the optimization of the design of an interlocking device shows its potential for truly experimental work. This capability is very important when considering rapid prototyping, as well as additive manufacturing projects such as those involving 3D printing. Indeed, in this 3D printing problem, the proposed strategy showed multicriteria improvement with less experimental runs that it would require for simple exploration with a full factorial experimental design.

In its current state, the method is an improvement over single-pass methods as well as the use of Data Envelopment Analysis models as previously proposed by our group. Manufacturing decisions regarding design, control and improvement of processes and systems can greatly benefit from using the proposed optimization strategy from the point of view of its capabilities - multicriteria, multifactorial- as well as its frugality in terms of the number of simulation runs.

9   Future Work

Future work along this line of research should focus on improving the enumeration embedded in the prediction stage, as it might be possible to cut on the number of solutions to be explored –even if it is inexpensive at this point-, as well as to maintain global optimality considerations when a large number of decision variables is at hand. Furthermore, when using the strategy to approach truly experimental projects, it should be investigated how to incorporate the analysis of experimental variability to the optimization problem.

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Appendix A

Table 7. Strategy Time for the Case Study I and Case Study II

Case Study I: Production Line with 50 Workstations		
Phase I: Initialization	Step 1	1327 runs – 30 min
	Step 2	5 min
	Step 3	15 min
Phase II: Main Iteration	Step 4	10 min
	Step 5	3 min
	Step 6	: 15 min
	Step 7	: 5min
	Step 8	: 10 min
Phase III: Termination	Iteration 1	: 93 min
	Iteration 2	: 53 min
	Iteration 3	: 53 min
	Iteration 4	: 53 min
	Iteration 5	: 53 min
	Iteration 6	: 53 min
	Iteration 7	: 53 min
Strategy Time=		**3 h 33 min**
Total Time		**7 h 48 min**

Case Study II: 3D Printing Prototype		
Phase I: Initialization	Step 1	15 runs – 38 h 75 min
	Step 2	5 min
	Step 3	15 min
Phase II: Main Iteration	Step 4	10 min
	Step 5	3 min
	Step 6	15 min
	Step 7	8 runs 16hrs
	Step 8	10 min
Phase III: Termination	Iteration 1	3353 min
	Iteration 2	38 min
Strategy Time=		**1 h 8 min**
Total Time		**55 h 38 min**

Appendix B

Metamodeling

In this work, the use of second-order regression metamodels is recommended due to their capability to succinctly represent –precisely- these three effects previously discussed. A second-order regression metamodel is a quadratic expression that relates a response or dependent variable, y, to L independent variables each represented by x_i as follows:

$$y = \beta_0 + \sum_{i=1}^{L} \beta_i x_i + \sum_{i=1}^{L} \beta_{ii} x_i^2 + \sum_{i=1}^{L-1} \sum_{j=i+1}^{L} \beta_{ij} x_i x_j + \varepsilon \tag{3}$$

where the β coefficients in this expression are the metamodel's parameters to be estimated, with β_0 representing an intercept to the $y-$axis, β_i and β_{ii} the linear and quadratic effect of each independent variable respectively, and β_{ij} the interaction between a pair of distinct independent variables. ε is a residual assumed to be a random variable following a normal distribution with a mean of 0 and a constant variance, σ^2. Following equation (3), the number of parameters needed to fully specify a second-order regression metamodel is given by the expression:

$$Par = 1 + 2L + \binom{L}{2} \tag{4}$$

To fit a second-order regression metamodel, a DOE of choice must then provide at least Par degrees of freedom, and be capable to estimate all parameters (Par). With this choice, a DOE matrix, **X,** containing $Par + 1$ independent runs (rows) and Par columns, together with a **Y** column vector containing the $Par + 1$ responses, will allow estimating the parameters of a second-order regression metamodel through the equation:

$$\hat{\beta} = (X^t X)^{-1} X^t Y \tag{5}$$